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Estimation of weibull parameters for wind energy application in Iran's cities

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Abstract. Wind speed is the most important parameter in the design and study of wind energy conversion systems. The weibull distribution is commonly used for wind energy analysis as it can represent the wind variations with an acceptable level of accuracy. In this study, the wind data for 11 cities in Iran have been analysed over a period of one year. The Goodness of fit test is used for testing data fit to weibull distribution. The results show that this data fit to weibull function very well. The scale and shape factors are two parameters of the weibull distribution that depend on the area under study. The kinds of numerical methods commonly used for estimating weibull parameters are reviewed. Their performance for the cities under study was compared according to root mean square and wind energy errors. The result of the study reveals the empirical, modified maximum likelihood estimate of wind speed with minimum error. Also, that the moment and modified maximum likelihood are the best methods for estimating the energy production of wind turbines.

Keywords: weibull distribution; numerical methods; Iran's cities; wind energy

1. Introduction

Many developed and developing countries have adopted policies to use to renewable energies such as wind and solar energies so as to reduce their dependence on fossil fuels (Mostafaeipour *et al.* 2014). Due to the global growth in the production of wind energy, various studies have been conducted in the last decade on wind properties and the potential of wind in many countries (Kwon 2010).

The first step to use wind power in every region is to assess the potential and feasibility of wind power (Mostafaeipour *et al.* 2014). In the assessment of wind resources, in addition to average wind speed, its distribution is also taken into account. Various distributions of wind power can cause differences to the energy generated by two turbines in two different locations with equal average wind speeds (Mathew and Philip 2011). Wind speed is changing continuously and one of the best ways of describing these changes is to use statistical methods (Johnson 2006). By

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calculating the probability of wind speed, it is possible to determine wind power potential accordingly. Hence, different Probability Density Functions (PDF) have been used in different studies to describe wind speed distribution. Some of these functions include the Weibull, Rayleigh, Gamma, beta, lognormal and logistical functions. Considering the presence of two flexible parameters in the Weibull function, this function has been used extensively in studies (Chang 2011a). This function is also highly used in the engineering, medical and biological fields, determining air quality and so forth (Kantar and Şenoğlu 2008).

Several numerical methods can be used to estimate the parameters of the Weibull function using wind speed data. Some of these numerical methods include the graphical method, moment method, maximum likelihood method, modified maximum likelihood method, empirical method and energy pattern factor method (Chang 2011b). Considering the minimal computation in the graphical method, this method is generally used to determine the Weibull parameters in the studies (Jamdade and Jamdade 2012). Seguro and Lambert (2000) used the graphical method and introduced it as a method with low precision. The poor performance of this method in comparison to other methods is shown in several studies (Dorvlo 2002, Jowder 2009). Chang (2011b) carried out a comparison between the performances of six numerical methods in the estimation of the Weibull function in three wind farms with different weather in Taiwan. It was found that the maximum likelihood method performed best followed by the modified maximum likelihood and moment methods. The graphical method produced the worst performance. Costa Rocha *et al.* (2012) collected wind data from two cities in Brazil. Following the analyses, they concluded that the energy pattern factor method and the graphical method are the least effective methods to fit Weibull distribution curves for wind speed data. Indhumathy *et al.* (2014) indicated that the energy pattern factor method is effective for estimating the Weibull function for wind speed data in the Kanyakumari region in India.

Different studies have also compared the performances of these methods. Considering the data sample size, the data location, the format of sample data, distribution of sample data, goodness of fit test, and statistical judgment criteria, one method can demonstrate a better performance than other methods (Akdağ and Dinler 2009).

The main objective of the present study is to propose a better method to estimate Weibull parameters for wind energy applications in Iran's cities. This study attempts to evaluate and compare Weibull parameters using 6 different methods for 11 cities in Iran with different weather. The deviation of the results of the Weibull function from the measured values of wind speed and the effect of this function on the estimation of annual energy generated by wind turbines using different methods in different cities were examined as well.

Wind speed data for this study were taken from Iran's Renewable Energy Organization (IREO). These data were collected every 10 minutes at 10, 30, and 40m heights. The data collection periods are given in Table 1. Fig. 1 shows the geographical location of the considered cities on an Iran map, and the location information is also summarized in Table 1.

