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Adaptive finite element wind analysis with mesh refinement and recovery

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Abstract. This paper deals with the development of variable-node element and its application to the adaptive h-version mesh refinement-recovery for the incompressible viscous flow analysis. The element which has variable mid-side nodes can be used in generating the transition zone between the refined and unrefined elements and efficiently used for the construction of a refined mesh without generating distorted elements. A modified Gaussian quadrature is needed to evaluate the element matrices due to the discontinuity of derivatives of the shape functions used for the element. The penalty function method which can reduce the number of independent variables is adopted for the purpose of computational efficiency and the selective reduced integration is carried out for the convection and pressure terms to preserve the stability of solution. For the economical analysis of transient problems in which the locations to be refined are changed in accordance with the dynamic distribution of velocity gradient, not only the mesh refinement but also the mesh recovery is needed. The numerical examples show that the optimal mesh for the finite element analysis of a wind around the structures can be obtained automatically by the proposed scheme.

Key words: FEM; variable-node element; adaptive; refinement; recovery; single-level rule; cavity-flow; bluff body; transition element.

1. Introduction

The wind effects on the structural behavior are very important to the engineers and designers as the structures become taller and larger. The practical application of numerical analysis of wind as the incompressible viscous flow has been limited due to the excessively high computational cost and the lack of feasible computational method and capacity for many years. In the recent years, the use of current finite element methodologies (FEM) along with the more sophisticated computer softwares and more capable hardwares have advanced the current state of technology of the computational wind engineering (Stathopoulos 1997) to make it a more feasible tool for the practical wind engineering problems. Although the computational method is not expected to replace all the wind tunnel testing in the near future, the method is likely to be used more extensively. Altough the finite element method does not have a superiority when compared with the finite difference method (FDM) in terms of the computational time, FEM can be applied easily to the problems with complex geometry where a singular problem may occur in case of the analysis by FDM. Furthermore, the use of adaptive mesh refinement techniques in FEM allows the analyst to treat the complex geometry more realistically and to capture the special localized features of the solution.

In order that the finite element analysis of wind effects on structures become more practical,

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there are three important conditions to be satisfied; (1) the accuracy of the solution, (2) the efficiency of analysis, and (3) the adequacy of modeling. Considering a large computational efforts required, it is essential to develop a more effective finite element which can be used in describing the behavior of wind properly and in modeling the analysis domain effectively. To generate the optimal mesh for the flow, the development of the adaptive mesh refinement-recovery strategy is also very important to achieve the effective analysis of wind around the structures. In the linear static analysis, the locations where the mesh refinement is needed do not change through the entire analysis and therefore, only the ordinary progressive refinement is needed to obtain the optimal mesh for the analysis. In the nonlinear analysis or in the dynamic analysis, however, the locations to be refined will be changed time to time in accordance with the dynamic distribution of particles and momentum of the flow. Therefore, in order to generate the optimal mesh in a domain at certain time, not only the refinement schemes but also the reliable recovery schemes are needed to be developed. Unfortunately, the systematic mesh refinement-recovery strategy for transient analysis using variable-node element can be seldom found in the published literature.

The four-node quadrilateral element has been frequently used in the two dimensional flow analysis due to its simplicity and easy availability. However, when an area of complicated geometry needs to be refined locally and reanalyzed due to the steep velocity gradient, the overall mesh should be reconstructed to be consistent with the local velocity gradient distribution. For such a mesh gradation, the use of a single type of elements, e.g., four-node elements only, often leads either to the meshes with highly distorted element shapes or to the meshes with too-many degrees-of-freedom that may result in an inefficient solution. When the four node elements generated by subdivision and the original coarse elements which do not have any mid-side nodes are connected, an irregular (or hanging) node will be inevitably created. In this conventional method, variables of an irregular node should be eliminated by the means of condensation before solving the equilibrium equation and are approximately determined by averaging the variables of neighboring nodes. The use of too many irregular nodes in the aforementioned way can cause the locking phenomena.(Lee 1993). The variable-node element which does not produce any hanging nodes can be effectively used for the refinement-recovery strategies.

This paper presents the development of a transition element for flow analysis which has a variable number of mid-side nodes and can be effectively used in the adaptive mesh refinement by connecting the locally refined mesh to the existing coarse mesh through a minimum mesh modification. The transition elements enable us to effectively refine the current mesh and/or recover the previous mesh without entire modification of the connections of all elements. Also, the aforementioned mesh distortion problems can also be avoided since this type of transition elements can keep the element shapes of the previous stage. In this paper, the development of refinement-recovery strategy is discussed in some detail.

Some numerical examples are presented to verify the behavior of the proposed transition element and to demonstrate its applicability to the newly developed adaptive refinement-recovery scheme in the flow analysis.

