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Probability distribution and statistical moments of the maximum wind velocity

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Abstract. This paper formulates a probabilistic model which is able to represent the maximum instantaneous wind velocity. Unlike the classical methods, where the randomness is circumscribed within the mean maximum component, this model relies also on the randomness of the maximum value of the turbulent fluctuation. The application of the FOSM method furnishes the first and second statistical moments in closed form. The comparison between the results herein obtained and those supplied by classical methods points out the central role of the turbulence intensity. Its importance is exalted when extending the analysis from the wind velocity to the wind pressure.

Key words: FOSM (First-Order Second-Moments); maximum distribution; probability theory; turbulence intensity; velocity gust factor; wind actions; wind velocity.

1. Introduction

The wind engineering literature generally defines the design wind actions on structures by two alternative methods (Solari 1993). The first method associates the maximum value of the pressure with the maximum value of the velocity given as the product of the mean velocity value by the related gust factor. This factor is a function of the duration of the gust peak, which depends on the exposed surface according to suitable spatial-temporal equivalence criteria.

The second method expresses the maximum value of the pressure as the product of the mean pressure value by the related gust factor. This factor takes into account the non-contemporaneity of the maximum local pressures on the exposed surface, using the theory of the stochastic time dependent fields. Both the above methods circumscribe the randomness of the physical phenomenon within the maximum values of the mean velocity and the mean pressure. The respective gust factors synthesize the role of the maximum turbulent fluctuations, which are random variables, according to the well-known pseudo-deterministic principles formulated by Davenport (1961, 1964).

This paper evaluates the distribution of the maximum instantaneous wind velocity by a so called *maximum summation method* (Schettini 1996). Section 2 provides the formulation of the model by means of which the instantaneous wind velocity is schematized. On the basis of considerations regarding the physics of the aeolian phenomenon, the maximum instantaneous

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wind velocity is identified with the maximum fluctuation achieved concomitant with a suitable neighbourhood of the maximum mean velocity. This method, illustrated in Section 3, utilizes the results of numerical experiments presented in Section 4. The first and second statistical moments of the maximum instantaneous wind velocity are evaluated in Section 5 using the FOSM ("First-Order & Second-Moment") technique. Section 6 illustrates some applications comparing the results of the procedure herein proposed with those supplied by the classical methods. Section 7 discusses the conclusions and the perspectives of this work, above all the extension of this formulation from the wind velocity to the wind pressure (Schettini 1996, Schettini and Solari 1998).

All studies herein carried out refer to well-behaved climates and extra-tropical cyclones. Extensions to mixed populations comprehending different phenomena such as tropical cyclones, tornadoes, downbursts, frontal and thunderstorm winds involve advanced approaches (Gomes and Vickery 1977, 1978). Analogous developments are necessary to take climate changes into account (Kasperski 1998).

2. Wind velocity

Neglecting the explicit dependence on height above ground for simplicity of notation, let V be the instantaneous wind velocity.

The mean wind velocity, or macro-meteorological component of V (Van der Hoven 1957), is defined as :

$$V_o(t)=\frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} V(\xi)\,d\xi \tag{1}$$

where t is the time and ΔT is the time interval (ranging between 10 minutes and 1 hour) where V is averaged. It is assumed that V_o varies so slowly over the time as to be considered as constant in ΔT.

The atmospheric turbulence, or micro-meteorological component of V, is defined as :

$$V'(t)=V(t)-V_o(t) \tag{2}$$

Unlike V_o, V' varies rapidly over the time.

The standard deviation of turbulence is defined as :

$$\sigma_{V'}(t)=\sqrt{\frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} V'^2(\xi)\,d\xi} \tag{3}$$

Like V_o, also $\sigma_{V'}$ varies so slowly over the time as to be considered as constant in ΔT.

It is assumed that the ratio between the standard deviation of turbulence and the mean wind velocity, defined as turbulence intensity, is independent of time :

$$I_V=\frac{\sigma_{V'}(t)}{V_o(t)} \quad (V_o>0) \tag{4}$$

Finally, the following nondimensional quantity :

$$\widetilde{V}'(t) = \frac{V'(t)}{\sigma_{V'}(t)} \quad (\sigma_{V'} > 0) \tag{5}$$

is defined as the reduced turbulence. Like V', also $\widetilde{V}'$ varies rapidly over the time.

Utilizing Eqs. (1)~(5), the instantaneous wind velocity assumes the form :

$$V(t) = V_o(t)[1 + I_V \widetilde{V}'(t)] \tag{6}$$

where $V(t)$, $V_o(t)$, $V'(t)$, $\widetilde{V}'(t)$ and $\sigma_{V'}(t)$ can be treated as mono-dimensional stochastic stationary processes (Davenport 1967, Gomes and Vickery 1977).

2.1. Mean wind velocity distributions

Using the Weibull model (1951) corrected by the hybrid technique (Takle and Brown 1978, Conradsen *et al.* 1984, Solari 1996a), the probability density function (pdf) and the cumulative distribution function (cdf) of the mean wind velocity V_o are given by :

$$f_{V_o}(v_o) = F_o\,\delta(v_o) + (1 - F_o)\frac{k}{c}\left(\frac{v_o}{c}\right)^{k-1}\exp\left[-\left(\frac{v_o}{c}\right)^k\right] \tag{7}$$

$$F_{V_o}(v_o) = F_o + (1 - F_o)\left\{1 - \exp\left[-\left(\frac{v_o}{c}\right)^k\right]\right\} \tag{8}$$

where v_o is the state variable of V_o; F_o is the probability that $V_o = 0$; $\delta(\,)$ is Dirac's function; c and k are model parameters.

The distribution of the maximum value of the mean wind velocity V_{oM} in a generic time interval $T \gg \Delta T$ is generally obtained by the asymptotic analysis or by the process analysis (Lagomarsino *et al.* 1992, Solari 1996b).

