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# Stochastic hygrothermoelectromechanical loaded post buckling analysis of piezoelectric laminated cylindrical shell panel

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**Abstract.** The present work deals with second order statistics of post buckling response of piezoelectric laminated composite cylindrical shell panel subjected to hygro-thermo-electro-mechanical loading with random system properties. System parameters such as the material properties, thermal expansion coefficients and lamina plate thickness are assumed to be independent of the temperature and electric field and modeled as random variables. The piezoelectric material is used in the forms of layers surface bonded on the layers of laminated composite shell panel. The mathematical formulation is based on higher order shear deformation shell theory (*HSDT*) with von-Karman nonlinear kinematics. A efficient $C^0$ nonlinear finite element method based on direct iterative procedure in conjunction with a first order perturbation approach (*FOPT*) is developed for the implementation of the proposed problems in random environment and is employed to evaluate the second order statistics (mean and variance) of the post buckling load of piezoelectric laminated cylindrical shell panel. Typical numerical results are presented to examine the effect of various environmental conditions, amplitude ratios, electrical voltages, panel side to thickness ratios, aspect ratios, boundary conditions, curvature to side ratios, lamination schemes and types of loadings with random system properties. It is observed that the piezoelectric effect has a significant influence on the stochastic post buckling response of composite shell panel under various loading conditions and some new results are presented to demonstrate the applications of present work. The results obtained using the present solution approach is validated with those results available in the literature and also with independent Monte Carlo Simulation (*MCS*).

**Keywords:** piezoelectric laminated cylindrical shell panel; random system properties; hygrothermoelectromechanical loading; finite element method

## 1. Introduction

The polymer-matrix fiber reinforced laminated composites are increasingly used in many engineering applications such as automotive, mechanical, structural and aerospace engineering industries due to its outstanding mechanical properties such as high strength and stiffness to weight ratio, very high fatigue characteristics, excellent corrosion resistance, tailoring ability of mechanical properties and high structural efficiency and durability.

In the recent research and development of smart materials embedded in composite layers have promised new design opportunities for future high performance for mechanical and structural systems

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and probably the most popular active material used in both sensor and actuator due to its properties of inherent capability to accurate measurements of traditional performance. Their buckling loads are of utmost importance in the design and development of high performance composite mechanical components. However, the composites display significantly variability in their material, geometric and structural properties due to complexity involved in its manufacturing and fabrication processes. This variability is attributed to the variations in the properties of fibers matrices and interfaces, fiber orientation, void content, and ply thickness and so on. These variables are statistical in nature. As a result, the buckling loads of geometrically nonlinear composite cylindrical shell panels become stochastic variables and should be quantify probabilistically. The statistics (mean and variance) of buckling load subjected to various types of loads are needed for accurate prediction of structural response for safe and reliable design. The capability to predict accurate structural response and enable better understanding and characterization of actual behavior of laminated composite structure in the presence of piezoelectric layers when subjected to hygro-thermo-electro-mechanical loading is one of the prime interests to structural analysis. One of the important problems deserving special attention for accurate prediction of structural response in sensitive applications is the study of their stability to geometrically non linear cylindrical shell panel considering system properties as random.

The considerable volume of literature are available on the initial and post buckling analysis of laminated beam, plate and shell panel with and without piezoelectric layers under various thermal, electrical and/or mechanical loading conditions. Numerous studies on the modeling and analysis of hybrid laminated cylindrical shells have been performed; these studies were mainly focused on buckling and/or vibration analysis subjected to various environmental conditions (for example, Tauchert *et al.* 1992, Xu and Noor 1996, Tzou and Bao 1997, Oh *et al.* 2000, Chattopadhyay *et al*. 2000, Tauchert *et al*. 2000, Lee and Saravanos 2000, Shen and Li 2002, Görnandt and Gabbert 2002, Kulkarni and Bajoria 2003, Ganesan and Kadoli 2003, Ray and Mallik 2004, Varelis and Saravanos 2004, Lee *et al*. 2004, Oh 2005, Kundu *et al*. 2007, Kundu and Han 2009, Alibeigloo and Madoliat 2009, Panda and Singh 2009, Akhras and Li 2010, Roy *et al.* 2010, and Shen 2010).

All the above literatures are based on the assumptions of complete determinacy of structural parameters. In the deterministic analysis of the structure, the variation in the random system parameters are neglected and mean value systems parameters are used in the analysis. Due to dependency of large number of parameters in the complex manufacturing and fabrication process of laminated composite panel as stated earlier, the system properties are statistical in nature resulting uncertainties in the response of the shell panel. Therefore, for accurate prediction the response, the response should be quantified probabilistically as the uncertainties in the system properties lead to uncertainties in the response behavior of the structure.

Relatively little efforts have been made in the past by researchers and investigators on the prediction of post buckling response of the laminated composite panel supported with and without foundation subjected to hygrothermomechanical loading acting simultaneously of individual with and without piezoelectric layers having random system properties. A detailed review for the studies related to randomness in system parameters are presented here. Chen *et al*. (1992) outlined the probabilistic method to evaluate the effects of uncertainties in geometric and material properties using semi-analytical based on *HSDT*. Englested and Reddy (1994) contributed metal matrix composites based on probabilistic micro mechanics non-linear analysis using semi-analytical based on *HSDT*. Kaminski and Kleiber (2000) developed the stochastic second order and second moment perturbation analysis for homogenization of the two-phase periodic composite structure using stochastic finite element method. Singh *et al.* (2001, 2001, 2002) analyzed the effects of random material properties on the elastic stability of laminated cylindrical panels using $C^0$ nonlinear finite element method in conjunction with *FOPT* based *HSDT*. Onkar *et al*. (2006, 2007) have used a generalized layer wise stochastic finite element formulation conjunction with *FOPT* based on *FSDT* to evaluate the second order statistics of initial buckling analysis of homogeneous and laminated plates with random material properties. Effect of random system properties on initial and post buckling of composite panels supported with and without elastic foundation in hygrothermomechanical environmental condition acting simultaneously or individually using direct iterative based stochastic nonlinear finite element method is studied by Lal *et al*. (2008, 2009, 2009, 2010) and Singh and Lal (2010). Thermal environmental effects on the buckling of laminated composite plates with random geometric and temperature independent material properties using stochastic finite element method based on *HSDT* are investigated by Singh and Verma (2009). Pandit *et al*. (2009) presented the stochastic perturbation technique for the analysis of composite plate considering randomness in system properties using stochastic finite element method based on layer wise theory.

Relatively very little work is available on hygrothermal initial buckling analysis of the structures made of composites with random system parameters using macro-mechanical approach by Singh and Verma (2009). Based on the higher order theory, Singh and Babu (2009) examined the sensitivity of randomness in material parameters on the thermal buckling of conical shells embedded with and without piezoelectric layer, where the first order perturbation technique employed to handle the randomness in the material properties. Recently, Lal *et al*. (2011) investigated the post buckling of the laminated composite shell panel subjected to hygrothermomechanical loading using direct iterative based stochastic finite method based on *HSDT* with von-Karman nonlinearity.

However, to the best of authors' knowledge, no literature covering the second order statistics of post buckling load of piezoelectric laminated cylindrical shell panel subjected to hygrothermoelectromechanical loadings with system randomness using an efficient $C^0$ nonlinear finite element method based on direct iterative based technique in conjunction with mean centered *FOPT* is reported.

In the present study, the post buckling load response of hygrothermoelectromechanical loaded piezoelectric laminated composite cylindrical shell panel in the presence of small random variations in the system parameters taking into account the transverse shear strain using the *HSDT* with von-Karman nonlinear strain displacement relations is studied. A direct iterative based $C^0$ nonlinear *FEM* in conjunction with the mean centered *FOPT* as developed by the authors is extended and employed to solve the generalized eigenvalue problems. Numerical illustration concerned with the effect of environmental conditions, amplitude ratios, piezoelectric layers, electrical voltages, shell thickness ratios, aspect ratios, curvature to side ratios, boundary conditions, lamination schemes and types of loadings with the variation of random system properties. It is observed that small amount of random system properties variations in the hygrothermoelectromechanical load of piezoelectric laminated composite laminated cylindrical shell panel significantly affects the post buckling load. The proposed probabilistic procedure would be valid for system properties with small random dispersion compared to their mean values. Fortunately, most of the engineering materials such as composite fall in this category.