2. Methods for determining Weibull parameters

It has been found that the weibull distribution can be used to describe the wind variations in a regime with an acceptable accuracy level (Sathyajith 2006). Weibull distribution can be described by its probability density function $f(v)$ and cumulative distribution function $F(v)$ given as (Manwell *et al.* 2002)

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

Fig. 1 Locations of the cities under study on Iran map

Table 1 Summary of regions information

City	Altitude (m)	Latitude (Degrees)	Longitude (Degrees)	Data collection period
Ardakan	1234	32.3º N	54.0 ºE	2006-2007
Bojnord	1070	37.2º N	57.2ºE	2006-2007
Chabahar	11	25.1ºN	60.3ºE	2008-2009
Khash	1400	28.2º N	61.2ºE	2006-2007
Divandareh	1850	35.5º N	47.0ºE	2006-2007
Kahrizak	1062	35.6º N	51.4ºE	2007-2009
Kish	10	26.3º N	53.5ºE	2006-2007
Langroud	25	37.1º N	50.1ºE	2008-2009
Marvdasht	1605	29.8º N	52.8ºE	2006-2007
Meshkinshahr	1394	38.3º N	47.6ºE	2008-2010
Moorchekhort	1570	31.8º N	51.6ºE	2006-2007

Where v is the wind speed, k is the dimensionless shape parameter, and c is the scale parameter having the same unit with v. Here, $f(v)$ represents the fraction of time (or probability) for which the wind blows with a velocity v and $F(v)$ indicates the fraction of time (or probability) that the wind velocity is equal or lower than v.

There are several methods for determining k and c from the site wind data. Some of the common methods are the graphical method, maximum likelihood method, modified maximum likelihood method, energy pattern factor method, moment method, and empirical method (Chang 2011b).

2.1 Graphical method

The Graphical method is implemented by fitting a straight line to wind speed data using the concept of least squares, where the time-series data must be sorted into bins. Taking a double logarithmic transformation, the equation of cumulative distribution function can be rewritten as (Deaves and Lines 1997)

$$\ln\left\{-\ln\left[1-F(v)\right]\right\} = k\ \ln(v) - k\ \ln(c) \tag{3}$$

Plotting $ln(v)$ against $ln[-ln[1-F(v)]]$, the slope of the straight line fitted best to data pairs is the shape parameter; the scale parameter is then obtained by the intercept with the y-ordinate.

2.2 Maximum likelihood method

In the maximum likelihood method, the shape and scale factors are given as (Seguro and Lambert 2000)

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right]^{-1} \tag{4}$$

$$c = \left(\frac{1}{n}\sum_{i=1}^{n} v_i^k\right)^{\frac{1}{k}} \tag{5}$$

Where v_i is the wind speed in time step i and n is the number of nonzero wind speed data points. The maximum likelihood estimation method is difficult to solve, since numerical iterations are needed to determine the parameters of the Weibull distribution (Chang 2011b, Costa Rocha *et al.* 2012).

2.3 Modified maximum likelihood method

When wind speed data are available in a frequency distribution format, a variation of the maximum likelihood method can be applied. The Weibull parameters are estimated using the following two equations (Costa Rocha *et al.* 2012, Seguro and Lambert 2000)

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^{n} v_i^k f(v_i)} - \frac{\sum_{i=1}^{n} \ln(v_i) f(v_i)}{f(v \geq 0)} \right]^{-1} \tag{6}$$

$$c = \left(\frac{1}{f(v \geq 0)} \sum_{i=1}^{n} v_i^k f(v_i) \right)^{\frac{1}{k}} \tag{7}$$

Where v_i is the wind speed central to bin i, n is the number of bins, $f(v_i)$ is the frequency with which the wind speed falls within bin i, $f(v \geq 0)$ is the probability that the wind speed equals or exceeds zero. These equations must be solved iteratively.

2.4 Energy pattern factor method

The energy pattern factor is the ratio of the average of power available in the wind and the power available in the mean wind speed and is defined by the following equation (Sathyajith 2006)

$$E_{pf} = \frac{\frac{1}{n} \sum_{i=1}^{n} v_i^3}{\left(\frac{1}{n} \sum_{i=1}^{n} v_i \right)^3} \tag{8}$$

