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Optimal design of wind-induced vibration control of tall buildings and high-rise structures

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Abstract. The most common used control device on tall buildings and high-rise structures is active and passive tuned mass damper (ATMD and TMD). The major advantages of ATMD and TMD are discussed. The existing installations of various passive/active control devices on real structures are listed. A set of parameter optimization methods is proposed to determine optimal parameters of passive tuned mass dampers under wind excitation. Simplified formulas for determining the optimal parameters are proposed so that the design of a TMD can be carried out easily. Optimal design of wind-induced vibration control of frame structures is investigated. A thirty-story tall building is used as an example to demonstrate the procedure and to verify the efficiency of ATMD and TMD with the optimal parameters.

Key words: optimization; active structural control; tuned mass damper; wind-induced vibration; tall buildings.

1. Introduction

In general, structural control devices can be divided into three classes : passive control, active control and semi-active control. Devices needed external energy to provide control forces are called active control systems. There are three types of active control devices : active tendon control, active tuned mass control (or hybrid mass damper) and active

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aerodynamic appendages control. Devices which do not need external energy are called passive control systems which can be mainly divided into four types : passive tendon control, base isolation, passive aerodynamic control, passive damper control including tuned mass damper (TMD), tuned liquid damper (TLD) and mass pump (MP). Devices needed a little external energy to change structural parameters are called semi-active control systems which can be mainly divided into two types : active variable stiffness and active variable damper system. The most common used control devices are active and passive tuned mass damper (ATMD and TMD). TMD basically is a mass block attached with springs and dashpots. An

Table 1 Application of passive tuned mass damper

Year	Mass	Structure
1971	132.4 m^3 water tank	Centerpoint Tower, Sydney
1975	2×300 ton mass blcks	John Hancock Tower, Boston
1980	400 ton concrete mass	Citicorp Center, New York
1988	10.0/15.4 ton (in x/y axis)	Chiba Port Tower, Tokyo
1992	97.4 kg disc	Funade Bridge Tower, Osaka
1992	5 ton pendulum	Steel Stacks, Kimitsu City

Table 2 Application of active structural control systems

Year	Active Devices	Structures
1989	AMD(Active Mass Damper)	Kyobashi Seiwa Building
1990	AVS(Active Variable Stiffness)	Katri No. 21 Building
1991	AMD	Tokyo Port Bridge
1992	HMD(Hybrid Mass Damper)	Hakucho Bridge
1992	HMD	Tsurumi Fariway Bridge
1992	HMD	Akashi Kaikyo Bridge
1992	AMD	Sendagaya Intes Building
1992	AMD	Hankyu Chayamachi Building
1992	HMD	Kansai Airport Control Tower
1992	HMD	ORC200 Symbol Tower
1993	HMD(DUOX)	Ando Nishikicho Building
1993	HMD	Long Term Credit Bank
1993	HMD	KS project
1994	HMD(Trigon)	Shinjuku Park Tower
1994	HMD	Hamanatsm ACT Tower
1994	HMD(Avice-1)	Riverside Sumida
1994	HMD	MHI Yokohama Building
1994	HMD	Down Fire & Marine Ins.
1994	HMD	J. City
1994	HMD	Porte Kanazawa
1994	HMD	Osaka WTC Building
1994	HMD	Hotel Ocean 45
1995	HMD(DUOX)	Dowa Kasai Phoenix Tower
1995	HMD	Rinku Gate Tower
1995	HMD	Hirobe Miyake Building

active tuned mass damper is a combination of a passive tuned mass damper and an active mass damper. Table 1 and Table 2 show the application of those control devices to buildings, bridges, towers and other structures during the last three decades. The major advantages of active and passive tuned mass damper are summarized as follows :

1.1. High efficiency

As is well known, all of tall buildings are very heavy, the control force needed is a big problem to control such a building. However, TMD can use a small mass as energy absorber to passively reduce the response of the main structure. A relatively small active control force is needed for ATMD to significantly reduce structural response by 40% to 50% (Higashino and Aizewa 1994), which illustrates that the efficiency of ATMD is attractive.

1.2. Flexibility

Because of the mass of TMD is small, it can be installed in many places in various structures. Furthermore, the mass of TMD may be a part of the structure. In one case, an ice thermal storage tank was used as the mass of TMD. In another one, the heliport on the top of building was used as the mass for ATMD (Higashino and Aizewa 1994).

1.3. Adaptability

TMD and ATMD have been installed in many buildings and structures during the last three decades, their application have proven their adaptability.