Applying the asymptotic analysis, the pdf and the cdf of V_{oM} are expressed by the I type asymptotic distribution (Gumbel 1958) :

$$f_{V_{oM}}(v_{oM}) = a\exp\{-\exp[-a(v_{oM} - u)]\}\exp[-a(v_{oM} - u)] \tag{9}$$

$$F_{V_{oM}}(v_{oM}) = \exp\{-\exp[-a(v_{oM} - u)]\} \tag{10}$$

where v_{oM} is the state variable of V_{oM}; a, u are model parameters. The mean value and the standard deviation of V_{oM} are known in closed form (Benjamin and Cornell 1970) :

$$\mu_{V_{oM}} = u + \frac{0.5772}{a} \tag{11}$$

$$\sigma_{V_{oM}} = \frac{\pi}{6a} \tag{12}$$

The process analysis schematizes the mean wind velocity as a stochastic stationary process (Davenport 1967, Gomes and Vickery 1977). Assuming that the up-crossings of a sufficiently high velocity threshold are Poissonian events, the pdf and the cdf of V_{oM} are given by (Solari 1996b) :

$$f_{V_{oM}}(v_{oM}) = -\lambda_o \exp\{-\lambda_o f_{V_o}(v_{oM})\} \times$$
$$(1-F_o)\frac{k}{c}\exp\left[-\left(\frac{v_{oM}}{c}\right)^k\right]\left\{\left(-\frac{k}{c}\right)\left(\frac{v_{oM}}{c}\right)^{2k-2}+\frac{k-1}{c}\left(\frac{v_{oM}}{c}\right)^{k-2}\right\} \tag{13}$$

$$F_{V_{oM}}(v_{oM}) = \exp\{-\lambda_o f_{V_o}(v_{oM})T\} \tag{14}$$

where :

$$\lambda_o = \int_0^\infty \dot{v}_o f_{\dot{V}_o}(\dot{v}_o)d\dot{v}_o \tag{15}$$

$f_{\dot{V}_o}$ is the pdf of $\dot{V}_o$, $\dot{V}_o$ being the prime temporal derivative of V_o.

2.2. Reduced turbulence distributions

The reduced turbulence $\tilde{V}'(t)$ is a stochastic stationary Gaussian process with zero mean and unit variance.

Let $\tilde{V}'_m$ be the maximum of $\tilde{V}'$ in the time interval ΔT where $V_o = v_o$, v_o being a generic mean wind velocity. Applying the procedure formulated by Davenport (1964), the pdf and the cdf of $\tilde{V}'_m$ are given by :

$$f_{\tilde{V}'_m}(\tilde{v}'_m) = \nu_{\tilde{V}'}\Delta T\, \tilde{v}'_m \exp\left\{-\frac{\tilde{v}'^2_m}{2}\right\}\exp\left\{-\nu_{\tilde{V}'}\Delta T \exp\left[-\frac{\tilde{v}'^2_m}{2}\right]\right\} \tag{16}$$

$$F_{\tilde{V}'_m}(\tilde{v}'_m) = \exp\left\{-\nu_{\tilde{V}'}\Delta T \exp\left[-\frac{\tilde{v}'^2_m}{2}\right]\right\} \tag{17}$$

where $\tilde{v}'_m$ is the state variable of $\tilde{V}'_m$; $\nu_{\tilde{V}'}$ is the expected frequency of $\tilde{V}'$:

$$\nu_{\tilde{V}'} = \frac{1}{2\pi}\frac{\sigma_{\dot{V}'}}{\sigma_{V'}} \tag{18}$$

$\sigma_{\dot{V}'}$is the standard derivation of $\dot{V}'$, $\dot{V}'$ being the prime temporal derivative of V'. The mean value and the standard deviation of $\tilde{V}'_m$ depend on $\nu_{\tilde{V}'}$ through the expressions (Davenport 1964) :

$$\mu_{\tilde{V}'_m} = \sqrt{2\ln[\Delta T\, \nu_{\tilde{V}'}]} + \frac{0.5772}{\sqrt{2\ln[\Delta T\, \nu_{\tilde{V}'}]}} \tag{19}$$

$$\sigma_{\tilde{V}'_m} = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2 \ln [\Delta T \, \nu_{\tilde{V}'}]}} \tag{20}$$

Applying the closed form solution developed by Solari (1993) :

$$\sigma_{V'} = v_o I_V \frac{1}{\sqrt{1+0.56\left(\frac{\tau v_o}{L_V}\right)^{0.74}}} \tag{21}$$

$$\sigma_{\dot{V}'} = \frac{v_o^2 I_V}{L_V} \frac{1.124}{\sqrt{\left[1+0.56\left(\frac{\tau v_o}{L_V}\right)^{0.74}\right]\left(\frac{\tau v_o}{L_V}\right)^{1.44}}} \tag{22}$$

where L_V is the integral length scale of turbulence; τ is the duration of the gust peak.

Substituting Eqs. (21), (22) into Eq. (18), it follows :

$$\nu_{\tilde{V}'} = \frac{0.178 \; v_o^{0.28}}{\tau^{0.72} \; L_V^{0.28}} \tag{23}$$

which points out the dependence of $\nu_{\tilde{V}'}$, and therefore of the distribution of $\tilde{V}'_m$ (Eqs. (16), (17)), on the mean wind velocity v_o.

3. Maximum summation method

The maximum summation method (Schettini 1996) is based on the physical property that the turbulent fluctuations are large where the mean wind velocity is large. The method assumes that the maximum instantaneous wind velocity V_M during the time interval T occurs concomitant with the maximum atmospheric turbulence in the time interval $\Delta T = \Delta T_{oMR}$ where $V_o = V_{oMR}$, V_{oMR} being the reduced maximum mean wind velocity (Fig. 1). It follows that :

$$V_M = V_{oMR}\,(1 + I_V \tilde{V}'_{omR}) \tag{24}$$

$$V_{oMR} = B \; V_{oM} \tag{25}$$

in which $\tilde{V}'_{omR}$ is the maximum value of $\tilde{V}'$ in ΔT_{oMR}, called the maximum reduced fluctuation. V_{oM} is the maximum mean velocity during T. $B \in [0, 1]$ is a stochastic variable referred to as the noncontemporaneity factor; its distribution and statistical moments are defined in the following section.

