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Effect of blockage on the drag of a triangular cylinder

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Abstract. A method is presented to estimate the form drag and the base pressure on a triangular cylinder in the presence of blockage effect. The Strouhal number, which is found to increase with the flow constriction experimentally by Ramamurthy & Ng (1973), may be decoupled from the blockage effect when re-defined by using the velocity at flow separation and a theoretical wake width. By incorporating this wake width into the momentum equation by Maskell (1963) for the confined flow, a relationship between the form drag and the base pressure is derived. Independently, the experimental data of surface pressure from Ramamurthy & Lee (1973) are found to be independent of the blockage effect when expressed in terms of a modified pressure coefficient involving the pressure at separation. Using the potential flow model by Parkinson & Jandali (1970) and its subsequent development in Yeung & Parkinson (2000) for the unconfined flow, a linear relation between the pressure at separation and the form drag is formulated. By solving the two equations simultaneously with a specified blockage ratio and an apex angle of the triangular cylinder, the predictions of the drag and the base pressure are in reasonable agreement with experimental data. A new theoretical relationship for the Strouhal number, pressure drag coefficient and base pressure proposed in this study allows the confinement effect to be appropriately taken into consideration. The present approach may be extended to three-dimensional bluff bodies.

Keywords: blockage effect; bluff bodies; drag; base pressure.

1. Introduction

Wind tunnel testing has been a standard research tool to obtain aerodynamic data for almost a century. Notable examples include the six-foot wind tunnel built by Wright brothers in the 1890s to conduct systematic tests on wing models. In the community of wind engineering, it has long been an integral part and still remains as the most reliable method for tackling problems and providing useful information for flows around buildings, special structures and pedestrians as well as for modeling air pollution in the urban environment. When testing a scaled model in a low-speed wind tunnel, the requirements of geometric accuracy and maintaining flow similarity through matching Reynolds numbers, however, necessitate the use of larger models. Owing to the constraint from the rigid wind tunnel walls on the lateral displacement of streamlines, the test conditions around the larger models deviate from the prototype conditions. Therefore, it is generally agreed that wind tunnel data must be properly corrected to reduce or eliminate the blockage effect before they are utilized to provide useful information.

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For streamlined bodies, standard procedures such as that by Allen & Vincenti (1944), which is based on potential-flow theory, are elaborated in Barlow, Rae & Pope (1999). Although originally intended for use with stalled wings, Maskell (1963) proposed a method for bluff bodies based on momentum balance and the idea of wall constraint being equivalent to a simple increase in velocity of the undisturbed stream. It has been extensively used when making corrections to the total drag measurements on bluff bodies. Nonetheless, according to Holmes (2007), "No corrections are available for pressures, mean or fluctuating, in separated flow regions, such as those which occur on roofs or side walls of building models". To alleviate the constraint from rigid tunnel walls, tunnels with adaptive walls such as that in Sumner & Brundrett (1995) have been developed. An update of this technology has been recently reviewed by Meyer & Nitsche (2004). Alternatively, passive low-correction wind tunnels with ventilated walls were developed by Parkinson (1984). In this design, the solid walls of the test section are replaced by arrays of transverse symmetrical airfoil-shaped slats at zero incidence housed in plenum chambers. As demonstrated in Parkinson & Hameury (1990), if the slats are appropriately spaced, then models with blockage area ratios of up to 30% can be tested without correction. Unsteady-flow testing in such tunnel was reported in Kong, Hameury & Parkinson (1998).

For a number of years, techniques of computational fluid dynamics (CFD) have been developed and applied to wind engineering in the specific area of studying the blockage effect. Sahin & Owens (2004) investigated the wall effects on two-dimensional flow past a circular cylinder at low Reynolds numbers but high blockage ratios. Lasher (2001) used a commercially available CFD code (FIDAP) based on the finite-element method to solve the Reynolds-averaged Navier-Stokes (RANS) equations for the two-dimensional blocked flow normal to a flat plate. While the drag is under-predicted by the quasi-steady computations, regardless of the choice of turbulence models, the transient simulations are more accurate in drag prediction but also sensitive to the choice of turbulence model. Typically, quasi-steady simulations took 1 to 2 hours but transient simulations took 16 hours to 5 days of CPU time, depending on the turbulence model used, problem domain size, and number of time steps. To avoid the disadvantages of grid and turbulence-modeling dependence, meeting the criteria of convergence and lengthy computational time as experienced by numerical methods, an analytical method was recently developed by Yeung (2008) to predict the drag and base pressure of an inclined flat plate in the confined flow. It makes use of the momentum equation by Maskell (1963) without imposing limits on the wake size. And by incorporating the potential flow model for the unconfined flow upstream of separation, a relationship is derived between the pressure drag and the base pressure. The predictions obtained by solving the two equations are in reasonable agreement with well-documented experimental data in the literature.

