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Instability of pipes and cables in non-homogeneous cross-flow

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Abstract. The vibrations of bodies subjected to fluid flow can cause modifications in the flow conditions, giving rise to interaction forces that depend primarily on displacements and velocities of the body in question. In this paper the linearized equations of motion for bodies of arbitrary prismatic or cylindrical cross-section in two-dimensional cross-flow are presented, considering the three degrees of freedom of the body cross-section. By restraining the rotational motion, equations applicable to circular tubes, pipes or cables are obtained. These equations can be used to determine stability limits for such structural systems when subjected to non uniform cross-flow, or to evaluate, under the quasi static assumption, their response to vortex or turbulent excitation. As a simple illustration, the stability of a pipe subjected to a bidimensional flow in the direction normal to the pipe axis is examined. It is shown that the approach is extremely powerful, allowing the evaluation of fluid-structure interaction in unidimensional structural systems, such as straight or curved pipes, cables, etc, by means of either a combined experimental-numerical scheme or through purely numerical methods.

Key words: linearized equations of motion; uniform cross-flow; quasi static assumption; vortex excitation; fluidstructure interaction.

1. Introduction

Fluid flow past fixed boundaries, as well as the pressure excerted on those boundaries, may be significantly altered by motion of the boundary. Thus, as it displaces and oscillates in response to the interface pressures, a flexible structure immersed in fluid flow will influence the excitation, in a process known as fluid-structure interaction. This may give rise to enhanced dynamic response, or to hydro or aeroelastic instability.

Although the phenomenon has wide implications in engineering, from the design of long span bridges, to pipe bundles in heat exchangers or vibrations of iced conductors, not to mention some of the most important problems of aeronautical engineering, there is yet no comprehensive theory or global approach to the subject. The so called pseudo-static theory, as presented by Brito and Riera (1995) offers such possibility, at least for two-dimensional situations. In fact, the authors suggested a general procedure to obtain the displacement and velocity dependent interaction forces in case of a cylindrical body subjected to a nonuniform flow, from which the

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equations applicable to many well known problems, such as simple galloping, coupled vertical and horizontal galloping, vibration of conductor in the wind field modified by others conductors, may be obtained.

In this paper those general equations are reproduced. The equation of motion of a tube in a bidimensional flow is then obtained and the ensuing problem of dynamic instability investigated. It is shown that the approach, which so far has been shown to accurately predict at least the onset of dynamic instability, is quite appealing for design applications.

2. Aerodynamic forces in two-dimensional flow

Consider a cylindrical or prismatic body submerged in a fluid that flows with uniform velocity V_o in the free field, oriented in the direction normal to the shear axis of the body. In order to render the final equations applicable to thin-walled, open section or multiple cell beams, the so-called shear or flexure center is adopted as origin of the coordinate system. If the body presents two planes of symmetry, the shear center coincides with the center of gravit of the cross section. The Ox axis of the rectangular coordinate system is taken parallel to the velocity vector in the free field, while the Oz axis coincides with the axis of the body.

Under those conditions it may be assumed that, in the neighbourhood of the plane of interest z=constant, the flow remains bi-dimensional. In general, except for very low flow velocities, the resulting fluid pressures will fluctuate with time t. Hence, the forces per unit length of the cylinder in the Ox and Oy directions, as well as the torsional moment may be assumed to be, under quite general conditions, stationary random processes with expected values given by

$$F_x = \frac{1}{2}\rho V_o^2 b C_x(\alpha),\ F_y = \frac{1}{2}\rho V_o^2 b C_y(\alpha),\ M = \frac{1}{2}\rho V_o^2 b^2 C_M(\alpha), \tag{1}$$

in which ρ denotes the specific mass of the fluid, b is a characteristic cross sectional dimension, usually equal to the body's projection on a plane normal to the Ox axis, while C_x, C_y and C_M are nondimensional coefficients that depend on the shape of the cross section as well as on the angle of incidence α.

For any given cross-section, the coefficients C_x, C_y and C_M may be experimentally obtained in wind tunnel tests. It is well known, however, that under nonuniform flow conditions, the free-field turbulence may affect the coefficients under consideration, which may also vary with Reynolds Number Re. Hence, it will be further assumed that C_x, C_y and C_M can be measured under the flow conditions that correspond to the specific problem at hand, in which case they may be considered functions of α only.