The effectiveness of a TMD depends on its parameters : its mass, frequency and damping ratio. Thus, it is important to optimize the TMD parameters in structural control design. Den Hartog (1928) first investigated the optimal design of TMD. He studied the response of the main structure with TMD subjected to sinusoidal excitation, and developed the basic principles and procedure to select the parameters of TMD. These so called Den Hartog's values have been widely used until now to determine the optimal parameters of TMD, though these parameters were obtained based on an assumption that the main structure is undamped. Warburton and Ayorinde (1980) studied the optimal parameters of a TMD which minimizes dynamic response of a complex system that is treated as an equivalent single degree-of-freedom (SDOF) system, subjected to harmonic excitation. They found that Den Hartog's values are good in reduction of response of main structure if the natural frequencies of the main structure are well separated. On the other hand, when the frequency separation is small, the optimal parameters diverge from Den Hartog's values with this divergence increasing as the effective mass ratio increases. Warburton (1982) discussed optimal parameters of TMD subjected to white noise excitation for an equivalent SDOF system. Fujino and Abé (1993) presented a set of design formulas for TMD based on a perturbation technique. Chang and Yang (1994) further studied Warburtin's model and proposed formulae to determine the optimal parameters for an undamped complex system that can be treated as an equivalent SDOF system. Kareem and Kline (1995) recently examined the performance of multiple turned mass dampers under random loading.

Many formulas proposed to determine the optimal parameters of TMD were obtained based on undamped SDOF systems subjected to white noise excitation. However, in general, the damping of the main structure can not be neglected, and actual external dynamic loads such as wind, earthquake and wave loads can not be simplified as white noise. Therefore, a more comprehensive investigation for optimal TMD parameter design is needed.

2. Optimization of TMD parameters

Though several practical TMD and ATMD have been installed on buildings, bridges and towers to reduce structural responses, several important issues such as optimization of the parameters of TMD remain. In this section, a set of parameter optimization methods is proposed to determine optimal parameters of TMD under wind excitation. Simplified formulas for determining the optimal parameters are proposed so that the design of a TMD can be carried out easily.

A frame structure that is treated as a multi-degrees-of-freedom system (MDOF) subjected to wind loads is shown in Fig. 1, where a TMD is installed on the ith floor of this building. The equations of motion of the whole system can be written as

$$M\ddot{X}(t)+C\dot{X}(t)+KX(t)=W(t)+F(t) \tag{1a}$$

$$m_d\ddot{x}_d(t)+c_d\dot{x}_d(t)+k_dx_d(t)=-m_d\ddot{x}_i(t) \tag{1b}$$

in which M, C and K are the mass, damping and stiffness matrices of the main structure, and m_d, c_d and k_d are the mass, damping coefficient and stiffness of the TMD, respectively, $X(t)$, $W(t)$ and $F(t)$ are vectors of displacement of the main structure, wind loads and the forces acting on the main structure induced by the TMD, respectively. $x_i(t)$ is the displacement of the ith floor of the structure. x_d is the relative displacement of the TMD with respect to the ith floor.

Assume that wind loads are acted on each floor and can be expressed as

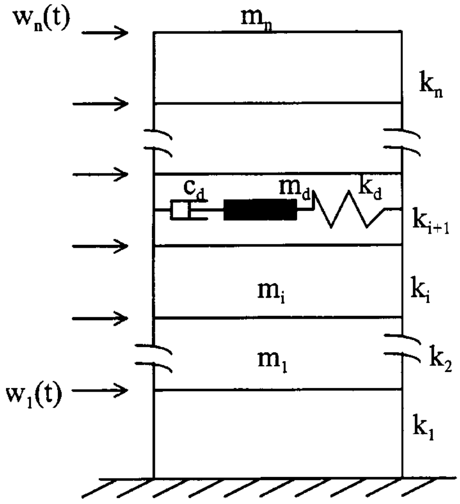

Fig. 1 Scheme of structure/TMD system

$$W(t) = \{s_j\}\, v(t) \tag{2}$$

where s_j is a parameter related to wind and structural characteristics at the j-th floor, $v(t)$ is wind velocity at a reference height.

If only one TMD is installed on the i-th floor, (in general, i=n), i.e.,

$$F(t) = \{f_j(t)\},\quad f_j(t) = \begin{cases} 0 & j \neq i \\ c_d \dot{x}_d + k_d x_d = i & j = i \end{cases} \tag{3}$$

If only the first m mode shapes of the structure are considered herein, according to the method of mode superposition, the displacement vector can be expressed as

$$X(t) = \sum_{j=1}^{m} \Phi_j y_j(t) \tag{4}$$

where Φ_j and $y_j(t)$ in Eq. (4) are the j-th mode shape function and generalized modal coordinate of the structure, respectively.