Skyscrapers, high-voltage-transmission towers and bridges are typical structures having sharp edges. The flow past such wedge-like body in a wind tunnel has been studied experimentally to understand the associated flow characteristics. Ranga Raju & Garde (1970) studied the drag of a sharp-edged plate at different inclinations when placed on a plane boundary. A semi-empirical relation has been developed for the blockage effect. Twigge-Molecey & Baines (1973) measured the unsteady pressure on and fluctuating velocity outside the wake of a two-dimensional equilateral triangular section, mounted symmetrically in a stream with the apex facing upstream. The wall effects on the surface pressure distribution, the aerodynamic forces and the Strouhal number of an equilateral triangular body were studied by Ramamurthy & Lee (1973) and Ramamurthy & Ng (1973). The drag coefficient normalized by the contracted jet velocity was found to be independent of the blockage ratio. Simmons (1975) measured the Strouhal number, the spacing ratio of vortices in the wake and the base pressure of a wedge having an arbitrary apex angle. His experimental results support the universal Strouhal number proposed by Bearman (1967). A semi-empirical relation based on curve fitting of the base pressure was developed by Ramamurthy, Balachandar and Vo (1989) for the drag force experienced by constrained bluff bodies with separating edges. As shown in Naudascher (1991), the drag increases non-linearly with the blockage ratio. Recently, Csiba and Martinuzzi (2008) investigated how asymmetry affects the vortex shedding of a triangular cylinder at Reynolds numbers of $10^4 - 3\times10^4$. The effects of an asymmetrical confined flow on a rectangular cylinder were reported by Cigada, Malavasi and Vanali (2006) at Reynolds numbers of $6\times10^3 - 4\times10^4$. Rehimi, Aloui, Nasrallah, Doubliez and Legrand (2008) conducted an experimental study of a confined flow (with the blockage ratio equal to 1/3) behind a circular cylinder at Reynolds number in the range of $30 - 277$.

By incorporating some of the ideas underlying the wake source model of Parkinson & Jandali (1970), Modi & El-Sherbiny (1977) presented a potential flow model for a normal flat plate and a circular cylinder under wall confinement. The method requires a condition that "seemed to give the best agreement with the experimental results for both the cases of a normal flat plate and a circular cylinder". El-Sherbiny (1980) proposed a potential flow theory for the flow past an inclined side wall (resembling a wedge) in constrained flow. The predicted drag which increases with the blockage ratio non-linearly is in good agreement with experimental data. Nonetheless, a common feature of all these models is that they require the empirical input of base pressure to predict the pressure drag. Such a requirement arises because the complex wake dynamics is ignored and replaced by a semi-infinite body of constant pressure. Recently, Yeung & Parkinson (2000) made use of the property of the Strouhal number behind an inclined flat plate to remove such an undesirable requirement for the unconfined flow. As such, the base pressure may be predicted theoretically.

In this paper, a simple method is developed to investigate the effect of blockage on the form drag and the base pressure on a triangular prism. Experimental measurements show that the Strouhal number increases non-linearly with the blockage ratio. Such dependence can be removed if the characteristic velocity is replaced by the velocity at flow separation and the characteristic length is replaced by a theoretical wake width. By equating this theoretical wake width to the characteristic wake width in the momentum equation for two-dimensional bluff sections for the confined flow, a non-linear equation between the pressure drag and base pressure is derived. Instead of following the approach in Modi & El-Sherbiny (1977) to construct a potential flow model under wall confinement, the experimental data of surface pressure distributions on the body surface are examined to reveal their similarity. After a proper re-normalization, the pressure distributions may be collapsed onto a single theoretical curve from the potential flow model by Parkinson & Jandali (1970) and its subsequent development in Yeung & Parkinson (2000) for the unconfined flow. Therefore, a linear relationship between the pressure drag and the base pressure can be formulated. Interestingly, such theoretical relationship derived from the potential model is consistent with Maskell's (1963) assumption that "the wall constraint can be regarded as equivalent to a simple increase in velocity of the undisturbed stream", although the connection between the relationship and the assumption is not obvious. By solving the two equations simultaneously, the form drag and the base pressure are found as functions of the blockage ratio and the apex angle. The predictions are in reasonable agreement with experimental data. Finally, a new theoretical relationship for the Strouhal number, pressure drag coefficient, base pressure coefficient and blockage ratio is proposed in this study and compared with existent experimental data. The agreement is satisfactory when compared with experimental data. Such an expression is useful in estimating any one of these three quantities.

2. Wake width

Consider orientation (I) in Fig. 1a where a symmetrical wedge-shaped section is placed midway between two parallel rigid boundaries at distance H apart. Its frontal surface has a width c and is perpendicular to a free stream of speed U, pressure p_∞ and density ρ to produce a wake of width D, which is defined as the lateral distance between the two shear layers. As such, the shear layers separate from the body at 90° measured with respect to the x axis. The size of its aft body may be characterized by the half wedge angle β. Orientation (II) in Fig. 1b is for the same wedge but with the apex facing the free stream so that the separation angle is also β. The blockage ratio in each case is $\varepsilon = c/H$.

