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A comparative study of along and cross-wind responses of a tall chimney with and without flexibility of soil

Piotr Górski and Tadeusz Chmielewski*

Opole University of Technology, ul. Katowicka 48, Opole, Poland

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Abstract. The paper is concerned with a comparative study of both the along and cross-wind responses of a tall industrial chimney with and without flexibility of soil. The along-wind response has been estimated by means of approaches presented in three Standards: the Polish, the ISO and the Eurocode and by random vibration approach which is outlined below. The cross-wind response has been estimated by means of the three models developed by Vickery and Basu, Ruscheweyh and Flaga and methods presented in Standards: the Polish, the ISO and the Eurocode (Approach 1 and 2). Computer programmes were developed to obtain estimates of responses of a six-flue, 250 m-tall chimney. The analytical results computed according to the methods presented in different standards and random vibration approaches have been compared. Some unexpected conclusions have been observed.

Keywords: numerical analysis; tall chimney; along and cross-wind responses; flexibility of soil.

1. Introduction

The along and cross-wind responses of the chimney with and without flexibility of soil have been computed in this paper by means of the existing models in codes and literature and the comparison of the results is presented. In carrying out these comparison large differences in the cross-wind predictions of the various models have been found. In order to identify the reasons for these differences the assumptions and basis for each of the models has been reviewed. It was found that Ruscheweyh's model is based on assumptions mainly applicable to steel chimneys and other steel structures (e.g., bridge hangers, antenna masts), where the mass distribution is relatively constant. Therefore it may have limitation when applied to concrete industrial chimneys for which mass distribution can vary considerably. In contrast, the Vickery/Basu's model is not affected by these some limitations and can be expected to give more realistic results for concrete chimneys.

2. The industrial chimney, its calculation model and free vibrations of this chimney

The description of the six-flue, 250 m-tall industrial chimney which is placed on the circular foundation slab – 50 m in diameter (lying directly on the soil) is given in paper Chmielewski, *et al.* (2005). In this paper the calculation model of the chimney and numerical results for the first four

* Corresponding Author, E-mail: t.chmielewski@po.opole.pl

natural vibration periods and mode shapes are also given.

3. Along-wind response of the chimney

At first, we will consider the along-wind dynamic response of the chimney which now is well established. The random vibration approach, which was outlined by Davenport (1962, 1964, 1967) and further developed by Harris (1965) has been applied.

3.1. Relationship between the spectrum of the along-wind force and the spectrum of the wind velocity

The chimney is a typical line-like structure, with a single spatial coordinate z. Let $W(z, t)$ be the force per a unit length along the chimney. It may be treated as the along-wind or cross-wind force. For along-wind forces, applying the quasi-steady and 'strip' assumptions, which relate the forces on a section of the chimney with the flow condition upstream of the section, one can write it as

$$W(z, t) = C_D(z)d(z) \cdot \frac{1}{2}\rho_a V^2(z, t) \tag{1}$$

where $C_D(z)$ is the local drag coefficient, $d(z)$ is the local diameter and $V(z,t)$ is the longitudinal velocity upstream. If the chimney moves, this should be a relative velocity, which generates an aerodynamic damping force. However, at this point we will assume that the chimney is stationary, in which case $V(z,t)$ can be written:

$$V(z, t) = \bar{V}(z) + \vartheta'(z, t) \tag{2}$$

where $\bar{V}(z)$ is the mean wind velocity which is represented by the mean wind velocity profile (by a logarithmic or by power law profile), and $\vartheta'(z, t)$ is a fluctuating component of the longitudinal velocity.

Then from Eq. (1)

$$W(z, t) = C_D(z)d(z)\rho_a\left[\frac{1}{2}\bar{V}^2(z) + \bar{V}(z)\vartheta'(z, t) + \frac{1}{2}\vartheta'^2(z, t)\right] \tag{3}$$

For $\vartheta(z, t) \ll \bar{V}(z)$ the third term within the square brackets may be neglected. The fluctuating along-wind force is given by

$$W'(z, t) = C_D(z)d(z)\rho_a\bar{V}(z)\vartheta'(z, t) \tag{4}$$

It is assumed that the fluctuating component $\vartheta'(z, t)$ is treated as a weakly stationary random process with respect to time t. Therefore, taking the means (time averages) of both sides of the joint moment of $W'(z_1, t) \cdot W'(z_2, t+\tau)$ we get

$$\overline{W'(z_1, t)W'(z_2, t+\tau)} = C_D(z_1)C_D(z_2)d(z_1)d(z_2)\rho_a^2\bar{V}(z_1)\bar{V}(z_2)[\overline{\vartheta'(z_1, t)\vartheta'(z_2, t+\tau)}] \tag{5}$$

This can be simplified for the uniform cross-section of the chimney, with $C_D(z)$ and $d(z)$ constants with z:

$$\overline{W'(z_1, t) \cdot W'(z_2, t+\tau)} = (C_D d\rho_a)^2\bar{V}(z_1)\bar{V}(z_2)[\overline{\vartheta'(z_1, t)\vartheta'(z_2, t+\tau)}] \tag{6}$$