2. Formulation

The differential equations of the boundary value problem for the 2-dimensional incompressible viscous flow is defined as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho \frac{du}{dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}$$

$$\rho \frac{dv}{dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{3}$$

where u and v are the velocities in the x- and y- direction, respectively, p is the pressure, ρ is the mass density, μ is the viscosity. By the conventional Galerkin formulation, matrix equation is obtained as

$$[M] \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \end{Bmatrix} + [C(u_n, v_n)] \begin{Bmatrix} u \\ v \\ p \end{Bmatrix} + [K] \begin{Bmatrix} u \\ v \\ p \end{Bmatrix} = \begin{Bmatrix} R_u \\ R_v \\ 0 \end{Bmatrix} \tag{4}$$

The element mass matrix $[M]$ is singular because of the null sub-matrix coefficients of $\dot{P}$ and the convection matrix $[C(u_n, v_n)]$ is a nonlinear asymmetric matrix that represents the convection of momentum. The viscous matrix $[K]$ is arising from the viscous term and represents the diffusion of momentum. In the penalty function formaulation, the pressure term P is replaced by an expression with the penalty parameter λ as

$$P = -\lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{5}$$

where λ is determined considering the presence of convection term (Huebner 1995)

$$\lambda = c \max\{\mu, \mu \mathrm{Re}\} \tag{6}$$

where μ is the viscosity, Re is the Reynolds number and c is 10^7. Thus, the incompressibility condition and the continuity equation are dropped out. Furthermore, since the primary variable P is eliminated, the total degrees of freedom of the problem is substantially reduced. From the penalty formulation, the element equation is obtained as

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} + \begin{bmatrix} 2K_{11} + K_{22} & K_{12} \\ K_{22} & K_{11} + 2K_{22} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} + \lambda \begin{bmatrix} [L_{11}] & [L_{12}] \\ [L_{12}]^T & [L_{22}] \end{bmatrix} = \begin{Bmatrix} f_u \\ f_v \end{Bmatrix} \tag{7}$$

where

$$M = \int_\Omega \rho \{N\} < N > d\Omega \qquad C = \int_\Omega \left(u \{N\} \left\langle \frac{\partial N}{\partial x} \right\rangle + v \{N\} \left\langle \frac{\partial N}{\partial y} \right\rangle \right) d\Omega$$

$$K_{ab} = \int_\Omega \mu \left\{ \frac{\partial N}{\partial x_b} \right\} \left\langle \frac{\partial N}{\partial x_a} \right\rangle d\Omega \qquad L_{ab} = \int_\Omega \left\{ \frac{\partial N}{\partial x_a} \right\} \left\langle \frac{\partial N}{\partial x_b} \right\rangle d\Omega$$

The penalty function formulation is known to be effective in solving the Navier-Stokes equations (Reddy 1982). The principal benefit of the use of penalty formulation is that there are fewer equations remained and the size of required memory is reduced accordingly. After the velocity fields has been calculated, the pressure could be obtained in a straight forward manner based on the velocity field.

Convection velocities u and v in the matrix $[C]$ may lead to the numerical instability in the convection dominated flow problem, and therefore, a special treatment is required. The upwind scheme is recognized as a useful tool to avoid such a problem (Zienkiewicz 1989, Kondo 1993). The matrix $[L]$ is related with the penalty function and the pressure.

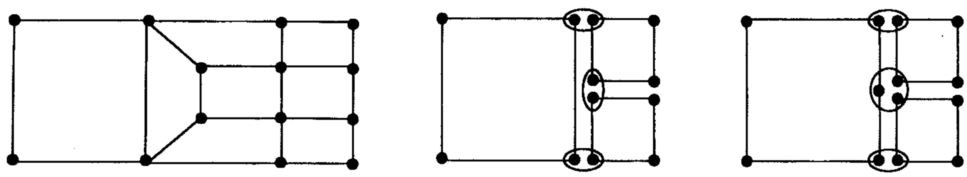
3. Variable-node element

Despite the significant advances made in the theory and algorithmic tools of the finite element method, the finite element modeling for a particular problem is still largely based on the intuition and experience of user. Moreover, the evaluation of reliability of the finite element solution remains to be the most difficult aspect of the finite element technology. Therefore, there is a strong necessity to develop a more effective adaptive mesh refinement algorithm and the element to be used for the optimum mesh generation.

The quadrilateral type elements generally produce better solutions than the triangular type elements but the hanging nodes which are not directly connected to the regular nodes are generated when the regular four-node elements are subdivided. The use of too many irregular nodes often cause a locking problem(Fig. 1(b)), or when only four node elements are used, there is a possibility of producing highly distorted elements(Fig. 1(a)). This problem can be overcome nicely by replacing the unrefined element by a transition element which has a variable number of mid-side nodes.(Fig. 1(c))

The performance of isotropic elements is generally best when they are used without distortion. The effect of distorted elements on the accuracy of solution depends to a large degree on the problem considered and the element used. Therefore, it is desired to refine the mesh locally with undistorted elements in the way that the corner nodes of refined elements are connected to the mid-side node of the coarse element as shown in Fig. 2. The refined elements can maintain the same shape as that of original or mother element when the variable-node elements are used for mesh refinement. Thus, if the well composed initial mesh is used, the mesh distortion will be minimized when the variable-node element is used as the transition element.

To discuss the development of the 5-node to 7-node transition elements for flow analysis, consider a variable-node quadrilateral element as shown in Fig. 3.



