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Control strategy of the lever-type active multiple tuned mass dampers for structures

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Abstract. The lever-type active multiple tuned mass dampers (LT-AMTMD), consisting of several lever-type active tuned mass dampers (LT-ATMD), is proposed in this paper to attenuate the vibrations of long-span bridges under the excitation directly acting on the structure, rather than through the base. With resorting to the derived analytical-expressions for the dynamic magnification factors of the LT-AMTMD structure system, the performance assessment then is conducted on the LT-AMTMD with the identical stiffness and damping coefficient but unequal mass. Numerical results indicate that the LT-AMTMD with the actuator set at the mass block can provide better effectiveness in reducing the vibrations of long-span bridges compared to the LT-AMTMD with the actuator set at other locations. An appealing feature of the LT-AMTMD with the actuator set at the mass block is that the static stretching of the spring may be freely adjusted in accordance with the practical requirements through changing the location of the support within the viable range while maintaining the same performance (including the same stroke displacement). Likewise, it is shown that the LT-AMTMD with the actuator set at the mass block can further ameliorate the performance of the lever-type multiple tuned mass dampers (LT-MTMD) and has higher effectiveness than a single lever-type active tuned mass damper (LT-ATMD). Therefore, the LT-AMTMD with the actuator set at the mass block may be a better means of suppressing the vibrations of long-span bridges with the consequence of not requiring the large static stretching of the spring and possessing a desirable robustness.

Keywords: damping; vibration control; lever-type active multiple tuned mass dampers (LT-AMTMD); lever-type multiple tuned mass dampers (LT-MTMD); lever-type active tuned mass dampers (LT-ATMD); long-span bridges; parameters.

1. Introduction

In recent years, with the rapid increase of bridge spans, the research on suppressing the buffeting response of long-span bridges has become a problem of great concern. By and large, the control

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strategies of the wind-induced vibrations of long-span bridges can be grouped into the structural countermeasures, aerodynamic countermeasures, and mechanical countermeasures. Significant strides have been made in recent years toward the development and application of the structural and aerodynamic countermeasures for controlling wind-induced vibrations of long-span bridges, especially those long-span bridges located in the regions where typhoon often occurs. It is worth noting, however, that there has been much interest in the research and development of the mechanical countermeasure for long-span bridges in the past few years. Studies on the mechanical countermeasures for the wind-induced vibrations of long-span bridges focus mainly on passive control devices. Among passive control devices, the tuned mass damper (TMD) is without doubt a simple, inexpensive, and reliable means of suppressing the undesirable oscillations of structures. Likewise, the TMD has been theoretically and experimentally shown to be effective in reducing the buffeting response of long-span bridges (Gu and Xiang 1992, Gu, *et al*. 1994). However, the traditional hanging-type tuned mass damper (HT-TMD) may not be fitted into the space available for installation within the bridge deck due to the large static stretching of the spring under the action of the mass block. Likewise, the large static stretching is likely to cause the spring to operate in a nonlinear range, thus rendering the unsatisfactory effectiveness. For instance, in the case of controlling the first vertical buffeting response of Yangpu Bridge with the first vertical bending natural frequency equal to 0.286 Hz, the static stretch of the spring under the action of the mass block of the HT-TMD equals 3.185 m in terms of the equation to be introduced next. However, the composite deck of the Yangpu Bridge can only provide a 2.7 m net height for housing the HT-TMD. In view of this, a passive lever-type tuned mass damper (LT-TMD) has been proposed to deal with the large static stretch of the spring of the traditional HT-TMD (Gu, *et al*. 1998). In terms of studies conducted by the authors, the LT-TMD and the traditional HT-TMD can approximately achieve the same effectiveness and stroke displacement. However, the LT-TMD needs lesser optimum damping ratio but significantly higher optimum tuning frequency ratio in comparison with the HT-TMD. It follows that the LT-TMD can not obviate the main disadvantage of the traditional HT-TMD (i.e. the sensitivity problem due to the fluctuation in tuning the natural frequency of the HT-TMD to the controlled frequency of a structure and/or that in the optimum damping ratio of the HT-TMD). Likewise, it has been demonstrated that both the dynamic responses and dynamic parameters of bridges in terms of field measurements and wind tunnel tests and theoretical analyses are usually different from each other because the phenomenon of the wind-induced vibrations of bridges is very complex (Conti, *et al*. 1996, Diana, *et al*. 1992). In view of these reasons, a more robust control device, which is able to control structural vibrations with variable natural frequencies, such as the multiple tuned mass dampers (MTMD), is needed for suppressing the buffeting response of long-span bridges.

The multiple tuned mass dampers (MTMD) with distributed natural frequencies were proposed by Xu and Igusa (1992) and also investigated by several researchers, such as Yamaguchi and Harnporchai (1993); Abe and Fujino (1994); Igusa and Xu (1994); Abe and Igusa (1995); Kareem and Kline (1995); Jangid (1995, 1999); Li (2000); Park and Reed (2001); Gu, *et al.* (2001); Chen and Wu (2001). The MTMD is shown to be more effective in suppressing the oscillations of structures with respect to a single TMD. Likewise, based on the various combinations available of the stiffness, mass, damping coefficient, and damping ratio in the MTMD, the five MTMD models have been recently presented by Li (2002). Through the implementation of min.min.max.displacement dynamic magnification factor (DDMF) and min.min.max.acceleration dynamic magnification factor (ADMF), it has been shown that the MTMD with the identical stiffness (i.e. $k_{T1}=k_{T2}=\cdots=k_{Tn}=k_T$)

