

New criteria to fix number of hidden neurons in multilayer perceptron networks for wind speed prediction

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Abstract. This paper proposes new criteria to fix hidden neuron in Multilayer Perceptron Networks for wind speed prediction in renewable energy systems. To fix hidden neurons, 101 various criteria are examined based on the estimated mean squared error. The results show that proposed approach performs better in terms of testing mean squared errors. The convergence analysis is performed for the various proposed criteria. Mean squared error is used as an indicator for fixing neuron in hidden layer. The proposed criteria find solution to fix hidden neuron in neural networks. This approach is effective, accurate with minimal error than other approaches. The significance of increasing the number of hidden neurons in multilayer perceptron network is also analyzed using these criteria. To verify the effectiveness of the proposed method, simulations were conducted on real time wind data. Simulations infer that with minimum mean squared error the proposed approach can be used for wind speed prediction in renewable energy systems.

Keywords: hidden neurons; mean squared error; multilayer perceptron; neural networks; wind speed prediction

1. Introduction

A Neural Network (NN) has been an explosion of interest over the last few decades and has been successfully applied in many fields such as prediction, pattern recognition, image processing and classification etc. The key factors of Neural Network are nonlinearity, ability to generalize, ability to learn, input and output mapping and fault tolerance. One of the major challenges in the design of neural network is the fixation of hidden neurons with minimal error and highest accuracy. The excessive hidden neurons will cause over fitting i.e. the neural network have over estimate the complexity of the target problem (Ke 2009). It greatly degrades generalization capability, to lead with significant deviation in prediction. In this sense, determining the proper number of hidden neurons to prevent over fitting is critical in prediction problem using NN.

The Neural Networks are effective for predicting events when the network has a large datasets to draw on. The quality of prediction made by the network is measured in terms of generalization error. Generalization performance varies over time as the network adapts during training. In this paper fixation of hidden neurons is discussed for Multilayer Perceptron (MLP) neural network as

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applied for wind speed prediction in renewable energy systems. The need for wind speed prediction is to assist with operational control of wind farm and planning development of power station.

Several researchers proposed many approaches for fixing hidden neuron in NN. (Li *et al.* 1995) proposed to estimate number of hidden neuron usage in prediction of time series. In 1997, Tamura *et al.* (1997) presented another method based on Akaike information criteria. (Zhang *et al.* 2003) implemented a set covering algorithm. (Ke *et al.* 2008) proposed method based on sensitivity analysis applied for stock prediction application. Stephen (2008) developed a formula for network. The key factors considered were simplicity, scalability and adaptively. (Xu *et al.* 2008) presented novel approach for data mining. (Doukim *et al.* 2010) implemented coarse to fine search. This search consists of binary search and sequential search. (Panchal *et al.* 2011) proposed method the number of hidden layer is indirectly proportional to epochs. (David *et al.* 2012) developed method used in proper NN architectures. The advantages of these approaches include are no trial and error method and generalization ability is preserved.

Thus various criteria were proposed for fixing hidden neuron by researchers during the last couple of decades. Most of researchers have fixed number of hidden neuron based on trials. In this paper, new criteria are proposed and are applied for MLP for wind speed prediction in renewable energy systems. The proposed various criteria are tested using convergence theorem which converges infinite into finite sequences. This proposed MLP model is stable and has fast convergence. The main objective is to minimize mean squared error (MSE), improve accuracy and stability of MLP neural network.

2. Problem definition

Wind Speed Prediction problem have been considered and fixing of number of hidden neurons in hidden layer have been analyzed for MLP neural network. The problem is analyzed with different criteria as mentioned. To fix hidden neurons of neural network to solve a specific task has been an important problem. With few hidden neurons, the network may not be powerful enough to meet the desired requirement including capacity, error precision and so on. With large number of hidden neurons, the training time and testing time may be long. Thus fixing the number of a hidden neuron is important for given problem. An important but difficult task is to determine the optimal number of parameters. In other words, it needs to measure of the discrepancy between neural network and an actual system. In order to tackle this, most researches have mainly focused on improving the performance. There is no way to find hidden neuron in neural network without try and test during the training and computing the generalization error. The hidden output connection weights becomes small as number of hidden neuron become large, and also that the trade off in stability between input hidden and hidden output connection exists. A trade off is formed that if the number of hidden neuron (N_h) become too large, the output neuron becomes unstable, and if the number of hidden neuron becomes too small, the hidden neuron becomes unstable again. The problems in the fixation of hidden neurons still exist.