## 2. Formulation

### 2.1 Geometrical consideration of cylindrical shell

The piezoelectric laminated cylindrical shell panel having length $a$, width $b$, thickness $h$, radius of

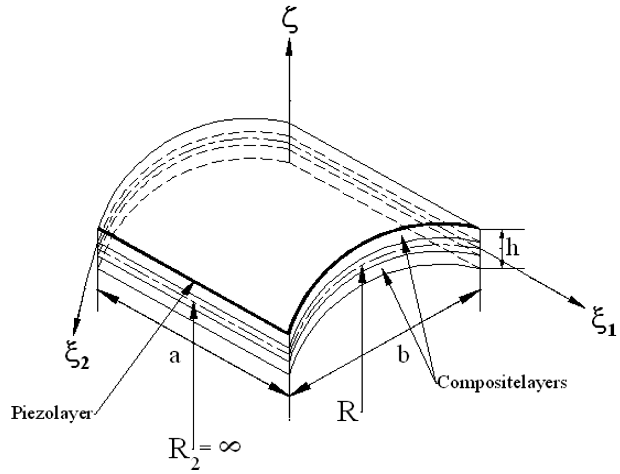


Fig. 1 Geometry of piezolaminated composite cylindrical shell panel

curvature of middle surface $R_1$ and $R_2$, located in the three dimensional curvilinear coordinate system ($\xi_1$, $\xi_2$, $\zeta$) is shown in Fig. 1. Where, $\xi_1$ and $\xi_2$ are in the axial and circumferential directions of shell panel and $\zeta$ is normal to the mid surface of shell panel, respectively. The shell panel consists of $N$ plies in which, the upper ply is piezoelectric layer bonded on the composite laminates. The panel is considered as cylindrical shell type when $R_2$ is taken as infinite. It is assumed that a perfect bonding exists between the layers so that no slippage can occurs at the interface and the strains experienced by the fiber, matrix and composite are equal. The fibers are assumed to be uniform in properties and diameter continues and parallel throughout the composite. It is also assumed that the composite behaves like homogeneous composite materials and the effects of the average constituent's materials (i.e., matrix and fiber) are detected simultaneously.

### 2.2 Displacement field model

In the present study, the assumed displacement is based on Reddy's higher order shear deformation shell theory Reddy and Liu (1985), which requires $C^1$ continuous approximation. The complexity and difficulty involved with making a choice of $C^1$ continuity (for solving partial differential equations) are well known. In order to avoid usual difficulties associated with these elements; the displacement field model has been slightly modified to make it suitable for $C^0$ continuous element. The $C^0$ continuous element permits easy isoperimetric finite element formulation and consequently can be applied for nonrectangular geometries as well. Thus five *DOFs* with $C^1$ continuity can be increased to 7 *DOFs* with $C^0$ continuity due to conformity with *HSDT*. In this change, the artificial constraints are imposed, which can be enforced variationally through a penalty approach, in order to satisfy the constraint emphasized. However, the literature Shankara and Iyenger (1996) demonstrated that the accurate results with $C^0$ continuity can be obtained without accounting for the energy due to these artificial constraints.

The modified displacement field along $\xi_1$, $\xi_2$ and $\zeta$ directions for arbitrary laminated composite cylindrical shell panel are now written as

$$\bar{u} = u + f_1(\zeta)\phi_1 + f_2(\zeta)\theta_1$$

$$\bar{v} = \left(1+\frac{\zeta}{R}\right)v + f_2(\zeta)\phi_2 + f_2(\zeta)\theta_2$$

$$\bar{w} = w \tag{1}$$

Where $\theta_1 = w,_{\xi_1}$ and $\theta_1 = w,_{\xi_2}$ and (,) denotes partial differential. $\bar{u}$, $\bar{v}$ and $\bar{w}$ denote the displacements of a point along the ($\xi_1$, $\xi_2$, $\zeta$) coordinate axes and $u$, $v$, and $w$ are the corresponding displacements of a point on the mid plane, $\phi_1$ and $\phi_2$ are the rotations at $\zeta = 0$ of normal to the mid-surface with respect to $\xi_1$ and $\xi_2$ axes, respectively. Here, $\theta_1$ and $\theta_2$ are the slope along $\xi_1$ and $\xi_2$ axes, respectively. The function $f_1(\zeta)$ and $f_2(\zeta)$ given in Eq. (1) can be written as

$$f_1(\zeta) = C_1\zeta - C_2\zeta^3 \text{ and } f_2(\zeta) = -C_4\zeta^3,\ C_1 = 1 \text{ and } C_2 = C_4 = 4/3h^2 \tag{2}$$

The displacement vector for the modified $C^0$ continuous model can be written as

$$q = (u \ \ v \ \ w \ \ \theta_2 \ \ \theta_1 \ \ \phi_2 \ \ \phi_1) \tag{3}$$

### *2.3 Strain displacement relations*

For the cylindrical shell panel considered here, the relevant strain vectors consisting of linear strain (in term of mid plane deformation rotation of normal and higher order terms), non-linear strain (von-Karman type), hygrothermal and piezoelectric strains vectors associate with the displacement for lamina layer are expressed as

$$\{\bar{\varepsilon}_{ij}\} = \{\bar{\varepsilon}_{ij}^{L}\} + \{\bar{\varepsilon}_{ij}^{NL}\} - \{\bar{\varepsilon}_{ij}^{TH}\} - \{\bar{\varepsilon}_{ij}^{V}\} \quad (i, j = 1,2,...,6) \tag{4}$$

Where $\{\bar{\varepsilon}_{ij}^{L}\}$, $\{\bar{\varepsilon}_{ij}^{NL}\}$, $\{\bar{\varepsilon}_{ij}^{TH}\}$ and $\{\bar{\varepsilon}_{ij}^{V}\}$ are the linear, non-linear, hygrothermal and piezoelectric strain vectors, respectively.

From Eq. (4) linear strain tensor using *HSDT* can be rewritten as (Singh *et al*. 2009).

$$\bar{\varepsilon}_{ij}^{L} = H_{mn}\varepsilon_{kl}^{L} \tag{5}$$

where, $H_{mn}$ is the function of $Z$ and unit step vector and $\varepsilon_{kl}^{L}$ is reference plain linear strain tensor defined as

$$\varepsilon_{kl}^{L} = \{\varepsilon_1^0 \ \ \varepsilon_2^0 \ \ \varepsilon_6^0 \ \ k_1^0 \ \ k_2^0 \ \ k_6^0 \ \ k_1^2 \ \ k_2^2 \ \ k_6^2 \ \ \varepsilon_4^0 \ \ \varepsilon_5^0 \ \ k_4^2 \ \ k_5^2\}^T \tag{6}$$

Assuming that the strains are much smaller than the rotations (in the von-Karman sense), one can rewrite nonlinear strain vector $\{\bar{\varepsilon}_{ij}^{NL}\}$ given in Eq. (4) as (Lal *et al*. 2009).

$$\bar{\varepsilon}_{ij}^{NL} = \frac{1}{2}[A_{nij}]\{\phi_{nij}\} \tag{7}$$

where $A_{nij} = \frac{1}{2}\begin{bmatrix} w,_s & 0 \\ 0 & w,_\theta \\ w,_\theta & w,_s \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\phi_{nij} = \begin{Bmatrix} w,_s \\ w,_\theta \end{Bmatrix}$

The hygrothermal strain vector $\{\bar{\varepsilon}_{ij}^{TH}\}$ given in Eq. (4) is represented as Lal *et al*. (2011).

$$\{\bar{\varepsilon}_{ij}^{TH}\} = \begin{Bmatrix} \alpha_{\xi_1}\Delta T + \beta_{\xi_1}\Delta C \\ \alpha_{\xi_2}\Delta T + \beta_{\xi_2}\Delta C \\ 0 \\ 0 \\ 0 \end{Bmatrix}; \quad (i, j = 1,2,..., 6) \tag{8}$$

Note that suffices 1 and 2 denote components along $\xi_1$ and $\xi_2$ directions, respectively.

Here, $\alpha_{\xi_1}$, $\alpha_{\xi_2}$ and $\beta_{\xi_1}$, $\beta_{\xi_2}$ are coefficients of thermal expansion and hygroscopic contraction along the $\xi_1$ and $\xi_2$ directions, respectively, which can be obtained from the thermal expansion and contraction coefficients due to temperature and moisture in the longitudinal ($\alpha_1$, $\beta_1$) and transverse ($\alpha_2$, $\beta_2$) directions of the fibers using transformation matrix and $\Delta T$ and $\Delta C$ are the change in temperature and moisture rise in the shell panel subjected with uniform temperature rise and moisture rise which can be defined as $\Delta T=T-T_0$ and $\Delta C=C-C_0$ (Chamis and Sinclai 1982, Chamis 1987, Singh and Verma 2009, Singh and Babu 2009, Singh *et al*. 2009, Pandey *et al*. 2010), where ($T_0$, $C_0$) and ($T$, $C$) are the uniform temperature and moisture rise at room temperature and at a given temperature and moisture, respectively. For the generated results in tables the values of $T_0$ and $C_0$ are taken as 25°C and $C_0$=0%.

The electric field vector $\{\bar{\varepsilon}_{ij}^{V}\}$ as given in Eq. (4) can be represented as (Shen 2002).

$$\{\bar{\varepsilon}_{ij}^{V}\} = \frac{V_k}{t_k}\begin{Bmatrix} d_{31} \\ d_{32} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

where $d_{31}$ and $d_{32}$ are the piezoelectric strain constants of a single ply along the $\xi_1$ and $\xi_2$ direction, respectively, which can be obtained from the electric coefficients in the longitudinal and transverse directions of the fibers using transformation matrix, here $V_k$ and $t_k$ are applied voltage across the *kth* ply and thickness of the piezoelectric layer, respectively in the shell panel subjected with uniform electric field rise.