and damping coefficient (i.e. $c_{T1}=c_{T2}=\cdots=c_{Tn}=c_T$) but unequal mass (i.e. $m_{T1} \neq m_{T2} \neq \ldots \neq m_{Tn}$) provides better effectiveness and wider optimum frequency spacing (i.e. higher robustness against the change or the estimation error in the structural natural frequency) with respect to the rest of the MTMD models (Li 2002). Recently, the studies conducted by Li and Liu (2002a) have disclosed further trends of both the optimum parameters and effectiveness and further provided suggestion on selecting the total mass ratio and total number of the MTMD with the identical stiffness and damping coefficient but unequal mass. More recently, based on the uniform distribution of system parameters, instead of the uniform distribution of natural frequencies, the eight new MTMD models have been, for the first time, proposed in order to seek for the MTMD models without the near-zero optimum average damping ratio. The six MTMD models then are found without the near-zero optimum average damping. The optimum MTMD with the identical damping coefficient (i.e. $c_{T1}=c_{T2}=\cdots=c_{Tn}=c_T$) and damping ratio (i.e. $\xi_{T1}=\xi_{T2}=\cdots=\xi_{Tn}=\xi_T$) but unequal stiffness (i.e. $k_{T1} \neq k_{T2} \neq \cdots \neq k_{Tn}$) and with the uniform distribution of masses is found able to render better effectiveness and wider optimum frequency spacing in comparison with the rest of the MTMD models (Li and Liu 2003). Likewise, it is interesting to know that the two MTMD models mentioned above can approximately reach the same effectiveness and robustness (Li and Liu 2003). Evidently, significant strides have been made in recent years as regards studies on the MTMD for structural control. However, if the traditional hanging-type multiple tuned mass dampers (HT-MTMD), consisting of several hanging-type tuned mass dampers (HT-TMDs), are directly used to mitigate the buffeting response of long-span bridges, each HT-TMD in the HT-MTMD possesses the large static stretching of the spring, h_j, which can be determined in terms of the equation $h_j = g/\omega_{Tj}^2$, where ω_{Tj} is the circular frequency of each HT-TMD and g is the acceleration due to gravity. For instance, if applying the HT-MTMD with the total number and the total mass ratio respectively equal to 5 and 0.01 to suppress the first vertical buffeting response of the Yangpu Bridge mentioned above, this HT-MTMD has the static stretches of the spring at 3.580 m (HT-TMD1), 3.349 m (HT-TMD2), 3.163 m (HT-TMD3), 2.971 m (HT-TMD4), and 2.814 m (HT-TMD5). It is worth noting that the maximum static stretching of the spring of the HT-MTMD is larger than the static stretching of a single HT-TMD with equal total mass ratio. The lever-type multiple tuned mass dampers (LT-MTMD) recently proposed by Li (2005) have dealt with this problem. However, the probability of great drift of the controlled natural frequency may occur due to the complexity of the wind-induced vibrations of long-span bridges mentioned above. Thus, a beforehand well-designed optimum LT-MTMD may render the unsatisfactory effectiveness. In view of the above situations, based on the concept of the active multiple tuned mass dampers (AMTMD) (Li and Liu 2002b), the lever-type active multiple tuned mass dampers (LT-AMTMD), consisting of several lever-type active tuned mass dampers (LT-ATMDs) with a uniform distribution of natural frequencies are proposed here for improving the performance of the LT-MTMD for long-span bridges under the excitation directly acting on the structure, rather than through the base. The main objective of the present study then is to evaluate the performance (including the stroke displacement of the LT-AMTMD) to demonstrate that the LT-AMTMD may be a better choice for suppressing the buffeting response of long-span bridges with the consequence of not requiring the large static stretching of the spring and possessing a desirable robustness.

2. Transfer functions of the LT-AMTMD structure system

The LT-AMTMD is taken into consideration here to control the specific mode of a structure, and

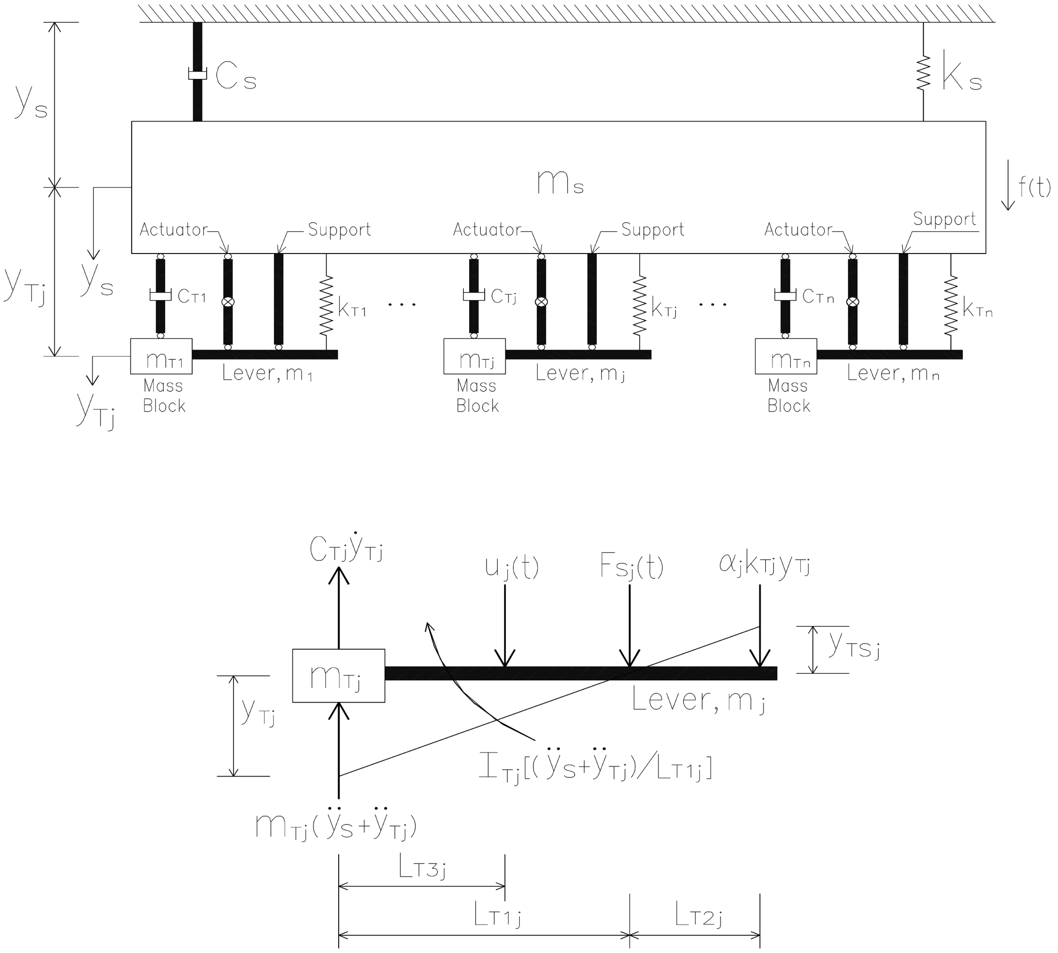

Fig. 1 Schematic diagram of the LT-AMTMD structure system

the modeling shown in Fig. 1, in which the structure is modeled as an SDOF structure using the mode reduced-order method, is adopted. Introducing the nondimensional parameters $\alpha_j = L_{T2j}/L_{T1j}$ and $\beta_j = L_{T3j}/(L_{T1j} + L_{T2j})$, in which L_{T1j} represents the distance between the mass block and support point of the jth LT-ATMD in the LT-AMTMD, L_{T2j} denotes the distance between the spring and support point of the jth LT-ATMD in the LT-AMTMD, and L_{T3j} is the distance between the active control force and mass block of the jth LT-ATMD in the LT-AMTMD, the dynamic stretching of each spring and the location of each active control force in the LT-AMTMD can be, respectively, determined by

$$y_{TSj} = \alpha_j y_{Tj} \tag{1a}$$

$$L_{T3j} = [(1+\alpha_j)\beta_j]L_{T1j} \qquad (j = 1, 2, 3, ..., n) \tag{1b}$$

where y_{Tj} is the displacement of each LT-ATMD in the LT-AMTMD with reference to the structure.