There is ample evidence, moreover, suggesting that Eqs. (1) are still valid if V_o and α are slowly varying functions of time. Theories based on this hypothesis are known as *quasi-static.* They have been successfully used in the study of galloping oscillations. Assume now that the flow is not uniform, but defined by the equations

$$V_x = \phi_x V_o\ ,\quad V_y = \phi_y V_o\ , \tag{2}$$

in which V_x and V_y denote the flow mean velocity components in the x and y coordinate directions and ϕ_x, ϕ_y being continuous, differentiable, slowly varying functions of x and y. The dimensions of the body are small in comparison with the length scale of the fluctuations of ϕ_x and ϕ_y. The velocity field (2) may be produced by an upstream obstacle, or group of bodies, or simply by the flow boundary conditions.

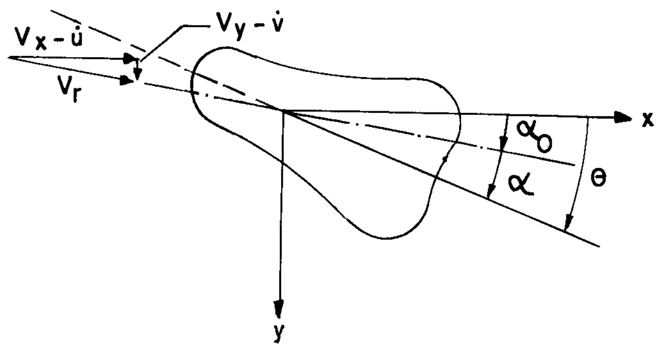

Fig. 1 Definition of relative velocity vector

Consider now that the body is displaced, u, v being the displacements in the coordinate directions, and θ the rotation around the shear center. The square of the modulus of the incident velocity V_r, relative to the body, will then be given by

$$V_r^2 = (V_x - \dot{u})^2 + (V_y - \dot{v})^2 \tag{3}$$

where dots represent derivatives with respect to time. Introducing Eqs. (2) into (3) we obtain

$$V_r^2 = V_o^2(\phi_x^2 + \phi_y^2) - 2V_o(\phi_x \dot{u} + \phi_y \dot{v}) + \dot{u}^2 + \dot{v}^2 \tag{4}$$

On the other hand, the angle of relative incidence, shown in Fig. 1, becomes

$$\alpha = \theta - \tan^{-1}\frac{V_y - \dot{v}}{V_x - \dot{u}} \tag{5}$$

The Eqs. (1), initially proposed for a stationary body in uniform flow, may now be extended, introducing a classical notion in aeronautical engineering, to the situation under consideration, by observing that the aerodynamic coefficients depend both on α as well as on the angular velocity of the body $\dot{\theta}$

$$F_x = \frac{1}{2}\rho V_r^2 bC_x(\alpha, \dot{\theta}),\ F_y = \frac{1}{2}\rho V_r^2 bC_y(\alpha, \dot{\theta}),\ M = \frac{1}{2}\rho V_r^2 b^2 C_M(\alpha, \dot{\theta}) \tag{6}$$

The expressions for the forces are nonlinear in the velocities $\dot{u}$ and $\dot{v}$. Admitting now that $V_y \ll V_x$ and expanding the coefficients C_x, C_y and C_M, as well as α and V_r^2, as power series around a reference angle $\bar{\alpha}$, after some algebraic manipulations and neglecting higher order terms, the following linearized equations may be obtained (Brito 1995, Brito and Riera 1995)

$$\begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix} = \frac{1}{2}\rho b V_o^2 \begin{Bmatrix} D_D \\ C_L \\ C_T \end{Bmatrix} + \frac{1}{2}\rho b V_o^2 \boldsymbol{A} \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix} + \frac{1}{2}\rho b V_o^2 \boldsymbol{B} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{\theta} \end{Bmatrix} \tag{7}$$

in which

$$C_D = (\phi_x^2 + \phi_y^2)C_x,\quad C_L = (\phi_x^2 + \phi_y^2)C_y,\quad C_T = b(\phi_x^2 + \phi_y^2)C_M \tag{8}$$

Morever, the coefficients of matrices ***A*** and ***B*** are given by :

$$A_{11} = 2C_x\,(\phi_x\,\phi_{x,x} + \phi_y\,\phi_{y,x}) - \phi_x\,C_{x,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,x} - \bar{\alpha}_o\,\phi_{x,x}) \tag{9}$$

$$A_{12} = 2C_x\,(\phi_x\,\phi_{x,y} + \phi_y\,\phi_{y,y}) - \phi_x\,C_{x,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,y} - \bar{\alpha}_o\,\phi_{x,y}) \tag{10}$$