### 2.4 Constitutive relations

The constitutive law of thermo-elasticity for material under consideration relates the stresses with strains in a in-plane stress state for the $k^{th}$ orthotropic lamina of a laminate consisting of $N$ layers, having fibers oriented in any arbitrary orientation with respect to the reference axes Shen (2002, 2010) is expressed as

$$\bar{\sigma}_{ij} = \bar{C}_{ijkl}\bar{\varepsilon}_{ij} - e_{kl}E_k \ \ (i, j = 1,2,..., 6) \text{ and } k, l = 1,2,3) \tag{10}$$

Where, $\overline{C}_{ijkl}$, $e_{kl}$ are transform reduced elastic constant matrix of shell material, matrix of piezoelectric coefficients and electric field vectors, respectively.

The electric displacement vector of the *kth* piezoelectric layer is given by Shen (2002, 2010).

$$D_k = e_{kj}(\bar{\varepsilon}_{ij}^{L}+\bar{\varepsilon}_{ij}^{NL})+\xi_{kl}E_l+p_k\Delta T \ \ (i, j = 1,2,..., 6) \text{ and } k, l = 1,2,3) \tag{11}$$

Where $\xi_{kl}$ and $p_k$ is the dielectric coefficient matrix and the pyroelectric constants vector, respectively (Akhras and Li 2010, Shen 2002, 2010).

From Eq. (10), the electric field vector $E_k$ is calculated based on the gradient of the electric potential $\varphi_p$ can be written as

$$E_k = H_{mn\phi}\, E_{kl} \tag{11a}$$

where, $H_{mn\phi}$ is the function of $z$ and unit step vector of piezoelectric layers and $\varepsilon_{kl}^{\phi}$ is reference plain piezoelectric strain tensor defined as

$$E_{kl} = \left\{E_{\xi_1}^{(0)} \quad E_{\xi_2}^{(0)} \quad E_{\xi_1}^{(1)} \quad E_{\xi_2}^{(1)} \quad E_{\xi_1}^{(2)} \quad E_{\xi_2}^{(2)} \quad X^{(1)} \quad X^{(2)}\right\}^T \tag{11b}$$

where, $E_{\xi_1}$, $E_{\xi_2}$ and $X$ are electric field vector along $s$, $\theta$ and $z$ direction, respectively,

### *2.5 Strain energy*

The strain energy of the piezoelectric laminated composite cylindrical shell panel subjected to hygrothermoelectromechanical loads undergoing large deformation can be expressed as Tzou and Tseng (1991).

$$\Pi_1 = \Pi_a - \Pi_b \tag{12}$$

Where $\Pi_a$ and $\Pi_b$ are the strain energy of laminated cylindrical shell panel and piezoelectric layer, respectively.

From Eq. (12), the strain energy ($\Pi_a$) of the laminated cylindrical shell panel can be expressed as

$$\Pi_a = U_L + U_{NL} \tag{13}$$

From Eq. (13), the linear stain energy ($U_L$) of the cylindrical shell panel is given by

$$U_L = \int_{\Omega} \frac{1}{2}\overline{C}_{ijkl}\ \bar{\varepsilon}_{ij}^{L}\bar{\varepsilon}_{kl}^{L} d\Omega = \int_{\Delta} \frac{1}{2}\varepsilon_{ij}\ D_{mn}\ \varepsilon_{kl} d\Delta \tag{14}$$

where $D_{mn}$ is the laminate elastic stiffness matrix and $\overline{C}_{ijkl}$ is reduced elastic material constant as defined earlier.

From Eq. (14), the nonlinear strain energy ($U_{NL}$) of the cylindrical shell panel can be rewritten as

$$U_{NL} = \int_{\Omega} \frac{1}{2}\bar{\varepsilon}_{ij}^{L}\ D_3\bar{\varepsilon}_{ij}^{NL} d\Omega + \frac{1}{2}\int_{\Omega}\bar{\varepsilon}_{kl}^{L}\ D_4\bar{\varepsilon}_{ij}^{L} d\Omega + \frac{1}{2}\int_{\Omega}\bar{\varepsilon}_{kl}^{NL}\ D_5\bar{\varepsilon}_{ij}^{NL} d\Omega \quad (i, j, k, l = 1,2,3) \tag{15}$$

Where $\Omega$, $\bar{\varepsilon}_{ij}^{L}$ and $\bar{\varepsilon}^{NL}$ denotes undeformed configuration of cylindrical shell, linear and nonlinear strain tensors, respectively.

Using Eq. (7), the Eq. (15) can be expressed as

$$U_{NL} = \frac{1}{2}\int_{\Omega} A_{nij}\phi_{nij}\, D_3 \bar{\varepsilon}_{ij}^{L}\, d\Omega + \frac{1}{2}\int_{\Omega} \bar{\varepsilon}_{ij}^{L}\, D_4 A_{nkl}\phi_{nkl} d\Omega \; + \frac{1}{2}\int_{\Omega} A_{nij}\phi_{nij}\, D_5 A_{nkl}\phi_{nkl}\, d\Omega \; (i, j, k, l = 1,2,3) \quad (16)$$

where $D_3$, $D_4$ and $D_5$ are the laminate stiffness matrices of the cylindrical shell panel.

Similarly from Eq. (12), the strain energy ($\Pi_b$) due to piezoelectric layers can be rewritten as

$$\Pi_b = U_{\phi L} + U_{\phi NL} \quad (17)$$

where, $U_{\varphi L}$ and $U_{\varphi NL}$ are the linear and nonlinear strain energy of piezoelectric layers.

From Eq. (15(a)), linear strain energy ($U_{\varphi L}$) due to piezoelectric layer can be expressed as

$$U_{\phi L} = \int_{\Omega} \frac{1}{2} \varepsilon_{kl}^{L} D_{1P} E_{ij}^{\phi} d\Omega + \int_{\Omega} \frac{1}{2} E_{kl}^{\phi} D_{1P} \varepsilon_{ij}^{L} d\Omega + \int_{\Omega} \frac{1}{2} k_{ij}^{\phi} D_{2P} E_{ij}^{\phi} d\Omega \quad (i, j, k, l = 1,2,3) \quad (18)$$

where, $D_{1P}$ and $D_{2P}$ are the linear piezoelectric stiffness matrices, respectively.

From Eq. (17), nonlinear strain energy ($U_{\varphi NL}$) due to piezoelectric layer can be expressed as

$$U_{\phi NL} = \frac{1}{2}\int_{\Omega} \{\phi_p\}^T \{A_{nij}\}^T D_6 \{L_{\phi}\}\{\phi_{nij}\} d\Omega + \frac{1}{2}\int_{\Omega} \{\phi_{nij}\}^T \{L_{\phi}\}^T D_7 \{A_{nij}\}^T \{\phi_p\} d\Omega \quad (19)$$

where, $D_6$, $D_7$ are the nonlinear piezoelectric stiffness matrices.

The potential energy ($\Pi_2$) storage due to hygrothermoelectromechanical loads (uniform change in temperature, moisture and electrical potential) is written as

$$\Pi_2 = \frac{1}{2}\int_{A} [N_{xMTHE}(w_{,x})^2 + N_{yMTHE}(w_{,y})^2 + 2N_{xyMTHE}(w_{,x})(w_{,y})] dA \quad (20)$$

Where, $N_{xMTHE}$, $N_{yMTHE}$ and $N_{xyMTHE}$ are the pre-buckling hygrothermoelectromechanical in-plane stresses along x, y and shear directions, respectively with $N_{xMTHE} = N_0^M - (N_0^T + N_0^V + N_0^H)$. The symbols $M$, $T$, $H$ and $V$ represent the mechanical, thermal, hygro and electrical, respectively.

### *2.6 Finite element model*

In the present study, a $C^0$ nine-node isoparametric finite element with 7 *DOFs* per node is employed. For this type of element, the displacement vector and the element geometry are expressed as

$$q = \sum_{i=1}^{NN} N_i q_i; \quad \xi_1 = \sum_{i=1}^{NN} N_i \xi_{1i}; \text{ and } \xi_2 = \sum_{i=1}^{NN} N_i \xi_{2i} \quad (21)$$

Where $N_i$ and $q_i$ are the interpolation function and vector of unknown displacements for the *ith* node, respectively, *NN* is the number of nodes per element and $\xi_{1i}$ and $\xi_{2i}$ are Cartesian coordinate of the *i*th node.

The linear mid plane strain vector as given in Eq. (6) can be expressed in terms of mid plane displacement field and then the energy is computed for each element and then summed over all the elements to get the total strain energy (Lal *et al.* 2009).

Following this and using finite element model as given in Eq. (21), Eq. (13) after summed over all the elements can be written as

$$\Pi_a = \sum_{e=1}^{NE} \Pi_a^{(e)} = \sum_{e=1}^{NE} (U_L^{(e)} + U_{NL}^{(e)}) \tag{22}$$

Where, $NE$ is the number of elements and $\Pi_a^{(e)}$ is the elemental potential energy of the shell panel.