It is assumed that the damping of the structure is Rayleigh one, then, we can obtain the equations of generalized coordinates, $y_j(t)$, and relative displacement of the TMD, $x_d(t)$, as follows :

$$\begin{gathered} \ddot{y}_j(t) + 2\xi_j\, \omega_j\, \dot{y}_j(t) + \omega_j^2 y_j(t) - \mu_j\, \Phi_j\, [2\xi_d\, \omega_d\, \dot{x}_d(t) + \omega_d^2 x_d(t)] = \Lambda_j\, v(t) \\ \ddot{x}_d(t) + 2\xi_d\, \omega_d\, \dot{x}_d(t) + \omega_d^2 x_d(t) + \sum_{j=1}^{m} \Phi_j\, \ddot{y}_j(t) = 0,\ j = 1, 2, \cdots, m \end{gathered} \tag{5}$$

where,

$$\begin{gathered} \hat{M}_j = \sum_{i=1}^{n} \Phi_{ij}^2 m_i, \qquad \mu_j = \frac{m_d}{\hat{M}_j} \\ \Lambda_j = \frac{1}{\hat{M}_j} \sum_{i=1}^{n} \Phi_{ij}\, s_i, \qquad \omega_d^2 = \frac{k_d}{m_d} \\ 2\xi_d\, \omega_d = \frac{c_d}{m_d} \end{gathered} \tag{6}$$

where ξ_d and ω_d are critical damping ratio and natural frequency of the TMD, Λ_j is the j-th mode participation factor for wind excitation and $\hat{M}_j$ is the jth modal generalized mass.

In general, the first mode shape is dominant in the wind-induced vibration of a structure. Thus, only the first mode shape is considered herein and taking the value of the first mode shape function at the TMD (the i-th) floor as one (Φ_{i1}=1), then, Eq. (5) becomes

$$M^* \ddot{Y}(t) + C^* \dot{Y}(t) + K^* Y(t) = F^*(t) \tag{7}$$

where

$$Y(t) = \begin{Bmatrix} y_1(t) \\ x_d(t) \end{Bmatrix},\quad F^*(t) = \begin{Bmatrix} \Lambda_1 \\ 0 \end{Bmatrix} v(t)$$

$$M^* = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\quad C^* = \begin{bmatrix} 2\xi_1\omega_1 & 2\mu\xi_d\,\omega_d \\ 0 & 2\xi_d\,\omega_d \end{bmatrix},\quad K^* = \begin{bmatrix} \omega_1^2 & -\mu\omega_d^2 \\ 0 & \omega_d^2 \end{bmatrix} \tag{8}$$

where $\mu = \mu_1$

The power spectral density (psd) of the response of the generalized coordinates can be expressed as

$$S_{YY}(\omega) = H^T(i\omega)\ S_{FF}(\omega)\ H^*(i\omega) \tag{9}$$

in which S_{FF} is the psd of the generalized force F^*.

$$S_{FF} = \begin{bmatrix} \Lambda_1^2 & 0 \\ 0 & 0 \end{bmatrix} S_{vv}(\omega) \tag{10}$$

where $S_{vv}(\omega)$ is the psd of wind velocity at the reference height.

$H(i\omega)$ is the frequency response function corresponding to Eq. (7)

$$H(i\omega) = \frac{1}{\Delta}\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{11}$$

in which

$$h_{11} = -\omega^2 + 2\xi_d\ \omega_d\ (i\omega) + \omega_d^2$$
$$h_{12} = 2\mu\,\xi_d\ \omega_d(i\omega) + \mu\omega_d^2 \tag{12}$$
$$h_{21} = \omega^2$$
$$h_{22} = -\omega^2 + 2\xi_1\ \omega_1(i\omega) + \omega_1^2 \tag{12}$$

and

$$\Delta = \sum_{j=0}^{4} a_j (i\omega)^j$$
$$a_0 = \omega_1^2\omega_d^2,\ a_1 = 2\omega_1^2\xi_d\ \omega_d + 2\omega_1\xi_1\omega_d^2$$
$$a_2 = \omega_1^2 + 4\xi_1\omega_1\xi_d\ \omega_d + (1+\mu)\,\omega_d^2$$
$$a_3 = 2\xi_1\omega_1 + 2(1+\mu)\,\xi_d\ \omega_d\ ,\ a_4 = 1 \tag{13}$$

Substituting Eqs. (10) and (11) into Eq. (9) gives

$$S_{YY}(\omega) = \begin{bmatrix} S_{Y_1Y_1}(\omega) & S_{Y_1Y_2}(\omega) \\ S_{Y_2Y_1}(\omega) & S_{Y_2Y_2}(\omega) \end{bmatrix} = \frac{\Lambda_1^2}{|\Delta|^2}\begin{bmatrix} h_{11}h_{11}^* & h_{21}h_{11}^* \\ h_{11}h_{21}^* & h_{21}h_{21}^* \end{bmatrix} S_{vv}(\omega) \tag{14}$$

The psd of Y_1 can be expressed as

$$S_{Y_1Y_1}(\omega) = \frac{\sum_{j=1}^{3} b_j(\omega^2)}{|\Delta|^2} = \frac{\Lambda_1^2}{|\Delta|^2}\left[\omega^4 + 2(2\xi_d^2 - 1)\,\omega_d^2\,\omega^2 + \omega_d^4\right] S_{vv}(\omega) \tag{15}$$