(a) transition zone (b) hanging node (c) variable-node element

Fig. 1 Element connections

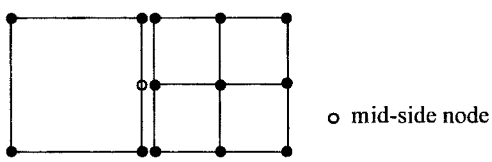


Fig. 2 Element connection between coarse mesh and finer mesh

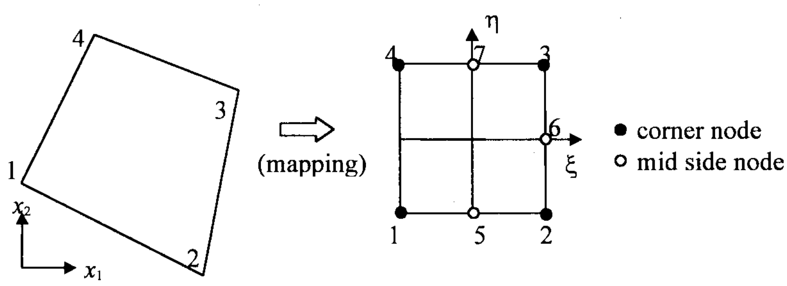


Fig. 3 Configuration of variable-node element

The independent velocity field of the variable-node element is interpolated as a continuous field over the element.

$$u = \sum_{i=1}^{n} N_i(\xi, \eta) u_i \tag{8}$$

The shape functions of the transition elements are written as

$$\begin{aligned}
N_1 &= N_1' - \frac{1}{2}(N_8 + N_5) & N_5 &= \frac{1}{2}(1-|\xi|)(1-\eta) \\
N_2 &= N_2' - \frac{1}{2}(N_5 + N_6) & N_6 &= \frac{1}{2}(1+\xi)(1-|\eta|) \\
N_3 &= N_3' - \frac{1}{2}(N_6 + N_7) & N_7 &= \frac{1}{2}(1-|\xi|)(1+\eta) \\
N_4 &= N_4' - \frac{1}{2}(N_7 + N_8) & N_8 &= \frac{1}{2}(1-\xi)(1-|\eta|)
\end{aligned}$$

$$N_i' = \frac{1}{4}(1+\xi_i \xi)(1+\eta_i \eta), \quad \text{for } i = 1, 2, 3, 4 \tag{9}$$

If the mid-side nodes do not exist, the corresponding shape functions become zeros. This type of elements have been successfully developed and used in the solid mechanics problems. (Gupta 1978, Choi 1989, 1992, 1993, 1995)

4. Numerical integration and upwind

In evaluation of the matrices of variable-nodel element, a normal numerical integration may not be applied directly over the entire element domain because the slope discontinuity of velocity assumed by the shape functions (Eq. 9) in the elements may cause a singular integral. Therefore, the

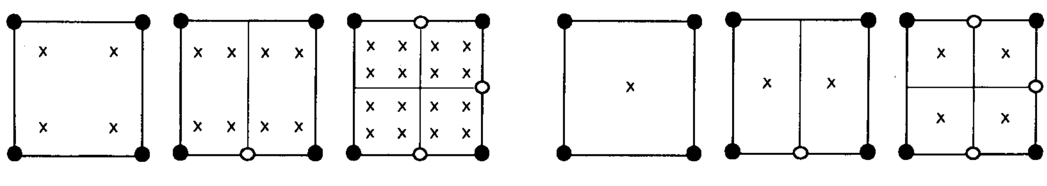


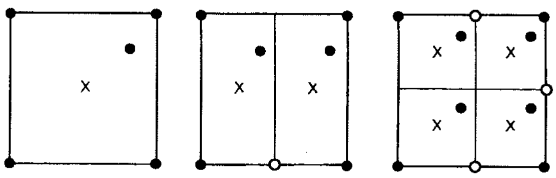
Fig. 4 Modified Gaussian quadrature points

Table 1 Modified full Gaussian quadrature for transition element

point	$\tilde{\xi}$, $\tilde{\eta}$	weight
1	-0.788675135	0.5
2	-0.211314865	0.5
3	0.211314865	0.5
4	0.788675135	0.5

Table 2 Modified reduced Gaussian quadrature for transition element

point	$\tilde{\xi}$, $\tilde{\eta}$	weight
1	-0.5	1.0
2	0.5	1.0



• upwind integral point
× original integral point

Fig. 5 Modified quadrature upwinding

Gaussian quadrature is carried out in the separate subdomains and then combined together. In case of 5-node element, the element is divided into two subdomains and the 6-node and 7-node elements into four subdomains as shown Fig. 4.

The coordinates and corresponding weight coefficients for the modified quadrature (Gupta 1978, Choi 1992) are listed in Table 1 and Table 2. While the mass matrix and the viscous matrix are constructed by the full Gaussian quadrature, the matrices of pressure $[L]$ and convection term $[C]$ are calculated by the reduced integration.

When the Galerkin formulation is applied to the convection term, numerical solutions are unstable and wiggle phenomena will be produced. To prevent the aforementioned problem, the quadrature upwind technique is used (Hughes 1979). Modifications to the upwind technique for the variable-node element is necessary as the domain of variable-node element is divided into two or four subdomains. The original upwind technique should be applied to each subdomain. Thus, the number of integral points in a variable-node element is identical with that of subdomains as shown in Fig. 5. when the reduced integration is used.

