

## Vortex excitation model. Part II. application to real structures and validation

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**Abstract.** This paper presents results of calculations performed according to our own semi-empirical mathematical model of critical vortex excitation. All calculations are carried out using own computer program, which allows the simulation of both the across-wind action caused by vortices and the lateral response of analysed structures. Vortex excitation simulations were performed in real time taking into account wind-structure interaction. Several structures of circular cross-sections were modelled using a FEM program and calculated under the action of critical vortex excitation. Six steel chimneys, six concrete chimneys and two concrete towers were considered. The method of selection and estimation of the experimental parameters describing the model are also presented. Finally, the results concerning maximum lateral top displacements of the structures are compared with available full-scale data for steel and concrete chimneys.

**Keywords:** across-wind load; vortex excitation; steel chimneys; concrete chimneys; towers; lateral vibrations; circular cross-section

### 1. Introduction

Mathematical model for critical vortex excitation elaborated in part 1 was applied to real slender structures of circular cross-sections. In this paper the influence of several experimental parameters ( $\alpha$ ,  $\sigma_w$ ,  $k$ ,  $B$ ,  $L_w$ ), which describe critical vortex excitation, on structure response was considered. Time histories of across-wind load caused by vortices as well as lateral displacements were calculated for characteristic (central) point  $z_0$  of the vortex excitation domain along the structure  $\Delta L$ . Several computations were performed for various values of model parameters. Sensitivity analysis was carried out on the obtained time histories of displacements in point  $z_0$ . Sensitivity analysis indicated the significance of particular experimental parameters for lateral response under vortex excitation. In practice, it means that some parameters are less important for final results and some freedom in their determination can be left, and on the other hand some parameters are crucial for the result and should be determined carefully. Ranges of parameters values in which the influence on lateral response is strong or weak were also investigated. Lateral top displacements of analysed structures were estimated for sets of

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experimental parameters that gave maximum response. These results were compared with full-scale data collected from literature.

Table 1 Basic parameters of analysed steel chimneys

	$H$ [m]	$D$ [m]	$d$ [mm]	$f_1$ [Hz]	$f_2$ [Hz]	$\Delta$ [-]	$\lambda$ [-]	$Sc$ [-]	$V_{crit}$ [m/sec]
Steel chimney 1	32.146	1.25	5-7	1.2767	7.4324	0.02	25.717	3.684	8.866
Steel chimney 2	38.174	1.4	5-7	0.8445	5.4065	0.02	27.267	4.37	6.569
Steel chimney 3	40	0.56-2	6-7	0.6495	2.4726	0.02	51.282	4.538	2.021
Steel chimney 4	60	1-4	10-16	0.5804	2.5983	0.02	32.952	5.699	3.224
Steel chimney 5	60	2.2	8-12	0.6412	3.5806	0.02	27.273	3.762	7.834
Steel chimney 6	83.5	3.06	6-40	0.4952	2.2204	0.026	27.288	2.947	8.418

Table 2 Basic parameters of analysed concrete chimneys

	$H$ [m]	$D_b$ [m]	$D_t$ [m]	$d$ [m]	$f_1$ [Hz]	$f_2$ [Hz]	$\Delta$ [-]	$\lambda$ [-]	$Sc$ [-]	$V_{crit}$ [m/sec]
Concrete chimney 1	120	11.56	6.76	0.2-0.4	0.54	2.49	0.15	13.10	52.36	20.28
Concrete chimney 2	150	7.2	4.2	0.3-0.44	0.20	0.92	0.15	26.32	127.56	4.67
Concrete chimney 3	200	15	5	0.2-0.38	0.29	1.09	0.15	20.00	41.81	8.06
Concrete chimney 4	250	24	24	0.3-0.7	0.22	1.09	0.15	10.42	49.19	29.33
Concrete chimney 5	260	15.8	7.7	0.15-0.7	0.21	0.69	0.15	22.13	46.38	8.98
Concrete chimney 6	300	27.8	21	0.25-0.9	0.30	3.88	0.15	12.30	24.72	35.00

## 2. Analysed structures

The following structural configurations were considered: cantilever, tower-like structures of circular cross-sections with constant diameter (steel and concrete chimneys), stepped-diameter (steel chimneys), tapered diameter (concrete chimneys) and structures of strongly varying cross-sections along the height (concrete towers). Six steel chimneys, six concrete chimneys and two concrete towers were analysed. The basic data of analysed structures are given in Table 1 and 2, where:  $H$  – height;  $D$  – outer diameter;  $D_t$ ,  $D_b$  – respectively outer top and base diameters;  $d$  – wall

thickness;  $f_1, f_2$  – natural frequencies of vibrations;  $\Delta$  – logarithmic decrement of damping;  $\lambda$  – slenderness ratio;  $Sc$  – Scruton number,  $V_{crit1}$  – approximate critical wind speed for the top diameter and Strouhal number equal to 0.18. Analysed structures are presented schematically in Fig. 1.

Moreover, two concrete towers: Ostankino Tower (in Russia) of the height 533.3 m, and Hornisgrinde Tower (in Germany) of the height 210 m were also analysed.