When the excitation is a white noise with unit intensity, $S_{vv}(\omega)$=1, the parameter b_j in Eq. (15) are

$$b_0 = \Lambda_1^2\omega_d^4,\quad b_1 = -2\Lambda_1^2\omega_d^2 + 4\Lambda_1^2\xi_d^2\omega_d^2,\quad b_2 = \Lambda_1^2,\quad b_3 = 0 \tag{16}$$

The variance of Y_1 is (Roberts and Spanos 1990)

$$\sigma_{Y_1}^2 = \frac{B_4 \pi}{A_4 a_4} \tag{17}$$

in which

$$\begin{aligned} A_4 &= a_0(-a_4 a_1^2 + a_3 a_2 a_1 - a_3^2 a_0) \\ B_4 &= b_3 b_0 (a_2 a_1 - a_3 a_0) + a_4 (b_2 a_1 a_0 + b_0 a_3 a_2 - b_0 a_4 a_1 + b_1 a_3 a_0) \end{aligned} \tag{18}$$

The optimal damping ratio can be obtained when the minimum of σ_{y_1} is reached (i.e., its derivative with respect to ξ_d become zero).

$$\frac{\partial}{\partial \xi_d}\left(\frac{B_4}{A_4}\right) = \frac{1}{A_4^2}\left[B_4 \frac{\partial A_4}{\partial \xi_d} - A_4 \frac{\partial B_4}{\partial \xi_d}\right] = 0 \tag{19}$$

Then,

$$B_4 \frac{\partial A_4}{\partial \xi_d} - A_4 \frac{\partial B_4}{\partial \xi_d} = 0 \tag{20}$$

Similarly, the optimal TMD frequency and mass ratio can be obtained from the following two equations, respectively.

$$B_4 \frac{\partial A_4}{\partial \omega_d} - A_4 \frac{\partial B_4}{\partial \omega_d} = 0 \tag{21}$$

$$B_4 \frac{\partial A_4}{\partial \mu} - A_4 \frac{\partial B_4}{\partial \mu} = 0 \tag{22}$$

Eqs. (20) to (22) are obtained based on the criterion of minimum displacement. If other criteria, such as minimum velocity and minimum acceleration of the main structure, are adopted, then the equations for determining the optimal parameters are as follows

$$\frac{\partial \sigma_{\dot{y}_1}^2}{\partial \xi_d} = 0, \quad \frac{\partial \sigma_{\dot{y}_1}^2}{\partial \omega_d} = 0, \quad \frac{\partial \sigma_{\dot{y}_1}^2}{\partial \mu} = 0 \tag{23}$$

or

$$\frac{\partial \sigma_{\ddot{y}_1}^2}{\partial \xi_d} = 0, \quad \frac{\partial \sigma_{\ddot{y}_1}^2}{\partial \omega_d} = 0, \quad \frac{\partial \sigma_{\ddot{y}_1}^2}{\partial \mu} = 0 \tag{24}$$

In practice, Davenport spectrum and some other spectra of wind velocity rather than white noise are widely used for describing wind excitation, thus, Davenport spectrum is adopted herein to determine the optimal parameters. It is difficult to obtain the analytical solution for this case, a numerical method is thus proposed. Davenport spectrum has a form

$$\left.\begin{aligned} S_{vv}(\omega) &= 4K\overline{V}_{10}\frac{x^2}{(1+x^2)^{4/3}} \\ x &= \frac{600\omega}{\pi\overline{V}_{10}} \end{aligned}\right\} \tag{25}$$

where $\overline{V}_{10}$ is the mean wind velocity at 10 meters height, K is the coefficient related to ground roughness.

When the excitation is not white noise, the variation Y_1 can be given by

$$\sigma_{Y_1}^2 = \int_{-\infty}^{\infty} F(\omega_d, \xi_d, \mu) S_{vv}(\omega)\, d\omega \tag{26}$$

in which

$$F(\omega_d, \xi_d, \mu) = \frac{1}{|\Delta|^2} A_1^2 [\omega^4 + 2(2\xi_d^2 - 1)\omega_d^2 \omega^2 + \omega_d^4] \tag{27}$$

It is assumed that $S_{vv}(\omega)$ is independent of ω_d, ξ_d and μ, so, we have

$$\frac{\partial \sigma_{Y_1}^2}{\partial \omega_d} = \int_{-\infty}^{\infty} S_{vv}(\omega) \frac{\partial F}{\partial \omega_d}\, d\omega = 0 \tag{28}$$

The optimal frequency or frequency ratio can be obtained numerically from Eq. (28). In the same way, the optimal damping and mass ratio can be obtained from the following equations, respectively

$$\frac{\partial \sigma_{Y_1}^2}{\partial \xi_d} = \int_{-\infty}^{\infty} S_{vv}(\omega) \frac{\partial F}{\partial \xi_d}\, d\omega = 0 \tag{29}$$

$$\frac{\partial \sigma_{Y_1}^2}{\partial \mu} = \int_{-\infty}^{\infty} S_{vv}(\omega) \frac{\partial F}{\partial \mu}\, d\omega = 0 \tag{30}$$

Similarly, the optimal parameters based on other criteria, such as minimum velocity and minimum acceleration of the main structure, can also be obtained numerically based on the Davenport spectrum or other wind velocity spectra.