After applying the Fourier transformation to Eq. (6), the fluctuation along wind-force spectrum $S_{W'}(z_1, z_2, f)$ may be expressed in the form:

$$S_{W'}(z_1, z_2, f) = (C_D d \rho_a)^2 \bar{V}(z_1) \bar{V}(z_2) S_{\vartheta'}(z_1, z_2, f) \tag{7}$$

where $S_{\vartheta'}(z_1, z_2, f)$ is the cross spectrum of the fluctuating component of the longitudinal velocity, which can be expressed in terms of the power spectrum of the fluctuating component and the coherence function

$$S_{\vartheta'}(z_1, z_2, f) = \sqrt{S_{\vartheta'}(z_1, f)}\sqrt{S_{\vartheta'}(z_2, f)}\mathrm{Coh}(z_1, z_2, f) \tag{8}$$

If we assume that the fluctuating component $\vartheta'(z, t)$ is a weakly homogeneous random process with respect to *z*, then Eq. (8) can be written as

$$S_{\vartheta'}(z_1, z_2, f) = S_{\vartheta'}(f)\exp\left(\frac{-2kf|z_2 - z_1|}{\bar{V}(z_1) + \bar{V}(z_2)}\right) \tag{9}$$

where *k* is the empirical constant, used to fit to the measured data; a typical range of values for the atmospheric turbulence is 8 to 20 (Holmes 2001), $S_{\vartheta'}(f)$ is the spectral density function for $\vartheta'(t)$.

3.2. Along-wind response of the chimney – random vibration approach

The dynamic response of the chimney due to the dynamic forces (along and cross-wind) can be evaluated by the modal analysis. The complete displacement response *q*(*z*, *t*) expands as a summation of components associated with each of the natural modes of vibration:

$$q(z, t) = \sum_{i=1}^{N} y_i(t)\phi_i(z) \tag{10}$$

where $\phi_i(z)$ is a mode shape for the *i*th mode; $y_i(t)$ is a time-varying generalized coordinate; *z* is a spatial coordinate on the chimney.

The modal analysis of the chimney with and without flexibility of soil has been evaluated and is presented in book (Chmielewski and Zembaty 1998). The equation of motion for the *i*th generalized coordinate is as follows:

$$\ddot{y}_i(t) + 2\xi_i\omega_i\dot{y}_i(t) + \omega_i^2 y_i(t) = \frac{P_i'(t)}{M_i} \tag{11}$$

where M_i is the generalized mass equal to $\int_0^L m(z)\phi_i^2(z)dz$, $m(z)$ is the mass per a unit length along the chimney, L is the height of the chimney, ξ_i is the damping ratio for the *i*th mode, ω_i is the natural undamped circular frequency for the *i*th mode, K_i is the modal stiffness, $P_i'(t)$ is the generalized force equal to $\int_0^L W'(z, t)\phi_i(z)dz$ fluctuating along wind force.

The spectral density of $P_i'(t)$ can be obtained in an analogous way to the mean square value of $W'(z, t)$, it is as follows

$$S_{P_i}(f) = \int_0^L \int_0^L S_{W'}(z_1, z_2, f)\phi_i(z_1)\phi_i(z_2)dz_1dz_2 \quad (12)$$

Based on the random vibration theory (Chmielewski and Zembaty 1998), the spectral density of the generalized coordinate $y_i(t)$ is given by

$$S_{y_i}(f) = \frac{1}{K_i^2}|H_i(f)|^2 S_{P_i}(f) \quad (13)$$

where the mechanical admittance for the ith mode is

$$|H_i(f)|^2 = \frac{1}{\left\{\left[1-\left(\frac{f}{f_i}\right)^2\right]^2 + 4\xi_i^2\left(\frac{f}{f_i}\right)^2\right\}} \quad (14)$$

The mean square value of $y_i(t)$ can be obtained by integrating Eq. (13) with respect to frequency

$$\overline{y_i^2} = \int_0^\infty S_{y_i}(f)df \quad (15)$$

Applying Eq. (10), the mean square displacement is obtained from

$$\overline{q^2}(z) = \sum_{i=1}^{N}\sum_{k=1}^{N}\overline{y_iy_k}\phi_i(z)\phi_k(z) \quad (16)$$

The cross-coupling between modes for the chimney can be neglected, the above equation becomes

$$\overline{q^2}(z) = \sum_{i=1}^{N}\overline{y_i^2}\phi_i^2(z) \quad (17)$$

The mean square value of any other response (e.g. bending moment, stress) can similarly be obtained.

4. Cross-wind response of the chimney

A great deal of effort has been made during recent decades (approximately the last 30 years) to improve the analytical models used for predicting vibrations due to vortex shedding. The important pioneering research contributions have come from Vickery and Basu (1983), Ruscheweyh (1990) and Flaga (1996, 1997). Their analytical models and models existing in three codes: the Polish, the ISO, and the Eurocode (Approach 1 and 2) to calculate the cross-wind chimney response have been applied.

4.1. Vickery and Basu's model

4.1.1. Cross-wind force and spectrum of vortex-induced force

The phenomena of separating shear layers and vortex shedding for a bluff body with the circular cross-section is well described. For the chimney with the Reynolds number generally in excess of 10^7 the flow regimes for a circular cylinder are called post-critical. These flow regimes are turbulent and the alternate shedding of vortices induces a random cross-wind force on the chimney. A random excitation model, for the vortex shedding response prediction, which was developed by Vickery and Basu (1983), has been applied. According to this model, the vortex shedding wind force per a unit length may be written as

$$F_v(z,t) = \frac{1}{2}\rho_a d(z)\bar{V}^2(z)C_L(z,t) \tag{18}$$

where $C_L(z, t)$ is a non-dimensional, normalized lift coefficient. $C_L(z, t)$ is a weakly stationary random process with zero mean.