The experimental data of the conventional Strouhal number $S = fc/U$ (where f is the vortex shedding frequency) plotted in Fig. 2a from Ramamurthy & Ng (1973) at Reynolds numbers of 10^4 – 3×10^5 and those in Fig. 2b from Toebes (1969) are for the orientation (I) where $7.1\% \le \varepsilon \le 42.3\%$. In general, the Reynolds-number influence on the Strouhal number is negligible when the location of flow separation is fixed by a sharp edge. As shown, S increases non-linearly with ε. In Fig. 2, the data are expressed in terms of a modified Strouhal number

$$S^{**} = \frac{fD^{**}}{U\sqrt{1-c_{pb}}} \tag{1}$$

where the characteristic wake width is

$$D^{**} = c\sqrt{1-c_{pb}}(1-\varepsilon)^{7/4} \tag{2}$$

Here, $U\sqrt{1-c_{pb}}$ is the velocity near separation and c_{pb} is the coefficient of constant base pressure p_b defined as $c_{pb} = 2(p_b - p_\infty)/(\rho U^2)$. As S^{**} remains almost constant over this range of blockage ratio and has a mean value of 0.13, it is related to S through

$$S = \frac{S^{**}}{(1-\varepsilon)^{7/4}} \approx \frac{0.13}{(1-\varepsilon)^{7/4}} \tag{3}$$

Eq. (2) is similar to that for the flat plate normal to the free stream, namely $D^{**} = c\sqrt{1-c_{pb}}(1-\varepsilon)^{6/4}$, which was found by Yeung (2008). The presence of the aft body is reflected by the modification of exponent in $(1-\varepsilon)$. It is noted that the crucial parameter $(1-\varepsilon)$ for the confined flow was deduced from Continuity. Then, its exponent n was determined next. Surprisingly, this functional dependence, $(1-\varepsilon)^n$, where n varies with the geometrical shape, leads to realistic results for the bluff bodies

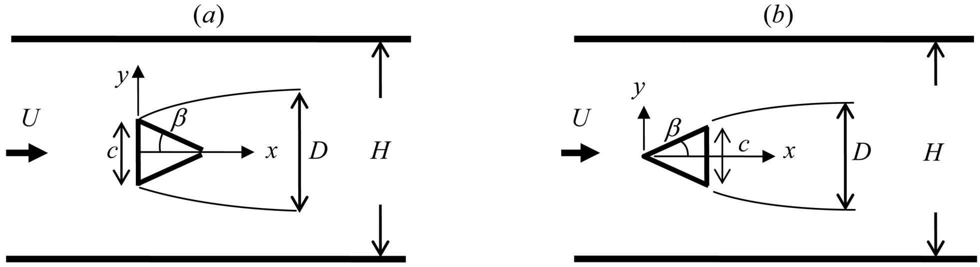

Fig. 1 Definition sketches for confined flow past a wedge having two different orientations

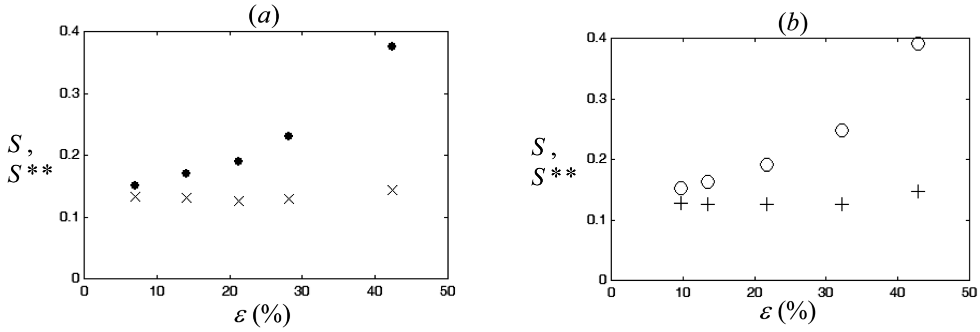

Fig. 2 Variations of Strouhal numbers S and S^{**} with ε for a wedge: orientation (I).
(a) • : S from Ramamurthy & Ng (1973), × : S^{**} , (b) **O** : S from Toebes (1969), + : S^{**}

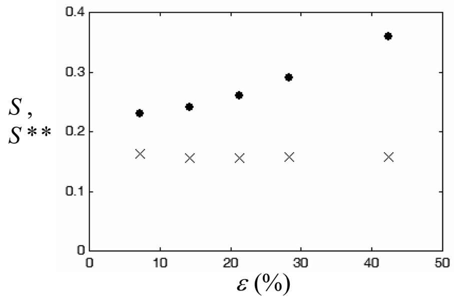

Fig. 3 Variations of Strouhal numbers S and S^{**} with ε for a wedge ($\beta = 30^{\circ}$): orientation (II). • : S from Ramamurthy & Ng (1973), × : S^{**}

examined presently.

In Yeung & Parkinson (2000), the experimental data of S from Simmons (1975) for wedges with $10^{\circ} \leq \beta \leq 90^{\circ}$ in the unconfined flow become independent of β, if they are converted to S^{**} as defined in Eq. (1) with

$$D^{**} = c\sqrt{1-c_{pb}}(1-0.16-0.16\sin(2\beta+\pi/2)) \tag{4}$$

S^{**} has a mean value of 0.161, which is in close agreement with 0.163 from Simmons (1975). For orientation (II), the experimental data of S from Ramamurthy & Ng (1973) for $\beta = 30^{\circ}$ over $7.1\% \leq \varepsilon \leq 42.3\%$ are depicted in Fig. 3. Similar to those in Fig. 2, they also exhibit a distinct, nonlinear increase in the Strouhal number with ε, although the gradient of variation is less than those in Fig. 2. Based on