Substituting Eqs. (14) and (16) in Eq. (22), The Eq. (22) can be further expressed as

$$\Pi_a = \sum_{e=1}^{NE} \left[ \{q_i\}^{T(e)} \left[ K_{l_{ij}} + K_{nl_{ij}}(q_j) \right]^{(e)} \{q_j\}^{(e)} \right] = \{q_i\}^T \left[ K_{l_{ij}} + K_{nl_{ij}}(q_j) \right] \{q_j\} \tag{23}$$

with $[K_{nl_{ij}}] = \frac{1}{2}[K_{nl_{1ij}}] + [K_{nl_{2ij}}] + \frac{1}{2}[K_{nl_{3ij}}]$

where $[K_{l_{ij}}]$, $[K_{nl_{ij}}]$ and $\{q_i\}$ are defined as global linear, nonlinear stiffness matrices of shell panel and global displacement vector, respectively.

Using finite element model Eq. (21), Eq. (17) after summing over the entire element can be written as

$$\Pi_b = \sum_{e=1}^{NE} \Pi_b^{(e)} = \sum_{e=1}^{NE} (U_{\phi L}^{(e)} + U_{\phi NL}^{(e)}) \tag{24}$$

where $\Pi_b^{(e)}(U_{\phi L}^{(e)} + U_{\phi NL}^{(e)})$ is the elemental linear and nonlinear potential energy of the piezoelectric layers.

Using Eqs. (18) and (24) for linear potential energy for piezoelectric layer can be expressed as

$$\begin{aligned} \Pi_{bl} &= \sum_{e=1}^{NE} \left( \{q_i\}^{(e)T} \left[ K_{1lp_{ij}} \right]^{(e)} \{q_{\phi i}\}^{(e)} + \{q_{\phi i}\}^{(e)T} \left[ K_{1lp_{ij}} \right]^{(e)} \{q_i\}^{(e)} + \{q_{\phi i}\}^{(e)T} \left[ K_{2lp_{ij}} \right]^{(e)} \{q_{\phi i}\}^{(e)} \right) \\ &= \{q_i\}^T \left[ K_{1lp_{ij}} \right] \{q_{\phi i}\} + \{q_{\phi i}\}^T \left[ K_{1lp_{ij}} \right] \{q_i\} + \{q_{\phi i}\}^T \left[ K_{2lp_{ij}} \right] \{q_{\phi i}\} \end{aligned} \tag{25a}$$

Similarly, using and Eqs. (19) and (24) for nonlinear potential energy for piezoelectric layer can be expressed as

$$\begin{aligned} \Pi_{bnl} &= \frac{1}{2} \sum_{e=1}^{NE} \left( \{q_i\}^{(e)T} \left[ K_{1nlp_{ij}} \right]^{(e)} \{q_{\phi i}\}^{(e)} + \{q_{\phi i}\}^{(e)T} \left[ K_{2nlp_{ij}} \right]^{(e)} \{q_i\}^{(e)} \right) \\ &= \{q_i\}^T \left[ K_{1nllp_{ij}} \right] \{q_{\phi i}\} + \{q_{\phi i}\}^T \left[ K_{2nllp_{ij}} \right] \{q_i\} \end{aligned} \tag{25b}$$

where $[K_{1lp_{ij}}]$, $[K_{2lp_{ij}}]$, $[K_{1nllp_{ij}}]$, $[K_{2nllp_{ij}}]$ and $\{q_{\phi i}\}$ are defined as global linear coupling matrix between elastic mechanical and electrical effects, linear dielectric stiffness matrix, nonlinear coupling matrices between elastic mechanical and electrical effects and global electric field vector, respectively.

Using finite element model Eq. (21), Eq. (20) after summing over the entire element can be written as

$$\Pi_2 = \sum_{e=1}^{NE} \Pi_2^{(e)} = \frac{1}{2}\sum_{e=1}^{NE} \{q_i\}^{T(e)} \lambda \left[K_{(G)ij}\right]^{(e)} \{q_j\}^{(e)} = \frac{1}{2}\lambda \{q_i\}^T \left[K_{(G)ij}\right]\{q_j\} \tag{26}$$

Where, $\lambda$ and $[K_{(G)_{ij}}]$ are defined as the hygrothermoelectromechanical buckling load parameters and global geometric stiffness matrix (arises due to hygrothermoelectromechanical loadings), respectively.

Adopting Gauss quadrature integration numerical rule, the linear, non linear shell panel stiffness matrices, linear coupling, dielectric stiffness matrix, nonlinear coupling matrices and displacement and dielectric load vectors, respectively can be obtained by transforming expression in $\xi_1$, $\xi_2$ coordinate system to natural coordinate system $\xi$, $\eta$.

It is evident from the finite element formulation presented in the previous paragraphs that the model does not include a full coupling model among electrical, mechanical and thermal fields.

## 2.7 Governing equation

The governing equation for hygrothermoelectromechanical induced post buckling load of piezoelectric laminated cylindrical shell panel can be derived using Variational principle, which is generalization of the principle of virtual displacement. For the displacement field of the buckling, the minimization of $\Pi(\Pi_a+\Pi_{bl}+\Pi_{bnl}+\Pi_2)$ with respect to generalized displacement vector and after simplification for critical post buckling state corresponding to the neutral equilibrium using condition of minimization of second variation of total potential energy is zero, using Eqs. (23), (25(a)), (25(b)) and (26), the standard eigenvalue problem can be represented as (Reddy 1984).

$$[K_{ij}]\{q_i\} = \lambda_i[K_{(G)_{ij}}]\{q_i\} \tag{27}$$

where, $[K_{ij}] = \{[K_{L_{ij}}]+[K_{NL_{ij}}]\}$ with $\left[K_{L_{ij}}\right]=\frac{1}{2}\left[K_{l1_{ij}}\right]-\frac{1}{2}\left[K_{lp1_{ij}}\right]-\frac{1}{2}\left[K_{lp2_{ij}}\right]$

and $\left[K_{NL_{ij}}\right]=\frac{1}{2}\left[K_{nl1_{ij}}\right]+\left[K_{nl2_{ij}}\right]+\frac{1}{2}\left[K_{nl3_{ij}}\right]-\frac{1}{2}\left[K_{1nlp_{ij}}\right]-\frac{1}{2}\left[K_{2nlp_{ij}}\right]$

The plate stiffness matrix $[K_{ij}]$ consists of linear and nonlinear stiffness matrices; elective field stiffness, coupling matrix between elastic mechanical and electrical effects, dielectric stiffness matrix and geometric stiffness matrix, respectively are random in nature, being dependent on the system properties of the structure. Consequently, the eigenvalue and eigenvectors obtained by Eq. (27) are random in nature. In deterministic environment, the solution of Eq. (27) can be obtained using standard solution procedure such as direct iterative, incremental and/or Newton-Raphson method etc. However, in random environment it is not possible to obtain the solution using above mentioned methods. Further analysis is required to obtain the complete solution of Eq. (27).

For this purpose, the direct iterative finite element method is successfully combined with mean cantered *FOPT* i.e., direct iterative based stochastic finite element method (*DISFEM*), developed by authors (Lal *et al.* 2011) for composite plate is extended for this problem in the present work, which is presented below, to obtain the solution of the random nonlinear governing equation.

## 3. Solution approach

### 3.1 A DISFEM for hygro-thermo-electro-mechanical post buckling problems

The nonlinear eigenvalue problem as given in Eq. (27) is solved by employing a *DISFEM* assuming that the random changes in eigenvector during iterations does not affect the nonlinear stiffness matrices with the following steps.

(i) By setting amplitude to zero, the random linear eigenvalue problem $[K_{L_{ij}}]\{q_{L_i}\} = \lambda_i[K_{(G)_{ij}}]\{q_{L_i}\}$ is obtained from Eq. (27) neglecting the nonlinear stiffness matrices. Then the random linear eigenvalue problem is broken up into zeroth and first order equations using perturbation technique. The zeroth order linear eigenvalue problem is solved by normal eigen solution procedure to obtain the linear critical load parameters l and the linear eigenvector $\{q_{L_i}\}$ The first order perturbation equation is used to obtain the standard deviation of the hygrothermoelectromechanical post buckling load which is presented in next sub-section of perturbation technique. The computation of buckling load and eigenvector under initial hygrothermoelectromechanical stress condition for the zeroth order deterministic eigenvalue problem requires as following.

A linear hygrothermoelectromechanical problem by assuming nonlinear stiffness matrix as zero is solved first for the reference hygrothermoelectromechanical load $\{q_i\}$ for a given constraints of the shell panel. The linear solution is used for computing the initial hygrothermoelectromechanical stress tensor $\overline{\sigma}_{ij}$. The hygrothermoelectromechanical stress tensor $\overline{\sigma}_{ij}$ at each integration point is used to compute the geometric stiffness matrix. After the geometric stiffness matrix is computed, the system stiffness matrix is modified by geometric stiffness matrix to solve the deterministic eigenvalue problem using proposed by Eq. (27). This is generalizes eigenvalue problem where $[K_{ij}]$ and $[K_{(G)_{ij}}]$ are symmetric matrices and are generally found to be positive definite. In some cases $[K_{(G)_{ij}}]$ can be positive semi-definite which can overcome by shifting invert transformation. In such cases the right most eigenvalue gives the minimum value of the mean post buckling load parameter. The critical or minimum mean post buckling load of the structure is obtained by multiplying the load parameter $\lambda_i$ with reference load (consists of hygrothermoelectromechanical).