The Vickery and Basu's model is a semi-empirical mathematical model which has been presented for predicting the cross-wind response of tall slender structures of circular cross-section to the wind. In this model the forces caused by vortex shedding are characterized by four aerodynamic parameters: the lift coefficient $C_L(z, t)$, the spectral bandwidth, the Strouhal number and a measure of the spanwise correlation. The following section concerns with the definition of the key parameters for circular cross-sections in large scale turbulence and at the Reynolds numbers consistent with full scale structures.

The spectrum of the normalized lift force $S_{CL}(z, f)$ per a unit length is expressed as

$$\frac{f S_{CL}(z,f)}{\sigma_{CL}^2(z)} = \frac{f}{\sqrt{\pi}B(z)f_s(z)}\exp\left[-\left[\frac{1-\frac{f}{f_s(z)}}{B(z)}\right]^2\right] \tag{19}$$

where f_s is the shedding frequency, B is the bandwidth parameter which is expressed by the relationship $B(z) = 0.1 + 2.0\ I(z)$, $St = f_s d(z)/\bar{V}(z)$ is the Strouhal number, σ_{CL} is the rms of the normalized lift force per a unit length equal to Vickery and Basu (1983), and Waldeck (1992).

$$\sigma_{CL}(z) = [0.15 + 0.55I^*(z)] - [0.09 + 0.55I^*(z)]e^{-(20I^*(z))^3} \tag{20}$$

where $I^*(z)$ is the modified turbulence intensity given by

$$I^*(z) = I(z)\left(\frac{d(z)}{L(z)}\right)^{\frac{1}{3}} \tag{21}$$

where $L(z)$ is the integral length scale of turbulence and $I(z)$ is the turbulence intensity defined as

$$I(z) = \frac{\sigma_{g'}(z)}{\bar{V}(z)} \tag{22}$$

The integral length scale $L(z)$ at heights of z in the range of 10-240 m has been suggested by Counihan (1975) in the form:

$$L(z) = Cz^m \tag{23}$$

where C and m depend on the roughness length z_0.

4.1.2. Cross-wind response of the chimney – random vibration approach

The dynamic cross-wind response of the considered chimney was evaluated in the same manner as described in section 3.2, i.e., by the modal analysis.

On the basis of the random vibration approach the mean square value of the displacement $\bar{q}_v^2(z)$ becomes

$$\sigma_{qv}^2(z) = \frac{\rho_a^2}{4}\sum_{i=1}^{N}\left(\frac{\phi_i(z)}{m_i}\right)^2\int_0^{\infty}S_v(f)|H_i(f)|^2 df \tag{24}$$

where

$$S_v(f) = \int_0^H\int_0^H d(z_1)d(z_2)\bar{V}^2(z_1)\bar{V}^2(z_2)S_{CL}(z_1, z_2, f)\phi_i(z_1)\phi_i(z_2)dz_1dz_2 \tag{25}$$

where $S_{CL}(z_1, z_2, f)$ is the cross spectrum of the vortex-induced force suggested by Vickery and Basu (1983) in the form:

$$S_{CL}(z_1, z_2, f) = \sqrt{S_{CL}(z_1, f)}\sqrt{S_{CL}(z_2, f)}R(z_1, z_2) \tag{26}$$

$$R(z_1, z_2) = \cos\left(\frac{2r}{3}\right)\exp\left\{-\left(\frac{r}{3}\right)^2\right\} \tag{27}$$

and r is the dimensionless distance

$$r = \frac{2(z_1 - z_2)}{d(z_1) + d(z_2)} \tag{28}$$

4.2. Ruscheweyh's model

The basis of this model is an effective exciting action caused by vortex – shedding which is uniformly distributed at so called the correlation length L_i (i = 1, 2,.., limited to one or more length). The largest displacement $y_{F,\max}$ of the structure can be calculated using Eq. (29)

$$\frac{y_{F,\max}}{b} = \frac{1}{St^2}\cdot\frac{1}{Sc}\cdot K\cdot K_w\cdot C_{lat} \tag{29}$$

where

St is the Strouhal number,
Sc is the Scruton number,
K_w is the effective correlation length,
K is the mode shape factor,
C_{lat} is the lateral force coefficient.

NOTE. The aeroelastic forces are taken into account by the effective correlation length factor K_w.

The Eq. (29) is the basis of Approach 1 for the calculation of the cross-wind amplitudes given in the Eurocode Standard (2003). In this Standard there is also Approach 2 for calculating the vortex excited cross-wind amplitudes. According to this Approach the characteristic maximum displacement at the point with the largest movement is given in Eq. (30)

$$y_{\max} = \sigma_y \cdot k_p \tag{30}$$

where

σ_y is the standard deviation of the displacement which can be calculated by using expression given in Eurocode Standard (2003),

k_p is the peak factor – see Eurocode Standard (2003).