$$D^{**} = c\sqrt{1-c_{pb}}(1-0.16-0.16\sin(2\beta+\pi/2))(1-\varepsilon) \tag{5}$$

the modified Strouhal number S^{**} as depicted in Fig. 3 is no longer a function of ε. After the elimination of $\sqrt{1-c_{pb}}$ in S^{**}, it is found that

$$S \approx \frac{S^{**}}{(1-0.16-0.16\sin(2\beta+\pi/2))(1-\varepsilon)} \tag{6}$$

where S^{**} remains as 0.161. The results in Figs. 2 and 3 clearly indicate that Eqs. (2) and (5) are the appropriate choices of the characteristic wake widths because the modified Strouhal number in each case becomes independent of the blockage ratio. The fact that the functional forms of D^{**} are distinct in the two orientations may be attributed to the differences in (a) the angle at which the flow separates from the body and (b) the existence of the aft body.

While parameter $(1-\varepsilon)$ is derived from kinematics (i.e. the continuity equation), β is a geometrical property of the wedge. Surprisingly, the dynamic parameter c_{pb} is not involved in expressions (3) and (6). It is worth mentioning that in the absence of blockage, the conventional Strouhal number (based on base width) of a triangular cylinder in an asymmetric flow studied by Csiba & Martinuzzi (2008) collapsed onto a single curve having a mean value of 0.182, when the base width was replaced by the projected width, the angle of attack was within $0 \le \alpha \le 30^\circ$ and the Reynolds number range was of $10^4 - 3\times10^4$. This lends support to the present finding in the sense that the modified Strouhal number is related to the conventional Strouhal number by kinematic as well as geometric parameters.

In Maskell (1963), an equation was derived by balancing the forces and the rate of change of momentum around a three-dimensional bluff body and the constant-pressure surface bounding the effective wake in a wind tunnel. It has been modified to suit a two-dimensional bluff section of width c with a downstream wake at a constant base pressure c_{pb} in a wind tunnel of height H . In terms of a wake width D , the sectional form drag F_d (or in coefficient form $c_d = 2F_d/(\rho U^2 c)$) is given by

$$c_d = (1-c_{pb})\frac{D}{c} - \frac{D}{c}\left(1-\frac{D}{c}\frac{c}{H}\right)^{-1} \tag{7}$$

By equating D in (7) to D^{**} in (2) or (5), the drag coefficient of a triangular section becomes a function of c_{pb}, blockage ratio ε and angle of flow separation β.

It should be emphasized that Maskell's (1963) method has been widely accepted as a suitable means of correcting wind tunnel data for the blockage effect on bluff bodies created by the rigid walls. In fact, it is actually based on a first order approximation of Eq. (7); that is, $(1-D/H)^{-1} \approx 1 + D/H$. Furthermore, through the same order of approximation, the wake size is related to the drag and the base pressure coefficients without resorting to the wake dynamics which is characterized by vortex shedding. Such approximation is well justified because the experimental data quoted in that study to validate the method correspond to low blockage ratios (i.e. $\varepsilon < 8\%$). In the present study, such approximation is not made because the purpose of this investigation is to study how the drag is affected by the confinement effect. Therefore, it becomes unnecessary to impose any stringent condition on the range of blockage ratio over which the prediction is valid. Eq. (7) is utilized in the present form with the wake size deduced from the Strouhal number which is closely linked to the physics of the vortices in the wake.

3. Invariance of surface pressure

The surface pressure distributions expressed in terms of usual pressure coefficient $c_p = 2(p-p_\infty)/(\rho U^2)$ vs. y/c at $\varepsilon = 18.6\%$, 24.3% and 32.4% from Ramamurthy & Lee (1973) at Reynolds numbers of $4\times10^4 - 2\times10^5$ for orientations (I) and (II) are shown in Figs. 4a and 5a, respectively. As found experimentally, the pressure distribution on a bluff body is insensitive to Reynolds numbers, if flow separation takes place at sharp edges on the body. Axis y is perpendicular to the free stream, as indicated in Fig. 1. From Figs. 4a and 5a, it is observed that the pressure distributions upstream of separation are less sensitive to ε in comparison with those downstream. And, c_{pb} becomes more negative when the blockage ratio is larger. Except within a small region near the central portion where the sharp edge is located, c_p in most part of the aft body in Fig. 4a may be considered

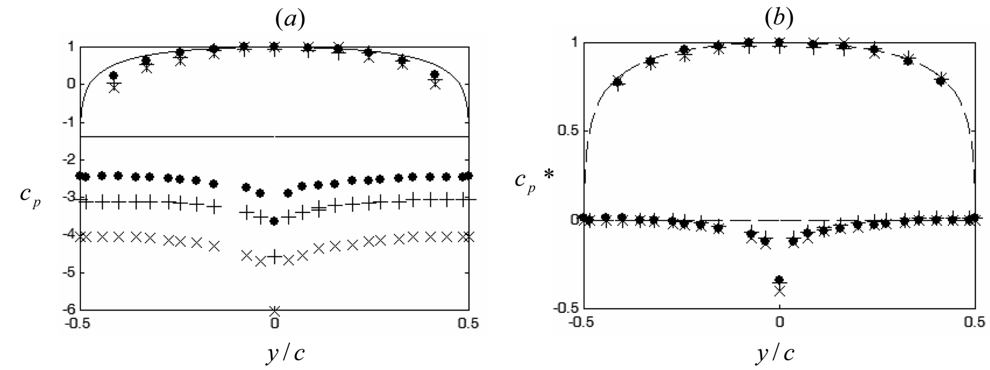

Fig. 4 Pressure distributions on a wedge: orientation (I). (a) c_p vs. y/c, (b) c_p * vs. y/c. • : $\varepsilon = 18.6\%$, + : 24.3%, × : 32.4% from Ramamurthy & Lee (1973), – : $\varepsilon = 0$ from Parkinson & Jandali (1973) and Yeung & Parkinson (2000), - - - : present