The equations of motion of the LT-AMTMD structure system can be given by

$$I_{Tj}[(\ddot{y}_s+\ddot{y}_{Tj})/L_{T1j}] + [m_{Tj}(\ddot{y}_s+\ddot{y}_{Tj}) + c_{Tj}\dot{y}_{Tj}]L_{T1j} + \alpha_j k_{Tj} y_{Tj} L_{T2j} = u_j(t)(L_{T1j}-L_{T3j}) \quad (j = 1, 2, 3,..., n) \tag{2}$$

$$\alpha_j k_{Tj} y_{Tj} + F_{sj}(t) + u_j(t) = c_{Tj}\dot{y}_{Tj} + m_{Tj}(\ddot{y}_s+\ddot{y}_{Tj}) \quad (j = 1, 2, 3,..., n) \tag{3}$$

$$m_s\ddot{y}_s + c_s\dot{y}_s + k_s y_s = f(t) + \sum_{j=1}^{n} F_{cj}(t) \tag{4}$$

$$F_{cj}(t) = c_{Tj}\dot{y}_{Tj} - u_j(t) - F_{sj}(t) - \alpha_j k_{Tj} y_{Tj} = -m_{Tj}(\ddot{y}_s+\ddot{y}_{Tj}) \quad (j = 1, 2, 3,..., n) \tag{5}$$

$$u_j(t) = m_{tj}\ddot{y}_{Tj} - c_{tj}\dot{y}_{Tj} - k_{tj} y_{Tj} \quad (j = 1, 2, 3,..., n) \tag{6}$$

where m_s, c_s, and k_s are, respectively, the mode-generalized stiffness, damping coefficient, and mass; m_{Tj}, c_{Tj}, and k_{Tj} represent the mass, damping coefficient, and stiffness of the jth LT-ATMD in the LT-AMTMD, respectively; y_s is the structural displacement; $f(t)$ represents the external excitation; $F_{sj}(t)$ is the interaction force between the structure and the support point of each LT-ATMD in the LT-AMTMD; $I_{Tj} = [3(1-\alpha_j)^2 + (1+\alpha_j)^2]m_j L_{T1j}^2/12$ $(j = 1, 2, 3,..., n)$ is the mass moment of inertia of each lever in the LT-AMTMD, where m_j is the mass of each lever. It is assumed that the stiffness and damping coefficient of each LT-ATMD in the LT-AMTMD are same and the natural frequencies of the LT-AMTMD are uniformly distributed. As a result, the LT-AMTMD is manufactured by keeping the stiffness and damping constant but mass unequal (i.e. $k_{T1} = k_{T2} = \cdots = k_{Tn} = k_T$; $c_{T1} = c_{T2} = \cdots = c_{Tn} = c_T$; $m_{T1} \neq m_{T2} \neq \ldots \neq m_{Tn}$). Likewise, the nondimensional parameter α_j and β_j are, respectively, assumed to maintain constant (i.e. $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$; $\beta_1 = \beta_2 = \cdots = \beta_n = \beta$).

In the formulation of the transfer functions, the following parameters are introduced: $\eta_j = [3(1-\alpha_j)^2 + (1+\alpha_j)^2]/12$ (in physical terms $\eta_1 = \eta_2 = \cdots = \eta_n = \eta$); $\omega_s = \sqrt{k_s/m_s}$; $\xi_s = c_s/2m_s\omega_s$; $\omega_{Tj} = \sqrt{[\alpha_j^2 k_{Tj} + k_{tj}(1-\beta_j-\alpha_j\beta_j)]/m_{Tj}}$; $\xi_{Tj} = [c_{Tj} + c_{tj}(1-\beta_j-\alpha_j\beta_j)]/2m_{Tj}\omega_{Tj}$; $r_j = m_{tj}/m_{Tj}$ (the normalized acceleration feedback gain factor, referred to as NAFGF, letting $r_1 = r_2 = \cdots = r_n = r$); $\mu_{Tj} = m_{Tj}m_s$ (the mass ratio of each LT-ATMD in the LT-AMTMD); $\mu_j = m_j m_{Tj}$ (the lever to block mass ratio of each LT-ATMD in the LT-AMTMD).

Let it be supposed that the lever to block mass ratio of each LT-ATMD in the LT-AMTMD is held constant (i.e. $\mu_1 = \mu_2 = \cdots = \mu_n = \mu$). With the hypothesis of $f(t) = m_s e^{-i\omega t}$, $y_{Tj} = H_{y_{Tj}}(-i\omega)e^{-i\omega t}$ $(j = 1, 2, 3,..., n)$ and $y_s = H_{y_s}(-i\omega)e^{-i\omega t}$, and setting these in Eqs. (2)-(6), the transfer functions of the LT-AMTMD structure system can then be given by

$$H_{y_s}(-i\omega) = \frac{1}{\omega_s^2[\mathrm{Re}(\omega) + i\mathrm{Im}(\omega)]} \tag{7}$$

$$H_{y_{Tj}}(-i\omega) = \frac{(1+\eta_j\mu_j)\omega^2}{\omega_s^4[\bar{\mathrm{R}}\mathrm{e}(\omega) + i\bar{\mathrm{I}}\mathrm{m}(\omega)][\mathrm{Re}(\omega) + i\mathrm{Im}(\omega)]} \quad (j = 1, 2, 3,..., n) \tag{8}$$

in which Re(ω), Im(ω), $\bar{\mathrm{R}}\mathrm{e}(\omega)$, and $\bar{\mathrm{I}}\mathrm{m}(\omega)$ will be given next.

3. Performance criteria of the LT-AMTMD

Let ω_T be the average frequency of the LT-AMTMD (i.e. $\omega_T = \Sigma_{k=1}^{n}\omega_{Tk}/n$). The natural frequency of each LT-ATMD in the LT-AMTMD can be derived as follows:

$$\omega_{Tj} = \omega_T\left[1 + \left[j - \frac{n+1}{2}\right]\frac{F_S}{n-1}\right] \qquad (j = 1, 2, 3,..., n) \tag{9}$$

in which the nondimensional parameter F_S is defined to be the frequency spacing of the LT-AMTMD (used for reflecting the robustness of the LT-AMTMD) determined by $F_S = \frac{\omega_{Tn} - \omega_{T1}}{\omega_T}$.