5. Adaptive refinement and recovery strategy

As the first step of the adaptive strategy for the mesh refinement, the evaluation of discretization error which represents the difference between the exact solution and the finite element solution of the mathematical model should be carried out. The L_2 norm of the velocity gradient is often used in the error estimation and given as

$$\| \nabla a \| = \left(\int_{\Omega} \nabla u \cdot \nabla u d\Omega \right)^{1/2} \tag{10}$$

Then, the L_2 norm of the error is defined as,

$$\| e \| = \left(\int_{\Omega} (\nabla u - \nabla \hat{u}) \cdot (\nabla u - \nabla \hat{u}) d\Omega \right)^{1/2} \tag{11}$$

∇u : real value of the velocity gradient

$\nabla \hat{u}$: calculated value of the velocity gradient

Since the exact velocity gradient is not readily known in the most engineering problems, the error norm can not be obtained directly. However, if the smoothed velocity gradient ∇u^* is introduced, L_2 norms of the velocity gradient and the error of solution may be approximately obtained (Zienkiewicz 1987). The error estimation is performed by these approximated norms. If the velocity gradient error $e^*_{\nabla u}$ is defined as the difference between the smoothed velocity gradient and the velocity gradient obtained by the finite element analysis, it can be written as the following equation

$$e^*_{\nabla u} = \nabla u^* - \nabla \hat{u} \tag{12}$$

where the continuous velocity gradient field ∇u^* is defined using the same shape functions as used for the velocity field assumptions as

$$\nabla u^* = \sum_{i=1}^{n} N_i \nabla u_i^* \tag{13}$$

Then, the smoothed nodal velocity gradient ∇u_i^* can be obtained by the minimization of the following functional

$$\Pi = \sum_{i=1}^{nel} \left[\int_{\Omega_i} e^{*}_{\nabla u}{}^{T} e^*_{\nabla u} d\Omega \right] \tag{14}$$

The smoothing is performed on each velocity gradient component and the overall error in the entire domain is defined as

$$\eta = \frac{\| e^* \|}{\| \nabla a^* \|} \tag{15}$$

and the following error indicator in an element is used to judge whether the element should be refined or not.

$$\eta_i = \left(\frac{\| e^* \|_i^2}{\| \nabla a^* \|^2 / nel} \right)^{1/2} \tag{16}$$

If the error indicator of an element is larger than the user specified error criterion for refinement, the element will be divided into sub-elements. On the other hand, the elements will be merged into one element to recover the original element when all the error indicators of the elements concerned are smaller than the specified error criterion for recovering. The optimal mesh which produce a relatively uniform error distribution in the entire domain can be obtained after some iterations.

The adaptive mesh refinements using variable-node elements have been reported to be effective (Choi 1997 and Park 1990). However, the application of their refinement algorithms has been limited to the static problems or steady state problems. Because of the time dependent characteristics of a flow, the optimal mesh can not be fixed during the entire analysis process but it should be changed continuously in accordance with the change of error distribution. Therefore, if the refinement continues without recovery process, the elements refined in the past will remain and accumulated, and as a consequence, the computational inefficiency may result.

During the actual refinement process of the initial mesh, various types of variable-node elements can be utilized. Since the size of the element should not be changed abruptly from the neighboring elements, the 'singel-level-rule' which prevents a sudden change of the element sizes and provides the systematic mesh refinement algorithm is used in this study. The level number of an element denotes the number of refinements which the element has experienced. In the single-level-rule, the difference of level numbers between two neighboring elements should not exceed 'one'. This means that an element already refined can not be refined again without prior refinement of the neighboring element.

Fig. 6 demonstrates the adaptive refinement-recovery strategy developed in this study. In Fig. 6(a) the initial mesh is composed of four elements. At this initial step (or step 1), all the elements are of the level 1. The number in an element denote the element identification number and the level numbers are given in the parentheses. At the step 2, elements 1, 3 and 4 are divided based on the error indicator of each element and the user specified error criteria for refinement and as the consequence, new elements 5 thru 13 are generated. The elements 14 thru 19 are then generated in a similar way by the refinement at the step 3 (Fig. 6(b) and 6(c)). As