Where v_i is the wind speed in time step i and n is the number wind speed data points. Weibull parameters can be estimated with the following equations (Akdağ and Dinler 2009)

$$k = 1 + \frac{3.69}{(E_{pf})^2} \tag{9}$$

$$c = \frac{\bar{v}}{\Gamma(1 + 1/k)} \tag{10}$$

Where $\bar{v}$ the mean wind speed and Γ is is the gamma function and defined by

$$\Gamma(X) = \int_0^{\infty} t^{X-1} \exp(-t) dt \tag{11}$$

2.5 Moment method

When the mean wind speed $\bar{v}$ and standard deviation σ are available, shape and scale parameters can be estimated with this method using the numerical iteration of the following two equations (Chang 2011b, Costa Rocha *et al.* 2012, Yildirim *et al.* 2012)

$$c = \frac{\bar{v}}{\Gamma(1 + 1/k)} \tag{12}$$

$$\sigma = c \left[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k) \right]^{1/2} \tag{13}$$

2.6 Empirical method

For $1 \leq k < 10$, an empirical approximation for shape parameter is (Manwell *et al.* 2002)

$$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086} \tag{14}$$

Where $\bar{v}$ is the mean wind speed and σ is the standard deviation. The scale parameter is obtained as

$$c = \bar{v} / \Gamma(1 + 1/k) \tag{15}$$

3. Statistical analysis

3.1 Statistical testing

The χ^2 test is a popular form of hypothesis testing. The test is used in this work as a so-called goodness-of-fit test to check whether a sample could have been drawn from a weibull distribution. Hence, the null hypothesis is "H0=the given random sample x_i drawn from a Weibull distribution with specified k and c". The basic idea of the χ^2 test is to compare observed frequencies, y_i, with the theoretically expected frequencies, x_i, and provide criteria to decide if they show significant differences. The random variable (D'Agostino 1986, Focken and Lange 2006)

$$\chi_s^2 = \sum_{i=1}^{n} \frac{(y_i - x_i)^2}{x_i} \tag{16}$$

is defined which compares the empirically found frequencies, y_i, with the expected frequencies based on Weibull probabilities, x_i. The variable χ_s^2 is approximately χ^2 distributed with $n-m$ degrees of freedom so that n is the number of non-zero observations and m is the number of estimated parameters for weibull distribution plus 1. A prerequisite for this approximation is $x_i \geq 5$. The χ^2 distribution is defined as the distribution of the sum of the squares of independent standard normal variables and its values are usually tabulated.

The probability of wrong rejection α is selected. It implicitly defines the interval in which realisations of χ_s^2 according to (16) are rejected. Hence, the condition

$$F_{x^2}(\chi^2 \geq \chi^2_{\alpha,n-m}) = \alpha \tag{17}$$

Where F_{x^2} is the cumulated χ^2 distribution, gives a critical point, $\chi^2_{\alpha,n-m}$, and the probability to find values beyond this point is α. Typically, α is set to 0.10, 0.05 or 0.01 and is called the significance level. The corresponding values of $\chi^2_{\alpha,n-m}$ are tabulated. After $\chi^2_{\alpha,n-m}$ is determined, the last step is to check whether the realisation of χ_s^2 is smaller than this critical point; i.e., if

$$\chi_s^2 \leq \chi^2_{\alpha,n-m} \tag{18}$$

There is no objection to the assumption that the sample stems from a Gaussian distribution (D'Agostino 1986, Focken and Lange 2006).

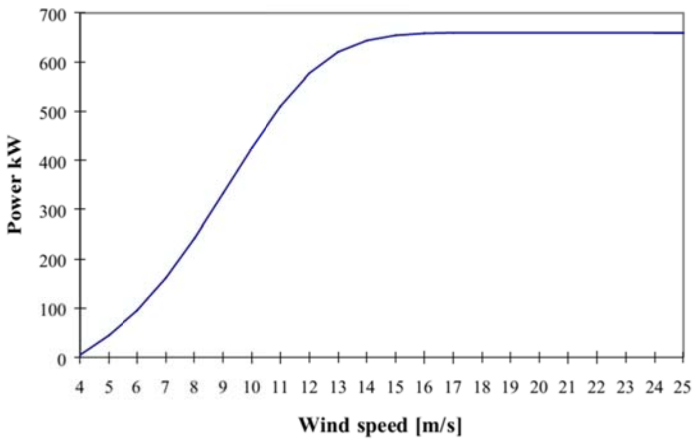

Fig. 2 V47-660 kW wind turbine power curve (Vestus)

3.2 Statistical criteria

To show how a theoretical weibull function corresponds with the observation data, two kinds of statistical errors are considered as the judgment criteria. The first is root mean square error (RMSE) and is defined as

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^{n} (y_i - x_i)^2 \right]^{\frac{1}{2}} \tag{19}$$

Where y_i is the frequency of observations in wind speed interval i, x_i is the frequency of Weibull, and n is number of intervals.