Then, the ratio of the natural frequency of each LT-ATMD in the LT-AMTMD to the controlled frequency of the structure can be written as follows:

$$r_{Tj} = \frac{\omega_{Tj}}{\omega_s} = f\left[1 + \left[j - \frac{n+1}{2}\right]\frac{F_S}{n-1}\right] \qquad (j = 1, 2, 3,..., n) \tag{10}$$

in which f is defined to be the tuning frequency ratio of the LT-AMTMD calculated by $f = \frac{\omega_T}{\omega_s}$.

The average damping ratio of the LT-AMTMD is defined as follows:

$$\xi_T = \sum_{j=1}^{n} \frac{\xi_{Tj}}{n} \tag{11}$$

The ratio of the total mass of the LT-AMTMD to the mode-generalized mass of the structure is referred to as the total mass ratio of the LT-AMTMD, which has the form

$$\mu_T = \sum_{j=1}^{n} \frac{m_{Tj}}{m_s} = \sum_{j=1}^{n} \mu_{Tj} \tag{12}$$

Employing the above assumptions and derived expressions, the total mass ratio of the LT-AMTMD and the damping ratio of each LT-ATMD in the LT-AMTMD can be, respectively, determined as [Li 2000, 2002]

$$\mu_T = \mu_{Tj}\left[r_{Tj}^2 \sum_{j=1}^{n} \frac{1}{r_{Tj}^2}\right] \qquad (j = 1, 2, 3,..., n) \tag{13}$$

$$\xi_{Tj} = \frac{r_{Tj}\xi_T}{f} \qquad (j = 1, 2, 3,..., n) \tag{14}$$

Defining the ratio of the external excitation frequency to the controlled frequency of the structure (i.e. $\lambda = \frac{\omega}{\omega_s}$) and taking advantage of Eqs. (7) and (8), the dynamic magnification factors of the structure with the LT-AMTMD and each LT-ATMD in the LT-AMTMD can then be respectively calculated by

$$DMF = \left|\omega_s^2[H_{y_s}(-i\lambda)]\right| = \frac{1}{\sqrt{[\mathrm{Re}(\lambda)]^2 + [\mathrm{Im}(\lambda)]^2}} \tag{15}$$

$$DMF_j = \left|\omega_s^2[H_{y_{Tj}}(-i\lambda)]\right| = \frac{(1 + \eta_j\mu_j)\lambda^2}{\sqrt{[\bar{\mathrm{R}}\mathrm{e}(\lambda)]^2 + [\bar{\mathrm{I}}\mathrm{m}(\lambda)]^2}\sqrt{[\mathrm{Re}(\lambda)]^2 + [\mathrm{Im}(\lambda)]^2}} \qquad (j = 1, 2, 3,..., n) \tag{16}$$

in which

$$\mathrm{Re}(\lambda) = 1 - (1 + \mu_T)\lambda^2 - \sum_{j=1}^{n} \frac{[\mu_{Tj}(1 + \eta_j\mu_j)\lambda^4]\bar{\mathrm{R}}\mathrm{e}(\lambda)}{[\bar{\mathrm{R}}\mathrm{e}(\lambda)]^2 + [\bar{\mathrm{I}}\mathrm{m}(\lambda)]^2},$$

$$\mathrm{Im}(\lambda) = -2\xi_s\lambda + \sum_{j=1}^{n} \frac{[\mu_{Tj}(1+\eta_j\mu_j)\lambda^4]\bar{\mathrm{I}}\mathrm{m}(\lambda)}{[\bar{\mathrm{R}}\mathrm{e}(\lambda)]^2 + [\bar{\mathrm{I}}\mathrm{m}(\lambda)]^2},$$

$$\bar{\mathrm{R}}\mathrm{e}(\lambda) = r_{Tj}^2 - [1+\eta_j\mu_j + r_j(1-\beta_j-\alpha_j\beta_j)]\lambda^2$$

$$\bar{\mathrm{I}}\mathrm{m}(\lambda) = -2\xi_{Tj}r_{Tj}\lambda$$

Now studies can be conducted on the performance (including the stroke displacement) of the LT-AMTMD through the implementation of the following two criteria

$$R_I = Min.Min.Max.DMF, \quad (17a)$$

$$R_{IIj} = Max.DMF_j \quad (j = 1, 2, 3,..., n) \quad (17b)$$

It is interesting to know that the LT-AMTMD model shown in Fig. 2 possesses the same transfer functions [i.e. Eqs. (7) and (8)] as that shown in Fig. 1. However, the damping ratio of each LT-ATMD in the LT-AMTMD shown in Fig. 2 takes the following form.

$$\xi_{Tj} = \frac{\varphi_j^2\alpha_j^2 c_{Tj} + c_{tj}(1-\beta_j-\alpha_j\beta_j)}{2m_{Tj}\omega_{Tj}} \quad (j = 1, 2, 3,..., n) \quad (18)$$

in which φ_j is an adjustable parameter varying from 0.0 to 1.0, but not including 0.0 and 1.0. Normally, this adjustable parameter is held constant (i.e., $\varphi_1 = \varphi_2 = \cdots = \varphi_n = \varphi$) for the convenience of the practical implementation of the LT-AMTMD.

It follows that the above two LT-AMTMD models possess the identical optimum frequency spacing (the robustness), optimum average damping ratio, optimum tuning frequency ratio, effectiveness, and stroke displacement, but unequal damping coefficient. Obviously, the LT-AMTMD shown in Fig. 1 needs lesser optimum damping coefficient.

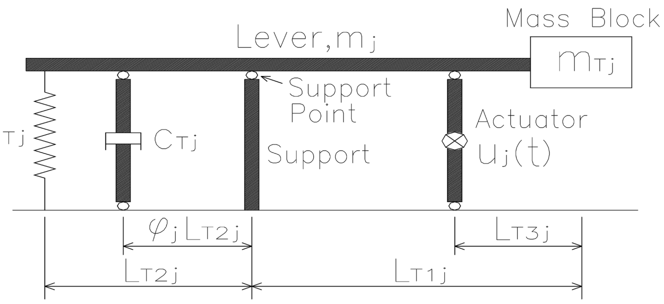

Fig. 2 Schematic diagram of the jth LT-ATMD in the LT-AMTMD with the damping set at the position between the spring and the support

4. Performance assessment of the LT-AMTMD

Displayed in Figs. 3-6 are the numerical results of the present research, in which the damping ratio of the structure is set equal to 0.02 and the ratio (λ) of the external excitation frequency to the structural controlled frequency is set within the range from 0.4 to 3.4. The superscript opt denotes the optimum values of the LT-AMTMD system parameters.