4.3. Flaga's model

Details of the model are given in Flaga (1996, 1997). According to this model (its main formula) the standard deviation of the displacement of the chimney with circular cross-section σ_y may be estimated from Eq. (31)

$$\sigma_y \cong \frac{\pi^{\frac{1}{4}}}{2} \frac{q_c \sigma_w \varepsilon \left(1 + \alpha \frac{\sigma_y}{D}\right)^3}{4 m \pi^2 f_i^2 \sqrt{B \gamma}} \tag{31}$$

where

q_c is the critical wind velocity pressure for the motionless chimney,

D is the diameter of the chimney,

f_i is the i-th natural frequency,

γ is the critical damping ratio,

m is the mass per unit length,

ε is the parameter limiting the domain of the vortex shedding on the height of the structure, it is also called the dimensionless coefficient of the spectral density function of the global across-wind load caused. It should be determined from the formula

$$\varepsilon = \sqrt{2 \frac{L(\sigma_y) D}{H} \left\{ 1 - \frac{L(\sigma_y) D}{H} \left[1 - \exp\left(\frac{H}{L(\sigma_y) D} \right) \right] \right\}} \tag{32}$$

where

$L(\sigma_y)$ is the dimensionless spanwise correlation length scale of across-wind load,

H is the height of the chimney.

5. Numerical analysis of the chimney response

The along-wind response of the chimney has been computed by means of the methods presented in

- the Polish Standard (1977),
- the ISO International Standard (1997),
- the Eurocode Standard (2003),
- the random vibration approach described in this paper.

In the computations the following data have been used:

z_0 = 0.03 m (the roughness length),

K = 0.005 (the terrain factor depending on the roughness length),

α = 0.16 (the power law exponent for the mean wind speed profile),

k = 8.0 (the empirical constant for the coherence function),

C_D = 0.63 (the drag coefficient),
g = 3.30 (the peak factor),
ξ = 0.02 (the structural damping ratio for each mode),
ξ_{a1} = 0.002, ξ_{a2} = 0.00035 (the aerodynamic damping for the first and second mode, respectively).

The comparison of the along-wind response (the displacements and bending moments) for the chimney for $\overline{V}_{10}$ = 24 m/s computed by means of these four methods mentioned above is shown in Fig. 1. Fig. 2. depicts the comparison of the top deck chimney displacement for different values of $\overline{V}_{10}$. The comparison of the along-wind displacements of the chimney with and without flexibility of soil is shown in Fig. 5.

The cross-wind response of the chimney has been computed by means of the methods presented in
- the Polish Standard (1977),
- the ISO International Standard (1997),
- the Eurocode Standard (Approach 1 and 2) (2003),
- the Vickery and Basu's model (1983),
- the Flaga's model (1996, 1997).

In the computations the following data have been used:
St = 0.2,
ρ = 1.25 kg/m^3 (the air density),
v = 0.145 · 10^{-4} m^2/s (the kinematic viscosity of the air),
f_0 = 0.216 Hz (the first natural frequency of the chimney),

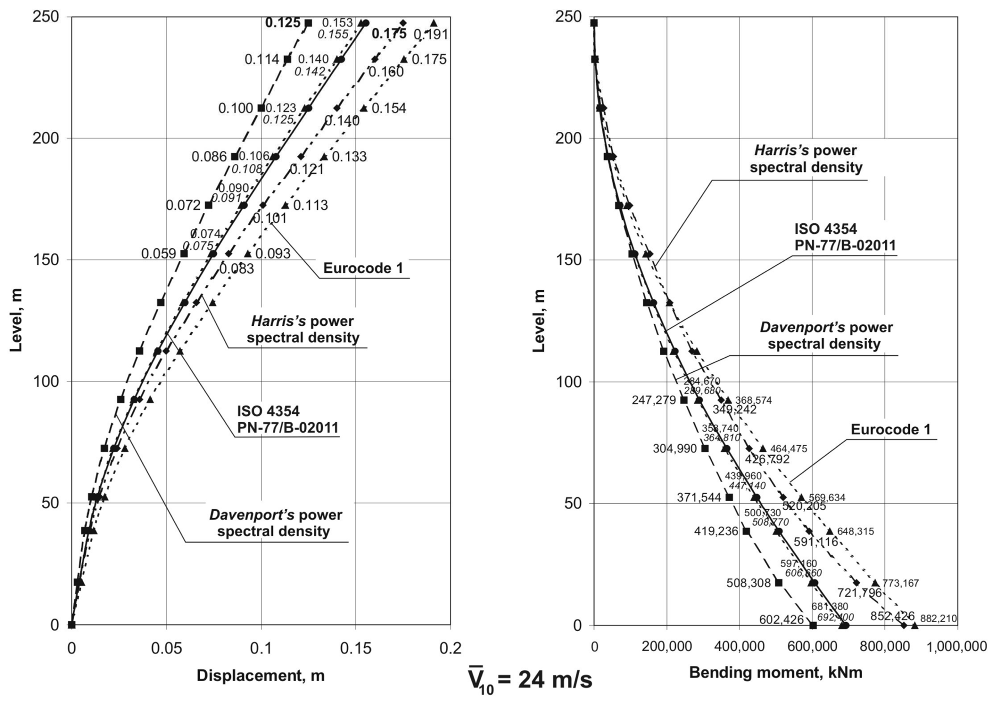

Fig. 1 Comparison of the along-wind displacements and bending moments of the chimney for $\overline{V}_{10}$ = 24 m/s