Assuming V_{oMR} and $\tilde{V}'_{omR}$ as statistically independent, the cdf of V_M is given by :

$$F_{V_M}(v_M) = \int_0^{v_M} f_{V_{oMR}}(v_{oMR}) F_{\tilde{V}'_{omR}}\left(\frac{v_M}{I_v \, v_{oMR}} - \frac{1}{I_v}\right) dv_{oMR} \tag{26}$$

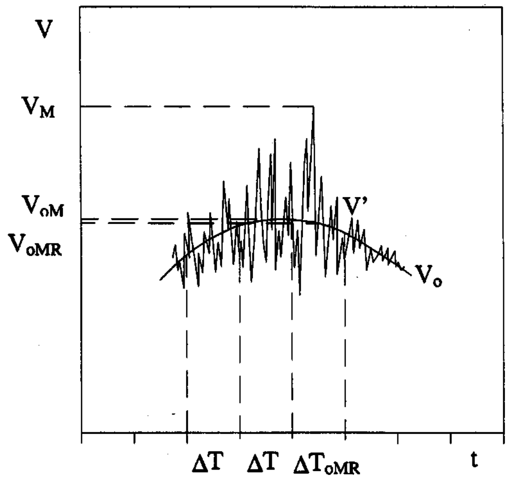


Fig. 1 Maximum instantaneous velocity

where v_M and v_{oMR} are the state variables of V_M and V_{oMR}; $f_{V_{oMR}}$ is the pdf of V_{oMR}; $F_{\widetilde{V}'_{omR}}$ is the cdf of $\widetilde{V}'_{omR}$ The latter is given by Eqs. (17) and (23) assuming $v_o = v_{oMR}$.

Considering V_{oM} and B as statistically independent, $f_{V_{oMR}}$ is provided by :

$$f_{V_{oMR}}(v_{oMR}) = \int_0^1 f_B(b)\, f_{V_{oM}}\left(\frac{v_{oMR}}{b}\right)\frac{1}{b}\, db \qquad (27)$$

where b is the state variable of B; f_B and $f_{V_{oM}}$ are the pdf of B and V_{oM}.

Defining :

$$f_B(b) = \delta(b-1) \qquad (28)$$

is equivalent to assume that the maximum instantaneous wind velocity V_M occurs concomitant with the maximum atmospheric turbulence in the time interval where the maximum mean wind velocity V_{oM} is achieved. Eq. (28) obviously provides an upper limit of the maximum real value.

Furthermore, disregarding the randomness of the maximum turbulence means postulating :

$$F_{\widetilde{V}'_{om}}(\widetilde{v}'_{om}) = H(\widetilde{v}'_{om} - \mu_{\widetilde{V}'_{om}}) \qquad (29)$$

where $\widetilde{V}'_{om}$ is the maximum reduced fluctuation in the time interval ΔT_{oM} where $V_o = V_{oM}$; $F_{\widetilde{V}'_{om}}$ and $\widetilde{v}'_{om}$ are the cdf and the state variable of $\widetilde{V}'_{om}$; $H(\ )$ is Heaviside function; $\mu_{\widetilde{V}'_{om}}$ is the mean value of $\widetilde{V}'_{om}$ and represents a peak factor (Davenport 1964). It is given by Eqs. (19), (23) assigning $v_o = v_{oM}$. The substitution of Eqs. (28), (29) into Eqs. (26), (27) leads to the following relationship :

$$F_{V_M}(v_M) = F_{V_{oM}}\left(\frac{v_M}{G_V}\right) \qquad (30)$$

which, consistent with classical methods, implies :

$$V_M = V_{oM}\, G_V \tag{31}$$

$$G_V = 1 + \mu_{\tilde{V}'_{om}}\, I_V \tag{32}$$

where G_V is referred to as the velocity gust factor (Solari 1993).

4. Noncontemporaneity factor

The distribution of noncontemporaneity factor B is evaluated by numerical experiments. Analyses are performed based on mean wind velocity measurements (over $\Delta T = 10$ minutes) carried out at five Italian meteorological stations (Ballio *et al.* 1997).

Table 1 lists the stations examined showing, for each one, the time interval t_1-t_2 of the available data, the latitude Φ, the longitude Θ, the height a_s above sea level, the characteristics of the site (H = hilly terrain, C = coast, P = plain), the height h above ground of the anemometer, the acquisition system. Data measured every 3 hours have been suitably interpolated with the aim of realizing a base of continuous measurements extending over several years.

In order to ensure a homogeneous treatment, measured velocities have been transformed into reference velocities corresponding to various conventional sites characterized by different values of the roughness coefficient z_o and of the turbulence intensity I_V.

For each reference site and for each transformed v_o value, assumed as constant in ΔT, the maximum instantaneous wind velocity in ΔT has been determined by generating an artificial occurrence of the maximum fluctuation using the Monte Carlo technique.

The maximum mean velocity v_{oM} and the maximum instantaneous velocity v_M have been initially obtained for data blocks corresponding to T=1 year. The maximum reduced mean velocity v_{oMR} is the value of v_o in correspondence of which v_M occurs; $b = v_{oMR} / v_{oM}$ is the occurrence of B. In order to obtain enough occurrences of B to derive its distribution, this criterion has been applied 1000 times per each year of available data at the stations and reference sites examined.

Fig. 2 shows some typical histograms of noncontemporaneity factors referred to single stations and single years. It is apparent that B assumes values averagely close to 1. Furthermore, its statistical moments do not seem to depend on the position of the station considered and on the year examined (Schettini 1996).

Fig. 3 collects all the results of the numerical experiments showing the mean value and the variance of B as functions of the turbulence intensity I_V for T=1 year. The expressions :

$$\mu_B = 1 - 1.82 I_V^2 + 6.12 I_V^3 - 6.44 I_V^4 \tag{33}$$

$$\sigma_B^2 = 0.08 I_V^2 - 0.23 I_V^3 + 0.19 I_V^4 \tag{34}$$

Table 1 Characteristics of the meteorological stations

stations	t_1-t_2	Φ	Θ	a_s [m]	h [m]	site	acquisition
Campomarino	1985~1992	41° 57'	15° 01'	15	90	P	continuous
Catania	1951~1990	37° 28'	15° 03'	15	10.5	P	every 3 hours
Fiume Santo	1981~1985	40° 50'	8° 17'	15	40	P	continuous
Macerata	1984~1989	43° 18'	13° 25'	15	300	H	continuous
Santa Caterina	1981~1988	39° 05'	8° 29'	15	1	CP	continuous