(ii) For a specified maximum deflection *C* at a centre of the shell panel, the linear normalized eigenvector is scaled up by *C* times, so that resultant vector will have a displacement *C* at the maximum deflection point.

(iii) Using the scale-up eigenvector, the nonlinear terms in the stiffness matrix $[K_{NL_{ij}}]$ can be obtained. The problem may now be treated as a linear eigenvalue problem with new updated stiffness matrices. The random eigenvalue problem can again be broken up into zeroth and first order equation using perturbation technique. The deterministic zeroth order can be used to obtain critical post buckling load $\lambda_{NL_i}$ and eigenvector $\{q_{NL_i}\}$ and the random first order equations can be used to obtain the standard deviation (*SD*) of the eigen solutions using the first order perturbation technique as presented in the next section.

(iv) Steps (ii)-(iii) are repeated by replacing $\{q_{L_i}\}$ by $\{q_{NL_i}\}$ in the step (ii) to obtain the converged mean and standard deviation of the nonlinear critical buckling load $\lambda_{NL_i}$ to a prescribed accuracy ($10^{-3}$)

(v) Steps (i) to (iv) are repeated for various value of *C*.

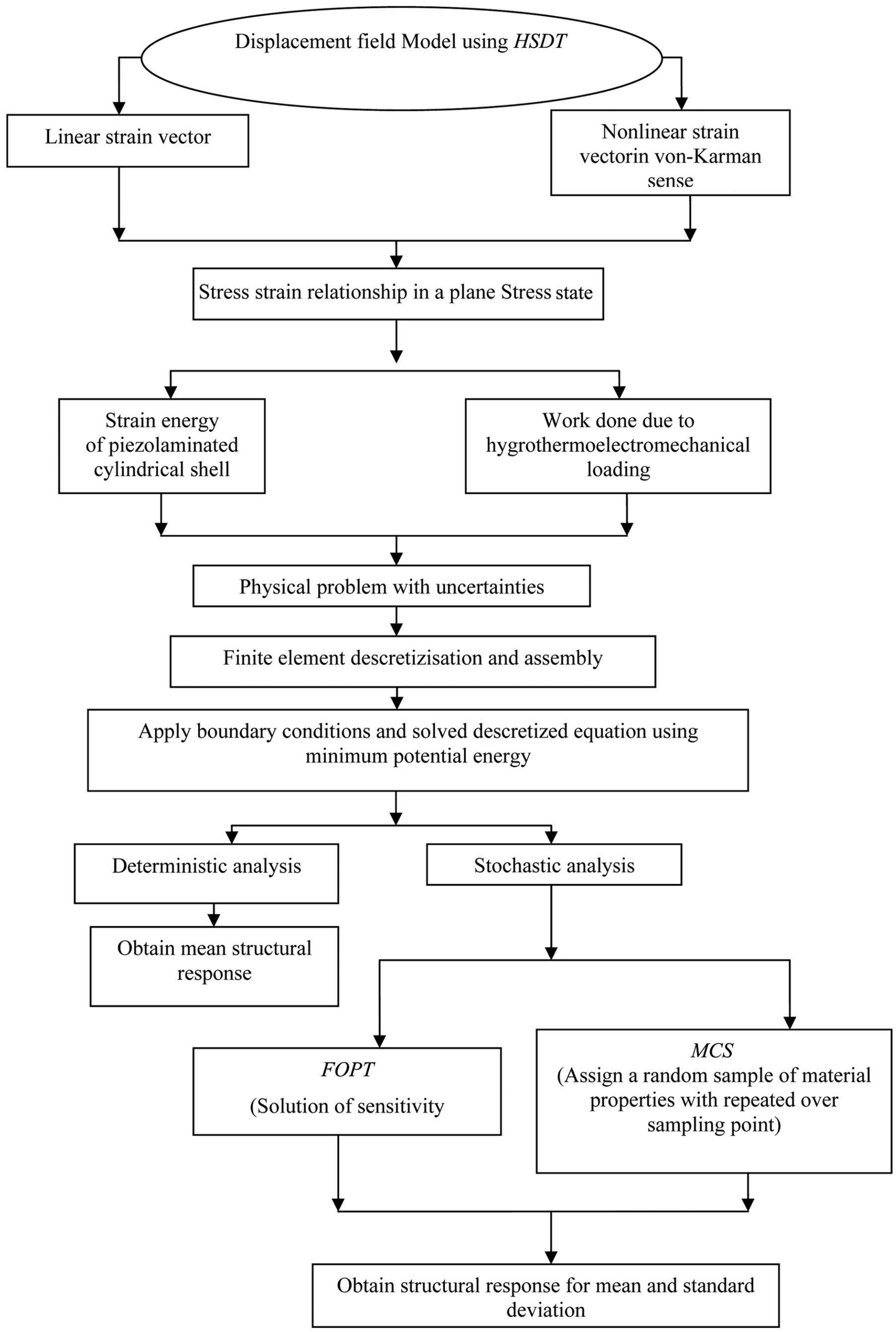


Fig. 2 Schematic of overview of present study

### *3.2 Solution: perturbation technique*

Consider a class of problems where the random variation is very small as compared to the mean part of random system properties. Further it is quite logical to assume that the dispersions in the derived quantities like $[K_{ij}]$, $\lambda_i$ etc. are also small with respect to their mean values. In the present analysis, the lamina material properties, thermal expansion coefficients and the geometric properties (lamina shell panel thickness) are treated as independent random variables (*RVs*). However, the formulation can be easily extended for even dependent random variables. The summery of the schematic overview of present study is shown by Fig. 2.

In general, without any loss of generality any arbitrary random variable can be represented as the sum of its mean and zero mean random part, denoted by superscripts 'd' and 'r', respectively (Lal *et al.* 2009).

$$\left[K_{ij}\right]=\left[K_{ij}^{d}\right]+\left[K_{ij}^{r}\right],\ \left[K_{(G)_{ij}}\right]=\left[K_{(G)_{ij}}^{d}\right]+\left[K_{(G)_{ij}}^{r}\right],\ \lambda_i=\lambda_i^{d}+\lambda_i^{r},\ \{q_i\}=\left\{q_i^{d}\right\}+\left\{q_i^{r}\right\} \tag{28}$$

where, $[K_{ij}^{d}]$ and $[K_{(G)_{ij}}^{d}]$ are the mean elastic (linear and nonlinear) and geometric stiffness matrices of the structures, respectively. Correspondingly $[K_{ij}^{r}]$ and $[K_{(G)_{ij}}^{r}]$ are the first order derivatives of elastic and geometric (arises due to shell panel thickness) matrices, respectively with respect to the $r^{\text{th}}$ basic random variables (*BRV*) with "*R*" is the total number of random input variables chosen for the analysis.

By substituting Eq. (28) in Eq. (27) and expanding it using Taylor's series expansion keeping up to the first order terms and neglecting the second and higher order terms, since first order approximation is sufficient to yield results with the desired accuracy having low variability as in the sensitive applications. After comparing the zeroth and first order terms once obtained as

Zeroth-order perturbation equation

$$\left(\left[K_{ij}^{d}\right]+\lambda_i^{d}\left[K_{(G)_{ij}}^{d}\right]\right)\left\{q_i^{d}\right\}=0 \quad (i=1,2,3,..,n: \textit{no sum over } k) \tag{29}$$

First order perturbation equation

$$\left[K_{ij}^{d}\right]\left\{q_i^{r}\right\}+\left[K_{ij}^{r}\right]\left\{q_i^{d}\right\}=\lambda_i^{d}\left[K_{(G)_{ij}}^{r}\right]\left\{q_i^{d}\right\}+\lambda_i^{d}\left[K_{(G)_{ij}}^{d}\right]\left\{q_i^{r}\right\}+\lambda_i^{r}\left[K_{(G)_{ij}}^{d}\right]\left\{q_i^{d}\right\} \quad (i=1,2,3,..,n: \textit{no sum over } k) \tag{30}$$

Here, Eq. (29) is the deterministic equation relating to the mean eigenvalue and corresponding mean eigenvectors, which can be determined by conventional eigen solution procedures. Eq. (30) is the random equation, defining the stochastic nature of the hygrothermoelectromechanical post buckling which cannot be solved using conventional method. For this a further analysis is required as discussed below.

### *3.3 Variance of post buckling load*

The first order equation is used to obtain the first order partial derivatives of eigenvalue problem with respect to the basic random variables which are then used to the post buckling load covariance.