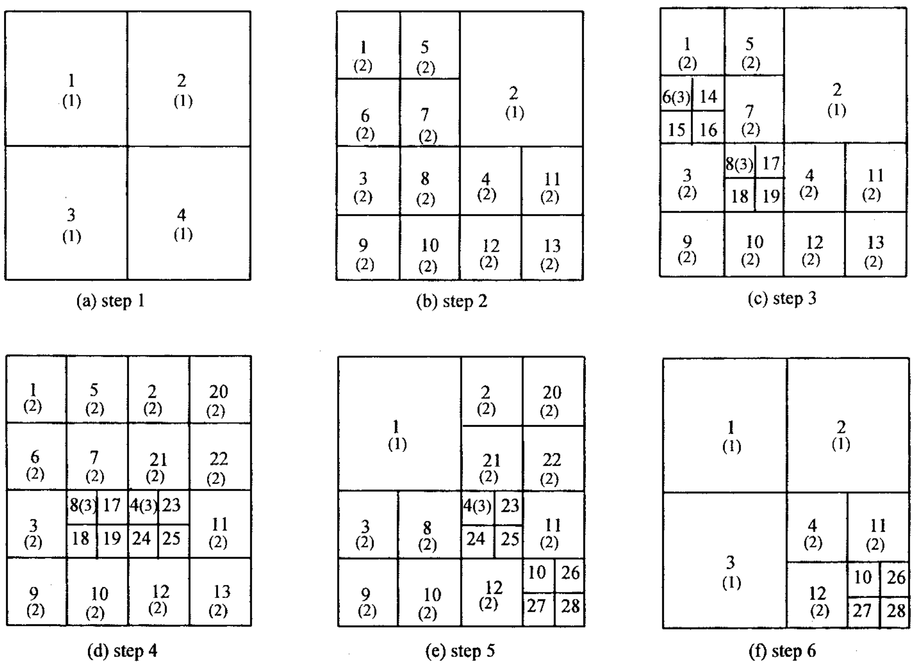


Fig. 6 History of refinement and recovery

the refinements proceed with the steps 2 and 3, the level numbers of these newly generated elements are increased by one at a time. It should be noted that there is no need of mesh recovery so far.

If the error indicator of element 4 is found to be larger than the error criterion for refinement at the next step (step 4), the element 4 needs to be subdivided. Prior to the refinement of the element 4, the element 2 should be refined in accordance with the single-level-rule. Simultaneously, the elements 6, 14, 15 and 16 are merged into one element to recover the previous element 6 as shown in Fig. 6(d) and discussed in the following paragraph. The information on the refined elements and recovered elements is saved in the storage array for the future retrieval.

For the mesh recovery in which the subdivided(refined) elements merge into one element to recover the original element, the procedures should be basically in the reverse order of mesh refinement. If any element is found to be recovered based on the error indicators and specified error criteria, it should be determined if the element can be combined with three sister elements which were generated previously by subdividing the mother element. If all these elements do not violate the single-level-rule and the error indicators of the elements are all less than the error criteria for recovery, the four elements (e.g. elements 6, 14, 15 and 16 in Fig. 6(c)) can be combined to form one element (element 6).

At the step 5, elements 1, 5, 6 and 7 are combined to recover the original element 1 in the similar manner as the element 6 is recovered. However, because elements 8 and 17 violate the single-level-rule, the elements 8, 17, 18 and 19 should be combined prior to the recovery of element 1. Otherwise, the element 1 can not be recovered at this time. After elements 8, 17, 18 and 19 are combined to recover element 8, the elements 1, 5, 6 and 7 are also combined and the original element 1 is recovered as shown in Fig. 6(e). As the refinement-recovery procedures proceed, elements 2, 3, and 4 are recovered and so on.

6. Numerical examples

6.1. Cavity flow

The cavity-flow problem is solved as a basic bench mark test to confirm the validity and to evaluate the effectiveness of the algorithm discussed in the previous sections. The analysis has started with a mesh which consists of 9 nodes and 4 elements as shown in Fig. 7. Reynolds number is 100, the size of time step is 0.02 seconds, and the error criterion for refinement of an element is selected to be 20%. To prevent generating too many elements, the maximum level of