In order to examine the impact of wind speed estimation on wind turbine energy production; the researchers assumed that a wind turbine with 660 kW rated capacity is installed in the cities under study. Fig. 2 depicts the turbine power curve (Vestus). The energy produced by the turbine during one year is calculated with the actual and estimated wind speed and the wind energy error is defined as follows

$$e_T = \frac{(E_{actual} - E_{weibull})}{E_{actual}} \times 100 \tag{20}$$

Where E_{actual} and $E_{weibull}$ are the annual energy production of the wind turbine using measured and weibull data, respectively.

4. Result and discussion

The weibull parameters are determined using the aforementioned methods for the cases under study and are examined with statistical testing (χ^2) and judgment criteria ($RMSE$ and e_T). These results are presented in Tables 2-12.

4.1 Analysis of Weibull parameters

Figs. 3-13 show how the calculated Weibull function, for each numerical method, corresponds with the observed wind speed histogram for the cases under study.

The Weibull density function gets relatively narrower and more peaked as *k* gets larger. The peak also moves in the direction of higher wind speeds as *k* increases (Johnson 2006). For example in Bojnord, since the determined shape parameter (*k*) using the graphical method and energy pattern method are greater than other methods, the peaks of the probability density curve were larger and occurred at higher wind speeds in these two methods (Fig. 4).