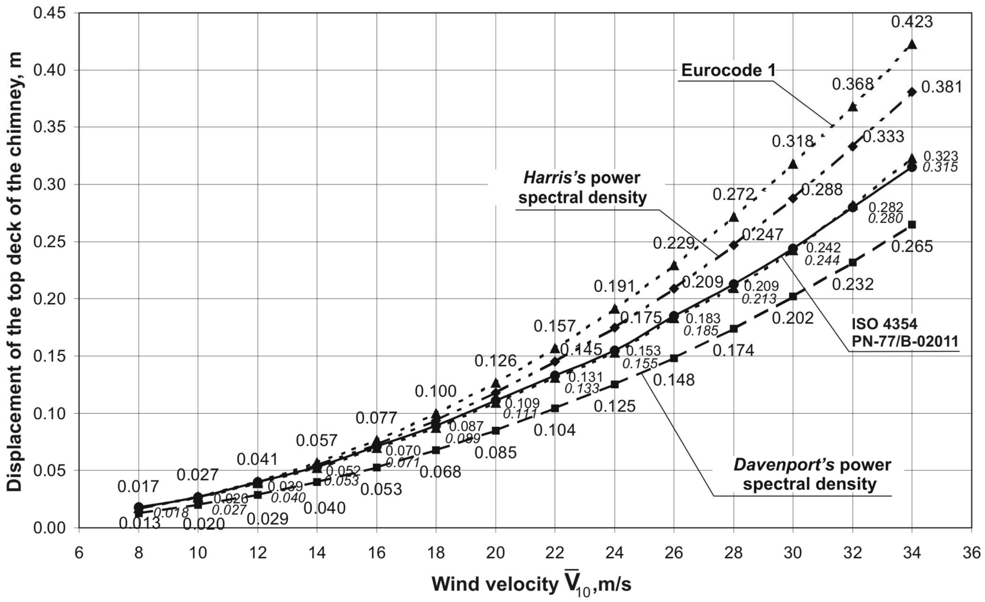

Fig. 2 Comparison of the top deck chimney along-wind displacements for different values of $\overline{V}_{10}$

$$V_{cr} = \frac{f_0 \cdot d}{St} = \frac{0.216 \cdot 24}{0.2} = 25.9 \text{ m/s} \text{ (the critical wind velocity)},$$

$$\mathrm{Re} = \frac{d \cdot V_{cr}}{\nu} = \frac{24 \cdot 25.9}{0.145 \cdot 10^{-4}} = 4.29 \cdot 10^{7} \text{ (the Reynolds number)},$$

$$Sc = \frac{2 \cdot \delta \cdot m}{\rho \cdot d^2} = \frac{2 \cdot 0.03 \cdot 123141.48}{1.25 \cdot 24^2} = 10.26 \text{ (the Scruton number)},$$

$$I(z = 245\ m) = 2.45 \cdot \sqrt{K} \cdot \left(\frac{z}{10}\right)^{-\alpha} = 2.45 \cdot \sqrt{0.005} \cdot \left(\frac{245}{10}\right)^{-0.16} = 0.104 \text{ (the turbulence intensity}$$

at height of the top of the chimney).

The comparison of the cross-wind displacements for the chimney for V_{cr} = 25.9 m/s computed by means of these five methods mentioned above is shown in Fig. 3. Fig. 4 depicts the comparison of the along and cross-wind chimney displacements for V_{cr} = 25.9 m/s. The comparison of the cross-wind displacements of the chimney with and without flexibility of soil is shown in Fig. 6.

6. Discussion on possible reasons for the large differences observed in the cross-wind response between Ruscheweyh's model and the Vickery/Basu model

There are two classes of calculation methods for vortex shedding response of slender structures, like chimneys, TV-towers, tubes, etc.: (1) those based on sinusoidal excitation; (2) those based on random excitation.

A sinusoidal excitation model was developed by Ruscheweyh, *et al.* (1982, 1996, 1998); this

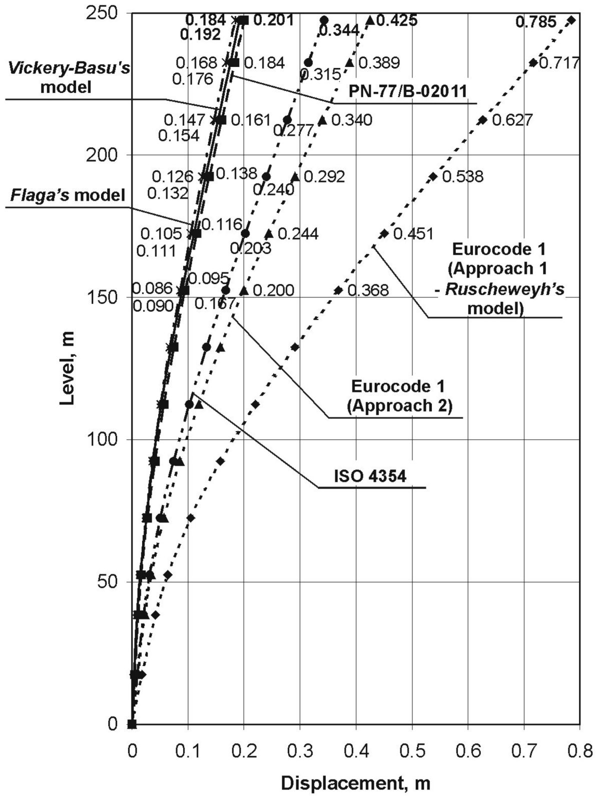

Fig. 3 Comparison of the cross-wind chimney displacements for $V_{cr} = 25.9$ m/s ($St = 0.2$)