The formulas for determining the optimal parameters of TMD proposed by Warburton (1982) are

$$\alpha_{opt} = \frac{\omega_{opt}}{\omega_1} = \frac{\sqrt{1+0.5\mu}}{1+\mu} \tag{31}$$

$$\xi_{opt} = \sqrt{\frac{\mu(1+0.75\mu)}{4(1+\mu)(1+0.5\mu)}} \tag{32}$$

where α_{opt} and ξ_{opt} are optimal frequency ratio and damping ratio of TMD.

When mass ratio μ is very small, it is clear that optimal damping ratio ξ_{opt} can be expressed as

$$\xi_{opt} = 0.5\sqrt{\mu} \tag{33}$$

The optimal frequency ratio $\alpha_{opt} = \dfrac{\omega_{opt}}{\omega_1}$ computed from Eq. (21) for white noise excitation and from Eq. (28) based on the Davenport spectrum with different mass ratios and damping ratios are shown in Fig. 2 and Fig. 3, respectively. Unstable results are found for the undamped main structure for Davenport spectrum as shown in Fig. 3. The results calculated by use of the approximated formula, Eq. (31), are also plotted in these figures for comparison purposes. The approximated formula gives results with good accuracy when the damping ratio of the main structure and the mass ratio are relatively small. However, it can be seen from these figures that the optimal frequency ratios calculated from Eq. (21) and Eq. (28) are slightly larger than those obtained from the approximated formula Eq. (31). Thus, the following approximated formula for determining the optimal frequency ratio is proposed to best fit the calculated results presented in Figs. 2 and 3.

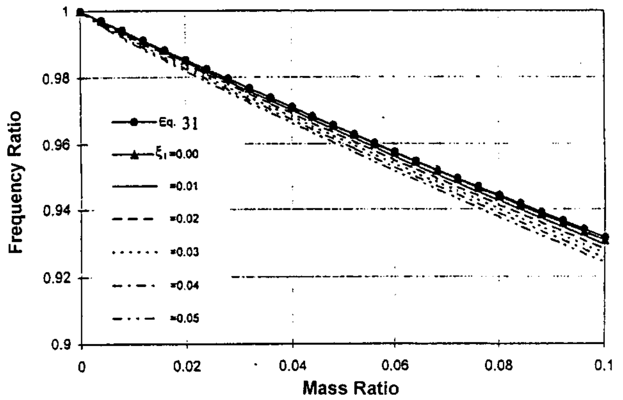

Fig. 2 Optimal frequency ratio (wind excitation-white noise)

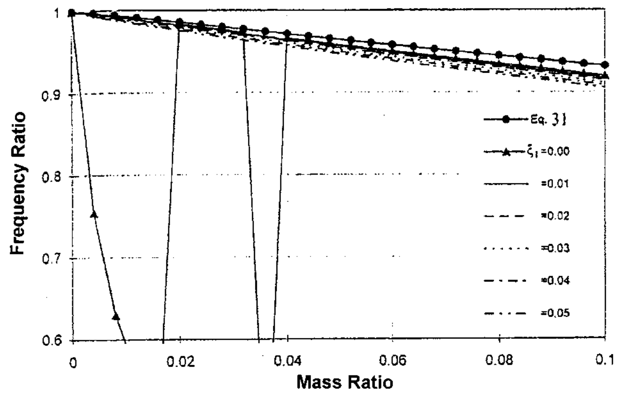

Fig. 3 Optimal frequency ratio (wind excitation-Davenport spectrum)

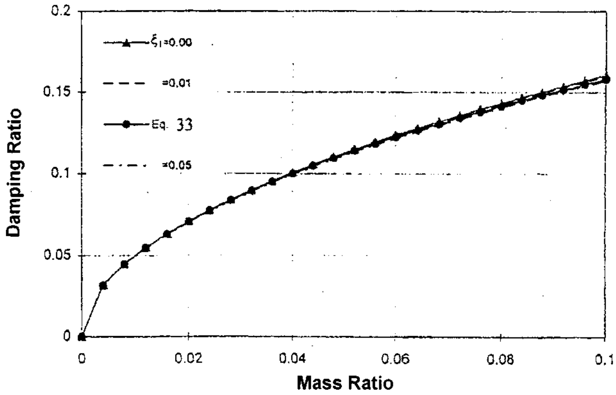

Fig. 4 Optimal damping ratio (wind excitation-white noise)