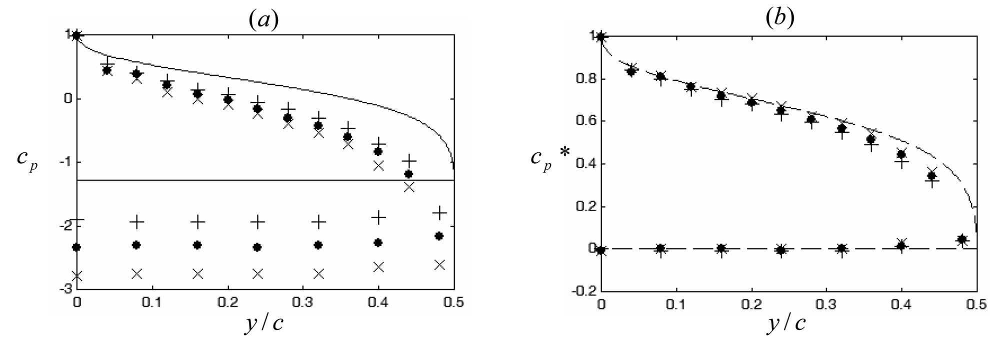

Fig. 5 Pressure distributions on a wedge: orientation (II). (a) c_p vs. y/c, (b) c_p * vs. y/c. + : $\varepsilon = 18.6\%$, • : 24.3%, × : 32.4% from Ramamurthy & Lee (1973), –: $\varepsilon = 0$ from Yeung & Parkinson (2000), - - - : present

effectively constant and equal to c_{pb} at any given value of ε.

For orientation (I), as the surface upstream of separation is normal to the free stream, its pressure distributions in Fig. 4a resemble that of a normal flat plate. The theoretical pressure distribution for a normal flat plate in the unconfined flow (i.e. $\varepsilon = 0$) by Parkinson & Jandali (1970) and its subsequent development in Yeung & Parkinson (2000) (i.e. the value for $c_{pb} = -1.385$ was found theoretically) is compared with the experimental data in Fig. 4a. Because of the difference in ε values, the discrepancy between the experimental data and the theoretical prediction is anticipated. The unconfined flow past a symmetrical wedge with an arbitrary half-apex angle β was also considered in Yeung & Parkinson (2000). The prediction of surface pressure with the theoretical value of $c_{pb} = -1.28$ is included in Fig. 5a for orientation (II). Similar to Fig. 4a, the discrepancy is caused by the blockage ratio.

It is interesting to note that the experimental data and the theoretical prediction in Fig. 4a are collapsed onto a single curve in Figs. 4b, if c_p is re-normalized to form the modified pressure coefficient

$$c_p^* = \frac{c_p - c_{pb}}{1 - c_{pb}} \tag{8}$$

where individual values of c_{pb} (i.e. c_p at $y/c = \pm 0.5$ in Fig. 4a, see also Fig. 8a) are used. The similar result is also found for orientation (II), although the single curve in Fig. 5b is not the same as that in Fig. 4b. This general result implies that the pressure distributions, when properly rescaled, are invariant under the influence of blockage effect. While the theoretical curves in Figs. 4b and 5b do not provide any clue of the values of c_{pb} as functions of the blockage ratio in the two configurations, they are useful for the calculation of form drag.

4. Prediction of form drag

The form drag coefficient for the normal flat plate and a wedge of half-apex angle β is given by integrating the surface pressure distribution as

$$c_d = 2\int_0^{1/2} (c_p - c_{pb})d\left(\frac{y}{c}\right) \tag{9}$$

which may be expressed in terms of c_p^* with the help of Eq. (8) as

$$c_d = 2(1-c_{pb})\int_0^{1/2} c_p{}^*d\left(\frac{y}{c}\right) \tag{10}$$

where c_{pb} is considered as an unknown. Using the theoretical c_p distributions in Figs. 4a and 5a, it is found that

$$c_d = 0.897(1-c_{pb}) \tag{11}$$

for the flat plate and

$$c_d = 0.644(1-c_{pb}) \tag{12}$$

for the wedge with $\beta = 30^\circ$. Generally, $c_d/(1-c_{pb})$ is a function of the angle of flow separation measured with respect to the x – axis in Fig. 1. To find out how realistic the two linear relationships are, the pressure data in Figs. 4a and 5a are integrated by using Eq. (9) numerically to produce c_d for the two orientations. When plotted against c_{pb} in Fig. 6, they are in reasonable agreement with the theoretical variations.