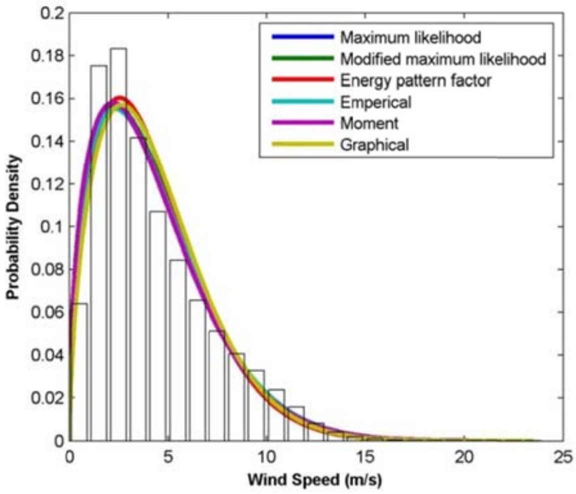

Fig. 3 Comparison of probability density distributions for Ardakan

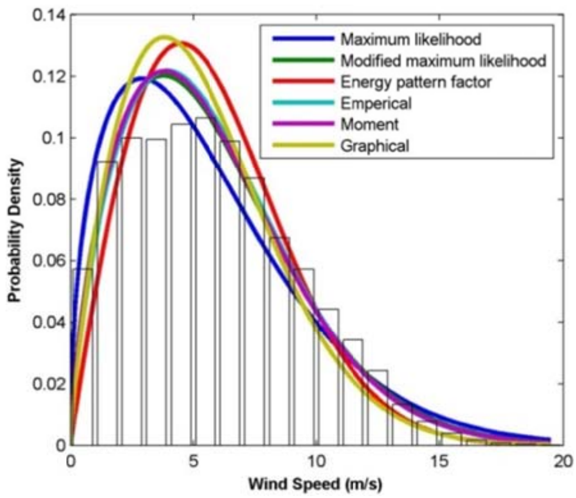

Fig. 4 Comparison of probability density distributions for Bojnord

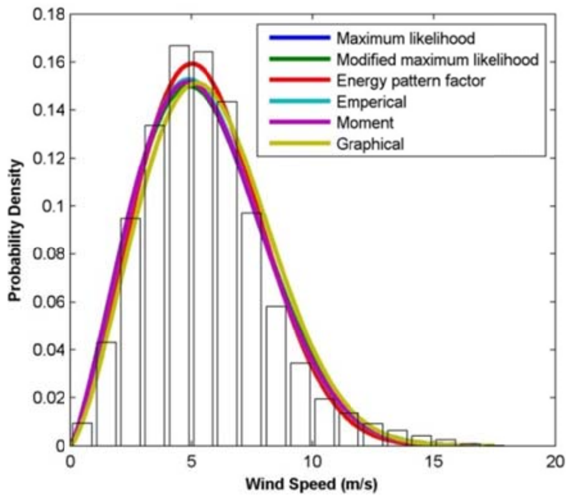

Fig. 5 Comparison of probability density distributions for Chabahar

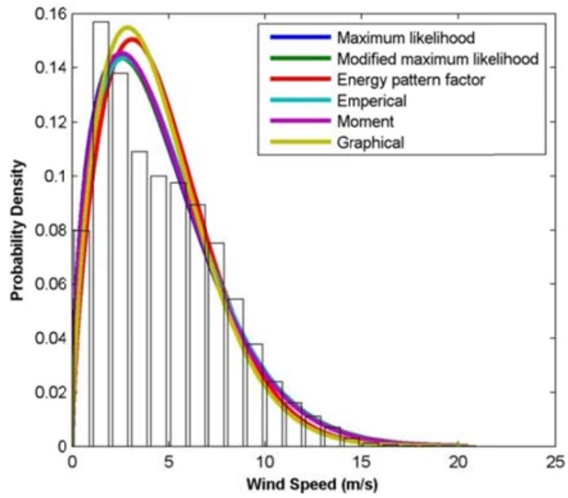

Fig. 6 Comparison of probability density distributions for Khash

As shown in Fig. 12, the Weibull distributions in Meshkinshahr are different from other cases under study. This is because in this city, the calculated shape parameter (*k*) from the different methods was less than or equal to unity (Table 11), whereas this parameter in the other cities was greater.

The Weibull parameters in the cities under study based on minimum *RMSE* are given in Table 13. As shown in this table, the values of *k* range from 1.05 in Meshkinshahr to 2.45 in Chabahar.

The probability density curve compresses vertically and expands horizontally as *c* increases. In the cities under study, the maximum shape parameter (*k*) was due to Chabahar. Because of its large scale parameter (*c*), the peak of the probability density curve in this city was not higher than other cities (Fig. 5). As illustrated in Table 13, scale parameter (*c*) varies from 3.77 to 6.23. The average values of *k* and *c* for cities under study are 1.65 and 5.01, respectively.