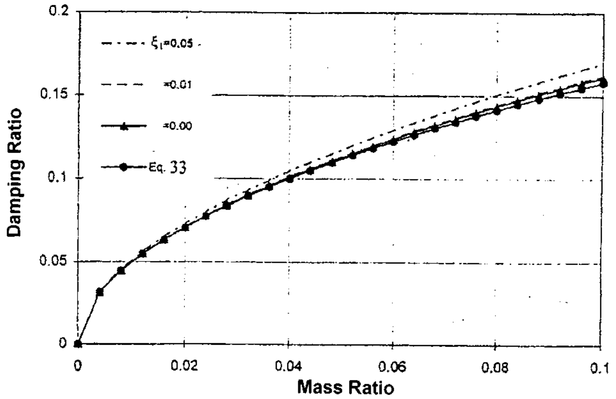

Fig. 5 Optimal damping ratio (wind excitation-Davenport spectrum)

$$\alpha_{opt} = \frac{\omega_{opt}}{\omega_1} = \frac{\sqrt{1+0.4\mu}}{1+\mu} \tag{34}$$

Optimal damping ratio ξ_{opt} as a function of mass ratio μ for wind excitation (white noise and Davenport spectrum) is shown in Fig. 4 and Fig. 5. It is clear that the optimal damping ratio of a TMD is dependent on the damping ratio of the main structure, implying that the higher the damping ratio of the main structure, the higher the optimal TMD damping ratio. It is noted that the approximated formula Eq. (33) gives results with good accuracy, suggesting the simplified formula is applicable to determining the optimal damping ratio of TMD.

It can be seen from Figs. 2 to 5 that the difference between the optimal parameters based on white noise excitation and those determined by Davenport spectrum is not significant.

3. Numerical example 1

Realistic structural parameters taken from a communication tower are considered in this numerical example. The first natural frequency and critical damping ratio of the tower are 1.53 rad/sec and 0.02%, respectively. The first modal mass is 5356 Ton and the mass of TMD is chosen as 60 Tons ($\mu = 1.06\%$). The optimal frequency ratio and damping ratio for wind

Table 3 Optimal TMD parameters

Optimal parameters	White noise	Davenport spectrum	Approximated method
	μ=0.0106		
Optimal frequency (rad/sec)	1.514 (1.529)	1.510 (1.525)	1.517
Optimal damping ratio	0.051 (0.052)	0.052 (0.052)	0.051
	μ=0.0424		
Optimal frequency (rad/sec)	1.477 (1.521)	1.466 (1.510)	1.480
Optimal damping ratio	0.102 (0.108)	0.103 (0.105)	0.101

Note : The results in parentheses are obtained based on minimum acceleration criterion.

excitation (white noise and Davenport spectrum) are presented in Table 3. The results obtained based on the minimum acceleration criterion are also listed in Table 3 (if it is not specified in the figures and tables presented in this paper, the minimum displacement criterion is used). It can be seen that the optimal parameters obtained from different methods are also the same, illustrating again that the simplified formulas proposed in this paper are applicable to determining the optimal parameters of TMD.

4. Optimal control of frame structures

After the optimal parameters of TMD are determined, optimal control of frame structures by using active TMD is investigated in this section.

It is assumed that a frame structure with an active TMD installed at the top floor of the building subjected to wind excitation is treated as a multi-degrees-of-freedom system. Only along-wind response control of the structure is considered herein. In this case, the equation of motion of the whole system, Eq. (1), can be rewritten as

$$M\ddot{X}(t)+C\dot{X}(t)+KX(t)=DW+U \tag{35}$$

in which,

$$M=\begin{bmatrix} m_1 & 0 & \cdots & & 0 \\ 0 & m_2 & & & \\ \vdots & & \ddots & & \vdots \\ & & & m_n+m_d & 0 \\ 0 & \cdots & & 0 & m_{n+1} \end{bmatrix} \qquad C=\begin{bmatrix} c_1 & 0 & \cdots & & 0 \\ 0 & c_2 & & & 0 \\ \vdots & & \ddots & & \\ & & & c_n+c & -c \\ 0 & \cdots & & -c & c \end{bmatrix}$$

$$K=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & \cdots & 0 \\ -k_2 & k_2+k_3 & -k_3 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & \cdots & -k_n & k_n+k & -k \\ 0 & \cdots & & -k & k \end{bmatrix} \qquad D=\begin{bmatrix} D_1 & 0 & \cdots & & 0 \\ 0 & D_2 & & & 0 \\ \vdots & & \ddots & & \vdots \\ & & & D_n & \\ 0 & \cdots & & & D_{n+1} \end{bmatrix}$$

$$D_q = A_q \mu_s(x_q), \quad U = [0, 0 \cdots, 0, -u, u], \quad W = [w_1, w_2, \cdots w_n, w_{n+1}]^T$$

where c_q is the viscous damping coefficient, A_q is the area of windward face, $\mu_s(x_q)$ is the drag Coefficient and w_q is the wind pressure at x_q, m_d, c and u are the mass and the viscous damping coefficient of the TMD as well as the control force acting on the TMD, respectively.