It is worth mentioning that Maskell (1963) arrived at a relationship between c_d and $1-c_{pb}$ for a square plate similar to Eq. (11) by assuming the exact equivalence between wall constraint and a simple increase in velocity of the undisturbed stream. In the absence of any theoretical pressure

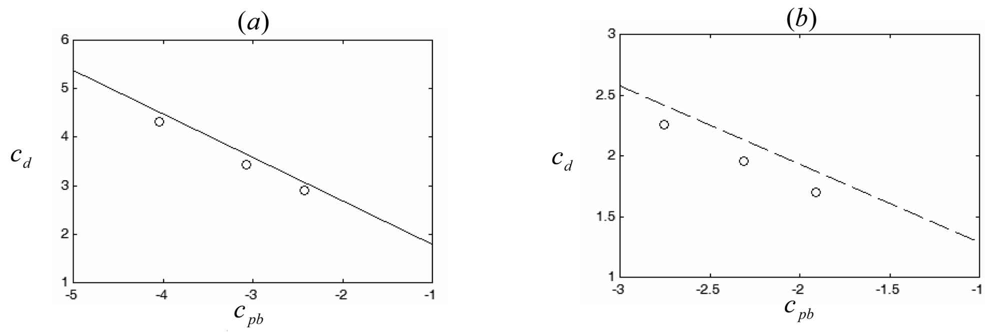

Fig. 6 Relationships between pressure drag and base pressure. (a) orientations (I) and (b) orientation (II). O : Ramamurthy & Lee (1973), – : Eq. (11), - - : Eq. (12)

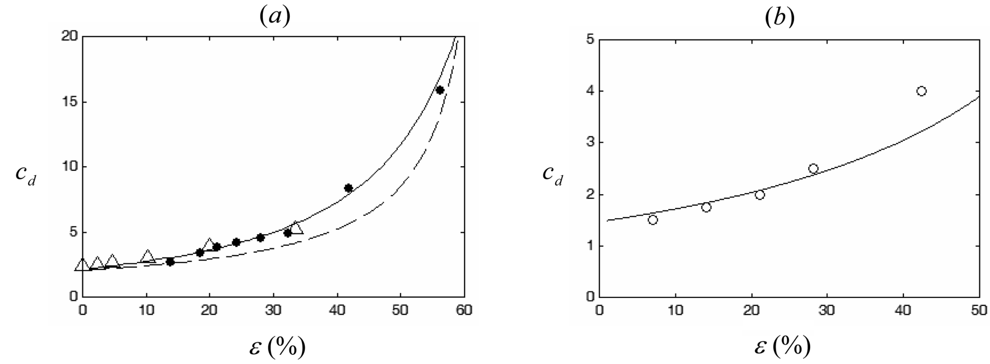

Fig. 7 Variations of c_d for (a) normal flat plate and (b) wedge ($\beta = 30^o$) with ε. •, **O** : Ramamurthy & Ng (1973), - - - : Ota, *et al.* (1994), Δ : Lasher (2001), – : present

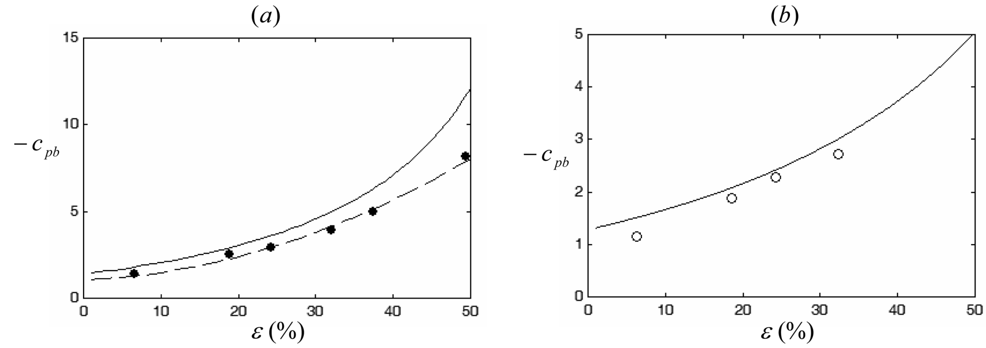

Fig. 8 Variations of c_{pb} for (a) normal flat plate and (b) wedge ($\beta = 30^o$) with ε. •, **O** : Ramamurthy & Lee (1973), - - - : Ramamurthy, *et al.* (1989), – :present

distribution on a square plate, $c_d/(1 - c_{pb}) = 0.837$ was deduced from experimental data over $0.19\% \leq \varepsilon \leq 4.5\%$. The fact that the numerical constant is of the same order of magnitude as those in Eqs. (11) and (12) provide further support to the invariance of pressure distributions under the blockage effect proposed herein.