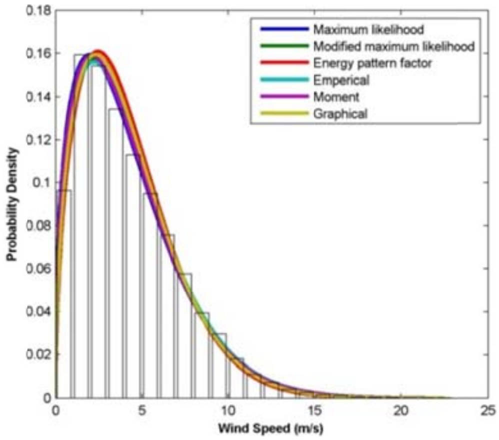

Fig. 7 Comparison of probability density distributions for Divandareh

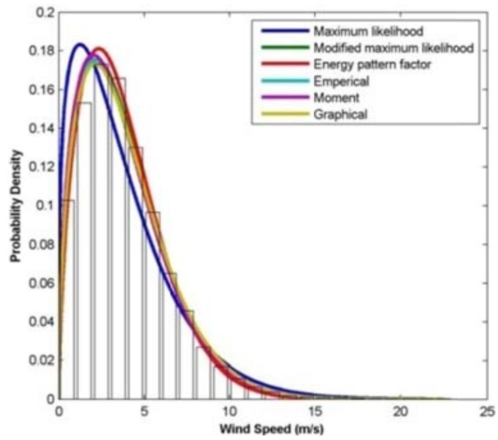

Fig. 8 Comparison of probability density distributions for Kahrizak

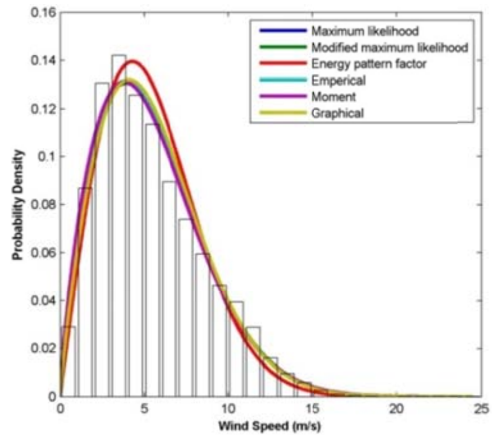

Fig. 9 Comparison of probability density distributions for Kish

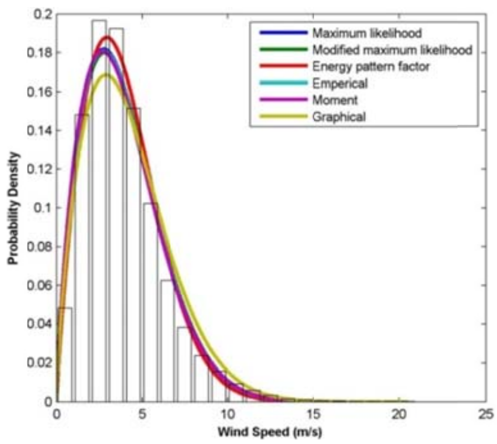

Fig. 10 Comparison of probability density distributions for Langroud

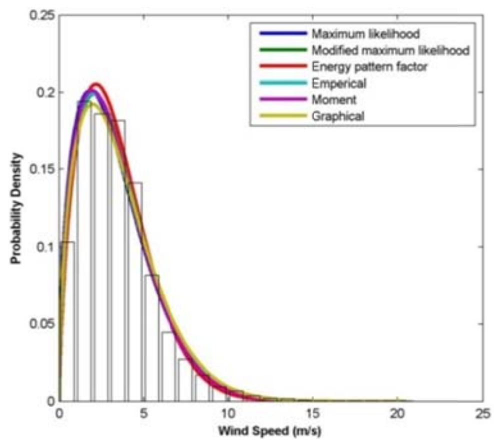

Fig. 11 Comparison of probability density distributions for Marvdasht

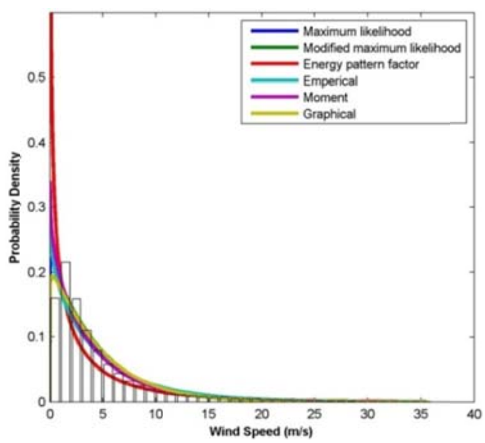

Fig. 12 Comparison of probability density distributions for Meshkinshahr

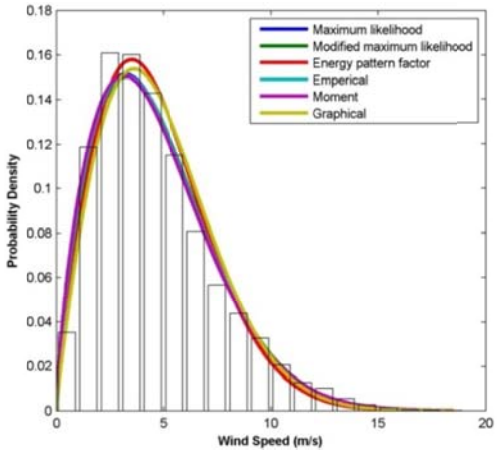

Fig. 13 Comparison of probability density distributions for Moorchekhort

4.2 Statistical analysis

The critical value at 99% confident level in a χ^2 test is 6.4. As shown in Tables 2-12, χ^2 test values in all of cases under studies are much smaller than the critical value. This means that the Weibull distribution is applicable to describe the probability density of wind speed data.

In the cases under study, *RMSE* and e_T varies between 0.003–0.016 and -29.39–17, respectively. The methods for determining k and c for each city are ranked based on *RMSE* and $|e_T|$ in Tables 14 and 15, respectively. Higher position in these tables is interpreted as less precision. For example, as shown in Table 14, in Ardakan, the maximum likelihood method and the energy pattern method have ranked 1 and 6, respectively. This means that based on *RMSE*, the maximum likelihood method is more accurate than the others and the energy pattern method is the worst method.

The general rank of each method based on *RMSE* and $|e_T|$ is presented in Tables 16 and 17, respectively. As shown in Table 16, based on *RMSE*, the empirical method is the best and the energy pattern method is the worst method. As shown in Table 17, based on $|e_T|$, the moment method is more accurate than the others and the graphical method gives the worst performance.

According to the obtained results of Tables 16 and 17, generally, the modified maximum likelihood is recommended for use when a greater precision in both judgment criteria is required.

5. Conclusions

In this paper, Weibull parameters are estimated using different methods for 11 cities in Iran with different climates. Using the Statistical test, it was found that the Weibull distribution is applicable to describe the probability density of wind speed.