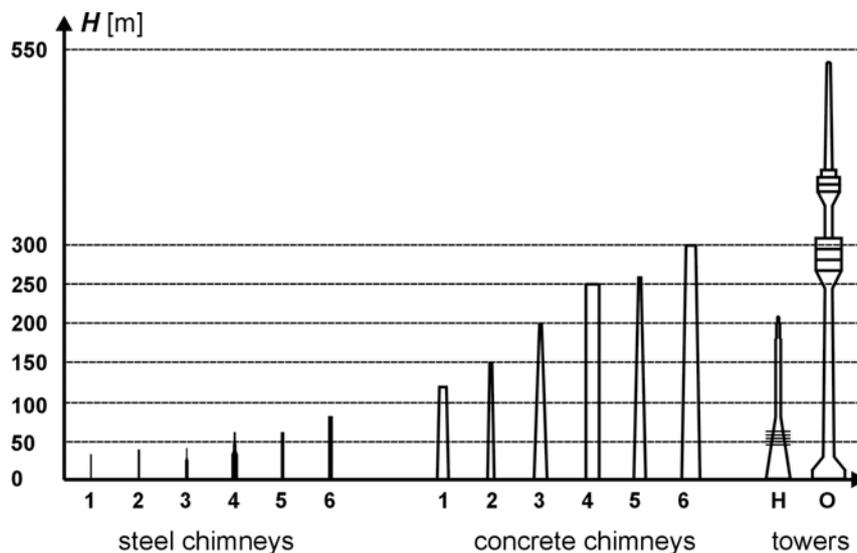


Fig. 1. Analysed structures

### 3. Discussion of preliminary results

#### 3.1. Range of time steps

WAWS method was used in all simulations. Range of time steps is the crucial value for load simulation. Both time of computations and computer power limitations cause that the algorithm of calculations should utilise displacement history for short time interval  $\chi_1 T_1$  to generate next steps of load ( $T_1$  is the first period of natural vibrations of analysed structure;  $\chi_1$  – parameter  $> 1$ ). So, several lateral responses of the structures were computed taking into account displacement histories for different time intervals (different  $\chi_1$ ). Results presented in Fig. 2 are related to estimators ( $\bar{\eta}$  – mean value [m],  $\sigma_\eta$  – standard deviation [m],  $\eta^{max}$  – maximum value [m],  $g$  – peak factor [-], calculated from  $\eta^{max} = g \cdot \sigma_\eta$ ) that were obtained in simulation of time steps  $\Delta t$  of load on the basis of varying time interval  $\chi_1 T_1$ . For short time intervals of displacements the increase of the value  $\eta^{max}$  is caused by feedback between lateral vibrations of the structure and vortex shedding.

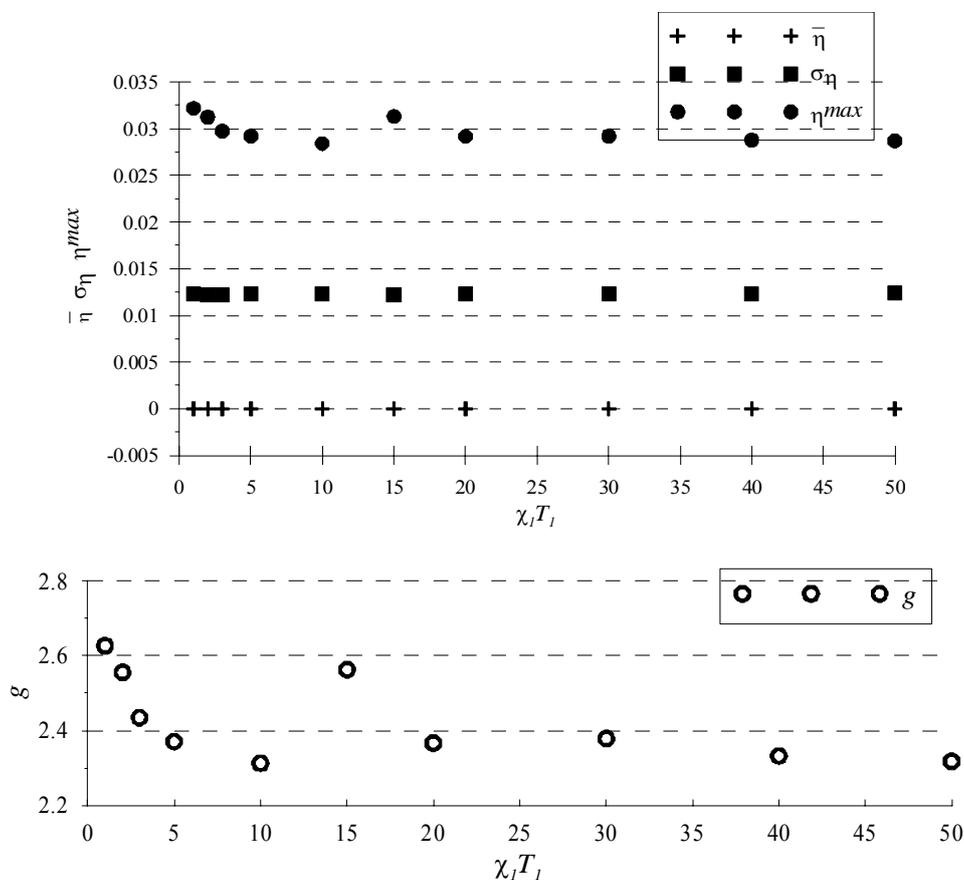
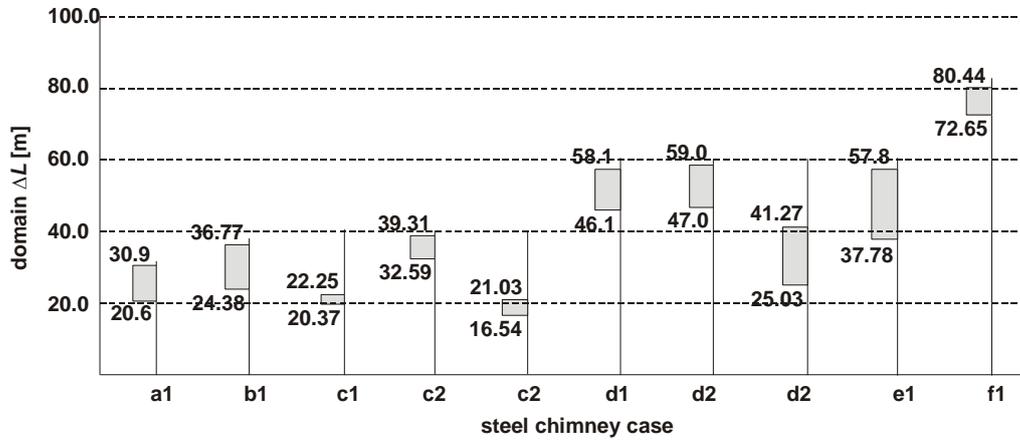