To obtain the statistics of critical post buckling load, multiply both sides of Eq. (30) by mean eigenvector $\{q_i^{d}\}$ computed from Eq. (29) for minimum mean eigenvalue $\lambda_{cr}^{d}$. This gives

$$\left\{q_i^{d}\right\}\left(\left[K_{ij}^{d}\right]+\lambda_{cr}^{d}\left[K_{(G)_{ij}}^{d}\right]\right)\left\{q_i^{r}\right\}=-\lambda_{cr}^{r}\left(\left\{q_i^{d}\right\}\left[K_{(G)_{ij}}^{d}\right]\left\{q_j^{d}\right\}\right)-\left\{q_i^{d}\right\}\left(\left[K_{ij}^{r}\right]+\lambda_{cr}^{d}\left[K_{(G)_{ij}}^{r}\right]\right)\left\{q_j^{d}\right\} \tag{31}$$

Since both $[K_{ij}^{d}]$ and $[K_{(G)_{ij}}^{d}]$ are symmetric therefore the left hand side of Eq. (31) equals zero by definition of the zeroth order equation. By employing $[K_{(G)_{ij}}^{d}]$ orthonormality conditions (using orthogonality properties), the first term on the right hand side equation reduces to $\lambda_{cr}^{r}$.

The expression for the first order derivative of the eigenvalue is then written as

$$\lambda_{cr}{}^{r} = -\{q_i^d\}\left(\left[K_{ij}^{r}\right]+\left[K_{(G)_{ij}}^{r}\right]\right)\{q_j^d\} \qquad (i,j=1,2,...,n;\ r=1,2,.....,R) \tag{32}$$

Using Eq. (32), the variances of the eigenvalue can now be expressed as Lal and Singh (2009)

$$Var(\lambda_{cr}) = \sum_{j=1}^{p}\sum_{k=1}^{p}\lambda_{i,j}^{d}\lambda_{i,k}^{d}Cov(b_j^r, b_k^r) \tag{33}$$

Where $Cov(b_j^r, b_k^r)$ is the cross variance between $b_j^r$ and $b_k^r$. The standard deviation (*SD*) is obtained by the square root of the variance. As revealed by the expression, the post buckling load dispersion of the shell panel exhibits linear variation with all random input variables.

## 4. Results and discussion

In the present study, a computer program in MATLAB 9.0 (R2009b) software has been developed to compute second-order statistics of the hygrothermoelectromechanical induced post buckling load of piezoelectric laminated composite cylindrical shell panel with random system properties, using *DISFEM*. The validation and efficacy of the proposed algorithm is examined by comparing the result with those available in literatures and independent *MCS*. The influence of randomness in system properties on hygrothermoelectromechanical post buckling behavior of piezoelectric laminated composite shell panel with varying amplitude ratios, material properties, applied voltage, boundary conditions, piezoelectric laminated shell panel geometry and environmental conditions are investigated in detailed. A nine nodded Lagrange isoparametric element with 63 degree of freedoms (*DOFs*) per element for the present *HSDT* model has been used for discretizing the laminate and (8×8) mesh based on convergence has been used throughout the study. For computation of the results, full integration scheme (3×3) is used for thick shell panel and selective integration scheme (2×2) for thin shell panel.

The second order statistics (mean and *SD* (Standard deviation)/mean known as *COV*) of the hygrothermoelectromechanical post buckling load are obtained considering the random material input variables, thermal expansion coefficients and lamina shell panel thickness taking combined as well as separately as basic random variables (*RVs*) as stated earlier. Being a linear nature of variation as of Eq. (33) and passing by through the origin, the results are only presented by *COV* (*SD*/mean) of the system properties equal to 0.10. However, the obtained results reveal that the stochastic results would be valid up to *COV*=0.20, Zang and Chen (1991). Moreover, the presented results would be sufficient to extrapolate the results for other *COV* value keeping in mind the limitation of *FOPT*. The basic random variables such as $E_1$, $E_2$, $G_{12}$, $G_{13}$, $G_{23}$, $\upsilon_{12}$, $\alpha_1$, $\alpha_2$ and $h$ are sequenced and defined as

$$b_1 = E_{11},\ \ b_2 = E_{22},\ \ b_3 = G_{12},\ \ b_4 = G_{13},\ \ b_5 = G_{23},\ \ b_6 = \alpha_{11},\ \ b_7 = \alpha_{22},\ \text{ and }\ b_8 = h$$

The following dimensionless hygrothermoelectromechanical post buckling load has been used in present analysis Lal *et al*. (2009).

$$\lambda_{crnl} = \frac{N_{crnl} b^2}{E_{22} h^3}$$

Where, $N_{crnl}$ is the dimensionalized hygrothermoelectromechanical post buckling load and $E_{22}$, $h$ and $b$ are defined in notation.

In the present analysis, various boundary conditions of laminate composite cylindrical shell panel such as simply supported, clamped and combination of both are used and are expressed as

All edges simply supported (*SSSS*)

$$v = w = \theta_2 = \phi_2 = 0, \text{ at } \xi_1 = 0, a; \; u = w = \theta_1 = \phi_1 = 0 \text{ at } \xi_2 = 0, b$$

All edges clamped (*CCCC*)

$$u = v = w = \phi_1 = \phi_2 = \theta_1 = \theta_1 = 0, \text{ at } \xi_1 = 0, a \text{ and } \xi_2 = 0, b$$

Two opposite edges clamped and other two simply supported (*CSCS*)

$$u = v = w = \phi_1 = \phi_2 = \theta_1 = \theta_1 = 0, \text{ at } \xi_1 = 0 \text{ and } \xi_2 = 0$$

$$v = w = \theta_2 = \phi_2 = 0, \text{ at } \xi_1 = a; \; u = w = \theta_1 = \phi_1 = 0, \text{ at } \xi_2 = b$$

The materials used for present investigation are Graphite/epoxy and *PZT*-5A and the material properties of these materials are shown in Table 1. The shell panel geometry used for computation are characterized by namely aspect ratios ($a/b$), side to thickness ratios ($b/h$), curvature to side ratios ($R_1/b$), amplitude ratio ($W_{max}/h$), voltage ($V_k$), piezoelectric layer ($P$) and change in the environmental condition ($\Delta T$, $\Delta C$), respectively having various numerical values. It is assumed that the uniform

Table 1 Material Properties of Graphite/ Epoxy and PZT-5A layer used in the present study

| Properties | Graphite/Epoxy | *PZT*-5A |
|---|---|---|
| $E_{11}$ (GPa) | 150 | 63 |
| $E_{22}$ (GPa) | 9 | 63 |
| $G_{12}$ (GPa) | 7.1 | 24.2 |
| $\nu_{12}$ | 0.3 | 0.3 |
| $\alpha_{11}$ ($10^{-6}$ $^{o}C^{-1}$) | 1.1 | 0.9 |
| $\alpha_{22}$ ($10^{-6}$ $^{o}C^{-1}$) | 25.2 | 0.9 |
| $d_{31}$ ($10^{-12}$ m/V) | - | 254 |
| $d_{32}$ ($10^{-12}$ m/V) | - | 254 |
| $e_{31}$=$e_{32}$ ($10^{-12}$ m/V) | - | -122.0 |
| $e_{33}$ ($10^{-12}$ m/V) | - | -285.0 |
| $\varepsilon_0$ (Farad/m) | - | $8.85\times10^{-12}$ |
| $k_{11}/\varepsilon_0$ | - | 1475 |
| $k_{22}/\varepsilon_0$ | - | 1475 |
| $k_{33}/\varepsilon_0$ | - | 1300 |

variation in temperature, moisture and electrical voltage change through thickness have been used for computation of results. In the present analysis lamina lay-ups are taken in degree.

### 4.1 Convergence and validation study

In order to validate the accuracy and efficacy of results obtained through present outlined, in terms of mean and *COV* of hygrothermoelectromechanical induced post buckling load of piezoelectric laminated cylindrical shell panel using *DISFEM*, six test examples have been illustrated and the results are compared with those available in the literature and independent *MCS*.

#### 4.1.1 Validation results for mean dimensionless thermo-electro buckling temperature

The accuracy of the present solution methodology is examined by comparing the results of the post buckling temperature of the cylindrical shell subjected to the thermoelectrical loadings with those available in the literature of Shen (2002) using semi analytical approach under three electrical loadings which is based on the large strain linear elastic theory with different piezoelectric laminated lay-up sequences with $R_1/a$=300 and $b^2/R_1h$=300, and shown in Table 2. It is observed that the present results for various applied voltage obtained by $C^0$ nonlinear finite element method are in good agreement with the results obtained by semi analytical approach of Shen (2002). The percentage differences are approximately less than 2.5% which shows accuracy of present outlined approach. The difference in results may be due to application of boundary condition and solution methodology. From the obtained results, it can be seen that when the negative control voltage ($V$=-100) is applied, it makes the shell panel contract so that the buckling temperature increases and in contrast, the positive control voltage ($V$=+100) decreases the buckling temperature.

#### 4.1.2 Validation results for nonlinear thermo-electro buckling temperature

Table 3 shows the validation of present solution methodology for mean dimensionless post buckling temperature of perfect piezoelectric laminated [P/0/90/0/90] simply supported cylindrical shell under three sets of electrical loading condition with various amplitude ratios. From the table it is clear that present results obtained for various electrical loading and amplitude ratios using finite element based *HSDT* model are in good agreements with available semi analytical results of Shen (2002). It is also seen that buckling temperature decreases with the increase in applied voltage due to the degradation of stiffness of the shell.