The average values of k and c for the cities under study are 1.65 and 5.01, respectively. The values of k ranged from 1.05 in Meshkinshahr to 2.45 in Chabahar and scale parameter (c) varied from 3.77 to 6.23. The results of the study reveal that the empirical, modified maximum likelihood and moment methods estimated the wind speed with minimum *RMSE*. Also the moment and modified maximum likelihood are the best while the graphical method is the worst method for estimating the energy production of wind turbines. The modified maximum likelihood method is recommended for use when a greater precision in both judgment criteria is required.

Table 2 Weibull parameters and statistical analysis in Ardakan

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	1.49	4071	0.011	4.26	0.031
Modified max likelihood	1.52	4.79	0.0113	2.3	0.031
Energy pattern factor	1.59	4.74	0.0115	9.95	0.037
Empirical	1.5	4.8	0.0113	0.15	0.031
Moment	1.48	4.7	0.011	3.87	0.031
Graphical	1.61	4.86	0.0124	4.57	0.037

Table 3 Weibull parameters and statistical analysis in Bojnord

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	1.48	6.23	0.01	6.03	0.039
Modified max likelihood	1.68	6.23	0.0077	3.43	0.039
Energy pattern factor	1.95	6.47	0.0133	9.32	0.075
Empirical	1.73	6.47	0.0084	4.4	0.025
Moment	1.7	6.43	0.0082	5.12	0.023
Graphical	1.79	6.04	0.0126	19.46	0.056

Table 4 Weibull parameters and statistical analysis in Chabahar

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	2.29	6.24	0.0092	-5.24	0.045
Modified max likelihood	2.29	6.29	0.0096	-7.83	0.043
Energy pattern factor	2.45	6.23	0.0065	-1.56	0.103
Empirical	2.32	6.23	0.0085	-4.35	0.051
Moment	2.3	6.24	0.0089	-4.88	0.047
Graphical	2.42	6.51	0.0109	-15.8	0.052

Table 5 Weibull parameters and statistical analysis in Khash

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	*c*	*RMSE*	e_T	χ^2
Maximum likelihood	1.5	5.14	0.0103	4.41	0.025
Modified max likelihood	1.52	5.2	0.0105	2.39	0.025
Energy pattern factor	1.71	5.2	0.0158	12.41	0.053
Empirical	1.55	5.24	0.0112	2.62	0.026
Moment	1.53	5.15	0.0109	5.38	0.026
Graphical	1.66	4.99	0.0149	19.71	0.054

Table 6 Weibull parameters and statistical analysis in Divandareh

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	*c*	*RMSE*	e_T	χ^2
Maximum likelihood	1.42	4.63	0.0034	0.09	0.004
Modified max likelihood	1.47	4.72	0.0032	-1.13	0.003
Energy pattern factor	1.57	4.69	0.0062	7.55	0.011
Empirical	1.47	4.76	0.0034	-2.88	0.003
Moment	1.45	4.65	0.0031	1.11	0.003
Graphical	1.52	4.69	0.0045	4.25	0.005

Table 7 Weibull parameters and statistical analysis in Kahrizak

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	*c*	*RMSE*	e_T	χ^2
Maximum likelihood	1.28	4.02	0.0158	-17.22	0.05
Modified max likelihood	1.55	4.29	0.0035	-5.32	0.004
Energy pattern factor	1.62	4.23	0.0038	6.17	0.016
Empirical	1.52	4.27	0.0042	-6.56	0.004
Moment	1.5	4.19	0.0053	3.15	0.006
Graphical	1.52	4.32	0.0042	-10.83	0.005

Table 8 Weibull parameters and statistical analysis in Kish

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	1.78	6.09	0.0053	1.24	0.013
Modified max likelihood	1.79	6.15	0.0056	-0.77	0.013
Energy pattern factor	1.98	6.11	0.0086	6.37	0.045
Empirical	1.8	6.11	0.0054	0.99	0.013
Moment	1.77	6.08	0.0053	1.2	0.013
Graphical	1.86	6.21	0.0065	-1.08	0.015

Table 9 Weibull parameters and statistical analysis in Langroud

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	1.77	4.39	0.0084	-1.67	0.032
Modified max likelihood	1.78	4.45	0.0087	-5.46	0.03
Energy pattern factor	1.88	4.39	0.0064	6.25	0.112
Empirical	1.77	4.4	0.0087	-2.99	0.031
Moment	1.74	4.38	0.0093	-3.35	0.029
Graphical	1.75	4.7	0.0124	-29.39	0.035