$$A_{13} = C_{x,\alpha}\,(\phi_x^2 + \phi_y^2) \tag{11}$$

$$A_{21} = 2C_y\,(\phi_x\,\phi_{x,x} + \phi_y\,\phi_{y,x}) - \phi_x\,C_{y,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,x} - \bar{\alpha}_o\,\phi_{x,x}) \tag{12}$$

$$A_{22} = 2C_y\,(\phi_x\,\phi_{x,y} + \phi_y\,\phi_{y,y}) - \phi_x\,C_{y,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,y} - \bar{\alpha}_o\,\phi_{x,y}) \tag{13}$$

$$A_{23} = C_{y,\alpha}(\phi_x^2 + \phi_y^2) \tag{14}$$

$$A_{31} = b\,[2C_M\,(\phi_x\,\phi_{x,x} + \phi_y\,\phi_{y,x}) - \phi_x\,C_{M,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,x} - \bar{\alpha}_o\,\phi_{x,x})] \tag{15}$$

$$A_{32} = b\,[2C_M\,(\phi_x\,\phi_{x,y} + \phi_y\,\phi_{y,y}) - \phi_x\,C_{M,\alpha}(1+\bar{\alpha}_o^2)\,(\phi_{y,y} - \bar{\alpha}_o\,\phi_{x,y})] \tag{16}$$

$$A_{33} = bC_{M,\alpha}(\phi_x^2 + \phi_y^2) \tag{17}$$

$$B_{11} = [-2\,C_x\,\phi_x - C_{x,\alpha}(\phi_y + \phi_x\,\bar{\alpha}_o^3)]/V_o \tag{18}$$

$$B_{12} = [-2\,C_x\,\phi_y + C_{x,\alpha}\phi_x\,(1+\bar{\alpha}_o^2)]/V_o \tag{19}$$

$$B_{13} = C_{x,\theta}(\phi_x^2 + \phi_y^2) \tag{20}$$

$$B_{21} = [-2\,C_y\,\phi_x - C_{y,\alpha}(\phi_y + \phi_x\,\bar{\alpha}_o^3)]/V_o \tag{21}$$

$$B_{22} = [-2\,C_y\,\phi_y - C_{y,\alpha}\phi_x\,(1+\bar{\alpha}_o^2)]/V_o \tag{22}$$

$$B_{23} = C_{y,\theta}(\phi_x^2 + \phi_y^2) \tag{23}$$

$$B_{31} = b\,[-2\,C_M\,\phi_x - C_{M,\alpha}(\phi_y + \phi_x\,\bar{\alpha}_o^3)]/V_o \tag{24}$$

$$B_{32} = b\,[-2\,C_M\,\phi_y + C_{M,\alpha}\phi_x\,(1+\bar{\alpha}_o^2)]/V_o \tag{25}$$

$$B_{33} = bC_{M,\theta}(\phi_x^2 + \phi_y^2) \tag{26}$$

in which $\alpha_0 = \phi_y / \phi_x$ and $\phi_{y,x} = \partial\phi_y / \partial x$, etc. The constants C_D, C_L, C_T and the elements of the matrices ***A*** and ***B*** are function of coefficients ϕ_x, ϕ_y, $\phi_{x,x}$, etc. as well as the aerodynamic constants C_x, $C_{x,\alpha}$, etc., and are evaluated at the origin of the coordinate system ($u = v = \theta = 0$). The former characterize the flow at the location of the body, while the latter depend on the cross section of the body. The important issue is that both sets of values may be determined in independent wind tunnel tests; it is also possible to combine numerical predictions of the flow functions with experimentally determined aerodynamic coefficients.

3. Body with two degrees of freedom: restricted torsional motion

In many cases of practical interest, torsional vibration are negligible, either because the torsional exitationis small or the torsional stiffness high. The aerodynamic forces may be obtained from Eqs. (7), by setting $\theta = 0$. For example, Jones (1992) studied the coupled vertical and horizontal galloping oscillations of iced conductors. With the notation of this paper, the interaction forces in such case ($\phi_x = 1$, $\phi_y = 0$) are :

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{2} \rho b V_o^2 \begin{Bmatrix} C_D \\ C_L \end{Bmatrix} + \frac{1}{2} \rho b V_o^2 B' \begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} \tag{27}$$

in which :

$$B' = \frac{1}{V_o} \begin{bmatrix} -2C_D & -(C_L - C_{D,\alpha}) \\ 2C_L & -(C_D + C_{L,\alpha}) \end{bmatrix} \tag{28}$$

the aerodynamic loads given by Eqs. (27) coincide with the velocity-dependent forces considered by Jones (1992).