Fig. 2 Estimators ( $\bar{\eta}$ ,  $\sigma_{\eta}$ ,  $\eta^{max}$ ,  $g$ ) for different time intervals of displacements histories  $\chi_1 T_1$  (where  $T_1$  – first period of natural vibrations) used in simulation of next time steps of load (on the example of steel chimney no. 6. of the height 83.5 m)

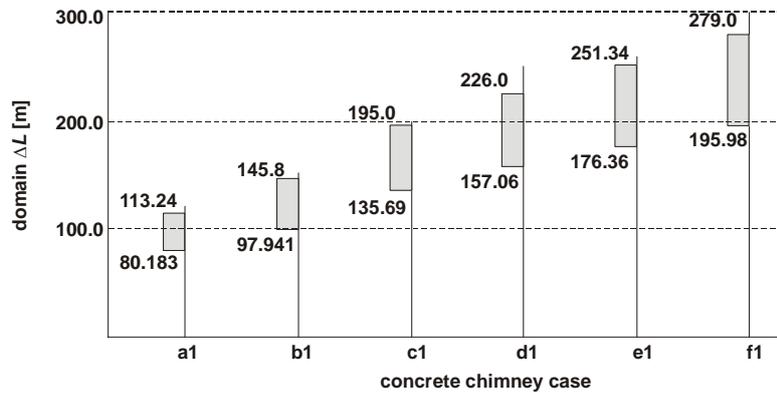
### 3.2. Domain $\Delta L$ , characteristic point $z_0$

The range of vortex excitation  $\Delta L$  has a crucial influence on lateral displacements. Its range is limited only to a short part of the analysed structures (Fig. 3). Moreover, there are only small changes of this range in time.

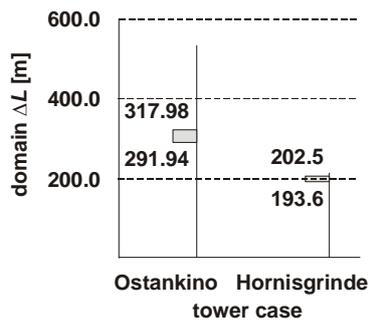
In the case of steel chimneys, the limitation of  $\Delta L$  is caused by a small value of correlation length scale  $L_w$  (mainly for chimneys with small diameters). In the case of concrete chimneys the limitation is caused by small value of area under the curve of the mode shape which is computed in the procedure of assuming the final domain  $\Delta L$ . In case of towers the limitation is also caused by a small value of area under the curve of the mode shape. Also the shapes of towers make vortex excitation almost impossible to occur in large parts of their height. There are many additional elements like antennas existing on the tower surface that disturb wind flow around them.



(a)



(b)



(c)

Fig. 3 Limits of the domain  $\Delta L$  in the case of: (a) steel chimneys (a – no. 1, b – no. 2, c – no.3, d – no. 4, e – no. 5, f – no. 6, 1 – first mode shape, 2 – second mode shape), (b) concrete chimneys (a – no. 1, b – no. 2, c – no.3, d – no. 4, e – no. 5, f – no. 6, 1 – first mode shape) and (c) concrete towers

#### 4. Discussion of the influence of experimental parameters on lateral response

Several computations were performed for various sets of model parameters in order to choose the most proper values of these parameters. Sensitivity analysis was carried out on the obtained results of displacements in point  $z_0$ . Sensitivity analysis should: 1. determine the significance of particular experimental parameters for lateral structure response under vortex excitation; 2. indicate ranges of parameter values in which the influence is strong or weak; 3. finally, indicate the most appropriate numerical procedures for those parameters. The lateral response of the structures due to vortex excitation was computed for various values of model parameters in the variant of sensitivity analysis used in calculations. Then, changes in result values were compared with changes in input parameter values. In that way the influence of one varying parameter, with others left constant at the average level was investigated. All considerations were related to point  $z_0$  of the domain  $\Delta L$ .

Sensitivity analysis was carried out for model parameters such as:  $\alpha$ ,  $B$ , and  $k$ . Moreover, the influence of equivalent surface roughness  $k_s$  was investigated.

The influence of the parameter  $\sigma_w$  was not computed because it depends on the determined value of Reynolds number  $Re$  and effective surface roughness  $k_s/D$ , as well. So, for the given structure changes of the value  $\sigma_w$  were small or similar to  $k_s$ . The influence of turbulence intensity  $I_v$  was non-evidently checked but according to the Vickery formula used in computations ( $B = 2I_v + 0.1$ ) its changes were similar to  $B$ .

The most detailed results of sensitivity analysis concerned steel chimney no. 6 of the following basic data: height  $H = 83.5$  m, outer diameter  $D = 3.06$  m, steel width  $g = 6 - 40$  mm. In case of that chimney computations were carried out in the wide range of experimental input parameters. Sensitivity analysis for this chimney indicated the significance of particular parameters for final results. The obtained results were confirmed by calculations conducted for other structures, but in narrower ranges of experimental input parameters. Next figures concern steel chimney no. 6.