Using the method of mode superposition and substituting Eq. (4) into Eq. (35) obtain

$$\ddot{y}_j(t) + 2\xi_j \omega_j \dot{y}_j(t) + \omega_j^2 y_j(t) = f_j + U_j u \tag{36}$$

Eq. (36) can be expressed in state form as follows

$$\dot{z}_j = A_j z_j + B_j u + F_j \tag{37}$$

in which

$$A_j = \begin{bmatrix} 0 & 1 \\ -\omega_j^2 & -2\xi_j \omega_j \end{bmatrix}, \quad B_j = \begin{Bmatrix} 0 \\ U_j \end{Bmatrix} \quad z_j = \begin{Bmatrix} y_j \\ \dot{y}_j \end{Bmatrix} \quad F_j = \begin{Bmatrix} 0 \\ f_j \end{Bmatrix} w(t)$$

$$\hat{M}_j = \Phi_j^T M \Phi_j \quad f_j = \frac{1}{\hat{M}} \Phi_j^T D W \quad U_j = \frac{1}{\hat{M}} (\Phi_{j,n+1}^T - \Phi_{j,n}^T)$$

The LQR control strategy, due to its simplicity and inherent stability, is the most widely used control strategy in civil engineering, especially in the application of ATMD. Thus, the LQR control strategy is adopted herein.

Using the following criterion

$$J_1 = \frac{1}{2} \int_0^t [z_j Q_j z_j + \gamma_j u^2] dt \longrightarrow \min \tag{38}$$

yields

$$u^* = -\frac{1}{\gamma_j} B_j^T P_j Z_j \qquad P_j = \begin{bmatrix} P_{1j} & P_{2j} \\ P_{2j} & P_{3j} \end{bmatrix} \tag{39}$$

where u^* is the optimal control force.

The matrix P can be determined by solving the following Riccati equation

$$\dot{P}_j + P_j A_j + A_j^T P_j - \frac{1}{\gamma_j} P_j B_j B_j^T P_j + Q_j = 0 \tag{40}$$

If set

$$\gamma_j = \gamma, \qquad Q_j = Q \tag{41}$$

then

$$u^* = -\frac{1}{\gamma_j} U_j (P_{2j} y_j + P_{3j} \dot{y}_j) \tag{42}$$

The equation of the j-th generalized coordinate, Eq. (36), can be rewritten as

$$\ddot{y}_j(t)+\left(2\xi_j\,\omega_j+\frac{1}{\gamma}U_j^2P_{3j}\right)\dot{y}_j(t)+\left(\omega_j^2+\frac{1}{\gamma}U_j^2P_{2j}\right)y_j(t)=f_j \tag{43}$$

The relationship between the spectrum of the generalized coordinate y and that of the generalized force, f, can be derived as follows

$$S_{y_jy_k}(\omega)=H_j^*(i\,\omega)H_k(i\,\omega)S_{f_j\,f_k}(\omega) \tag{44}$$

in which

$$H_j(i\,\omega)\frac{1}{\left(\omega_j^2+\frac{1}{\gamma}U_j^2P_{2j}\right)-\omega^2+\left(2\xi_j\,\omega_j+\frac{1}{\gamma}U_j^2P_{3j}\right)i\,\omega} \tag{45}$$

$$S_{f_j\,f_k}(\omega)=\frac{1}{\hat{M}_j\,\hat{M}_k}\sum_{l=1}^{n+1}\sum_{i=1}^{n+1}\Phi_{ij}\,\Phi_{lk}\,D_i\,D_l\,\rho^2\,\overline{V}_i\,\overline{V}_l\,S_v(\omega)\,\gamma(h_i\,,h_l) \tag{46}$$

in which $\overline{V}_i$ is the mean wind velocity at the coordinate x_i, and $\gamma(h_i, h_l)$ that is the coefficient of correlation of wind velocity is given by (Cao, 1997)

$$\gamma(h_i\,,h_l)=\exp[-0.232495\,|\,h_i-h_l\,|/\sqrt{h_i+h_l}\,] \tag{47}$$

The spectrum of structural response is

$$S_{x_i}(\omega)=\sum_{l=1}^{n+1}\sum_{k=1}^{n+1}\Phi_{il}\,\Phi_{ik}\,S_{y_jy_k}(\omega) \tag{48}$$