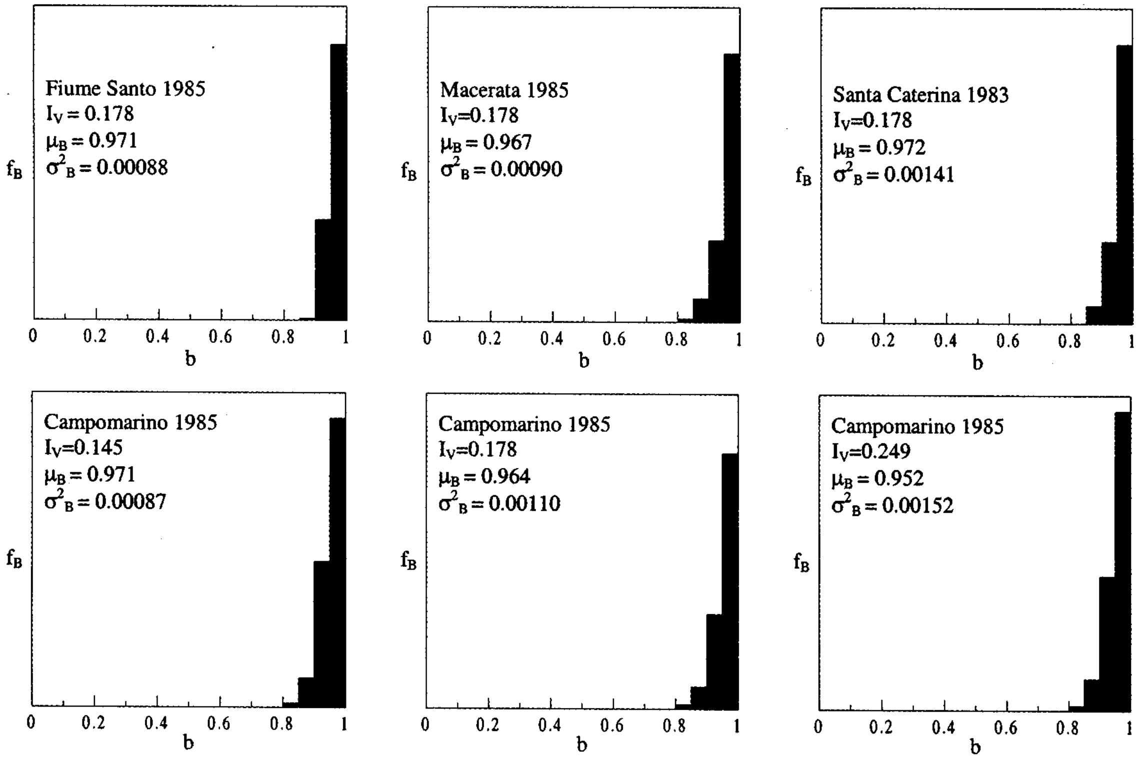


Fig. 2 Histograms of B

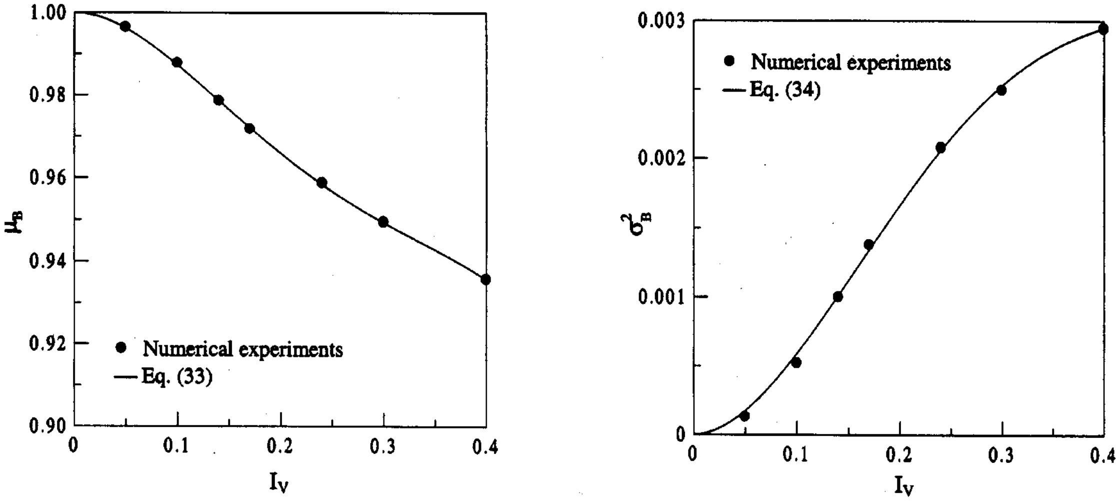


Fig. 3 Mean values and variances of B as functions of I_V ($T = 1$ year)

constitute adequate representations of the data.

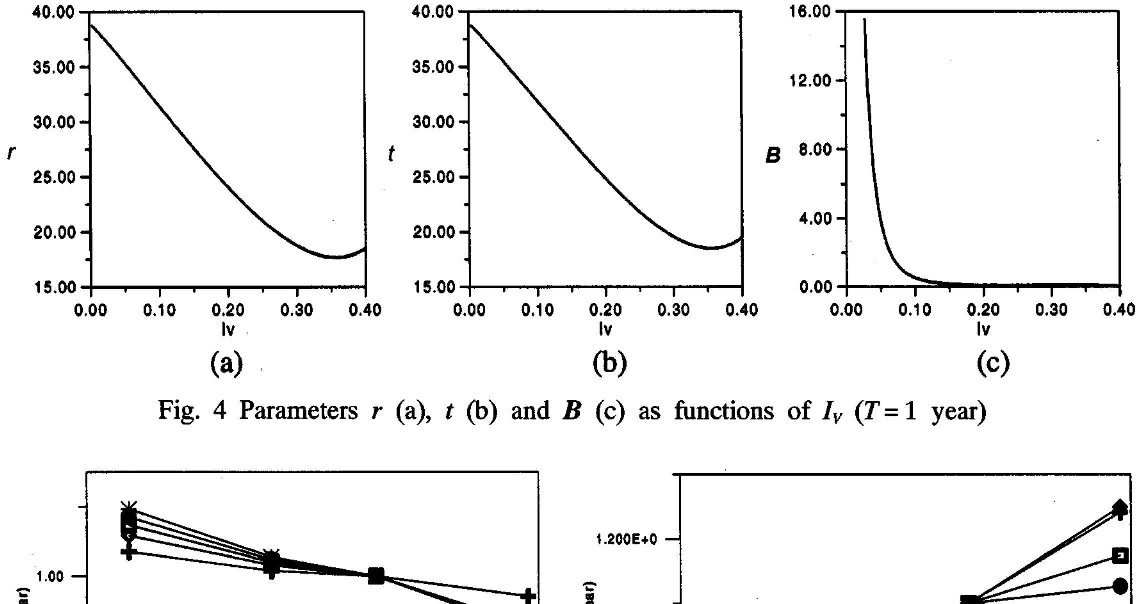
In accordance with these graphs, when I_V approaches zero the maximum instantaneous velocit V_M coincides with the maximum mean velocity V_{oM}; in this case the mean of B approaches on and the standard deviation approaches zero. When, on the other hand, I_V increases, so does the probability that the maximum instantaneous velocity does not occur concomitant with the maximum mean; this is confirmed by the fact that the standard deviation of B increases while, more significantly, its mean decreases.