There is no generally accepted theory to determine how many hidden neuron are needed to approximate any given function in single hidden layer that can be useful in certain architectures. If it has few numbers of hidden neurons, there might have a large training error due to under fitting. If it has more number of hidden neurons, there might have a large training error due to over fitting. An exceeding number of hidden neurons made on the network deepen the local minima problem.

The proposed new criteria which shows stable performance on training despite of large number of hidden neuron .The aim is to fix hidden neurons in designing the MLP and minimizing the MSE for wind speed prediction in renewable energy systems.

3. Overview of multilayer perceptron

The MLP is one of the perceptron learning rule used computing models in artificial neural network. It is supervised learning network which can be used for many applications. The MLP networks possess a nonlinear activation function (Sivanandam *et al.* 2008). The widely used nonlinear activation function is hyperbolic sigmoid function. The hidden neurons make the MLP network active for complex tasks. This MLP model consists of input, hidden and output layer. Any layer that is formed between input and output layers is called hidden layer. This hidden layer is internal to the network and has no direct contact with the external environment. The input layer connects to hidden layer and hidden layer connects to output layer by weights. Initially inputs are assigned. Then the net input is computed. The net input is the sum of weighted inputs. The output is obtained by applying activation function over the calculated net input. The network is trained until stopping criterion is reached. The performance of neural network model is measured and evaluated based on the test data. The MLP network is formed by nonlinear transformation. It has mainly two steps. The first step is to find output of each layer. If expected output is not reached use second step. The second step is to update the weight and minimize global error.

4. Proposed approach for fixing number of hidden neuron in MLP

There exists various heuristics in the literature; amalgamating the knowledge gained from previous experiments on where a near optimal topology might exists .The objective is to devise criteria that estimate the number of hidden neurons as a function of input neurons and to develop the model for Wind Speed Prediction in Renewable Energy Systems. The estimate can take the form of a single exact topology to be adopted.

4.1 Proposed architecture

For the considered Wind Speed Prediction model, the inputs are temperature (T_w), wind direction (D_w) and wind speed (N_w). As a result, 3 input neurons were built in the output layer. The wind speed to be predicted forms the single output neuron in output layer. The proposed approach aims to fix the number of neurons in the hidden layer so as to achieve better accuracy and faster convergence. From Fig. 1,

The input and the output target vector pairs are,

$(X_1, X_2, X_3 : Y) = (\text{Temperature, Wind Direction, Wind Speed: Predicted Wind speed})$

$(X_1, X_2, X_3 : Y) = (T_w, D_w, N_w: N_{wp})$ where N_{wp} is the predicted wind speed