Table 2 Comparison of thermo-electro initial buckling temperature ($T_{cr}=\lambda_{cr}T\alpha_{11}\times10^3$) for perfect piezoelectric laminated cylindrical shell ($R_1/h$=300 and $b^2/R_1h$=100) under uniform temperature rise and three set of loading conditions. (where, $Tcr$, $T$ and $\alpha_{11}$ are initial buckling temperature, guessed temperature and thermal expansion coefficients, respectively)

| Lamina lay up | Voltage ($V$) | Shen (2002) | present | % Difference |
|---|---|---|---|---|
| | -100 | 428.96 | 433.8628 | 1.1429 |
| $[P/(0/90)_2]_S$ | 0 | 409.73 | 409.3059 | -0.1035 |
| | +100 | 390.47 | 387.3800 | -0.7913 |
| | -100 | 405.59 | 415.6017 | 2.4684 |
| $[0/P/90/0/90]_S$ | 0 | 386.35 | 392.0784 | 1.4826 |
| | +100 | 367.10 | 371.0753 | 1.0828 |

Table 3 Comparison of thermo-electro post buckling temperature for perfect piezoelectric laminated cylindrical shell ($R_1/h$=300, $b^2/R_1h$=100 and lamina lay-up $[0/P/90/0/90]_S$) under uniform temperature rise and three set of electrical loading conditions

| Voltage ($V$) | $W_{max}/h$ | present | Shen (2002) |
|---|---|---|---|
| -100 | 0.5 | 416.6892 | 405.87 |
| | 0.75 | 419.2845 | 406.53 |
| | 1.0 | 423.5158 | 411.92 |
| | 1.25 | 448.6489 | 429.07 |
| | 1.5 | 520.3683 | 522.39 |
| 0 | 0.5 | 395.9705 | 386.425 |
| | 0.75 | 400.3194 | 388.67 |
| | 1.0 | 406.5085 | 393.33 |
| | 1.25 | 416.2491 | 426.20 |
| | 1.5 | 519.3828 | 520 |
| +100 | 0.5 | 374.7680 | 360 |
| | 0.75 | 378.8838 | 366.67 |
| | 1.0 | 394.9510 | 390.33 |
| | 1.25 | 414.8730 | 420.66 |
| | 1.5 | 492.2923 | 480 |

Table 4 Comparison of buckling loads for piezolaminated cylindrical shell under thermal environments and three sets of electrical loading conditions ($R_1/h$=200, $b^2/R_1h$=100 and $h$=1.2)

| Lamina lay up | Voltage ($V$) | Shen (2010) | | present | |
|---|---|---|---|---|---|
| | | $\Delta T$=0 | $\Delta T$=200 | $\Delta T$=0 | $\Delta T$=200 |
| $P/[0/90]_{2s}$ | -100 | 220.33 | 209.78 | 228.65 | 211.31 |
| | 0 | 220.39 | 210.07 | 228.98 | 211.83 |
| | +100 | 220.43 | 210.34 | 229.25 | 212.49 |
| $[45/P/-45/45/-45]_S$ | -100 | 268.47 | 250.65 | 270.64 | 253.24 |
| | 0 | 268.72 | 250.93 | 270.94 | 253.36 |
| | +100 | 268.96 | 251.21 | 271.35 | 253.73 |

### *4.1.3 Validation results for mean dimensionless thermoelectrical buckling load*

In another comparison, the results of dimensionless buckling load for piezoelectric laminated simply supported square shell under two sets of applied voltage having $R_1/h$=200, $b^2/R_1h$=100 are shown in Table 4. From the table it is again confirm that the present results using $C^0$ nonlinear finite element method are in good agreement with the results obtained by semi analytical approach of Shen (2010). From the results, it is observed that the post buckling load of cylindrical shell panel decreases with the increase in thermal load while increases with increment of applied voltage. The decrease in the post buckling load with temperature is due to decreases of the stiffness of shell panel.

### *4.1.4 Validation results for mean dimensionless hygrothermal buckling load*

The effect of the hygrothermal environmental conditions on mean dimensionless post buckling load of simply supported square $[0/90]_s$ cross-ply symmetric laminated composite cylindrical shell

Table 5 Comparison of hygrothermal buckling loads for perfect cross-ply $[0/90^0]_s$ laminated cylindrical shell panels under different environmental conditions (*a/b*=1.0, *a/R*=1.0, *b/h*=20 and *b*=0.1 m)

| Environmental conditions | $V_f$=0.5 | | $V_f$=0.6 | | $V_f$=0.7 | |
|---|---|---|---|---|---|---|
| | Shen[200] | present | Shen[2002] | present | Shen[2002] | present |
| $\Delta T$=0°C, $\Delta C$=0% | 228.846 | 219.101 | 271.359 | 261.642 | 324.794 | 315.254 |
| $\Delta T$=100°C, $\Delta C$=1% | 218.049 | 196.623 | 259.249 | 234.715 | 311.288 | 291.844 |
| $\Delta T$=200°C, $\Delta C$=2% | 206.802 | 188.268 | 246.541 | 226.483 | 296.986 | 272.483 |
| $\Delta T$=300°C, $\Delta C$=3% | 195.033 | 174.847 | 233.143 | 209.016 | 281.765 | 264.346 |

Table 6 Validation of the *COV* (*SD* to mean ratio) with co efficient of correlation (*COC*) varying from 0 to 20% of normalized buckling load co-efficient of square simply supported cross ply $[0/90^0]$ laminated cylindrical shell with random change in only one material property $E_1$ other as deterministic to mean value

| *COC* | *COV* | |
|---|---|---|
| | Present | Singh *et al.* (2001) |
| 0.05 | 0.0312 | 0.0288 |
| 0.1 | 0.0624 | 0.0577 |
| 0.15 | 0.0935 | 0.0855 |
| 0.2 | 0.1247 | 0.1144 |

panel is compared with the results available in literature of Shen (2002) for various fiber volume fraction. The obtained results are shown in Table 5. Based on the convergence study conducted as discussed earlier, the obtained results are in good agreement with the published results. It is seen that the results due to present $C^0$ nonlinear finite element method are in good agreement with the results obtained by semi analytical approach of Shen (2002) and present solution methodology may be efficiently used for hygrothermal post buckling analysis of laminated shell panel subjected to different loading conditions. The material properties used to obtain the results for validation purpose are directly taken from the reference papers and are not mentioned here.

#### *4.1.5 Validation results for SD to mean ratio of normalized buckling co-efficient*

The validation of random thermal buckling load coefficient of cross ply [0/90] square simply supported laminated composite cylindrical shell is obtained by using outlined probabilistic approach have been compared with the available results of Singh *et al*. (2001) is shown in Table 6. For normalized standard deviation, *SD* (i.e., the ratio of the standard deviation (*SD*) to the mean value) of post buckling load versus the *SD* to the mean value, for the random change in only one material property (i.e., $E_{11}$) changing at a time keeping other as deterministic to mean value. The closed correlations between two results are obtained for various coefficient of correlation (*COC*) changing from 0 to 20%, *b*/*h*=10 and *b/a*=1.

#### *4.1.6 Validation study for post buckling load with random system properties*

The coefficient of variation (*COV*=*SD*/mean) of normalized buckling coefficient for simply supported square [0/90] cylindrical shell obtained using outlined *DISFEM* probabilistic approach have been compared with those obtained by an independent *MCS* approach. Fig. 3 plots the normalized standard deviation (*SDV*) (i.e., ratio of standard deviation to mean value) of the mechanical buckling

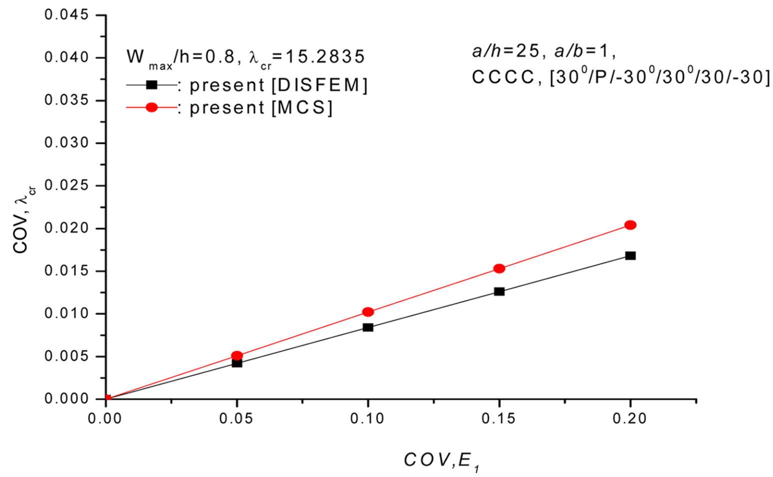


Fig. 3 Validation of the present *DISFEM* results from independent *MCS* results for the hygrothermoelectromechanical post buckling load of [30/P/-30/30/-30] laminated simply supported cylindrical shell panel with amplitude ratio ($W_{max}/h$)=0.8 with only one material property $E_{11}$ as random and keeping others as deterministic