Neglecting the correlation between the mode shapes, the variances of displacement of the structure is determined by

$$\sigma_{x_i}^2=\sum_{j=1}^{n+1}\Phi_{ij}^2\,\sigma_{y_j}^2 \tag{49}$$

in which

$$\sigma_{y_j}=\int_0^\infty|H_j(i\,\omega)|^2S_{f_j}(\omega)\,d\,\omega=g_j\,\lambda_{0j} \tag{50}$$

$$g_j=\frac{1}{\hat{M}_j^2}\sum_{l=1}^{n+1}\sum_{i=1}^{n+1}\Phi_{ij}\,\Phi_{lj}\,D_i\,D_l\,\rho^2\,\overline{V}_i\,\overline{V}_l\,\gamma(h_i\,,h_l) \tag{51}$$

$$\lambda_{0j}=\int_0^\infty|H_j(i\,\omega)|^2S_v(\omega)\,d\,\omega \tag{52}$$

If $S_v(\omega)$ is taken as the Davenport spectrum, then, the analytical expression of λ_{0j} is derived as

$$\lambda_{oj} = 6Kv_{10}^2/(b_0^4 b)\left\{1-\sum_{i=1}^{3}\left[0.5d_i \ln(1-2r\cos\theta_i + r^2) + \frac{r+3\cos\theta_i d_i}{2\sin\theta_i}\left(tg^{-1}\frac{1-r\cos\theta_i}{r\sin\theta_i} + \frac{\pi}{2} - \theta_i\right)\right]\right\} \tag{53}$$

where

$$b = \pi v_{10}/600, \quad x_j = \omega_j/b, \quad b_0 = (1+x_j^2)^2 - 4\xi_j^2 x_j^2$$

$$b_1 = (1+x_j^2 - 2\xi_j^2 x_j^2)/b_0, \ b_2 = 1/\sqrt{b} = r^3, \ \theta_1 = \arccos(b_1/b_2)/3$$

$$\theta_2 = \theta_1 + 2\pi/3, \ \theta_3 = \theta_1 + 4\pi/3, \ d_i = 2\sin\theta_i(b_2\cos\theta_i + b_1)/(3r^2\sin 3\theta_1)$$

4. Numeric example 2

A tall building considered herein has 30 stories, the mass, height, shear stiffness and the damping ratio of each story are as follows

$m_i = 1152$ Tons, $h_i = 3.1$ m, $k_i = 6.0\times 10^6$ KN/m, $\xi_i = 0.033$

The Davenport spectrum is adopted. The other parameters for determining wind loading on the building are : A = 124 m^2, $\mu_s(x_i) = 1.3$ The variation of mean wind velocity along height of the structure can be expressed as

$$\overline{V}_{x_j} = \left(\frac{x_j}{10}\right)^{0.33}\overline{V}_{10}$$

The first circular natural frequency of the main structure is found as : ω_1=3.1

The mass of TMD that is installed at the top floor of the building is chosen as 230.4 Tons ($\mu = 0.111$) due to geographical considerations. The optimal damping ratio and circular natural frequency calculated by the present method and the approximated formulae, Eq. (34) and Eq. (33), are as follows :

$\omega_{opt} = 2.867(2.852)$, $\xi_{opt} = 0.1764(0.1666)$

The values in parentheses are obtained from Eqs. (34) and (33).

Three cases, the structure without control device, with active TMD and passive TMD (the control force u = 0) are considered in this numerical example. Two set of parameters of the TMD : $\omega_d = 3.1$, $\xi_d = 0.1006$ and the optimal parameters determined, are adopted for the computation.

Table 4 The variance of structure response

Response	Without control device	Passive TMD	Active TMD
X_{30}	3.6 cm	2.2 cm (2.0)	1.6 cm (1.4)
$\ddot{X}_{30}$	45 cm/s^2	26.7 cm/s^2 (25.4)	1.5 cm/s^2 (1.3)

Note : The values in parentheses are calculated by used of the optimal parameters.

The results are calculated and listed in Table 4. It can be seen from the results presented in Table 4 that the control efficiency of active TMD with the optimal parameters is the best.

5. Conclusions

The active and passive tuned mass damper installed on tall buildings and high-rise structures have many advantages, such as high efficiency, flexibility and adaptability. A set of parameter optimization methods is proposed to determine optimal parameters of passive tuned mass dampers under wind excitation. The calculated results show that the difference between the optimal parameters calculated based on white noise excitation and those determined by Davenport spectrum is not significant. It is found that the optimal damping ratio of a TMD is dependent on the damping ratio of the main structure, but independent of the natural frequency of the main structure. Simplified formulas for the determination of the optimal parameters are proposed so that the design of a TMD can be carried out easily. The numerical example 1 shows that the optimal parameters determined by the approximated formulas are close to those calculated by the exact methods proposed in this paper, suggesting that the simplified formulas are applicable to the design of the optimal parameters of TMD. The present methods and simplified formulas provide not only the optimal parameters for the TMD design under wind excitation but also the good estimates or initial values for other excitations.

Optimal design of wind-induced vibration control for frame structures is investigated. A thirty-story tall building is used as an example to demonstrate the procedure and to verify the efficiency of ATMD and TMD with the optimal parameters. The numerical example shows that the control efficiency for both displacement and acceleration responses of a structure by use of ATMD and TMD is very good, especially, the control efficiency of ATMD with the optimal parameters is the best.

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