Fig. 3 [(a1) and (b1)] shows the variation of the optimum average damping ratio of the LT-

AMTMD (including the LT-MTMD and LT-ATMD) with respect to the total mass ratio with $n = 5$, $\alpha = 0.4$, $\mu = 0.01$, and various β values for two cases: (a1) NAFGF = -0.4 and (b1) NAFGF = -0.8. It is clearly shown in Fig. 3 [(a1) and (b1)] that increasing the total mass ratio increases the optimum average damping ratio of the LT-AMTMD (including that of the LT-MTMD and the optimum damping ratio of the LT-ATMD). It is interesting in observing Fig. 3 [(a1) and (b1)] to note that changing the location of the actuator (i.e. changing the value of β) makes little difference in the optimum average damping ratio of the LT-AMTMD. It is noted that the optimum average damping ratio of the LT-AMTMD is greater than that of the LT-MTMD, but significantly lower than the optimum damping ratio of the LT-ATMD. Also, it is noted that the NAFGF makes little difference in the optimum average damping ratio of the LT-AMTMD.

Fig. 3 [(a2) and (b2)] presents the variation of the optimum frequency spacing of the LT-AMTMD (including the LT-MTMD) with reference to the total mass ratio with $n = 5$, $\alpha = 0.4$, $\mu = 0.01$, and various β values for two cases: (a2) NAFGF=-0.4 and (b2) NAFGF = -0.8. Note that increasing the total mass ratio increases the optimum frequency spacing of the LT-AMTMD, which implies that the robustness of the LT-AMTMD is getting better. It is worth noting that the LT-AMTMD with the actuator set at the mass block offers higher robustness compared to the LT-AMTMD with the actuator set at other locations. Likewise, the LT-AMTMD with the actuator set at the mass block has higher robustness than the LT-MTMD. Meanwhile, it is also noted that increasing the absolute value of the NAFGF may further enhance the robustness of the LT-AMTMD.

Fig. 3 [(a3) and (b3)] presents the variation of the optimum tuning frequency ratio of the LT-AMTMD (including the LT-MTMD and LT-ATMD) with regard to the total mass ratio with n=5, α = 0.4, $\mu = 0.01$, and various β values for two cases: (a3) NAFGF=-0.4 and (b3) NAFGF = –0.8. It is interesting to see from Fig. 3 [(a3) and (b3)] that the optimum tuning frequency ratio of the LT-AMTMD (including the LT-MTMD and LT-ATMD) maintains constant with the increase of the total mass ratio. However, the influences of both the location of the actuator and the NAFGF are not insignificant on the optimum tuning frequency ratio of the LT-AMTMD. Fig. 3[(a3) and (b3)] also shows that the optimum tuning frequency ratio of the LT-AMTMD is surprisingly close to that of the LT-ATMD, but remarkably lower than that of the LT-MTMD. The latter above indicates that the LT-AMTMD needs significantly lesser stiffness of the spring with respect to the LT-MTMD.

Fig. 3 [(a4) and (b4)] shows the variation of the R_I value, used for measuring the effectiveness of the LT-AMTMD, with respect to the total mass ratio with $n = 5$, $\alpha = 0.4$, $\mu = 0.01$, and various β values for two cases: (a4) NAFGF = –0.4 and (b4) NAFGF = –0.8. Fig. 3 [(a4) and (b4)] clearly demonstrates that the influence of the location of the actuator become more important on the effectiveness of the LT-AMTMD at lesser NAFGF, such as –0.8, and the LT-AMTMD with the actuator set at the mass block renders higher effectiveness with reference to the LT-AMTMD with the actuator set at other locations. Likewise, the LT-AMTMD with the actuator set at the mass block can provide better effectiveness in comparison to the LT-MTMD and the LT-ATMD.

Fig. 4 shows the variation of the optimum parameters and effectiveness of the LT-AMTMD, respectively, with respect to the total mass ratio with $n = 5$ and $\mu = 0.01$ and the total number with $\mu_T = 0.01$ and $\mu = 0.01$. It is interesting in observing Fig. 4 to note that the location of the support makes little difference in the optimum parameters and effectiveness of the LT-AMTMD, corresponding to the respective NAFGF. This observation indicates that the static stretching of the spring in the LT-AMTMD may be freely adjusted in accordance with the practical requirements through changing the location of the support while practically maintaining the same optimum average damping ratio, optimum frequency spacing (the robustness), optimum tuning frequency