4. Aerodynamic forces for body with circular cross section

The general linearized Eqs. (7) may also be particularized to a situtaion of considerable pratical interest : the wake induced flutter in cables with circular corss-section. In fact, Eqs. (7) allow the treatment of cables with arbitrary cross-section, like ice-covered conductors and may also be applied to wind induced instability of cables and other applications by setting $\theta = 0$.

Denoting by b and C_D the cable diameter and drag coefficient, respectively, considering that C_L and C_T are zero, and assuming that ϕ_y, the lateral velocity coefficient as well as its derivatives are small quantities, the following equations may be obtained :

$$\begin{aligned} \begin{Bmatrix} F_x \\ F_Y \end{Bmatrix} &= \frac{1}{2} \rho b C_D V_o^2 \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} + \frac{1}{2} \rho b C_D V_o \begin{bmatrix} -2\phi_x & -\phi_y \\ -\phi_y & -\phi_x \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} + \\ &+ \frac{1}{2} \rho b C_D V_o^2 \begin{bmatrix} 2\phi_x \phi_{x,x} & 2\phi_x \phi_{x,y} \\ \phi_y \phi_{x,x} + \phi_x \phi_{y,x} & \phi_x \phi_{y,y} + \phi_y \phi_{x,y} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \end{aligned} \tag{29}$$

where

$$L_1 = (\phi_x^2 + \phi_y^2) \cos \bar{\alpha}_o , \qquad L_2 = (\phi_x^2 + \phi_y^2) \sin \bar{\alpha}_o \tag{30}$$

Note that there was an error in coefficients B_{12} and B_{21} in Eqs. (29) as given by Brito and Riera (1995). Eqs. (29) reduce to the equations due to Simpson (1971) and Price (1975) to analyze the vibrations induced in overhead conductors by the presence of parallel conductors in the upwind direction. The satisfactory results obtained in these applications also lend support to the validity of the basic assumptions upon which they are based.

5. Dynamic stability of spring-supported pipe in bidimensional flow

By way of illustration, two simple applications of the preceding equations will be presented in this section. Consider a pipe or tube parallel to the *Oz* axis, that is, normal to the plane of a bidimensional flow defined by the functions ϕ_x and ϕ_y. Let the pipe be located at the origin of the coordinate system *xy*, for the following cases :

a) Flow A : shear flow, presented in Fig. 2, where the nondimensional functions are

$$\phi_x = 1 - y/a \quad \text{and} \quad \phi_y = 0$$

b) Flow B : radial flow in direction to a sink of intensity *Q*, presented in Fig. 3, where the nondimensional functions are

$$\phi_x = Q\,(a - x)/[(x - a)^2 + y^2] \text{ and } \phi_y = -Q\,y/[(x - a)^2 + y^2]$$

The equations of motion for a two degrees of freedom system are :

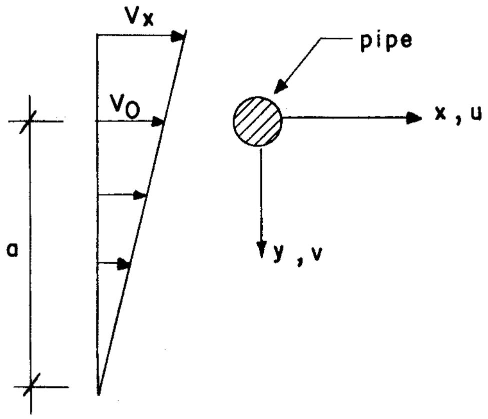

Fig. 2 Spring-supported pipe in flow A

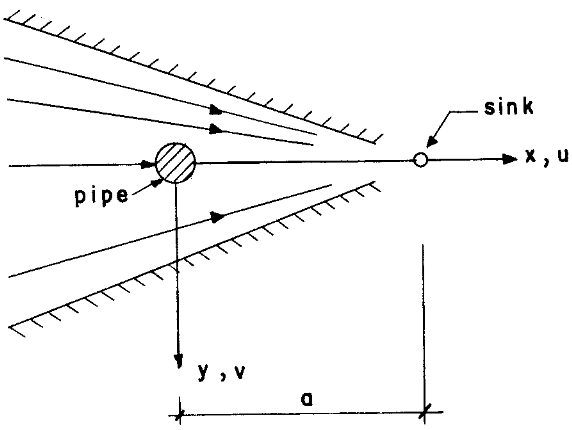

Fig. 3 Spring-supported pipe in flow B

$$m\,(\ddot{u}+2\,\xi_x\,\omega_x\,\dot{u}+\omega_x^2 u\,)=F_x \tag{31}$$

$$m\,(\ddot{v}+2\,\xi_y\,\omega_y\,\dot{v}+\omega_y^2 v\,)=F_y \tag{32}$$

in which m denotes the cylinder mass per unit length, ζ_x and ζ_y the damping ratios and ω_x, ω_y the natural vibration frequencies. F_x and F_y denote the total flow interaction forces, per unit length, in the coordinate directions.