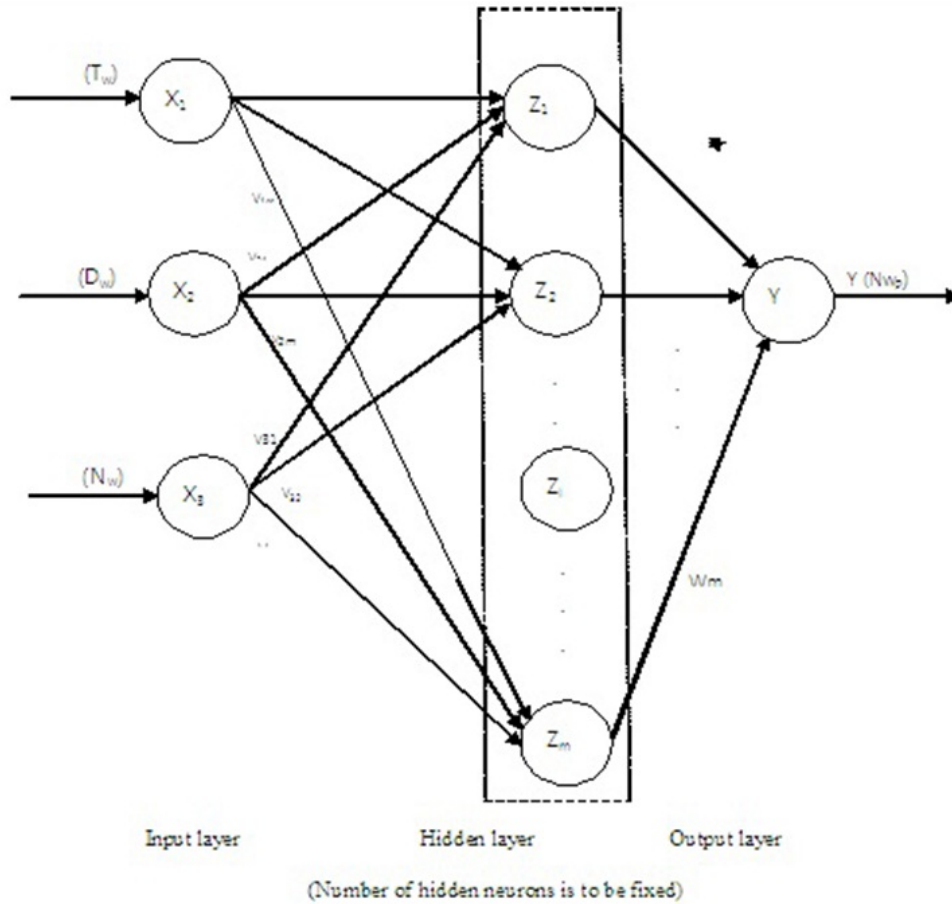


Fig. 1 Architecture of the proposed model

From Fig. 1, it can be observed that, the layers make independent computation on data that it receives and passes the results to another layer and finally, determine the output of the network. Each unit makes its computation based upon the weighted sum of its inputs. The hyperbolic tangent activation function is used to get the actual output. The hidden layer has hyperbolic sigmoid activation function and output layer has purelin activation function. The threshold value is fixed to the activation function of 1. The number of hidden neuron used in hidden layer is selected (fixed) based on new criteria. The proposed model is used for estimation and prediction. The key of the proposed criteria is to select the number of neurons in hidden layer.

Input vector, $X = [T_w, D_w, N_w]$

Output vector, $Y = [N_{wp}]$

Weight vector of input to hidden vector, $V = [V_{11}, V_{12}, \dots, V_{1n}; V_{21}, V_{22}, \dots, V_{2n}; V_{31}, V_{32}, \dots, V_{3n}]$

Weight vector of hidden to output vector, $W = [W_1, W_2, \dots, W_n]$

$$\text{Output of Hidden layer} \quad Z_j = f\left(\sum_{i=1}^n \sum_{j=1}^m X_i V_{ij}\right) \quad (1)$$

where V the weight between is input and hidden layer and X is the input value.

Then hyperbolic sigmoid activation function is applied over the net input to calculate the output. The output of the neural network is given by

$$Y = f\left(\sum_{j=1}^m Z_j W_j\right) \quad (2)$$

where $j = 1$ to m , 'm' is the number of hidden neurons which is fixed using the proposed approach.

4.2 Proposed block diagram

The block diagram of the proposed model is as shown in Fig. 2.

Generally, Neural Networks involves the process of training, testing and developing model at end stage for the past years in wind farms. The perfect design of NN model is important for challenge of better accuracy of model. The data required for input are wind speed, wind direction and temperature. The data are collected from wind farm. The higher valued collected data tend to suppress the influence of smaller variable during training. To overcome this problem, the normalization technique is used. And also it is required to improve accuracy of NN model for better results. Therefore, data are scaled within range [0 1]. The scaling is carried out to improve accuracy of subsequent numeric computation. The selection criteria to fix hidden neuron are important in prediction of wind speed in renewable energy system. The perfect design of NN computing model based on the selection criteria is substantiated using convergence theorem. The training can be learned from previous data after normalization. The performance of trained network is evaluated by two ways. First, the actual and predicted wind speed is compared and second, computes MSE of the network. Finally wind speed is predicted which is the output of the NN model.

The description on the block diagram of the proposed approach is as follows:

4.2.1 Data collection

The real time data is collected from Suzlon Energy Ltd., India Wind farm for a period from April 2011 to March 2012. The inputs are temperature, wind vane direction from true north and past wind speed in anemometer. The height of wind tower is 65 m. The predicted daily averaged wind speed is as an output of the model. The number of samples taken to develop a proposed model is 10000.