With the blockage ratio and half-apex angle specified, c_d and c_{pb} can be found numerically by solving Eqs. (2), (7) and (11) for orientations (I), and Eqs. (5), (7) and (12) for orientations (II). Fig. 7 compares the experimental data of c_d from Ramamurthy & Ng (1973) while Fig. 8 compares the experimental data of c_{pb} from Ramamurthy & Lee (1973) with the present predictions. Interestingly, the theoretical predictions may be approximated by the general forms $c_d = c_{d0}(1 - \varepsilon)^{-m}$ and $c_{pb} = c_{pb0}(1 - \varepsilon)^{-n}$, where c_{d0} and c_{pb0} are individually the values of c_d and c_{pb} when $\varepsilon = 0$. Together with m and n, they are functions of the angle of flow separation and the size of aft body. Based on a discrete vortex method, Ota, Okamoto & Yoshikawa (1994) proposed $c_d = 2.04(1 - 1.52\varepsilon)^{-1}$ for a flat plate. Ramamurthy, Balachandar and Vo (1989) obtained $-c_{pb} = 1.06 + 1.8\varepsilon + 24.35\varepsilon^2$ by curve fitting the measurements for a flat plate from a number of experimental studies. The two curves and the numerical results from FIDAP by Lasher (2001) for a flat plate are added in Figs. 6a and 7a for comparison. It is noted in Fig. 7b that the data point corresponding to about $\varepsilon = 42\%$ is above the prediction. According to Ramamurthy & Lee (1973), "the drag force is extremely sensitive to the relative incidence of the flow direction when θ (i.e. the half-apex angle) = 30°". Therefore, it is

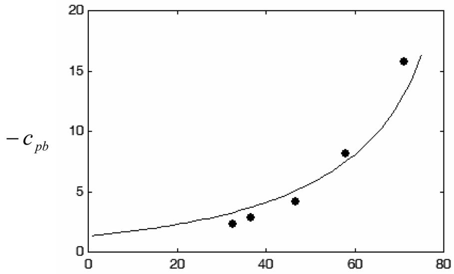

Fig. 9 Variation of c_{pb} for a wedge ($\beta = 45^{\circ}$) with ε. • : Chen (1967), – : present

possible that such sensitivity affects the drag measurements when the blockage ratio is as high as 42%.

Richter & Naudascher (1976) compared the separation velocity $U\sqrt{1-c_{pb}}$ as a function of the confinement ratio for a number of bluff bodies. The data of Chen (1967) for a wedge with $\beta = 45^{\circ}$, as quoted by them, have been converted to c_{pb} and plotted against ε in Fig. 9. Using the model from Yeung & Parkinson (2000) and Eq. (10), it is found that

$$c_d = 0.728(1 - c_{pb}) \tag{13}$$

for $\beta = 45^{\circ}$. By solving Eqs. (5), (7) and (13), the present prediction of the base pressure is included in Fig. 9. The agreement between the experimental data and the predictions in Figs. 7, 8 and 9 are generally reasonable. However, larger discrepancy is anticipated when

(a) the half-apex angle is smaller because the frictional drag may no longer be negligible, and

(b) the blockage ratio is large such that the shear layers from the body may interact with the boundary layers developed on the wind tunnel walls.

5. Relationship for Strouhal number, drag and base pressure

According to von Karman, the drag arising from the vortex street behind a bluff body may be theoretically related to the vortex speed and the spacing ratio of the vortices. Kronauer proposed a stability criterion corresponding to minimize the drag with respect to the spacing ratio at a given vortex velocity. It led Bearman (1967) to derive an analytical relationship between the product of the Strouhal number and pressure drag coefficient (i.e. Sc_d) and $k = \sqrt{1-c_{pb}}$ for two-dimensional bluff sections (independent of the fore-body shape) in the unconfined flow (i.e. $\varepsilon = 0$). The nonlinear variation of Bearman's relationship within $1 < k < 1.8$ is shown in Fig. 10, clearly indicating a maximum of Sc_d near $k \approx 1.55$. One of the main advantages of introducing this relationship is that any one of the quantities S, c_d and c_{pb} might be calculated from the measured values of the other two if such theoretical relationship exists. It is, however, noted that "the Kronauer criterion was not applicable for k greater than 1.5", according to Griffin (1981). And, the corresponding expression with the blockage effect taken into consideration is not available in the literature.

By setting $\beta = 45^{\circ}$ in Eq. (4) and substituting D^{**} into Eq. (1), it is found that $D = D^{**} = 0.84c\sqrt{1-c_{pb}}$, which leads to

$$S = \frac{S^{**}}{0.84} \tag{14}$$

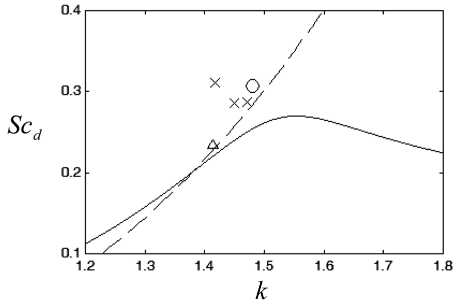

Fig. 10 Relationship between Sc_d and k for a triangular prism ($\beta = 45^o$) in unconfined flow. – : Bearman (1967), - - : Eq. (15), o : symmetrical flow, × : asymmetric flow from Csiba & Martinuzzi (2008), Δ : Roshko (1954)