##### 4.1. Equivalent surface roughness $k_s$

This value is constant for the given type of structures. Sensitivity analysis in that case can indicate type of structures that is more sensitive to any changes in surface roughness (for example corrosion of steel chimney pipe). Variations in value  $k_s$  were assumed in the range  $5 \cdot 10^{-6} - 5 \cdot 10^{-4}$ . Other input model parameters were assumed at constant, average level, respectively:  $k = 0.8$ ,  $B = 0.3$ ,  $\alpha = 1$ . Lateral response of the sample structure is more sensitive for small values of  $k_s$  (like for steel structures). There is a relation of estimators  $\sigma_y$ ,  $\eta^{max}$  in the function of  $k_s$  in Fig. 4. All discrete values were approximated using logarithmic and power-law functions. Fitting coefficient was almost equal to 1.0 for both functions. Sensitivity of results is a relative high value for smooth surfaces (lower values of  $k_s$ ) and decreases with increase in roughness. So, steel chimneys are more sensitive to any changes in surface roughness and proper maintenance of the surface is recommended.

##### 4.2. Parameter $\alpha$

Parameter  $\alpha$  used in calculation was assumed in the range: 0.9 - 1.4 with the step 0.1. Other values were constant and respectively equal to:  $k = 0.8$ ,  $B = 0.3$ . It was confirmed that for low values of lateral displacements the influence of  $\alpha$  on results is small. It means that lock-in is weak or does not appear in analysed cases. Moreover, increase of  $\alpha$  beyond 1.1 causes decrease in value

of estimator  $\eta^{max}$ . This can be explained by a self-limited character of vortex excitation. Parameter  $\alpha$  was established in further computations as  $\alpha = 1.0$ , and this is the correct assumption for small amplitudes. There are values of  $\sigma_\eta$  and  $\eta^{max}$  against  $\alpha$  collected together with approximation functions in Fig. 5.

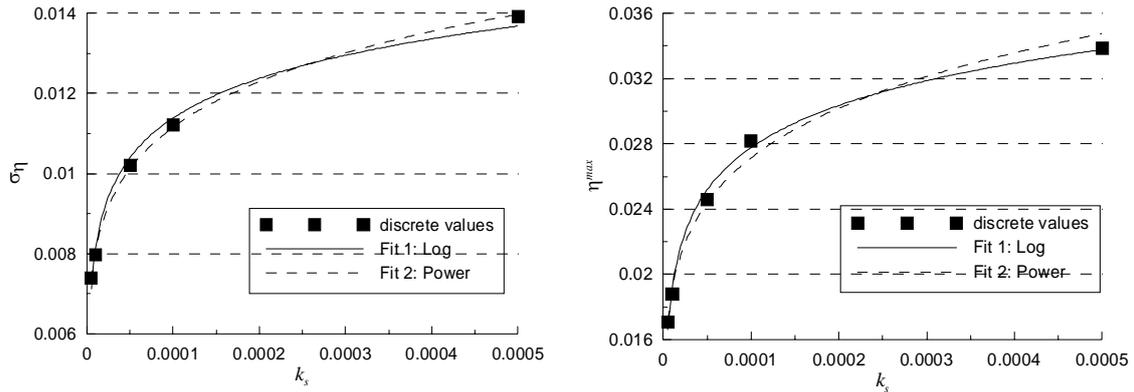


Fig. 4 Estimators  $\sigma_\eta$  and  $\eta^{max}$  against  $k_s$ . Case of steel chimney no 6

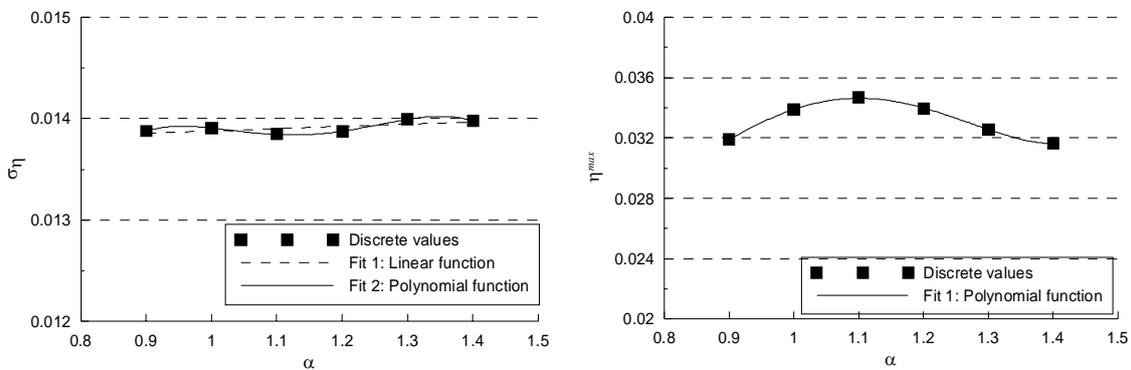


Fig. 5 Values of  $\sigma_\eta$  and  $\eta^{max}$  against  $\alpha$ . Case of steel chimney no 6

#### 4.3. Parameters B and k

Parameters  $B$  and  $k$  were investigated in a wide range. Values of other parameters were settled at the constant level and then  $B$  was assumed as equal to 0.05 (as for narrow-band process), while  $k$  was assumed as varying in the range 0.3 - 1.0. All calculations were repeated for increasing values of  $B$ , up to 0.6 (as for extremely wide-band process).