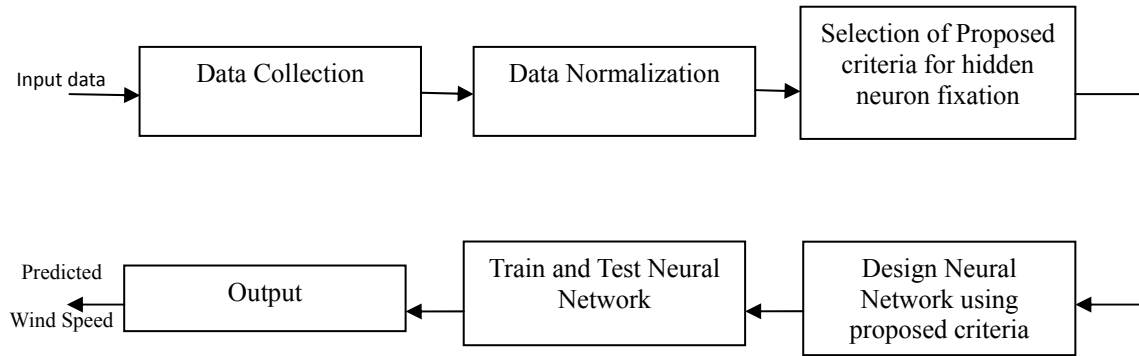


Fig. 2 Block Diagram of Proposed approach

Table 1 Input Parameters of the proposed model

S.NO	Input Parameters	Units	Range of the Parameters
1	Temperature	Degree. C	24-36
2	Wind direction	Degree	1-350
3	Wind speed	m/s	1-16

The above parameters are that are considered as input to the NN model are shown in Table 1. The sample inputs are as shown in Table 2.

Table 2 Sample Inputs

Temp Degree Celsius	Wind Vane direction from true north(degree)	Wind Speed (m/sec)
26.4	285.5	8.9
26.4	286.9	7.6
25.9	285.5	8.6
25.9	284.1	8.9
31.9	302.7	3
25.9	285.5	8.1
25.8	282.7	7.9
33.8	307.4	6.7
25.8	281.2	7.9
25.9	282.7	7.9
25.9	282.7	8.4
25.8	282.7	7.9

4.2.2 Data normalization

The Normalization of data is essential as the variables are of different units. Therefore, data are scaled within the range 0 to 1. The scaling is carried out to improve accuracy of subsequent numeric computation and obtain better output of model. The Min Max technique is used for normalization of input data. The advantage is preserving exactly all relationships in the data and it does not introduce bias. The normalization of data is obtained by the following transformation in Eq. (3)

$$\text{Normalized input, } X'_i = \left(\frac{X_i - X_{\min}}{X_{\max} - X_{\min}} \right) (X'_{\max} - X'_{\min}) + X'_{\min} \tag{3}$$

where X_i, X_{\min}, X_{\max} be the actual input data, minimum and maximum input data. X'_{\min}, X'_{\max} be the minimum and maximum target value.

4.2.3 Designing the network

Set up parameter includes learning rate, epoch and dimensions and so forth. The training can be learned from the past data after normalization. The dimensions like number of input, hidden and output neuron are to be designed. The three input parameters are such as temperature, wind direction and wind speed. The number of hidden layer is one. The number of hidden neuron is to be fixed based on proposed criteria. The input signal is processed and net input of model is computed. The net input is weighted sum of inputs. The activation function is applied over the net input to calculate the output of MLP network model. The parameters are used for design of MLP NN is shown in Table 3.

4.2.4 Selection of proposed criteria

For the proposed model, 101 various criteria were examined to estimate training process and MSE errors in MLP network. The input neuron is taken into account for all criteria. It is tested on convergence theorem. Convergence is infinite into finite sequence. All chosen criteria satisfied the convergence theorem. Initially, apply the chosen criteria to the MLP network for the development of proposed model. Then train the neural network and compute MSE. The result with the minimum estimated MSE is determined as the selection of neuron in hidden layer in the NN model.

Table 3 Parameter Selection of neural network

MLP Network	
Learning rate	= 0.25
Output Neuron	= 1 (N _{wp})
No. of hidden layer	= 1
Input Neurons	= 3 (T _w , D _w , N _w)
No. of Epochs	= 2000
Threshold	= 1

4.2.5 Training and evaluate performance of network

The collected data is divided into training and testing of network. The training data was used to develop models of wind speed prediction, while testing data was used to validate performance of models from training data. 7000 data is used for training and 3000 data is used for testing data. The training can be learned from past data after normalization. Apply testing data to evaluate the performance of the trained network.