With $\varepsilon = 0$, Eq. (7) is reduced to $c_d = -c_{pb}\frac{D}{c}$. Multiplying it by Eq. (14), it is obvious that $Sc_d = \frac{S^{**}}{0.84}(-c_{pb})\frac{D}{c} = S^{**}(-c_{pb})\sqrt{1-c_{pb}}$, which can be simplified to

$$Sc_d = 0.161(k^2 - 1)k \tag{15}$$

when using $S^{**} \approx 0.161$(see section 2) and $k = \sqrt{1-c_{pb}}$. In contrast to Bearman's curve in Fig. 10, Eq. (15) is a cubic equation in k without any maximum for $k > 1$. Furthermore, no restriction is imposed on the value of k. A data point (S, c_d, c_{pb}) = (0.18, 1.7, −1.19), which was estimated from Csiba & Martinuzzi (2008) for a triangular prism (i.e. $\beta = 45^o$) in a symmetric flow (i.e. angle of attack $\alpha = 0^o$), follows the present prediction in Fig. 10 but lies above Bearman's curve. Other data points from the same study but corresponding to asymmetric flow (i.e.$0 < \alpha \leq 30^o$) are found to be off the present prediction and Bearman's curve. Interestingly, the classical data point (S, c_d, c_{pb}) = (0.18, 1.3, −1.0) from Roshko (1954) is near the intersection of the two theoretical curves.

In the case of the confined flow, orientation (I) is taken as a typical case for consideration. When Eq. (2) is combined with Eq. (7),

$$c_d = k^3(1-\varepsilon)^{7/4} - \frac{k(1-\varepsilon)^{7/4}}{1-k\varepsilon(1-\varepsilon)^{7/4}} \tag{16}$$

Multiplying Eq. (16) by Eq. (3) gives

$$Sc_d = 0.13k^3 - \frac{0.13k}{1-k\varepsilon(1-\varepsilon)^{7/4}} \tag{17}$$

where $S^{**} = 0.13$ is chosen from the results shown in Fig. 2. Eq. (17) is compared with Bearman's curve in Fig. 11 for different blockage ratios. A close examination of the experimental data from Ramamurthy & Lee (1973) and Ramamurthy & Ng (1973) reveals that it is not possible to find a complete set of S, c_d and c_{pb} sharing the same value of ε. Therefore, the data from Parkinson & Hameury (1990), and Kong, Hameury & Parkinson (1998) for the flat plate at $\varepsilon = 8.3\%$, 19.4% and 33.3% are used instead and plotted in Figs. 10a, 10b and 10c, respectively. As the blockage ratio gets larger, the data points deviate more from Bearman's curve, which is for the unconfined flow but follow the trend of Eq. (17).

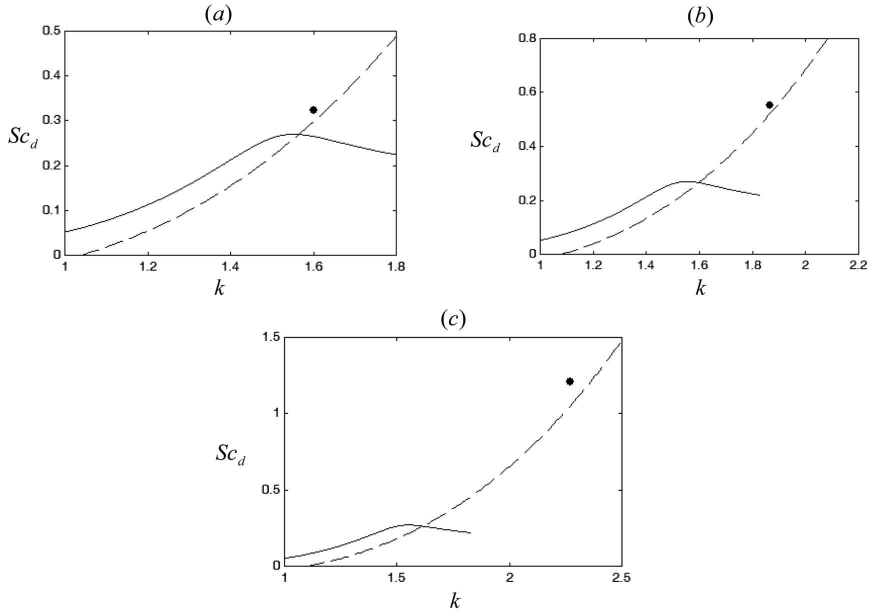

Fig. 11 Relationship between Sc_d and k for orientation (I) in confined flow. (a) $\varepsilon = 8.3\%$, (b) 19.4% and (c) 33.3%. – : Bearman (1967), - - : Eq. (17), • : data from Parkinson & Hameury (1990) and Kong, Hameury & Parkinson (1998)

6. Conclusion

This paper presents an analytical method of predicting the form drag and base pressure acting on a two-dimensional triangular bluff body in constricted flow. The method is based on (a) a theoretical wake width that allows the Strouhal number to be independent of the blockage ratio, (b) the momentum balance by Maskell (1963), (c) the invariance of pressure distribution upstream of separation under the blockage effect, and (d) the potential flow model by Parkinson and Jandali (1970) and its subsequent development in Yeung & Parkinson (2000). The prediction is reasonable when compared with experimental data. A new analytical relationship between the product of the Strouhal number and pressure drag coefficient, and the base pressure coefficient for the confined flow is proposed. In general, the agreement between experimental data and theoretical predictions are acceptable. Preliminary work has been carried to extend the present method to a three-dimensional bluff body such as a disk and a sphere at sub-critical Reynolds numbers in the present of blockage effect as the latter share similarities with the two-dimensional bluff bodies (i.e. a flat plate and a circular cylinder) as considered in Yeung (2008).

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