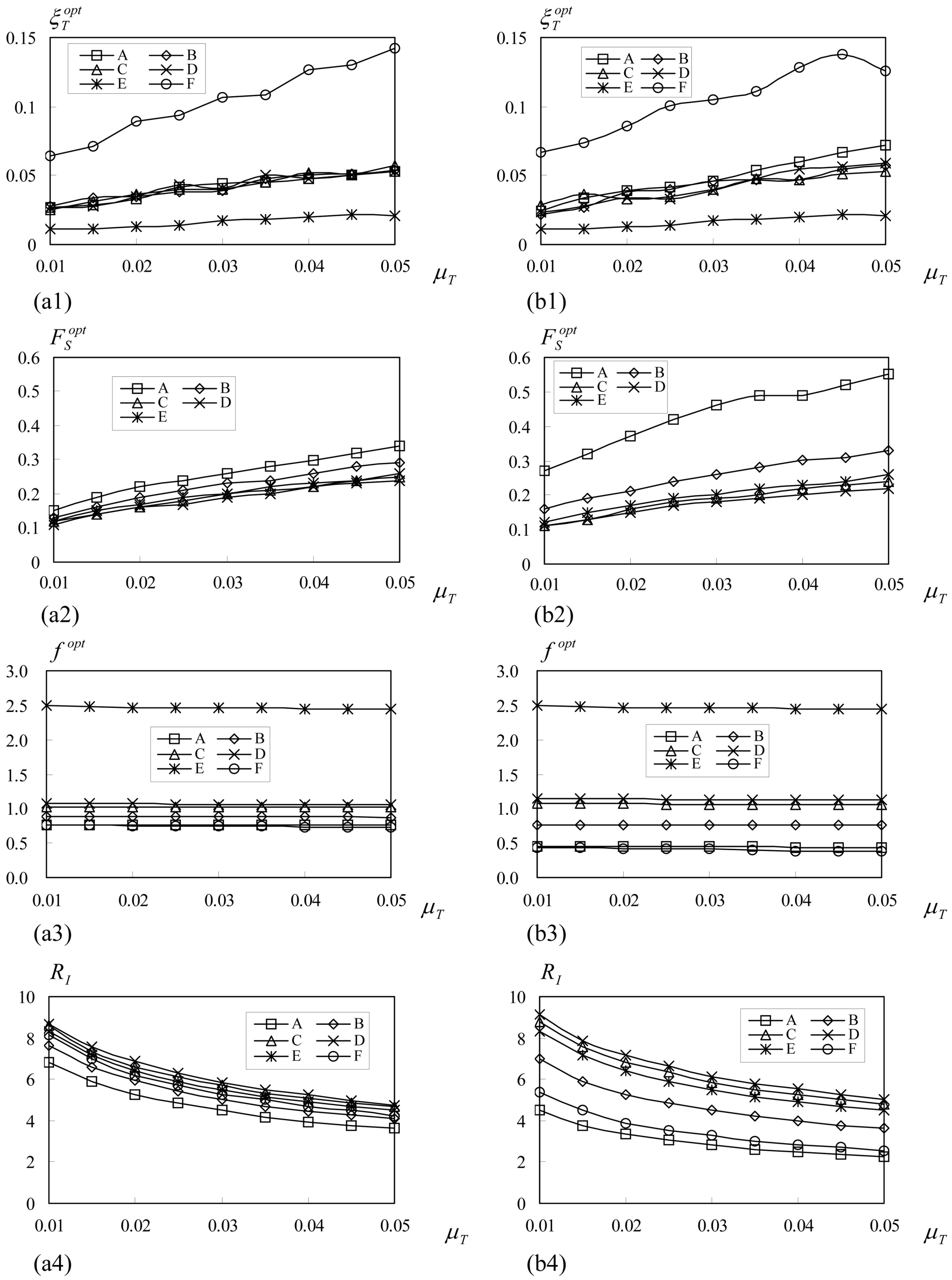

Fig. 3 Variation of the optimum parameters and effectiveness of the LT-AMTMD (including the LT-MTMD and LT-ATMD) [(a1): ξ_T^{opt} for r=−0.4; (b1): ξ_T^{opt} for r=−0.8; (a2):F_S^{opt} for r=−0.4; (b2):F_S^{opt} for r=−0.8; (a3): f^{opt} for r=−0.4; (b3):f^{opt} for r=−0.8; (a4):R_I for r=−0.4; (b4):R_I for r=−0.8] with respect to total mass ratio with n=5, α=0.4, μ=0.01, and various β values. [A=LT-AMTMD(β=0.0), B=LT-AMTMD(β=0.36), C=LT-AMTMD(β=0.86), D=LT-AMTMD(β=1.0), E=LT-MTMD, F=LT-ATMD (β=0.0)]

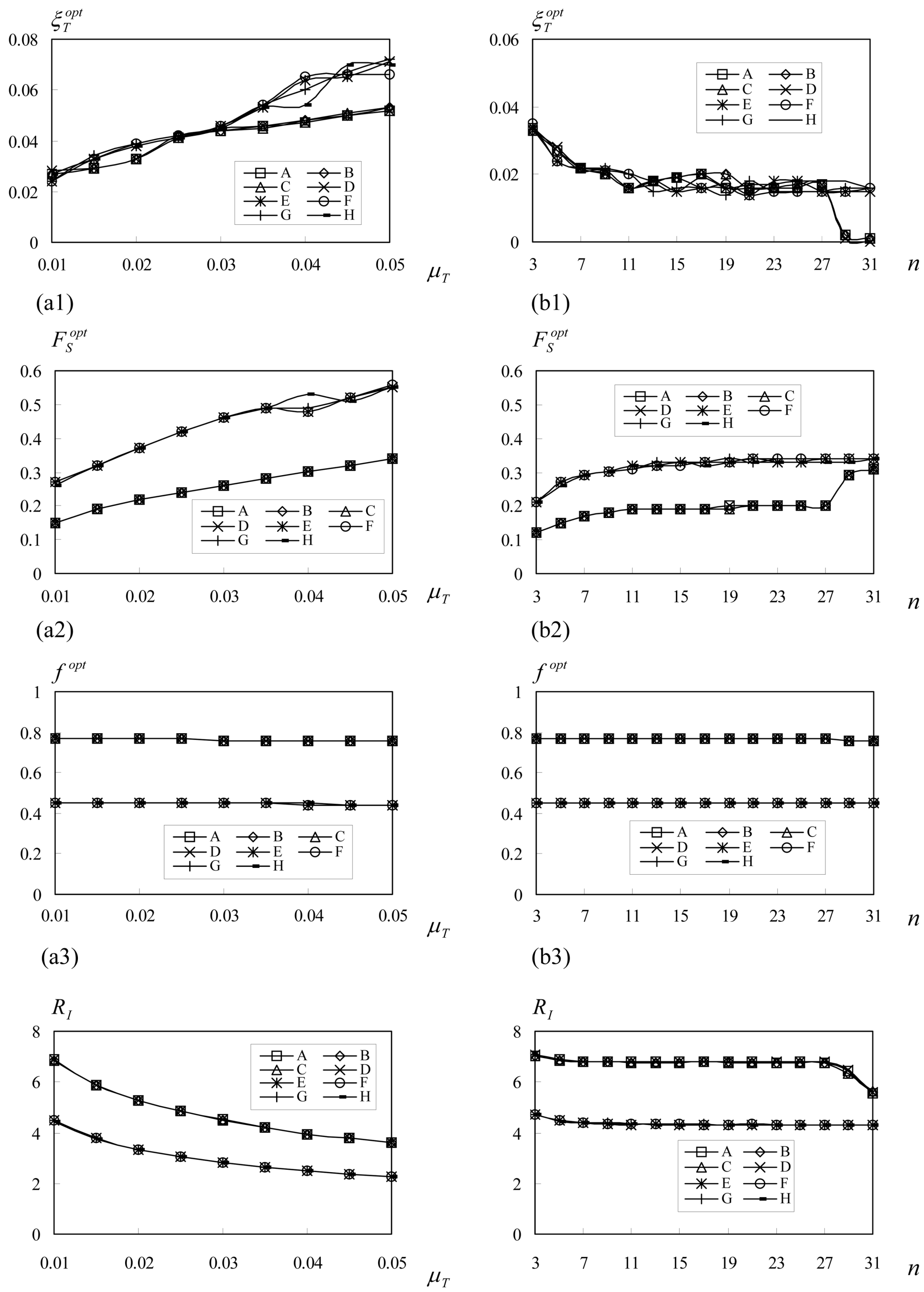

Fig. 4 Variation of the optimum parameters and effectiveness of the LT-AMTMD with respect to total mass ratio with n=5 and μ=0.01 [(a1): ξ_T^{opt}; (a2):F_S^{opt}; (a3):f^{opt}; (a4):R_I] and total number with μ_T=0.01 and μ=0.01 [(b1): ξ_T^{opt}; (b2):F_S^{opt}; (b3):f^{opt}; (b4):R_I] [A=LT-AMTMD(α=0.1, r=−0.4), B=LT-AMTMD (α=0.3, r=−0.4), C=LT-AMTMD(α=0.5, r=−0.4), D=LT-AMTMD(α=1.0, r=−0.4),E=LT-MTMD(α=0.1, r=−0.8), F=LT-AMTMD (α=0.3, r=−0.8), G=LT-AMTMD(α=0.5, r=−0.8), H=LT-AMTMD(α=1.0, r=−0.8)]