4.2.6 Performance of network

The performance of network is evaluated by MSE. The selection of proposed criteria is based on the lowest MSE. These proposed criteria are applied to MLP neural network for wind speed prediction. The network checks whether performance is accepted. Otherwise goes to next criteria then train and test performance of network. The MSE values are calculated for each criterion. The lowest error reflects the stability of network. The proposed criteria are used to fix neuron in hidden layer which is applied to the MLP network for implementation. The designed network model is used for predicted wind speed in renewable energy systems. To perform analysis of NN model, 101 cases with various hidden neuron are examined to estimate learning and generalization errors. The result with minimum mean squared error is determined as the best for selection of neurons in hidden layer.

The considered 101 various criteria for fixing the number of hidden neuron with MSE are shown in Table 4. The selected criteria for NN model is $4n/n-2$ which used 12 number of hidden neurons and obtained a minimal MSE value of 0.0042 in comparison with other criteria. So this criteria $4n/n-2$ is effective for wind speed prediction in renewable energy systems.

4.3 Proof for the chosen proposed criteria

Based on the discussion on convergence theorem in the Appendix, the proof for the selection criteria is established henceforth.

Lemma 1.1 is an estimate of sequence which proves the convergence of the proposed criteria

Lemma 1.1

Suppose a sequence $a_n = \frac{4n}{(n-2)}$, is converged and $a_n \geq 0$

It has limit l . If there exists constant $\varepsilon > 0$ such that $|a_n - l| < \varepsilon$

then $\lim_{n \rightarrow \infty} a_n = l$

Proof: The proof based on **Lemma 1.1**

According to theorem, parameter converges to finite value

$$a_n = \frac{4n}{(n-2)},$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{4n}{(n-2)} = 4, \text{ finite value}$$

Here 4 is limit of sequence as $n \rightarrow \infty$

If sequence has limit then it is a convergent sequence.

Table 4 Analysis of various criteria in MLP network

Considered criteria for fixing number of hidden neuron	No. of hidden neuron	MSE	Considered criteria for fixing number of hidden neuron	No. of hidden neuron	MSE	Considered criteria for fixing number of hidden neuron	No. of hidden neuron	MSE
$3n/n-1$	5	0.311	$6n+5/n$	74	1.204	$4n+5/n-1$	35	0.0623
$5n+4/n-2$	19	0.2268	$3n+4/n-2$	31	0.0186	$7n+15/n+1$	90	0.046
$3^n/n$	9	0.0395	$7(n^2+5)-1/n^2-8$	97	0.5299	$5(n^2+7)/n^2-8$	80	1.024
$3(n+1)/n$	4	0.1722	$8n-15/n+2$	99	0.0752	$7n+2/n-2$	23	0.1269
$7n+7/n+1$	88	0.1006	$4n+4/n-2$	68	0.0445	$5n+6/n$	44	0.6312
$4^n-5/n-2$	59	0.1173	$4n+3/n-2$	67	0.1892	$4n/n-2$	64	0.1644
$8n/n-2$	24	0.1211	$7(n^2+4)+1/n^2-8$	92	0.5559	$4n+5/n-2$	69	0.1808
$4^n/n+2$	13	0.0791	$3n+8/n-2$	17	0.1188	$7n-3/n+1$	85	0.0933
$4n^2+3/n^2-8$	39	0.2144	$6(n^2+5)-1/n^2-8$	83	0.2088	$7(n^2+4)+2/n^2-8$	93	0.5498
$n/n+1$	1	0.1843	$2n/n-1$	3	0.0779	$8n-20/n+2$	98	0.2613
$3^n/n-2$	27	0.2144	$5n+3/n$	43	0.2082	$3n+2/n-2$	29	0.2808
$5n^2+1/n^2-8$	46	0.0101	$6n+2/n$	73	0.5778	$4n-7/n-2$	57	0.276
$4^n+2/n-2$	66	0.0987	$5(n^2+6)/n^2-8$	75	0.2589	$7(n^2+2)/n^2-8$	77	0.3047
$5(n^2+6)+1/n^2-8$	76	0.5597	$8n-5/n+2$	101	0.245	$5n/n-2$	15	0.0319
$7^n+3/n+1$	87	0.0565	$6(n^2+4)+1/n^2-8$	79	0.2648	$8n+2/n-2$	26	0.0237
$5(n^2+1)+3/n^2-8$	53	2.5634	$4n^2+2/n^2-8$	38	0.0399	$5(n^2+7)+2/n^2-8$	82	1.4179
$4n^2+5/n^2-8$	41	0.0561	$7(n^2+5)-2/n^2-8$	96	0.8672	$4n+1/n-1$	33	0.1801
$3^n+1/n-2$	28	0.1286	$6n+2/n+1$	55	0.6098	$(1+11n)/n$	11	0.2279
$(1+10n)/n$	10	0.0592	$7(n^2+4)+3/n^2-8$	94	0.5976	$4n+7/n-2$	71	0.6392
$4n^2+1/n^2-8$	37	0.0517	$4n-6/n-2$	58	0.8598	$7n/n+1$	86	0.3536
$4n/(n-2)$	12	0.0042	$3n/n-1$	14	0.0133	$6n+1/n+1$	54	2.0144
$2n/n+1$	2	0.9745	$4n/n$	21	0.0655	$7(n^2+4)/n^2-8$	91	0.1877
$5n+7/n-2$	22	0.1334	$3n+3/n-2$	30	2.1985	$4n/n+1$	16	0.0314
$5(n^2+1)/n^2-8$	50	0.1712	$5n^2+2/n^2-8$	47	0.0812	$4n^2+4/n^2-8$	40	0.2895
$6n/n-2$	18	0.0077	$7n-6/n+1$	84	0.7853	$6(n^2+4)+3/n^2-8$	81	0.1354