For case a), the aerodynamic forces given by Eqs. (29) take the form :

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = C_o\,V_o^2 \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + C_o\,V_o \begin{bmatrix} -2 & 0 \\ -0 & -1 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} + C_o\,v_o^2 \begin{bmatrix} 0 & -2/a \\ o & o \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{33}$$

in which C_o=1/2 ρ b C_D. The first term in the right hand side represents mean values, giving rise to a static response. Hence, it needs not be considered in instability analysis. When the aeroelastic forces (33) are introduced in the equations of motion, the mass matrix $\boldsymbol{M}$, the effective damping matrix $[\boldsymbol{C}+\boldsymbol{C}']$ and the effective stiffness matrix $[\boldsymbol{K}+\boldsymbol{K}']$ can be show to be positive definite for any velocity V_o, indicating that the system is assyntotically stable.

Now, in case b), the aerodynamic forces also computed from Eqs. (29) results:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = C_o\,V_o^2\,\frac{Q^2}{a^2} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + C_o\,V_o\,\frac{Q}{a} \begin{bmatrix} -2 & 0 \\ -0 & -1 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} + C_o\,V_o^2\,\frac{Q^2}{a^3} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{34}$$

Again, introducing Eqs. (34) in the equations of motion (31-32) positive definite mass and effective damping matrices result. On the other hand, the effective stiffness turns singular for a *critical* value of the velocity V_o. In fact:

$$\boldsymbol{K}+\boldsymbol{K'} = \begin{bmatrix} m\,\omega_x^2 - 2\,C_o\,Q^2\,V_o^2/a^3 & 0 \\ 0 & m\,\omega_y^2 + C_o\,Q^2\,V_o^2/a^3 \end{bmatrix} \tag{35}$$

Consequently, it is sufficient to investigate the static problem, since in this case the instability is of the divergent type. It is evident that the effective stiffness matrix, given by Eqs. (35), becomes singular when its first diagonal term is zero, which leads to the critical velocity :

$$(V_o\,)_{cr} = \sqrt{\frac{m\,\omega_x^2\,a^3}{2\,C_o\,Q^2}} \tag{36}$$

6. Conclusions

An approach to evaluate fluid-structure interaction forces, applicable to any bidimensional problem in which the wave lengths of the fluctuations of the functions ϕ_x and ϕ_y that describe the flow are larger than the body cross-sectional dimensions, is described. A simple application to examine the stability of a spring-supported pipe in two different flow conditions is presented.

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Notations

a	a distance
A_{ij}	coefficientsof aerodynamic matris $\boldsymbol{A}$ in displacement-dependent term of Eqs. (7)
b	characteristic cross sectional dimension, usually the body's dimension normal to the flow
B_{ij}	coefficients of aerodynamic matrix $\boldsymbol{B}$ in velocity-dependent term of Eqs. (7)
C_x, C_y, C_M	nondimensional aerodynamic coefficients
C_D, C_L, C_T	drag, lift and torsion coefficients, given by Eqs. (8)
$\boldsymbol{C}$	viscous damping matrix
F_x, F_y	total force exerted by the fluid in the x and y direction, respectively, per unit length of body.
$\boldsymbol{K}$	elastic stiffness matrix
L_1, L_2	nondimensional coefficients in static component in Eqs. (30)
m	mass per unit span of the cylinder
M	torsional moment around body's axis, per unit length of body
$\boldsymbol{M}$	mass matrix
Q	intensity of a sink
u	displacement of the body in x direction
v	displacement of the body in y direction
V_o	wind velocity in the free field
V_r	incident velocity
V_x, V_y	mean flow velocity components
α	angle between instantaneous wind velocity vector and Ox axis
α	arbitrary reference angle
$\bar{\alpha}_o$	reference angle in Taylor's series expansion of aerodynamic coefficinest, taken equal to ϕ_y / ϕ_x
ζ_x, ζ_y	damping ratios, respectively, in the x and y directions
θ	rotation of the body around its shear center
ρ	specific mass of the fluid
ϕ_x, ϕ_y	nondimensional functions of the spatial coordinates that define the mean flow
ω_x, ω_y	natural frequencies, respectively, in the x and y directions of vibration