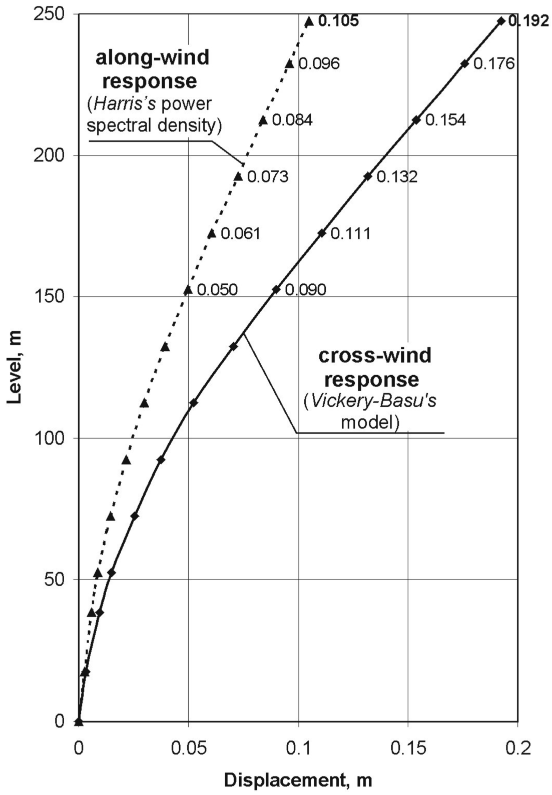

Fig. 4 Comparison of the along and cross-wind chimney displacements for $V_{cr} = 25.9$ m/s ($\overline{V}_{10} = 19$ m/s, $St = 0.2$)

model has been included into the draft of Eurocode prEN 1991-1-4.6 (2003). A random excitation model was developed by Vickery and Basu (1983, 1995). In the following section assumptions of these two methods and full scale measurements on some chimneys are described and assessed. In both methods chimneys are assumed as one – dimensional body (see Fig. 7)

In the sinusoidal excitation model Ruscheweyh assumed the excitation lift force per unit height as follows:

$$f(z,t) = \frac{1}{2}\rho\overline{V}^2_{crit}(z)\tilde{C}_y(z)\cdot d\cdot\sin(2\pi n_j\cdot t) \tag{33}$$

where ρ is the air density; $\overline{V}_{crit}(z)$ is the mean value of the critical wind velocity; $\tilde{C}_y(z)$ is the aerodynamic excitation force coefficient; d is the diameter of a slender structure. In this case, the excitation lift force is assumed to be harmonic with a frequency equal to the vortex shedding frequency n_v, which is equal to the natural frequency of the chimney, n_j.

The total amplitude of the excitation lift force is given by

$$\int_0^H f^{am}(z,t) = \frac{1}{2}\rho V^2_{crit}(z_e)d\int_0^H C_y(z)dz \tag{34}$$

which Ruscheweyh modified as

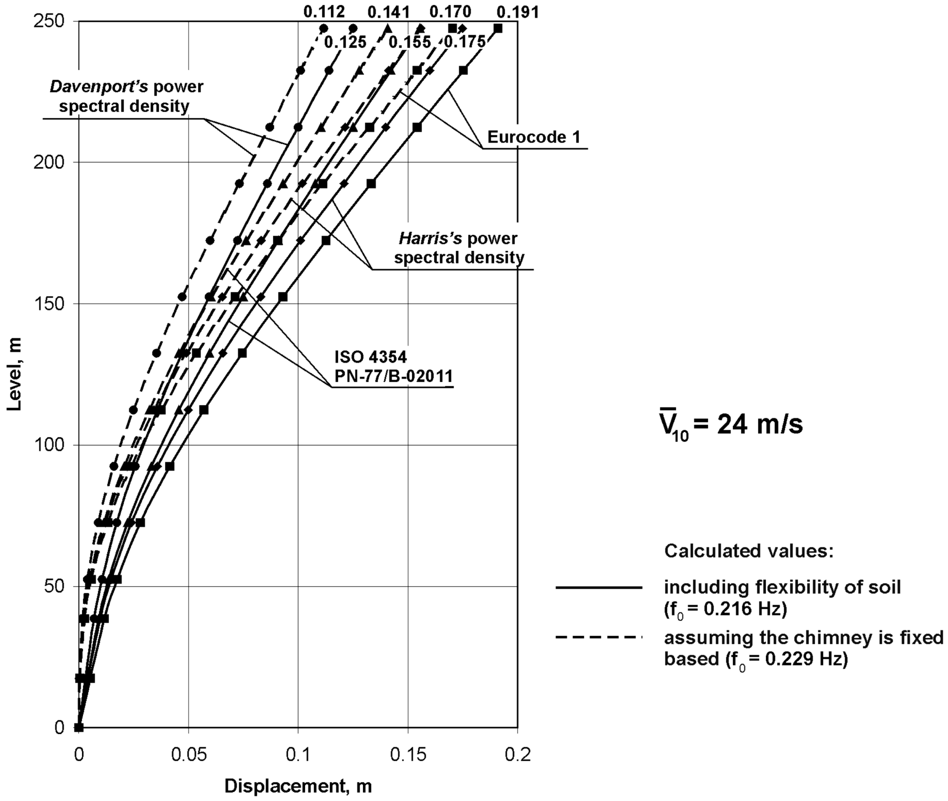

Fig. 5 Comparison of the along-wind displacements of the chimney with and without flexibility of soil for $\bar{V}_{10}$ = 24 m/s