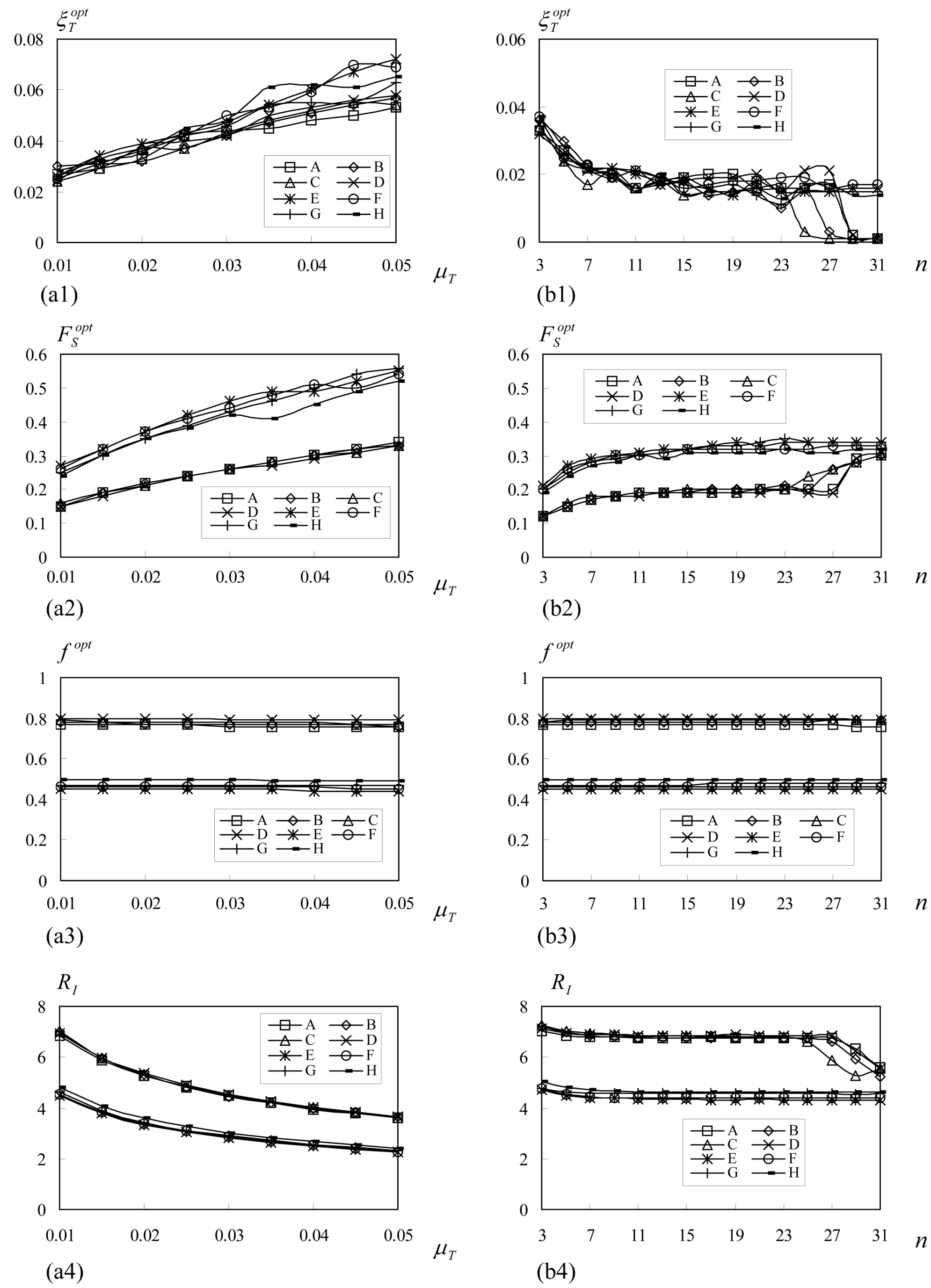

Fig. 5 Variation of the optimum parameters and effectiveness of the LT-AMTMD with respect to total mass ratio with n=5 and α=0.4 [(a1): ξ_T^{opt}; (a2): F_S^{opt}; (a3): f^{opt}; (a4): R_I] and total number with μ_T=0.01 and α=0.4 [(b1): ξ_T^{opt}; (b2): F_S^{opt}; (b3): f^{opt}; (b4): R_I] [A=LT-AMTMD(μ=0.01, r=−0.4), B=LT-AMTMD (μ=0.05, r=−0.4), C=LT-AMTMD(μ=0.1, r=−0.4), D=LT-AMTMD(μ=0.2, r=−0.4), E=LT-MTMD (μ=0.01, r=−0.8), F=LT-AMTMD (μ=0.05, r=−0.8), G=LT-AMTMD(μ=0.1, r=−0.8),H=LT-AMTMD (μ=0.2, r=−0.8)]