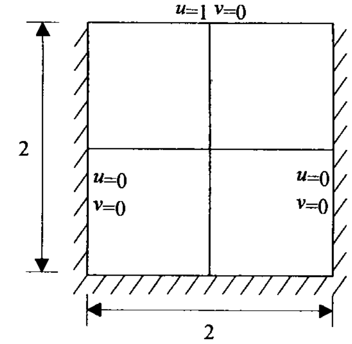


Fig. 7 Boundary conditions and initial mesh for the cavity-flow problem

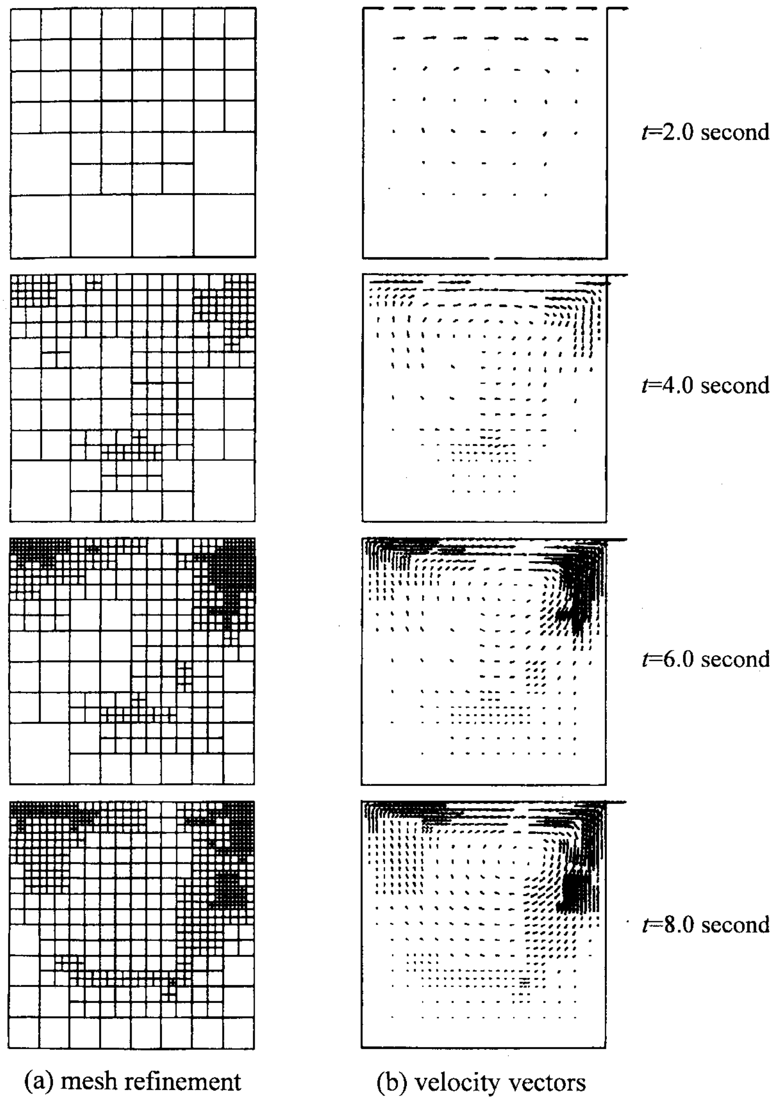
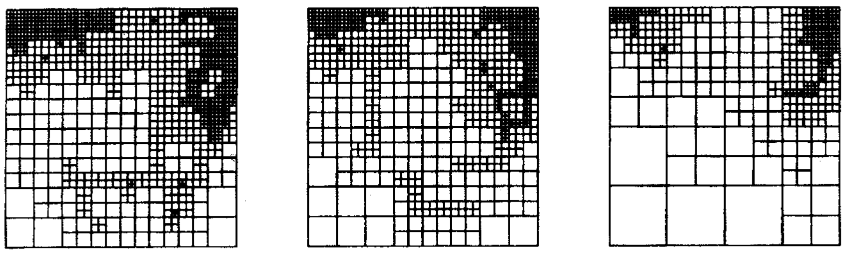


Fig. 8 Adaptive FE analysis of cavity-flow

an element is restricted to 6. The mesh refinements and recoveries have been performed every 0.5 seconds of analysis and the results are shown in Fig. 8. At t=8 seconds, the number of elements is increased to 718 and that of nodes to 845. As the initial conditions, the velocities of the newly generated nodes on the wall are zeroes ($u=v=0$) and the x-direction velocities at the top nodes of the cavity are unities ($u=1$, $v=0$).

The results of analysis using the adaptive mesh refinement strategy and variable-node rectangular elements are shown to be similar to those of the example using triangular elements for adaptive analysis (Sampio 1993). The refinement has been intensively performed near the left and right top corners where the velocity, pressure and vorticity were changed abruptly (Gresho 1984). The repulsive flow of the outlet at the right top leads to a sudden change of the properties of flow as indicated in the final mesh.

To evaluate the effect of the error criteria on the accuracy and convergence of the solution



(a) 10 % (b) 15% (c) 25%

Fig. 9 Obtained mesh with different error criteria at t=8 seconds

and to evaluate the adequacy of the selected criteria of 20% in this example, the problem is also solved with some selected upper (refinement) error criteria of 25%, 15%, and 10% and the lower (recovery) error criteria which are defined as 25% of the upper error criteria. The results at t=8 seconds are shown in Fig. 9. The overall errors (Eq. 15) of those solutions obtained are 17.3%, 13.6%, and 13.5% for the error criteria of 25%, 15% and 10%, respectively. Generally, the larger the selected error criteria is used, the bigger overall error is obtained.

The error criteria are found to be sensitive to the optimal mesh and therefore selection of the adequate error criteria is very important for the evaluation of overall error of the solution. Since the selection of the criteria largely depends on the intuition and experience of the user, some additional research on the selection of the criteria is needed to extend the capability of presented scheme in this study.

6.2 Bluff body

The problem statement of a bluff body in a crossflow and the boundaries of problem are described in Fig. 10. In the problem, the singularities exist at the corners of the bluff body and may cause a separation of flow which will increase the instability of the flow and therefore, the ac-

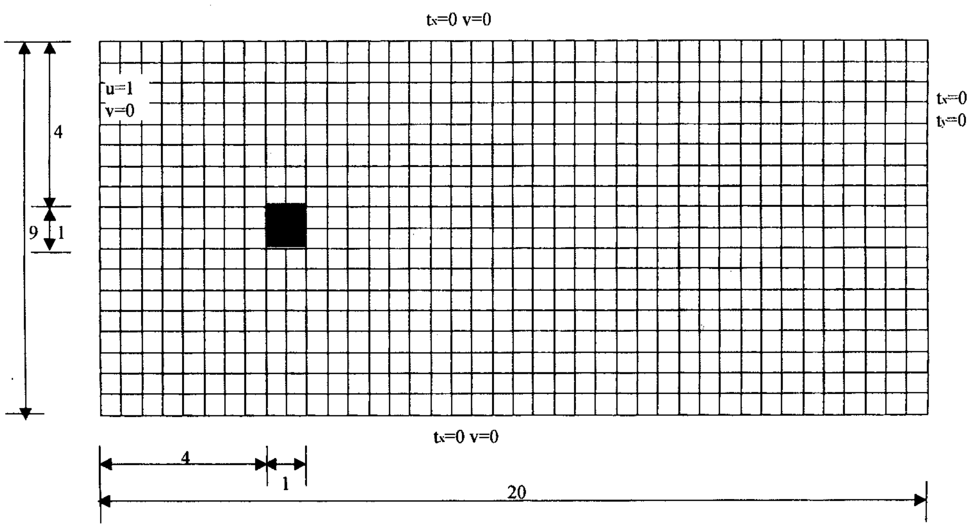


Fig. 10 Boundary conditions and initial mesh for the bluffbody in a crosswind problem