$4n-2/n-2$	62	0.1959	$8n-10/n+2$	100	0.2126	$4.5n/n-1$	7	0.01
$4n/n-1$	32	0.0838	$4n+1/n-2$	65	0.1018	$5n/n-1$	63	0.2438
$5n^2/n^2-8$	45	0.2214	$5n^2+4/n^2-8$	49	1.1039	$8n+1/n-2$	25	0.0346
$4n+6/n-2$	70	0.4621	$6n/n$	72	0.416	$4n+3/n-1$	34	0.0712
$5n/n-1$	8	0.0995	$6n+6/n+1$	56	0.5746	$7(n^2+5)-3/n^2-8$	95	0.346
$8n^2/n^2-7$	36	0.1363	$4n/n-1$	6	0.1611	$5(n^2+1)+1/n^2-8$	51	0.0238
$5n^2+3/n^2-8$	48	0.2578	$5n/n$	42	0.4042			
$4n-4/n-2$	60	0.1934	$5(n^2+1)+2/n^2-8$	52	0.4438			
$5n+5/n-2$	20	0.0256	$6(n^2+4)/n^2-8$	78	0.0908			
$4n-3/n-2$	61	0.1782	$7n+11/n+1$	89	0.3718			

5. Results

Several researchers proposed many approaches to fix number of hidden neurons in neural network. The approaches mainly classifies into constructive and pruning approach. The constructive approach, it start with undersized network and then add additional hidden neuron (Keeni *et al.* 1999, Han *et al.* 2008). The pruning approach, it starts with oversized network and then prunes the less relevant neuron and weights to find the smallest size. The problems of proper number of hidden neuron for a particular problem are to be fixed. The existing methods to determine number of hidden neuron is trial and error rule. This starts with undersized number of hidden neuron (N_h) and adds neurons to N_h . The disadvantage of this is the time consuming and there is no guarantee of fixing the hidden neuron. The NN model is useful tools for prediction application. Once an accurate NN model is developed, wind farm operator can easily apply this method to predict wind speed in renewable energy systems.

The salient points of the proposed approach discussed here. The result with minimum mean squared error is determined as best solution for fixing hidden neurons in neural networks. Simulation results are showing that predicted wind speed is in good agreement with the experimental measured values. Initially real time data are divided into training and testing set. The training set performs in neural network learning and testing set performs to estimating the error. The testing performance stops improving as number of hidden neuron continue to increase; training has begun to fit the noise in the training data, and over fitting occurs. From the results, it is observed that the proposed work given better results than the other approaches. In this paper, proposed new criteria are considered for designing a three layer neural networks. It is known that certain approaches produce large size network than necessary and others are expensive. The fixing of number of hidden layer neurons is important in the implementation of neural network. The analysis of wind speed prediction is carried out by the proposed new criteria. Table 5 shows that proposed model gives better value for MSE in comparison with other existing models.