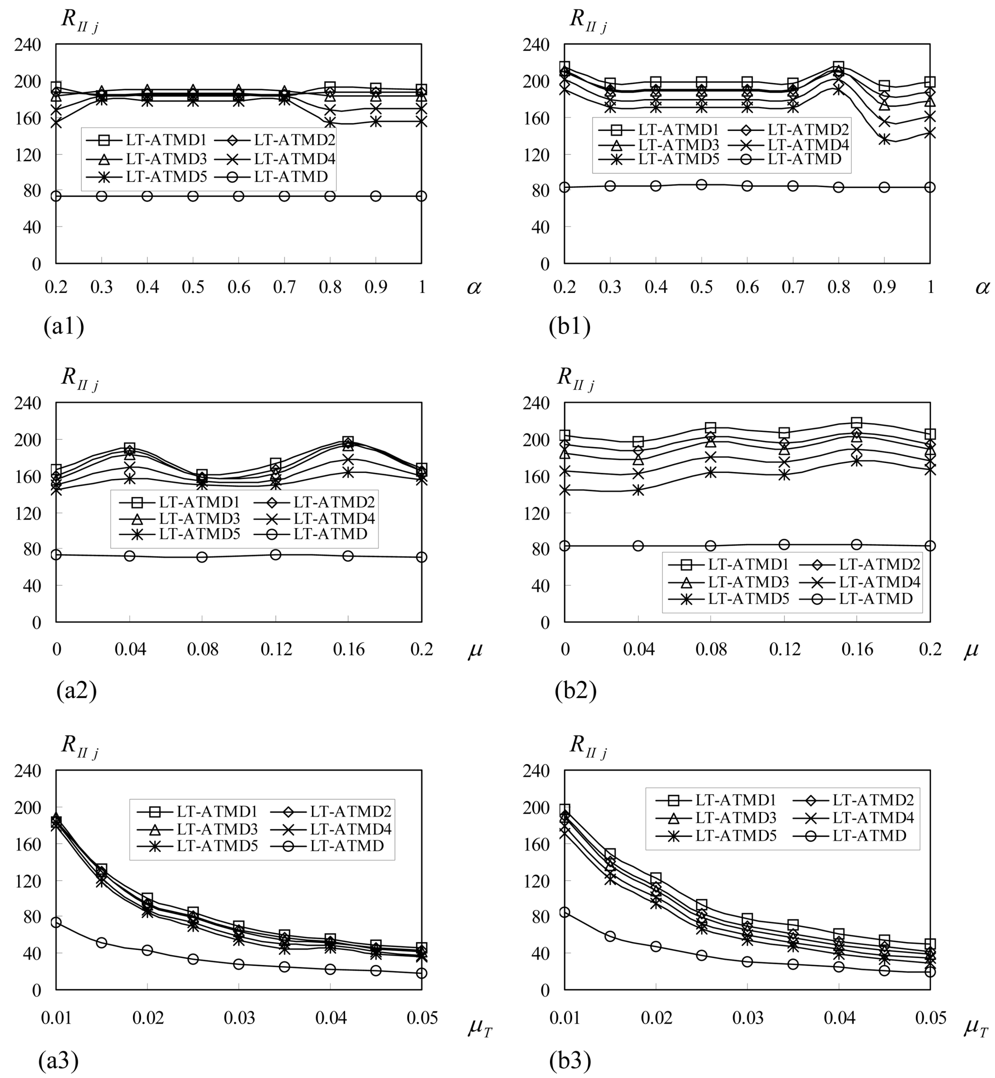

Fig. 6 Variation of the R_{IIj} value of the LT-AMTMD (including the LT-ATMD) with respect to the α value with n=5 and μ_T=0.01 and μ=0.03 [(a1): r=–0.4; (b1): r=–0.8] and the μ value with n=5 and u_T=0.01 and α=0.3 [(a2): r=–0.4; (b2): r=–0.8] and the u_T value with n=5 and α=0.3 and μ=0.03 [(a3): r=–0.4; (b3): r=–0.8]

ratio, and effectiveness. It is worth noting that in the situation where the NAFGF equals –0.4, the LT-AMTMD with the total number beyond 27 takes the near-zero optimum average damping ratio [see Fig. 4 (b1)]. The LT-AMTMD with the near-zero optimum average damping ratio is not meritorious for practical applications due to large stroke displacement, although significantly higher robustness and effectiveness (see Fig. 4 [(b3) and (b4)]).

Fig. 5 presents the variation of the optimum parameters and effectiveness of the LT-AMTMD, respectively, with respect to the total mass ratio with n = 5 and α = 0.4 and the total number with μ_T = 0.01 and α = 0.4. It is clear from Fig. 5 that the lever to block mass ratio makes little difference in the optimum parameters and effectiveness of the LT-AMTMD. This indicates the influences of the lever mass are rather negligible on the optimum parameter and effectiveness of the LT-AMTMD. However, it is noted that the lever to block mass ratio makes some difference in the near-zero optimum average damping ratio [see Fig. 5 (b1)].

Fig. 6 [(a1) and (b1)] presents the variation of the R_{IIj} value, used for measuring the stroke displacement of the LT-AMTMD (including the LT-ATMD), with respect to the α value with n=5, μ_T=0.01, and μ=0.03 for two cases: (a1) NAFGF=−0.4 and (b1) NAFGF=−0.8. Fig. 6 [(a1) and (b1)] clearly demonstrates that the influence of the α value within the range from 0.3 to 0.7 is rather negligible on the R_{IIj} value of the LT-AMTMD, but not insignificant out of the range. Thus, the static stretching of the spring in the LT-AMTMD may be freely adjusted in accordance with the practical requirements through changing the location of the support (i.e. changing the α value within the range from 0.3 to 0.7) while maintaining the same stroke displacement. This character is very useful for the implementation of the LT-AMTMD for long-span bridges.

Fig. 6 [(a2) and (b2)] presents the variation of the R_{IIj} value of the LT-AMTMD (including the LT-ATMD) with regard to the μ value with n = 5, μ_T = 0.01, and μ = 0.3 for two cases: (a2) NAFGF=-0.4 and (b2) NAFGF=−0.8. It is seen that the influence of the μ value is rather negligible on the R_{IIj} value of the LT-AMTMD at lesser NAFGF, such as -0.8, but not insignificant at higher NAFGF, such as −0.4. For better accuracy in the stroke displacement, the lever mass therefore, needs to be accounted for.

Fig. 6 [(a3) and (b3)] presents the variation of the R_{IIj} value of the LT-AMTMD with reference to the μ_T value with n = 5, α = 0.3, and α=0.03 for two cases: (a3) NAFGF=-0.4 and (b3) NAFGF=-0.8. It is seen that the R_{IIj} value of the LT-AMTMD decreases rapidly with the increase of the total mass ratio, which indicates that the stroke displacement of the LT-AMTMD is greatly reduced at higher total mass ratio. However, the gradient of stroke reduction becomes small in the case where the total mass ratio is beyond 0.03. It is important to emphasize that the NAFGF makes little difference in the stroke displacement of the LT-AMTMD.

It should be mentioned that the stroke displacement of the LT-AMTMD in terms of comparison is a little greater than that of the LT-MTMD (see Li and Li 2005), but significantly larger than that of the ATMD (see Fig. 6). Thus, with reference to the LT-ATMD, this is a disadvantage of the LT-AMTMD.

5. Conclusions

From the results presented, the following conclusions can be drawn:

(1) The LT-AMTMD with the actuator set at the mass block has been shown to be more effective in suppressing the vibrations of long-span bridges in comparison with the LT-AMTMD with the actuator set at other locations.

(2) The LT-AMTMD with the actuator set at the mass block can futher improve the performance of the LT-MTMD (i.e. further enhance the robustness and effectiveness of the LT-MTMD and greatly reduce the required stiffness of the spring in the LT-MTMD) and possesses higher effectiveness than the LT-ATMD. Likewise, increasing the absolute value of the NAFGF can further enhance the robustness and effectiveness of the LT-AMTMD.

(3) The static stretching of the spring in the LT-AMTMD with the actuator set at the mass block may be freely adjusted in accordance with the practical requirements through changing the location of the support within the viable range (i.e. changing the α value within the range from 0.3 to 0.7) while maintaining the same performance including the stroke displacement.

(4) For better accuracy in the stroke displacement, the lever mass needs to be accounted for in the designing the LT-AMTMD.

(5) The stroke displacement of the LT-AMTMD with the actuator set at the mass block is a little greater than that of the LT-MTMD, but significantly larger than that of the LT-ATMD. Thus, compared to the LT-ATMD, this is a disadvantage of the LT-AMTMD.

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