Table 10 Weibull parameters and statistical analysis in Marvdasht

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	c	$RMSE$	e_T	χ^2
Maximum likelihood	1.52	3.72	0.0091	-6.22	0.017
Modified max likelihood	1.6	3.82	0.0064	-6	0.015
Energy pattern factor	1.67	3.77	0.0054	6.69	0.036
Empirical	1.58	3.8	0.0068	-6.13	0.015
Moment	1.56	3.75	0.0075	-3.83	0.016
Graphical	1.54	3.9	0.0083	-21.89	0.017

Table 11 Weibull parameters and statistical analysis in Meshkinshahr

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	*c*	*RMSE*	e_T	χ^2
Maximum likelihood	1.01	4.31	0.0127	-6.68	0.056
Modified max likelihood	1.05	4.45	0.0117	-8.47	0.061
Energy pattern factor	0.67	3.25	0.0383	1.84	0.232
Empirical	0.92	4.85	0.016	-34.06	0.08
Moment	0.93	4.16	0.0163	-7.62	0.061
Graphical	1.04	4.54	0.0122	-13.28	0.058

Table 12 Weibull parameters and statistical analysis in Moorchekhort

Method	Weibull parameters		Statistical Criteria		Statistical test
	k	*c*	*RMSE*	e_T	χ^2
Maximum likelihood	1.75	5.23	0.0072	-0.47	0.016
Modified max likelihood	1.76	5.29	0.0076	-3.06	0.015
Energy pattern factor	1.89	5.24	0.0077	6.13	0.032
Empirical	1.74	5.24	0.0074	-1.45	0.016
Moment	1.72	5.21	0.0075	-0.99	0.016
Graphical	1.89	5.38	0.0094	-1.73	0.026

Table 13 Weibull parameters based on minimum RMSE

City	k	C
Ardakan	1.48	4.7
Bojnord	1.68	6.23
Chabahar	2.45	6.23
Khash	1.5	5.14
Divandareh	1.45	4.65
Kahrizak	1.55	4.29
Kish	1.77	6.08
Langroud	1.88	4.39
Marvdasht	1.67	3.77
Meshkinshahr	1.05	4.45
Moorchekhort	1.75	5.23

Table 14 Methods ranking based on minimum RMSE for cases under study.

City	Maximum likelihood	Modified maximum likelihood	Energy pattern factor	Empirical	Moment	Graphical
Ardakan	1	3	5	4	2	6
Bojnord	4	1	6	3	2	5
Chabahar	4	5	1	2	3	6
Khash	1	2	6	4	3	5
Divandareh	4	2	6	3	1	5
Kahrizak	6	1	2	3	5	4
Kish	2	4	6	3	1	5
Langroud	2	4	1	3	5	6
Marvdasht	6	2	1	3	4	5
Meshkinshahr	3	1	6	4	5	2
Moorchekhort	1	4	5	2	3	6

Table 15 Methods ranking based on minimum $|e_T|$ for cases under study

City	Maximum likelihood	Modified maximum likelihood	Energy pattern factor	Empirical	Moment	Graphical
Ardakan	1	3	5	4	2	6
Bojnord	4	1	6	3	2	5
Chabahar	4	5	1	2	3	6
Khash	1	2	6	4	3	5
Divandareh	4	2	6	3	1	5
Kahrizak	6	1	2	3	5	4
Kish	2	4	6	3	1	5
Langroud	2	4	1	3	5	6
Marvdasht	6	2	1	3	4	5
Meshkinshahr	3	1	6	4	5	2
Moorchekhort	1	4	5	2	3	6

Table 16 Methods performance based on minimum RMSE

Method	Minimum rank	Maximum Rank	Mean rank	Mean *RMSE*
Empirical	1	5	2.45	0.0078
Modified max likelihood	1	6	3	0.0102
Graphical	1	6	3.18	0.0088
Moment	1	6	3.82	0.0088
Maximum likelihood	1	6	4	0.0103
Energy pattern factor	2	6	4.55	0.0091

Table 17 Methods performance based on minimum $|e_T|$

Method	Minimum rank	Maximum rank	Mean rank	Mean $\|e_T\|$
Moment	1	4	2.64	3.68
Modified max likelihood	1	5	2.73	4.2
Empirical	1	6	2.82	6.05
Maximum likelihood	1	6	3.18	4.87
Energy pattern factor	1	6	4.45	6.75
Graphical	3	6	5.18	12.9

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