$$\int_0^H f^{am}(z,t) = \frac{1}{2}\rho V_{crit}^2(z_e)d \cdot C_{lat} \cdot L \tag{35}$$

where L is the excitation length which Ruscheweyh called as the correlation length; z_e is an average or effective height for the vortex shedding frequency; C_{lat} is the constant aerodynamic excitation force coefficient. According to this model the vortex shedding forces are applied over a height range less than the total height of the chimney.

Let us assume that the chimney vibrates in the first mode. The equation of this mode (for the first normal coordinates) can be written

$$\ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 = \frac{P_1(t)}{M_1} \tag{36}$$

where $P_1(t)$ is the generalized force, equal to

$$\int_0^H f(z,t)\phi_1(z)dz = \frac{\rho}{2}V_{crit}^2 d \cdot C_{lat}\int_L \phi_1(z)dz \sin 2\pi n_1 t \tag{37}$$

ω_1 is the natural undamped circular frequency for the first mode ($\omega_1 = 2\pi n_1$); ξ_1 is the modal damping ratio; M_1 is the generalized mass equal to $\int_0^H m(z)\phi_1^2(z)dz$; $m(z)$ is the mass per unit length along the

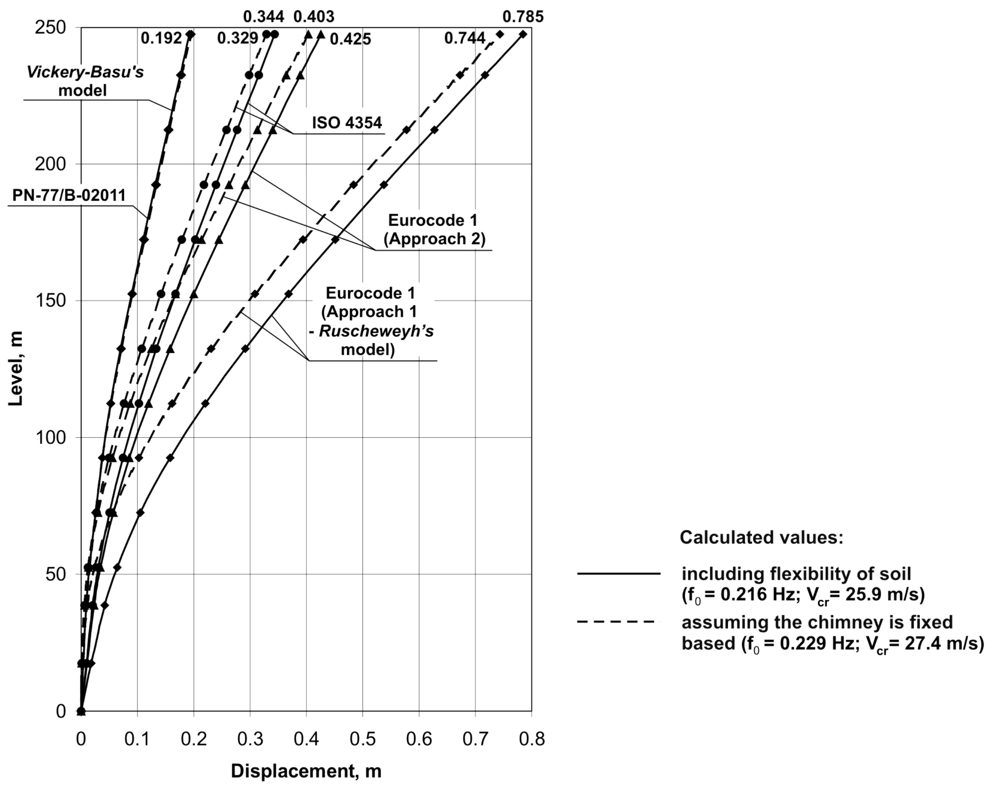

Fig. 6 Comparison of the cross-wind displacements of the chimney with and without flexibility of soil for $V_{cr} = 25.9$ m/s ($St = 0.2$)

chimney, and further Ruscheweyh assumed that M_1 may be written

$$M_1 = \int_0^H m(z)\phi_1^2(z)dz = M\int_0^H \phi_1^2(z)dz \tag{38}$$

where M is called an equivalent mass per unit length. This is an important assumption. It is possible to do that for steel chimneys, which generally have constant diameters and constant thickness of walls. The most extensive Ruscheweyh's measurements have been done with an experimental steel chimney of 28 m tall and 0.91 m diameter. This assumption can not be applied to industrial concrete chimneys, for one or multi-flue chimneys with non-uniform mass per unit height. Good examples for a non-uniform mass distribution are given by Müller and Nieser (1975/76) for the one-flue and in Chmielewski, *et al.* (2005) for the six-flue concrete chimney.