curacy of solution at those places are decreased. Many researchers have located the finer mesh near the bluff body to solve this problem of separation of flow (Yoshida 1985, Shimura 1993). The main purpose of this example is to show the capability of proposed scheme to form the adaptively refined meshes near the body and in the following vortex street.

The Reynold number used is 100. The initial mesh is composed of 778 nodes and 716 elements as shown in Fig. 10. As the local mesh refinement proceeds, the size of element decreases and accordingly, the size of time step to be used should be decided with more care. If time step is too large, the particles of the flow will pass over one or more elements during a single time step. Therefore, the size of time step should be kept small enough so that at least

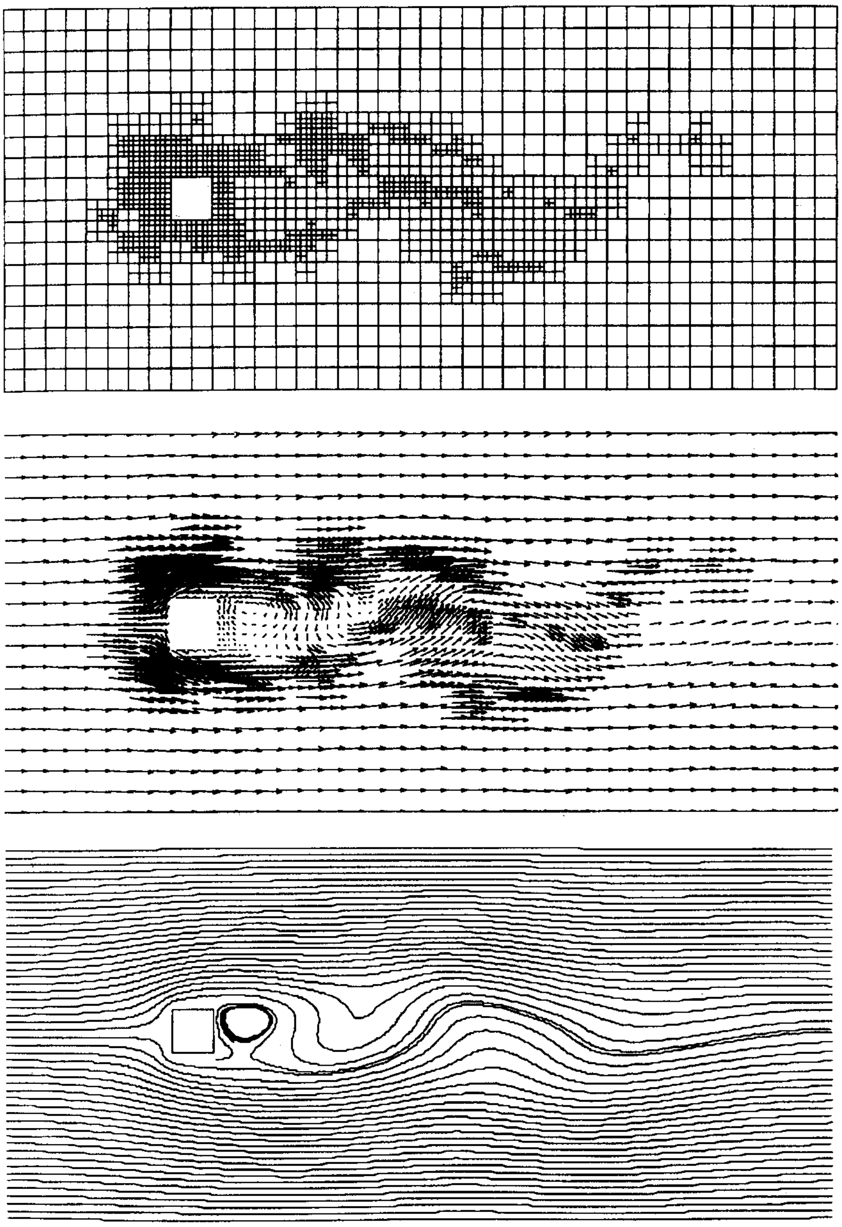


Fig. 11 Mesh, velocity vector and streamline at t=22.5 seconds

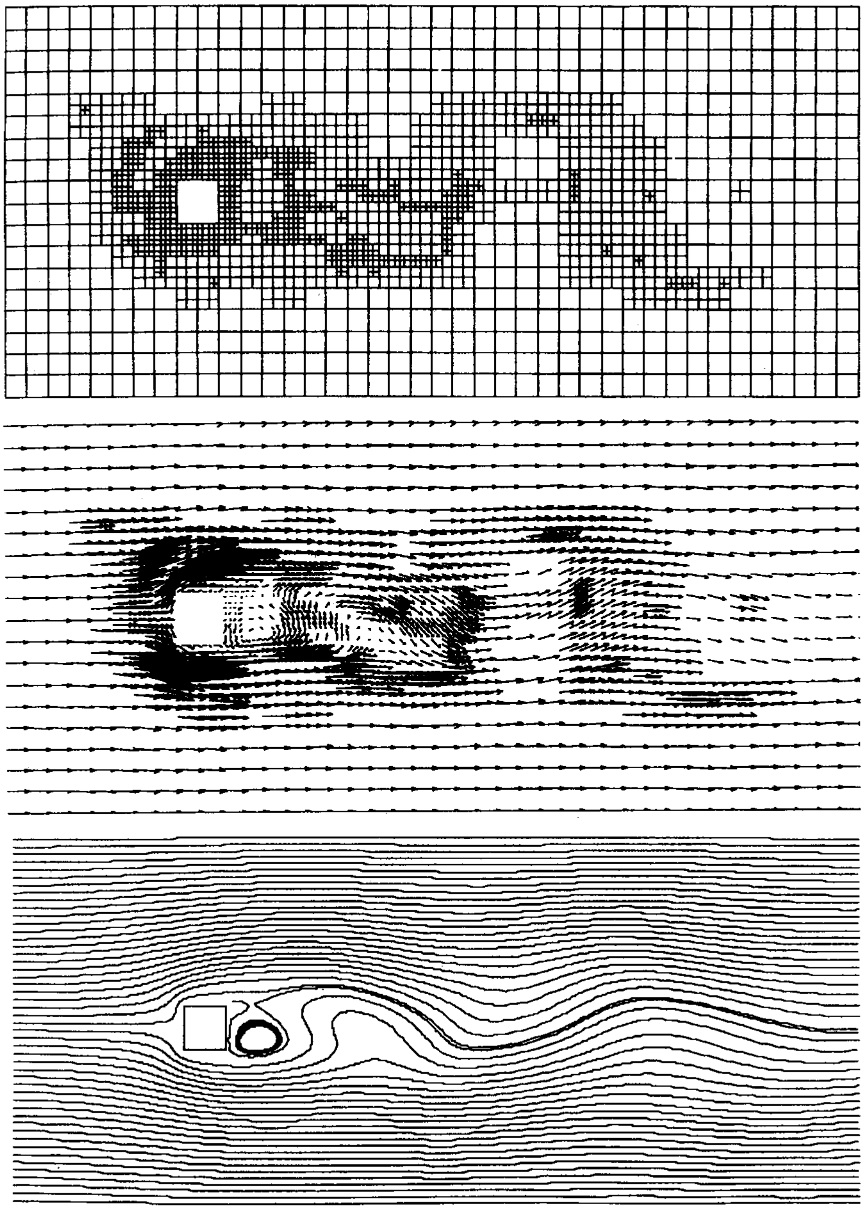


Fig. 12 Mesh, velocity vector and streamline at t=24.6 seconds

two time steps are needed for the particles of flow to pass through an element. Considering the above fact and some other factors, such as Courant number of element and computational cost, the time step for the transient analysis problem is decided to be 0.05 seconds. To initiate the vortex, an artificial disturbance which destroy the symmerty of flow is applied at t=1 second. The error critera for refinement and recovery are selected as 30% and 15%, respectively.

The reconstructed mesh, velocity vectors and streamlines at t=22.5 are shown in Fig. 11. The meshes are adaptively refined not only around the bluff body but also through the vortex street, which well agree with the meshes obtained by using triangular elements (Sampio 1993). By this time, the numbers of nodes and elements are gradually increased to 2155 and 1892, respectively.

The mesh, velocity vector and streamlines at a half period of the vortex shedding (at t=24.6) are shown in Fig. 12. The symmetry of the flow and the periodic nature of vortex are seen in Figs. 11 and 12.