Variable B is modelled by a beta distribution whose pdf is given by :

$$f_B(b) = \frac{1}{\boldsymbol{B}} b^{r-1}(1-b)^{t-r-1} \tag{35}$$

$$\boldsymbol{B} = \frac{\Gamma(r)\Gamma(t-r)}{\Gamma(t)} \tag{36}$$

where $\Gamma()$ is gamma function; r and t are model parameters linked with the mean value μ_B and with the variance $\sigma_B^{\,2}$ of B through the relationships :



(a) (b) (c)

Fig. 4 Parameters r (a), t (b) and $\boldsymbol{B}$ (c) as functions of I_V ($T = 1$ year)

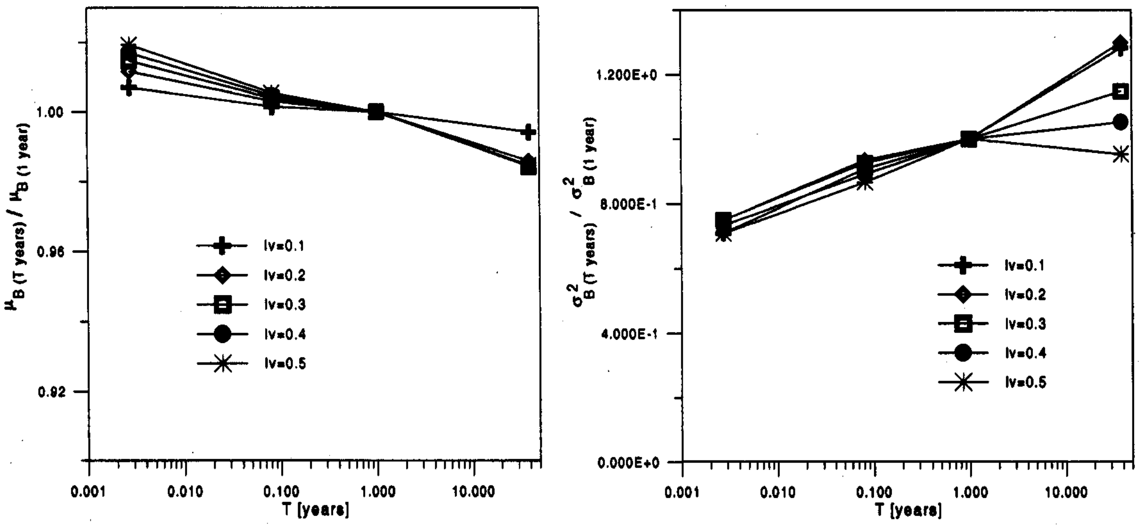


Fig. 5 Mean values and variances of the noncontemporaneity factor as functions of T

$$t = \frac{\mu_B\,(1-\mu_B\,)}{\sigma_B^2} - 1 \quad (\sigma_B \neq 0) \tag{37}$$

$$r = \frac{\mu_B^2\,(1-\mu_B\,)}{\sigma_B^2} - \mu_B \quad (\sigma_B \neq 0) \tag{38}$$

Fig. 4 shows the diagrams of r, t and $\boldsymbol{B}$ corresponding to different turbulence intensities.

Fig. 5 illustrates the results of some preliminary analyses (based on data referred to Catania) aimed at obtaining the dependence of μ_B and $\sigma_B{}^2$ on the period T over which the maximum is calculated. For T tending to the lower limit ΔT (outside the diagram) the maximum instantaneous velocity tends to become concomitant with the maximum mean velocity; as a consequence μ_B tends to one and $\sigma_B{}^2$ tends to zero. For T greater than one year, μ_B decreases and $\sigma_B{}^2$ increases; in this case the graphs seem to become relatively uncertain, above all due to the limited quantity of the available data.

5. Statistical moments of the maximum

The maximum summation method has the advantage that the maximum instantaneous wind velocity may be analytically expressed as a function of random variables whose distributions and statistical moments are known. By virtue of this property, the first and the second statistical moments of V_M may be conveniently evaluated by the FOSM (First-Order Second-Moment) method (Ditlevsen 1981, Kareem 1987, Solari 1996c, 1997).

On the basis of Eqs. (24) and (25) the maximum instantaneous wind velocity is a function of the three random variables V_{oM}, B and $\tilde{V}'_{omR}$. As such, it can be expressed in the form :

$$V_M = \boldsymbol{F}\,(X_1, X_2, X_3) \tag{39}$$

where $X_1 = V_{oM}$, $X_2 = B$ and $X_3 = \tilde{V}'_{omR}$.