The following diagrams present the obtained results, respectively: surface plots of  $\sigma_\eta$  and  $\eta^{max}$  against  $B$  and  $k$  – Fig. 6; approximation functions of  $\sigma_\eta$  and  $\eta^{max}$  against  $B$  in surface cross-sections for  $k = 0.8$  – Fig. 7 and against  $k$  for  $B = 0.1$  – Fig. 8.

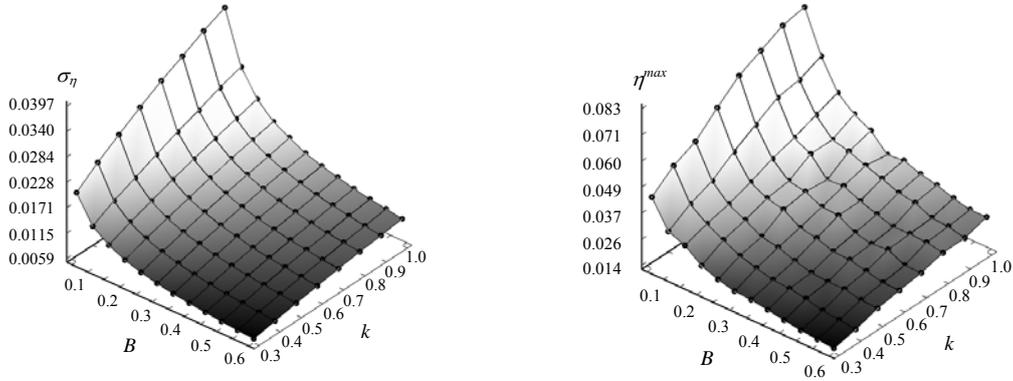


Fig. 6 Spatial distribution of  $\sigma_\eta$  and  $\eta^{max}$  for different  $B$  i  $k$ . Case of steel chimney no 6

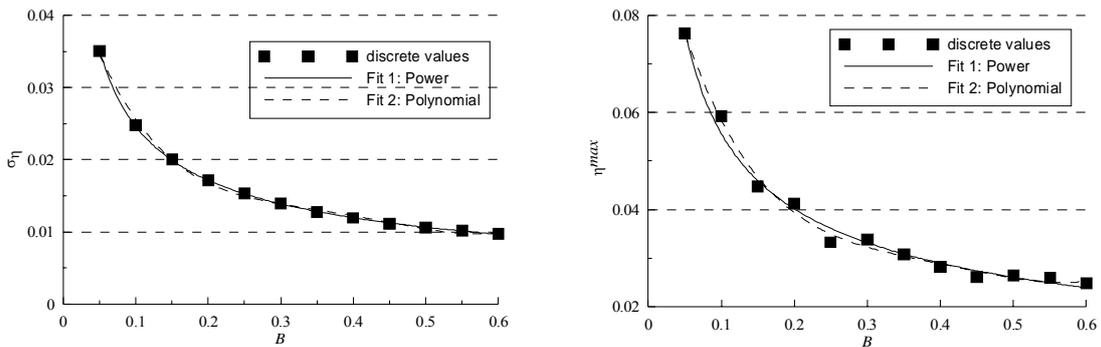


Fig. 7 Approximation functions of  $\sigma_\eta$  and  $\eta^{max}$  in one of the surface cross-sections ( $k = 0.8$ ). Case of steel chimney no 6

Power-law functions can be used as approximations for varying  $B$ . Results given in  $\sigma_\eta$  and  $\eta^{max}$  are more sensitive for low values of  $B$  in each surface cross-section. On the other hand linear and power-law functions could be assumed as approximations for varying  $k$ . Both estimators  $\sigma_\eta$  and  $\eta^{max}$  are similarly sensitive in each surface cross-section.

Considering results concerning sensitivity analysis of lateral displacements caused by vortex excitation for slender tower-like structures, some final conclusions can be formulated:

1. The version of sensitivity analysis used in this paper can be a useful tool in the determination process of influence strength of particular mathematical model parameters on across-wind load and lateral response of the structure. Such an analysis can also indicate ranges of parameters in which lateral response is more sensitive.

2. Lateral displacements of the chimney strongly depend on  $k$  and  $B$ . Computations of other steel chimneys, concrete chimneys and towers confirm that statement. The influence is the strongest in the case when  $k$  value is high and  $B$  value is low – it means that vortex excitation is decisive in across-wind load ( $k$ ) and moreover, vortex shedding is almost of harmonic character ( $B$ ).

3. Parameter  $\alpha$  has weak influence on lateral response, and its value can be accepted as equal to 1.0 in computations in cases of small vibrations. For  $\alpha > 1.1$  values of estimators  $\sigma_\eta$  and  $\eta^{max}$  decrease. In the authors' opinion it is caused by the self-limited character of vortex excitation.

4. There is a considerable influence of  $k_s$  on results, mainly for small values of  $k_s$ . It can be stated that the lack of proper maintenance of steel chimney surface can magnify lateral response under vortex shedding.