Next Ruscheweyh calculated the maximum amplitude at resonance for Eq. (4)

$$q_1^{am} = \frac{P_1^{am}(t)}{(2\pi n_1)^2 M\int_0^H \phi_1^2(z)dz} \cdot \frac{\pi}{\delta} = \frac{\frac{\rho}{2}V_{crit}^2 d \cdot C_{lat}\int_L \phi_1(z)dz}{4\pi^2 n_1^2 M\int_0^H \phi_1^2(z)dz} \tag{39}$$

where δ is the logarithmic decrement.

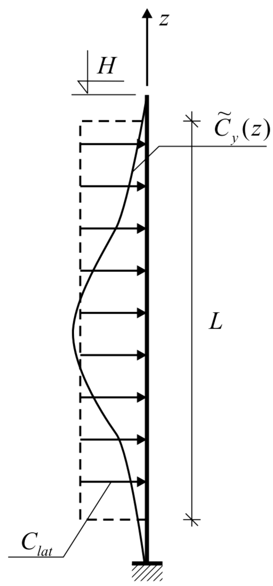

Fig. 7 Ruscheweyh's model for cross-wind excitation of a slender structure

The maximum amplitude of deflection at any height on a slender structure is given by

$$y_{\max}(z) = q_1^{am} \cdot \phi_1(z) \tag{40}$$

For a slender structure with a uniform mass per unit height, the maximum displacement at the tip ($z = H$), and where $\phi(H)$ is chosen as 1.0, is given by formula (29), where

$$K = \frac{\int_0^H \phi(z)dz}{4\pi \int_0^H \phi^2(z)dz}, \quad K_w = \frac{\int_L \phi(z)dz}{\int_0^H \phi^2(z)dz}$$

$$S_t = \frac{n_1 \cdot d}{V_{crit}}, \quad S_c = \frac{2\pi M\delta}{\rho d^2} \tag{41}$$

Vortex-excited vibrations are very important phenomena for slender structures or such elements as: chimneys (steel and concrete), TV-towers, bridge hangers of suspension cable stayed and arch bridges, antena masts, etc. The correct prediction of the vortex-induced vibration amplitude is rather complicated. The simple formula (29) derived by Ruscheweyh is based on some assumptions. Three of them are given by Holmes (2001). The fourth assumption requires a slender structure with a uniform mass per unit height. This assumption fulfils steel chimneys, bridge hangers and antena masts but not concrete one or multi-flue chimneys. A comparison of prediction with full scale measurements for steel chimneys or bridge hangers in the papers (Ruscheweyh, *et al.* 1996) are given. In the paper (Ruscheweyh, *et al.* 1996) (in the section 3.) there is the statement: "*the correlation length model" has been applied to slender structures since more than 15 years and the* *experience is very good. The procedure has been included in German standards for steel chimneys and antenna masts as well as in the new European wind load code proposal, ENV 1991-2-4*".

A random excitation model, for vortex shedding response prediction, was developed by Vickery and Basu (1983) and the idea of this model is presented in the section 4.1. In this model the lift forces caused by vortex shedding from a slender tower are characterized by four aerodynamic parameters. These parameters were calibrated from full-scale measurements (Vickery & Basu 1983, Sanada, *et al.* 1992, Waldeck 1992) of tall concrete chimneys. So, this model is specially applicable to these types of structures.

7. Conclusions

1. The procedures for calculating the along-wind response of line – like structures are similar in the ISO and Polish standards. Thus, the responses of the chimney differ slightly for these two codes. The response computed according to the Eurocode is higher about 25% than the ISO standard. The random vibration response is dependent on the power spectral density.
2. The cross-wind displacements of the chimney computed according to the Vickery-Basu's model, the Flaga's model and the Polish Standard differ also slightly (about 9%). The responses in accordance with the ISO and the Eurocode are much higher (the ISO about 70% and the Eurocode about 110% for the Approach 2 and about 400% for the Approach 1) than the first three results. For high Reynolds number the ISO and Eurocode procedures overestimate the response of structures like the chimney considered in the paper.
3. The simple formulae called "the correlation length model", derived by Ruscheweyh, has been applied to many different slender steel structures such as: steel stacks, bridge hangers, bridge bracings. The predicted values have been verified by full scale measurements of these type of structures. This model has been included into German DIN –standards (for steel stacks) (DIN 4133) and into Eurocode 1: Action on structures – without notice that model should not be applied to concrete chimneys. In particular we have demonstrated the limitations of Ruscheweyh's model for the analysis of concrete chimneys with non-uniform mass distribution.
4. The Vickery-Basu's and Flaga's models gave similar reasonable results for the cross-wind response of the analysed chimney. With appropriate input parameters these models are applicable to tall concrete chimneys.
5. Soil flexibility under the foundation of the chimney has different influence over the along and cross-wind responses. For the along-wind response the displacement of the top of the chimney is increased approximately in the range from 11% to 14% for $\bar{V}_{10} = 24\ m/s$. For the cross-wind response the displacement of the top of the chimney differs slightly for the methods presented in the Polish Standard, the Vickery-Basu's model and the Flaga's model. For the ISO Standard the response is bigger about 4.6%, and for the Eurocode Standard is bigger about 5.5% (for V_{cr}).

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