Expanding Eq. (39) in Taylor series around the mean values μ_{Xi} of the stochastic variables X_i (i=1,2,3) and retaining up to the first order derivative terms, it follows :

$$V_M = \boldsymbol{F}\,(\mu_{X_1}, \mu_{X_2}, \mu_{X_3}) + \sum_{i=1}^{3} (X_i - \mu_{X_i}\,) \left.\frac{\partial \boldsymbol{F}\,(X_1, X_2, X_3)}{\partial X_i}\right|^{o} \tag{40}$$

in which $\partial F() / \partial X_i |^o$ is the partial derivative of $\boldsymbol{F}$ with respect to X_i, calculated in $X_i = \mu_{Xi}$. Applying the mean and variance operators to Eq. (40) and assuming the variables X_i as statistically independent, the following relationships are obtained :

$$\mu_{VM} = \mu_{V_{oM}}\, \mu_B\, (1 + I_V\, \mu_{\tilde{V}'_{omR}}\,) \tag{41}$$

$$\sigma_{V_M}^2 = \mu_{V_M}^2 \left[\frac{1}{\mu_{V_{oM}}^2}\, \sigma_{V_{oM}}^2 + \frac{1}{\mu_B^2}\, \sigma_B^2 + \frac{I_V^2}{(1 + I_V\, \mu_{\tilde{V}'_{omR}}\,)^2}\, \sigma_{\tilde{V}'_{omR}}^2 \right] \tag{42}$$

where $\mu_{\tilde{V}'_{omR}}$ and $\sigma^2_{\tilde{V}'_{omR}}$ are the mean value and the variance of $\tilde{V}'_{omR}$.

As confirmed by the examples reported in next section, the precision of Eqs. (41) and (42) is usually so high as not to require the use of second order approaches (Solari 1997).

Furthermore, the possibility of utilizing known values and closed formulae of the quantities in Eqs. (41), (42) makes their application very simple and meaningful. As a rule, $\mu_{V_{oM}}$ and $\sigma^2_{V_{oM}}$ constitute *a* data of the problem; μ_B and σ^2_B are known as functions of I_V (Eqs. (33), (34)) and T (Fig. 5); $\mu_{\tilde{V}'_{omR}}$ and $\sigma^2_{\tilde{V}'_{omR}}$ are supplied by Eqs. (19), (20) and (23) assigning $v_o = \mu_{V_{oM}}\mu_B$.

Finally, imposing $B=1$, i.e., $\mu_B=1$ and $\sigma^2_B=0$ and neglecting the randomness of the maximum value of the turbulence, i.e., $\sigma^2_{\tilde{V}'_{om}}=0$, the classical formulae are obtained :

$$\mu_{V_M} = \mu_{V_{oM}} G_V \tag{43}$$

$$\sigma_{V_M} = \sigma_{V_{oM}} G_V \tag{44}$$

in which G_V is the velocity gust factor (Eq. (32)).

The comparison between Eqs. (43), (44) and Eqs. (41), (42) highlights the evolution of the method proposed with respect to the classical theory.

6. Applications and comparisons

The application and effectiveness of the above procedure are illustrated by using, as an example, the wind data registered at the meteorological station of Santa Caterina. Measured mean wind velocities are transformed into homogeneous velocities associated to conventional sites with height $z=30$ m and roughness coefficients $z_o=0.003$, 0.1 and 3 m, corresponding to turbulence intensities $I_V=0.133$, 0.178 and 0.327 (Solari 1993). Analyses are carried out assuming $T=1$ year.

The distribution of the yearly maximum mean wind velocity is evaluated by the asymptotic analysis (A) and by the process analysis (P). The parameters of the two distributions are summarized in Table 2. Table 3 provides the parameters of the noncontemporaneity factor. Table 4 lists the main parameters of the atmospheric turbulence (Solari 1993).

The study is carried out by calculating the distributions of the maximum instantaneous wind velocity with the aim of comparing the results provided by classical analyses reported in the literature (L) with those obtained applying the maximum summation method (MSM) and its simplification based on B=1 (MB1).

Table 2 Parameters of the mean wind velocity

z_o [m]	0.003	0.1	3
a [s/m]	0.369	0.466	0.908
u [m/s]	31.03	24.61	12.63
μ_{VoM} [m/s] (A)	32.59	25.85	13.27
σ_{VoM} [m/s] (A)	2.93	2.33	1.19
c [m/s]	6.622	5.253	2.696
k	1.445	1.445	1.445
F_o	0.0045	0.0045	0.0045
λ_o [m/s^2]	8.04×10^{-4}	6.38×10^{-4}	3.27×10^{-4}
μ_{VoM} [m/s] (P)	32.29	25.61	13.14
σ_{VoM} [m/s] (P)	2.91	2.31	1.17

Table 3 Parameters of the noncontemporaneity factor

I_V	0.133	0.178	0.327
μ_B	0.9802	0.9703	0.9457
σ^2_B	0.0009	0.0014	0.0026
t	19.80	19.11	18.11
r	19.41	18.55	17.13
B	0.711	0.302	0.061

Table 4 Parameters of the atmospheric turbulence

z_o [m]	0.003	0.1	3
I_V	0.133	0.178	0.327
L_V [m]	279	154	86
τ[s]	1	1	1
$\mu_{\tilde{V}_{om\,R}}$ (A)	2.85	2.88	2.87
$\sigma_{\tilde{V}_{om\,R}}$ (A)	0.450	0.444	0.445
$\mu_{\tilde{V}_{om\,R}}$ (P)	2.84	2.87	2.86
$\sigma_{\tilde{V}_{om\,R}}$ (P)	0.445	0.440	0.441

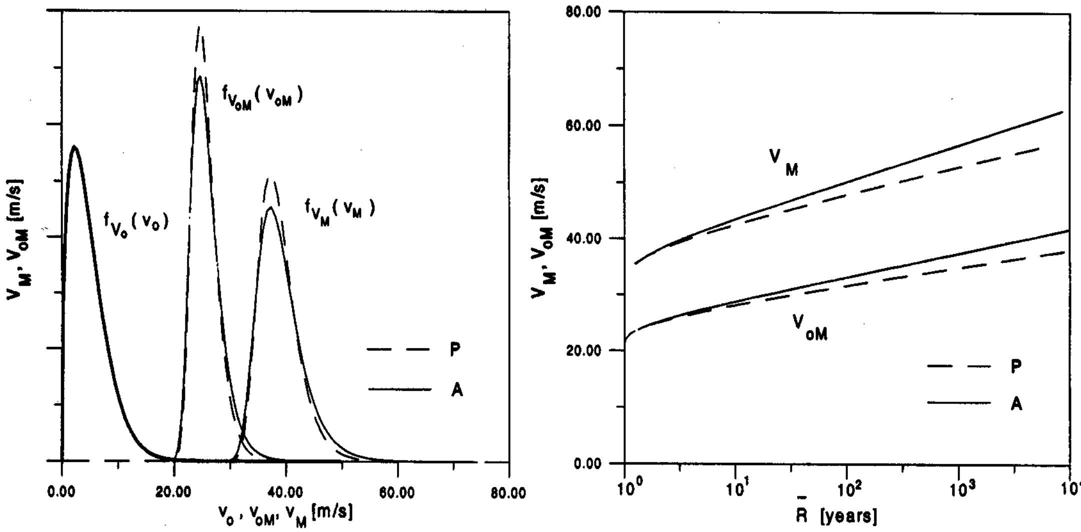


Fig. 6 Distributions of V_o, V_{oM} and V_M for I_V=0.178 using classical method (L)