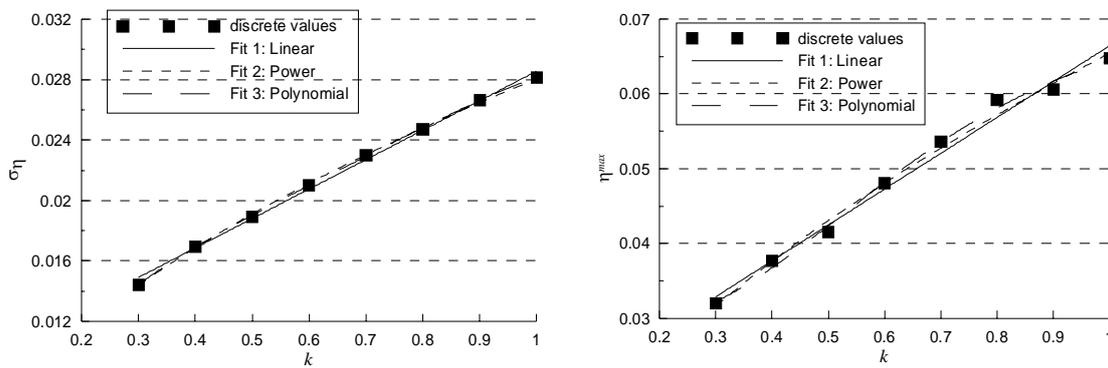


Fig. 8 Approximation functions of  $\sigma_\eta$  and  $\eta^{max}$  in one of the surface cross-sections ( $B = 0.1$ ). Case of steel chimney no 6

### 5. Lateral, maximum top displacement

Top lateral response is calculated for the most extreme cases of load, for parameters sets which were determined in sensitivity analysis. Maximum top displacement [m] in time  $T$  in one process and in  $N$  processes are evaluated according to relationship

$$\eta_{j,top}^{max} = \max \{ \eta_{j,i} \}, \quad \eta_{top}^{max} = \frac{1}{N} \sum_{j=1}^N \eta_{j,top}^{max} \tag{1}$$

Maximum top displacements were obtained for low value of  $B$  and high  $k$ , when power spectral density function is of narrow-banded character, and load process is almost similar to sine function. Results obtained in these circumstances for steel chimneys are presented in Fig. 9, as the maximum top displacement in the function of slenderness ratio  $\lambda$  and Scruton number  $Sc$ .

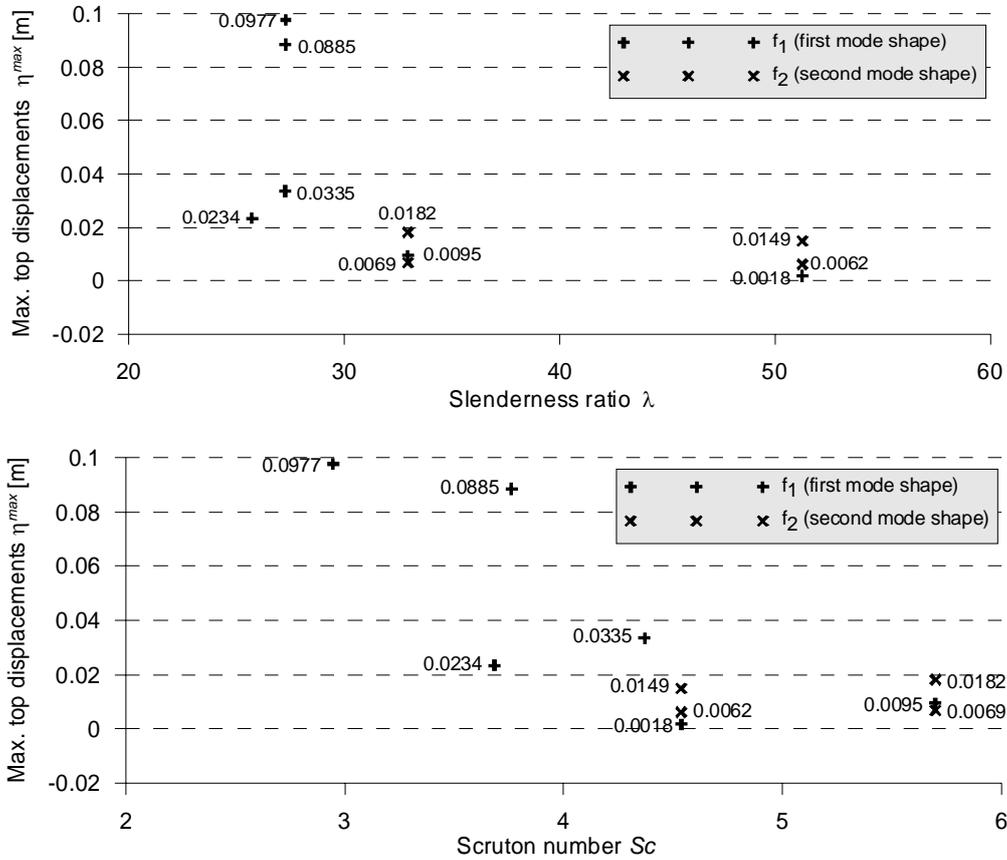


Fig. 9 Maximum lateral top displacement  $\eta^{max}$  against slenderness  $\lambda$  and Scruton number  $Sc$  for steel chimneys

If slenderness decreases, then  $\eta^{max}$  will increase. In general it is caused by higher values of diameter and also higher values of load. If  $Sc$  decreases, then  $\eta^{max}$  will increase because of lower damping.

The same conclusions can be formulated for concrete chimneys: when  $Sc$  is smaller, then displacements are higher (Fig. 10). The case of concrete chimney no. 3 ( $Sc = 41.812$ ) is an exception to this rule, and it is caused by strong convergence of that chimney (base diameter:  $D_b = 15$  m, top diameter:  $D_t = 5$  m, so convergence is 2.5 %). Dependence on  $\lambda$  is disturbed by the chimney no. 5 ( $\lambda = 22.128$ ) which is high but it has relatively small top diameter in comparison to chimneys no. 4 and 6.