From Fig. 3, the variation in the MSE values for the proposed 101 criteria can be noted. In these cases, the lowest error is estimated during the training process. The fixing of hidden neurons was carried out based on the value of lowest MSE. Then the selected criteria is applied to the MLP

network and observed predicted wind speed. From Fig. 4, the actual and predicted wind speed for the proposed model is observed. The advantage of the proposed approach includes minimal mean squared error, effective and ease of implementation for wind speed prediction in renewable energy systems. The proposed algorithm was simulated a minimal MSE of 0.0042 is obtained. The results show that good agreement between actual and predicted wind speed.

Table 5 Performance Analysis of various approaches

S. No.	Various approaches	Year	Number of hidden neuron	MSE
1	Li <i>et al.</i> method	1995	$N_h = [(\sqrt{1+8n}) - 1 / 2]$	0.0186
2	Tamura method	1997	$N_h = N - 1$	0.9745
3	Fujitha method	1998	$N_h = K \log (P_c Z / C) / \log s$	0.1722
4	Zhang <i>et al.</i> method	2003	$N_h = 2^n / n - 1$	0.9745
5	ke <i>et al.</i> method	2008	$N_h = (N_{in} + \sqrt{N_p}) / L$	0.0346
6	Trenn method	2008	$N_h = n + n_0 - m / 2$	0.9745
7	Xu <i>et al.</i> method	2008	$N_h = C_f (N / d \log N)^{1/2}$	0.01
8	Doukim <i>et al.</i> method	2010	$N_h =$ coarse to fine search	0.046
9	Hunter <i>et al.</i> method	2012	$N_h = N - 1$	0.01
10	Proposed approach		$N_h = 4n / (n - 2)$	0.0042

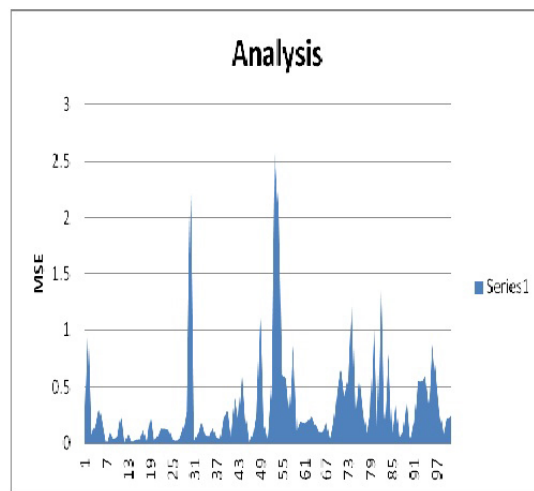


Fig. 3 Analysis of MSE of various criteria

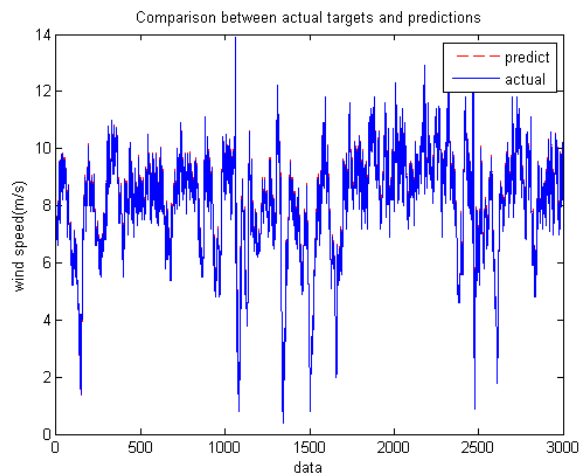


Fig. 4 Actual/ Predicted output waveform

6. Conclusions

A new criterion has been proposed in this paper to fix hidden neuron in MLP network for wind speed prediction in renewable energy systems. The usage of proposed approach effectively forms the MLP neural network model and this technique is compared with other approaches. The main advantage is less error and ease of implementation. Simulation result shows that new criteria is applied for MLP and resulted in minimum mean squared errors. A three layer neural network model is developed for predicting wind speed. The novelty of proposed approach is that it can determine number of hidden neuron with fewer errors. The experimental results show it reduces error and improves accuracy. The monotonic of error function and stability of neural network are improved.

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