Applying the classical methods for $I_V = 0.178$, Fig. 6 shows the pdf of V_o, V_{oM} and V_M. It also furnishes V_{oM} and V_M as functions of the mean return period $\bar{R}$. Since the parameter k of the population distribution (Eq. (7)) is greater than one, asymptotic analysis turns out to be conservative compared to process analysis (Lagomarsino *et al.* 1992).

Fig. 7 compares the distributions of the maximum calculated with the classical methods and those proposed in this paper for $I_V = 0.178$. Taking the randomness of the maximum turbulence into account, the step from L to MB1 involves an increase in the dispersion. Taking the randomness of B into account, the dispersion increases further moving from MB1 to MSM;

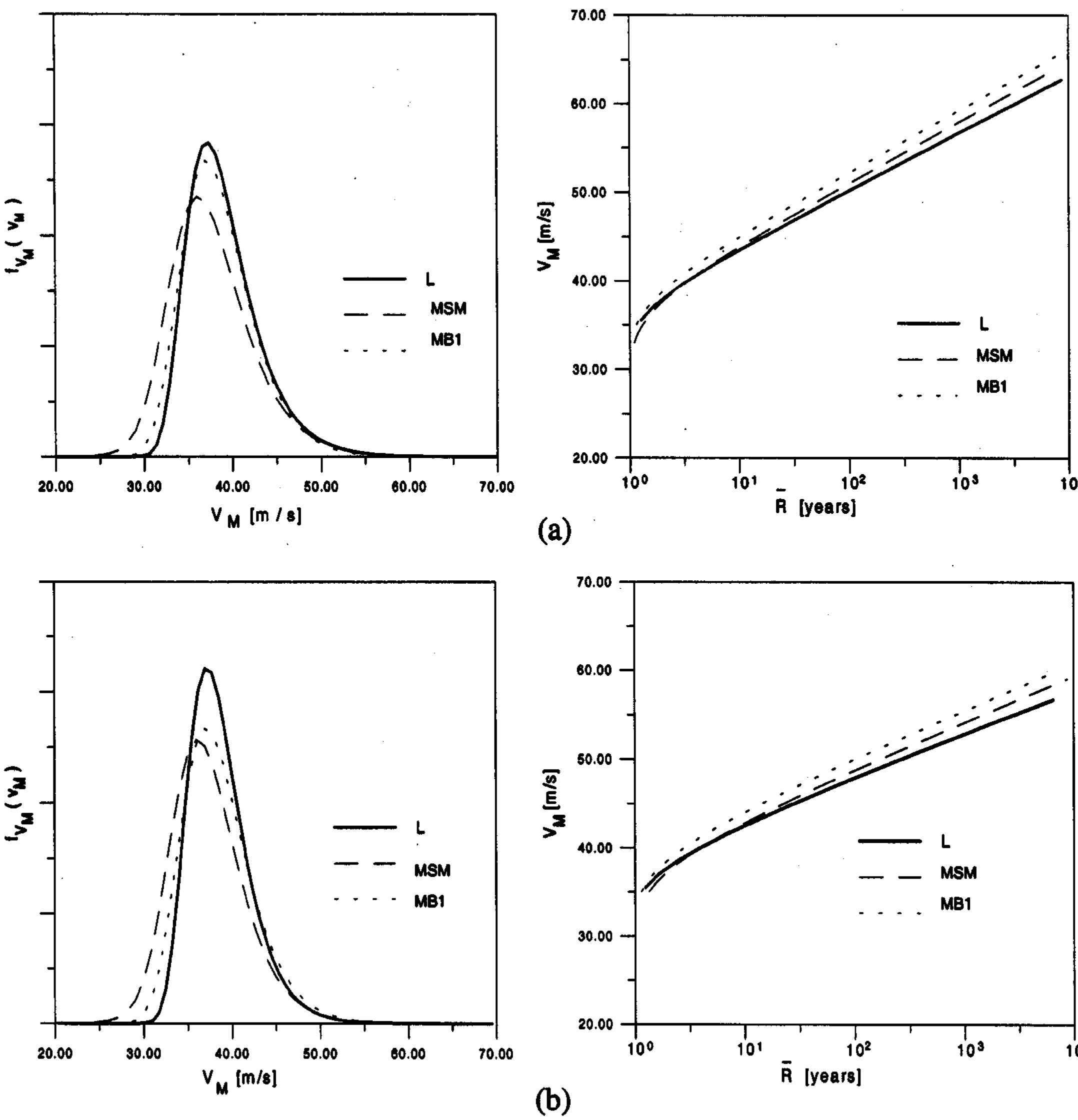


Fig. 7 Distributions of V_M for $I_V = 0.178$: (a) asympotic analysis (A); (b) process analysis (P)

furthermore, since $\mu_B < 1$, mean values associated with MSM are less than those obtained by MB1.

With reference to the process analysis only, Fig. 8 shows the distribution of V_M on varying the turbulence intensity I_V. The results demonstrate that classical methods decrease in accuracy as atmospheric turbulence grows in intensity. For example, in correspondence with $V_M = 35$ m/s and I_V=0.327 the classical methods estimate $\bar{R}$~1000 years, while the model proposed yields $\bar{R}$~300 years.

Table 5 compares the first and second statistical moments of V_M rigorously deduced from the distributions and obtained applying the FOSM technique. The precision associated with the use of FOSM is apparent. Classical methods yield adequate approximations of μ_{V_M} while underestimate σ_{V_M} to a degree which increases on increasing the turbulence intensity.