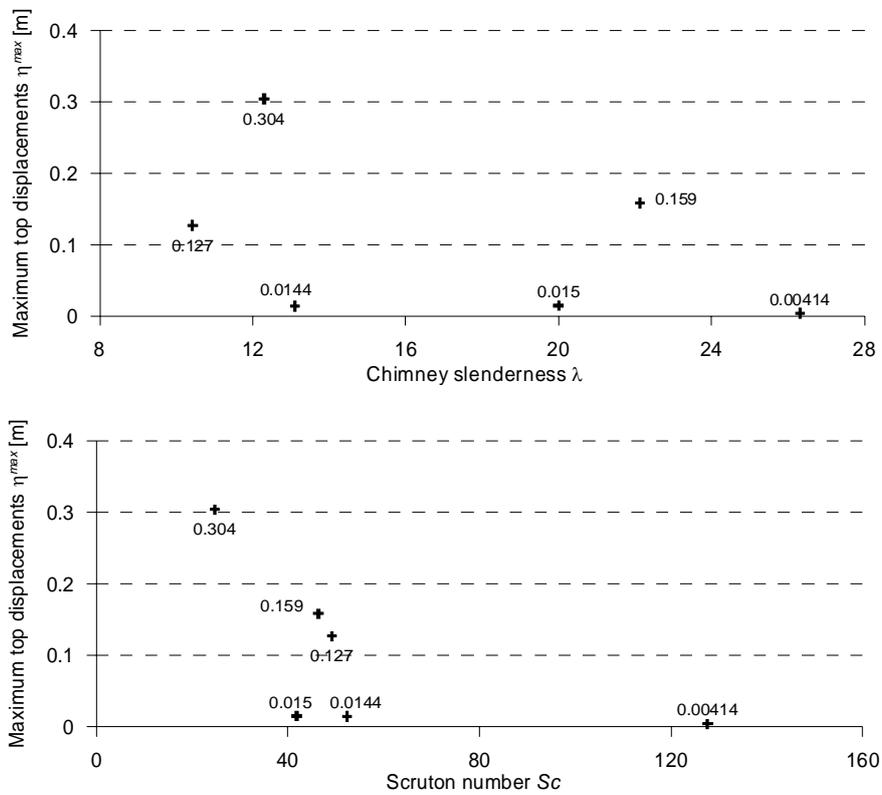


Fig. 10 Maximum lateral top displacement  $\eta^{max}$  against slenderness  $\lambda$  and Scruton number  $Sc$  for concrete chimneys

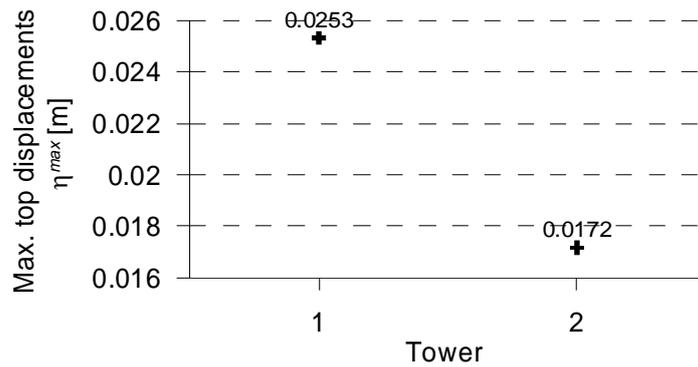


Fig. 11 Maximum lateral top displacement  $\eta^{max}$  for two analysed towers

Maximum top displacements are at the low value in cases of analysed concrete towers (Fig. 11). It is caused mainly by the small domain where vortex excitation can appear (comp. Fig. 3). The shape of towers themselves makes vortex excitation impossible to occur in large parts of height.

## 6. Comparison of calculation results with full-scale measurements

Some data of full-scale measurements concerning lateral response of steel chimneys can be found in the literature, e.g., Pritchard (1984) or more recently Kawecki and Żurański (2007).

To validate the model, calculations results were compared with full-scale measurements presented by Ruscheweyh and Sedlacek (1987), Ruscheweyh (1990) or Galemann and Ruscheweyh (1992). This comparison is presented in Fig. 12 as the value of normalized top displacement  $\eta^{max}/D$  in the function of non-dimensional Scruton number ( $Sc$ ). Steel chimneys of “average”  $Sc$  were considered in presented calculations. Non-dimensional value of  $Sc$  can be considered as the mass-damping ratio. Damping forces will be dominant if  $Sc$  is of the high value. In this case vortex excitation will probably not give large lateral displacements and rather transverse component of fluctuating wind will be more important. When damping forces are relatively to inertia forces low for low  $Sc$  values, then danger of large lateral displacements can occur. But in this case dampers are always used. The results for analysed steel chimneys are in a relatively good agreement with full-scale investigations. Moreover, it should be mentioned that there was no information about chimneys conditions when displacements were measured. It means that corrosion of the pipe was not clearly indicated. In calculations, the authors also took into account the degree of corrosion of the steel pipe inner side. Detailed considerations on corrosion were presented by Lipecki and Flaga (2007), Lipecki and Flaga (2011).