7. Conclusions

A new adaptive mesh refinement-recovery schemes using variable-node elements proposed in the paper is found to be very effective for the transient flow analysis to capture the dynamic characteristics of the flow.

When the initial coarse mesh is established to start the analysis, the optimal mesh for the flow analysis will be gradually constructed as the refinement-recovery process proceeds. When the elements in the analysis domain have the same aspect ratios since the refinement is achieved by bisecting the mother element, the mass matrix and viscous matrix of the fluid can be obtained by simple calculation without numerical integration. Since the nodes do not move the coordinates through analysis, the time history of properties at the observing node will be obtained without any approximation which may be needed in other analysis using moving node or remeshing algorithm.

The ability to create economical mesh by the adaptive mesh refinement, recovery schemes is confirmed by numerical examples. The finer elements are located at the area where a sudden change of the properties of the flow will occur. The new meshes at each time step in the example of the bluff body show a good agreement with previous studies in capturing the dynamic characteristics of flow, in particular the development of the vortex. Since the solution is sensitive to the error criteria, the technique for the selection of adequate error criteria remains as a subject of the future study.

The immediate application of the schemes developed in this study to the wind flow may not be promised because the analysis of turbulence and the 3-dimensional characteristics of the wind require a huge amount of computation. However, the proposed scheme can be extended to the adaptives finite element analysis of the real wind environment without requiring large modification of the basic philosophy used in this study.

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