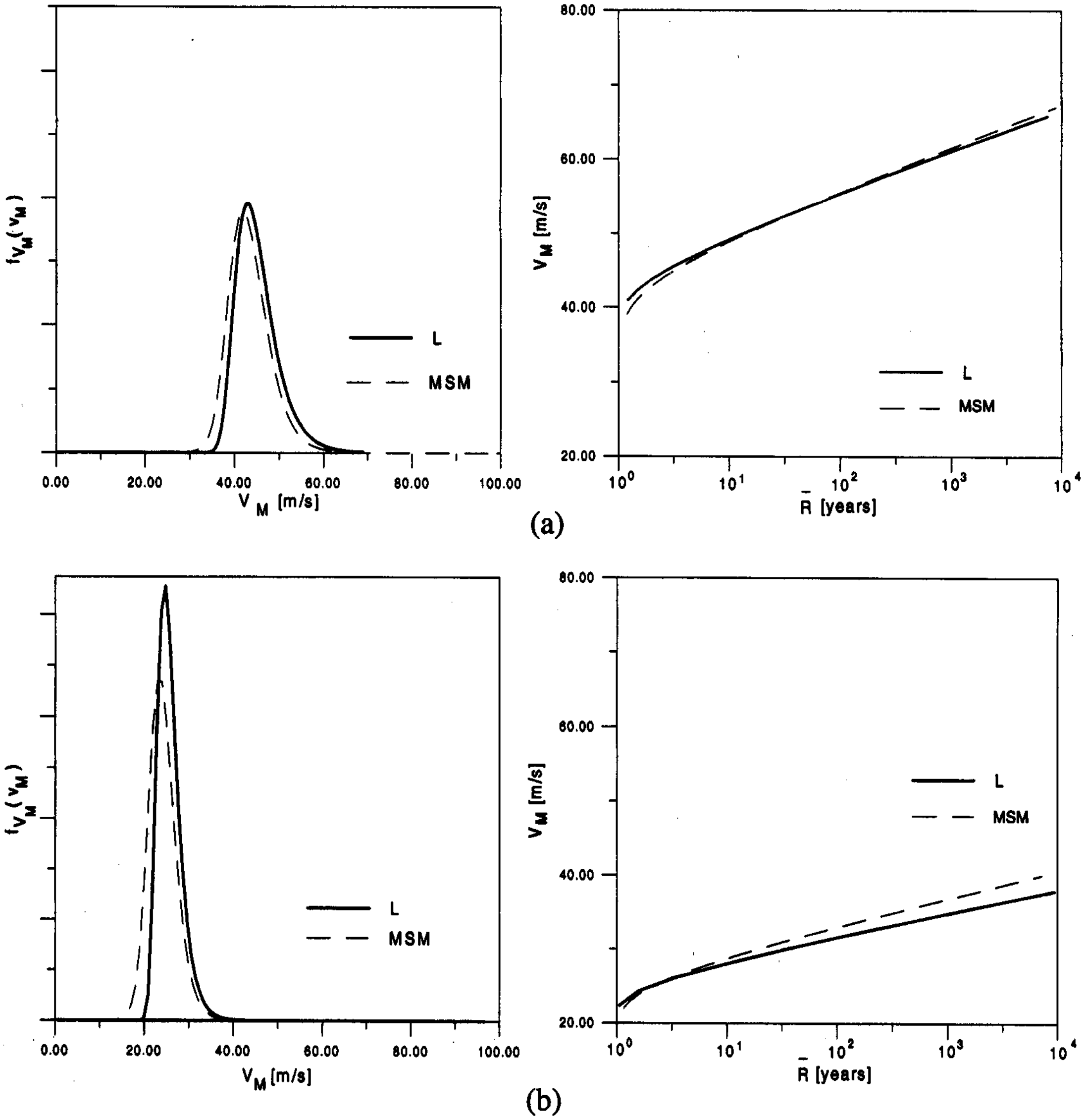


Fig. 8 Distributions of V_M using process analysis: (a) $I_V = 0.133$; (b) $I_V = 0.327$

7. Conclusions

This paper formulates a probabilistic model to represent the maximum value of the instantaneous wind velocity.

Unlike the classical methods, where the randomness of wind velocity is circumscribed within the mean maximum component, this model also takes into account the randomness of the fluctuating maximum component. Through the formulation of a series of physically realistic hypotheses, the distribution of the maximum is obtained by applying classical probability theorems. The use of the FOSM technique, favoured by the mathematical structure of the

Table 5 First and second statistical moments of V_M using the FOSM technique (1) and the probabilistic analysis (2)

z_o [m]		0.003		0.1		3	
I_V		0.133		0.178		0.327	
Process analysis		(1)	(2)	(1)	(2)	(1)	(2)
μ_{VM} [m / s]	L	44.75	44.75	38.71	38.71	25.52	25.52
	MSM	44.57	44.42	38.52	38.37	25.31	25.18
	MB1	45.48	45.33	39.73	39.58	26.42	26.28
σ_{VM} [m/s]	L	4.07	4.07	3.52	3.52	2.32	2.32
	MSM	4.67	4.66	4.29	4.28	3.21	3.19
	MB1	4.57	4.55	4.14	4.11	3.07	3.04
z_o [m]		0.003		0.1		3	
I_V		0.133		0.178		0.327	
Asymptotic analysis		(1)	(2)	(1)	(2)	(1)	(2)
μ_{VM} [m/s]	L	45.17	45.17	39.07	39.07	25.76	25.76
	MSM	44.99	44.84	38.88	38.74	25.56	25.42
	MB1	45.91	45.76	40.10	39.95	26.67	26.53
σ_{VM} [m/s]	L	4.80	4.80	4.15	4.15	2.74	2.74
	MSM	5.32	5.32	4.83	4.83	3.54	3.52
	MB1	5.26	5.25	4.73	4.71	3.43	3.41

model, furnishes the first and second statistical moments in closed form. It is shown that the gust factor method may be deduced from these expressions as a simplified particular case.

The comparison between the results supplied by the classical methods and those obtained by the proposed procedure points out increasing differences on increasing the turbulence intensity. At least with reference to the cases developed here, such differences appear as relatively moderate.

This method represents a starting point for expressing the distribution of design wind actions on structures in complete probabilistic terms. Operating in this context, the step from velocity to pressure exalts the differences observed above, underlining the conceptual and quantitative importance of the problem debated (Schettini and Solari 1998).

8. References

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