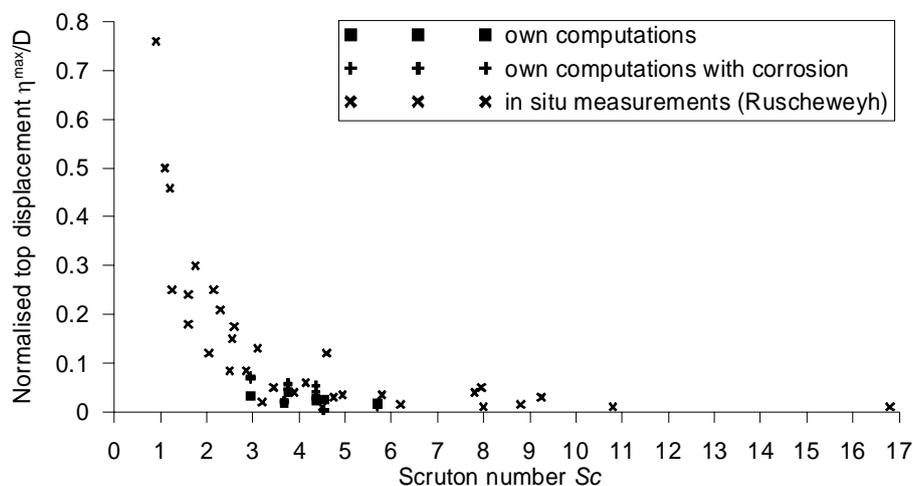


Fig. 12 Comparison of calculations results with full-scale measurements results

There are not too many results from full-scale measurements in the case of concrete chimneys. Accessible data are limited, so authors' comparisons are also limited. Some interesting information about GPS measurements performed on high industrial chimney are provided by Górski and Chmielewski (2008), Górski (2009). The following structures (the most widely described in the literature) were used in comparisons:

a) Chimney of 265 m height (Melbourne *et al.* 1983, Cheng and Kareem 1992). The r.m.s of maximum top displacement in relation to  $V_H/(fD_H)$  was analysed (Fig. 13(a)).  $V_H$  is the wind speed at the top,  $D_H$  – top diameter,  $f$  – frequency of vibrations.

b) Chimney of 130 m height (Christensen and Askegaard 1978, Cheng and Kareem 1992). Non-dimensional amplitude of displacements against  $V_H/(fD_H)$  was compared (Fig. 13(b)).

c) Four chimneys of 245 m, 200 m, 274 m, 180 m height respectively (ESDU 85038 1990, Vickery and Basu 1984), chimney of 200 m height (Sanada *et al.* 1992), chimney of 300 m height (Waldeck 1989, Waldeck 1992). Maximum top displacements against height and slenderness were compared (Fig. 13(c)).

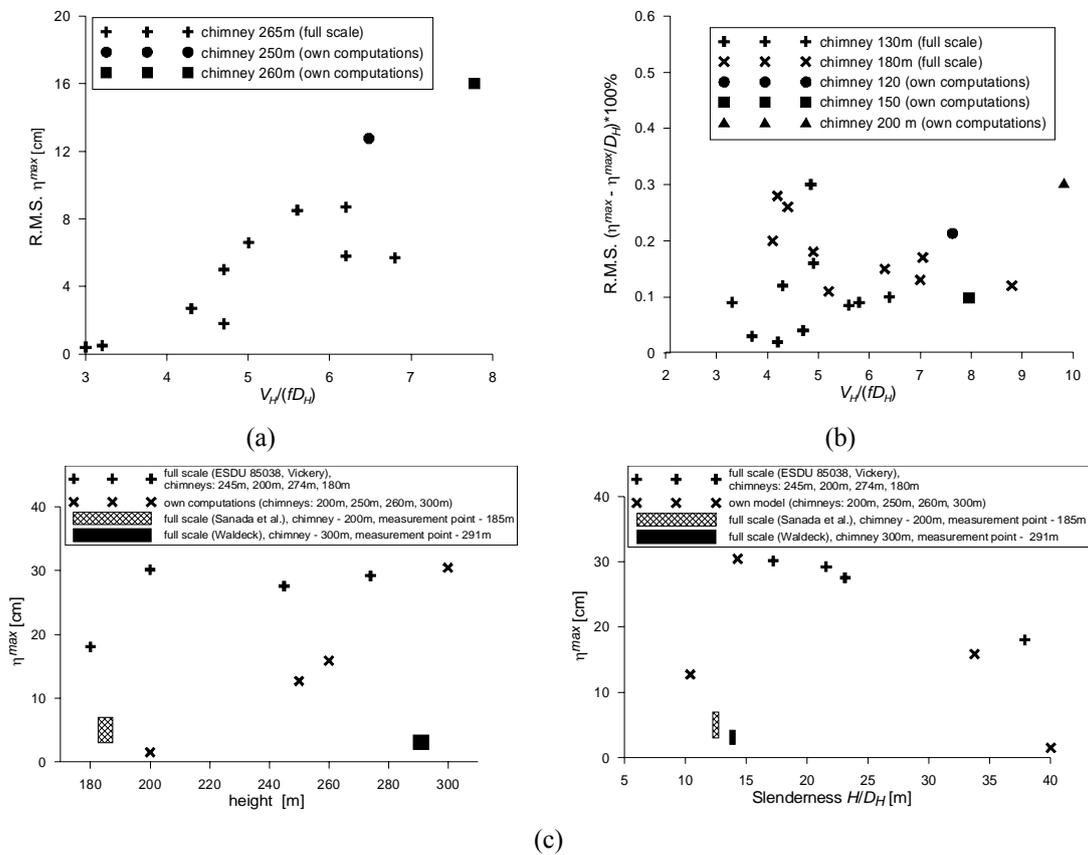


Fig. 13 Comparisons between concrete chimneys calculations results and collected full-scale measurements results

## 7. Conclusions

Some final conclusion can be formulated:

1. WAWS method used in calculations as the stochastic processes simulation method can be used directly to the vortex excitation simulation.

2. Results obtained according to our model of critical vortex excitation are in good agreement with full-scale data. Lateral response of steel chimneys caused by vortex excitation can be significantly influenced by corrosion.

3. This new approach of estimation of across-wind load caused by vortices as well as structure response (expressed in displacements) can be a useful tool in designing processes of cantilever structures with circular or compact cross-sections.

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