load varying from 0 to 20%. As mentioned earlier no results are available in the reported literature for the post buckling load of cylindrical shell subjected to hygrothermoelectrical loading conditions. The standard deviation results for the comparison have been computed by using independent *MCS* approach. In the present study independent *MCS* approach is used only for validation purpose because it is time consuming process and sometimes suffers from computationally inefficiency. In the Fig. 3 it is assumed that one of the material property (i.e., $E_{11}$) changes at a time keeping other as deterministic. The dotted line is the present *DISFEM* approach and the straight line represents the result of *MCS* approach. For the *MCS* approach the samples are generated using MATLAB software to fit the desired mean and *SD*. The samples are used in the response equation which is solved repeatedly, adopting conventional eigenvalue procedure to generate a sample of the hygrothermoelectromechanical post buckling load. The number of samples used for *MCS* approach is 12000 based on satisfactory convergence of the results. The normal distribution has been assumed for random number generation in *MCS*. However, the present *DISFEM* approach used in the present study doesn't have any limitation regarding probabilistic distribution of the system properties, which is advantageous over *MCS*. It is also observed that a *DISFEM* for present analysis is sufficient to give accurate results for two levels of variations considered in the basic random variable. Present *DISFEM* results obtained by using *FOPT* approach yields very close to the results obtained by the independent *MCS* as shown in Fig. 3.

### *4.2 Parametric study: second order statistic of post buckling load*

The effect of the piezoelectric layer with the environmental condition, amplitude ratio on the dimensionless post buckling load of simply supported square laminated cylindrical shell panel subjected to different electrical loading are shown in Table 7. It is observed that for the same number of layer, amplitude ratio and applied voltage dimensionless mean post buckling load decreases with the increase in the temperature and moisture concentration. It is because of environmental

Table 7 Influence of piezolaminated layer positions in different staking sequence, environmental conditions and amplitude ratios with applied voltage on dimensionless hygrothermoelectromechanical post buckling load of simply supported piezolaminated square cylindrical shell panel with $a/h$=10 and $R_1/a$=50

| Lay-up | Environmental conditions | $W_{max}$/h | Mean $\lambda_{crnl}$ | | |
|---|---|---|---|---|---|
| | | | Electric load $V$ | | |
| | | | +100 | 0 | -100 |
| [P/0/90/0/90] | $\Delta T$=0°C, $\Delta C$=0% | 0.4 | 17.8142 | 17.2886 | 16.2141 |
| | | 0.8 | 19.0103 | 18.9356 | 18.6856 |
| | $\Delta T$=100°C, $\Delta C$=1% | 0.4 | 11.7074 | 10.7417 | 9.7105 |
| | | 0.8 | 14.8362 | 14.5239 | 14.0567 |
| | $\Delta T$=200°C, $\Delta C$=3% | 0.4 | 10.8738 | 10.3838 | 9.5428 |
| | | 0.8 | 12.4244 | 12.1058 | 11.1432 |
| [0/90/0/90/P] | $\Delta T$=0°C, $\Delta C$=0% | 0.4 | 16.9753 | 16.5387 | 15.8541 |
| | | 0.8 | 18.5946 | 17.9492 | 17.2881 |
| | $\Delta T$=100°C, $\Delta C$=1% | 0.4 | 12.2679 | 12.0389 | 11.8996 |
| | | 0.8 | 15.7718 | 15.3745 | 15.1306 |
| | $\Delta T$=200°C, $\Delta C$=3% | 0.4 | 11.7597 | 11.0096 | 10.6929 |
| | | 0.8 | 13.6820 | 12.7455 | 12.2533 |
| [0/90/P/0/90] | $\Delta T$=0°C, $\Delta C$=0% | 0.4 | 12.8916 | 12.6551 | 12.0938 |
| | | 0.8 | 15.5395 | 15.3067 | 15.0373 |
| | $\Delta T$=100°C, $\Delta C$=1% | 0.4 | 12.3411 | 11.5553 | 11.2596 |
| | | 0.8 | 14.5948 | 13.7806 | 13.2396 |
| | $\Delta T$=200°C, $\Delta C$=3% | 0.4 | 11.9122 | 10.5725 | 10.2548 |
| | | 0.8 | 14.1669 | 13.6903 | 13.2260 |
| [P/0/90/0/90/P] | $\Delta T$=0°C, $\Delta C$=0% | 0.4 | 13.9246 | 12.6478 | 12.2516 |
| | | 0.8 | 15.0797 | 14.4062 | 14.2009 |
| | $\Delta T$=100°C, $\Delta C$=1% | 0.4 | 10.4279 | 10.0569 | 9.9810 |
| | | 0.8 | 12.0693 | 11.8021 | 11.0471 |
| | $\Delta T$=200°C, $\Delta C$=3% | 0.4 | 10.1426 | 9.3366 | 8.7566 |
| | | 0.8 | 11.7645 | 10.8304 | 10.1567 |

conditions makes the stiffness of shell panel lower. It is also observed that buckling load decreases when piezoelectric layer moves from top to middle. However, increases when number of piezoelectric layer increases. It is expected that increase the amplitude ratio the dimensionless post buckling increases. It is also observed that electrical voltage makes the post buckling load higher.

Table 8 shows the effect of individual random system property keeping others as deterministic, with amplitude ratio, environmental conditions and applied voltage on the dispersion of piezoelectric laminated cross ply [P/0/90/90/0] simply supported square cylindrical shell panel with $a/h$=10, $R_1/b$=40 having $COV$, $b_i$ ($i$=(1-8)=0.1). It is observed that shell panel is most sensitive to random change with system properties such as $E_{11}$, $E_{22}$, $G_{12}$ and $h$. The tight control of these parameters is

Table 8 Effects of individual random variables ($b_i$), amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, $\{b_i, (i=1 \text{ to } 8)=0.10\}$ of the post buckling load of cross-ply [P/0/90/90/0] piezolaminated simply supported square cylindrical shell panel in hygrothermal environment with $a/h$=10, $R_1/b$=40

| $b_i$ | $W_{max}/h$ | *COV*, $N_{crnl}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\Delta T$=0°C, $\Delta C$=0% | | | $\Delta T$=100°C, $\Delta C$=1% | | | $\Delta T$=200°C, $\Delta C$=3% | | |
| Electric load $V$ | | +100 | 0 | -100 | +100 | 0 | -100 | +100 | 0 | -100 |
| $E_{11}$ ($i$=1) | 0.4 | 0.0796 | 0.0793 | 0.0791 | 0.0638 | 0.0636 | 0.0633 | 0.0429 | 0.0426 | 0.0421 |
| | 0.8 | 0.0788 | 0.0786 | 0.0784 | 0.0621 | 0.0620 | 0.0619 | 0.0411 | 0.0410 | 0.0407 |
| $E_{22}$ ($i$=2) | 0.4 | 0.0339 | 0.0334 | 0.0332 | 0.0272 | 0.0270 | 0.0269 | 0.0168 | 0.0166 | 0.0163 |
| | 0.8 | 0.0322 | 0.0320 | 0.0318 | 0.0251 | 0.0249 | 0.0248 | 0.0143 | 0.0141 | 0.0139 |
| $G_{12}$ ($i$=3) | 0.4 | 0.0115 | 0.0112 | 0.0111 | 0.0056 | 0.0053 | 0.0052 | 0.0034 | 0.0031 | 0.0030 |
| | 0.8 | 0.0104 | 0.0103 | 0.0101 | 0.0048 | 0.0044 | 0.0041 | 0.0028 | 0.0026 | 0.0023 |
| $G_{13}$ ($i$=4) | 0.4 | 0.0367 | 0.0363 | 0.0362 | 0.0404 | 0.0403 | 0.0400 | 0.0489 | 0.0483 | 0.0481 |
| | 0.8 | 0.0356 | 0.0351 | 0.0349 | 0.0388 | 0.0384 | 0.0381 | 0.0461 | 0.0460 | 0.0457 |
| $G_{23}$ ($i$=5) | 0.4 | 0.0043 | 0.0042 | 0.0038 | 0.0045 | 0.0043 | 0.0040 | 0.0067 | 0.0066 | 0.0063 |
| | 0.8 | 0.0035 | 0.0031 | 0.0030 | 0.0036 | 0.0034 | 0.0032 | 0.0052 | 0.0050 | 0.0049 |
| $\alpha_{11}$ ($i$=6) $\times 10^{-5}$ | 0.4 | 2.2176 | 2.2170 | 2.2168 | 2.5583 | 2.5579 | 2.5576 | 2.6593 | 2.6592 | 2.6590 |
| | 0.8 | 2.1068 | 2.1061 | 2.1059 | 2.1857 | 2.1856 | 2.1851 | 2.4054 | 2.4050 | 2.4049 |
| $\alpha_{22}$ ($i$=7) $\times 10^{-5}$ | 0.4 | 4.2077 | 4.2078 | 4.2071 | 46.021 | 46.018 | 46.014 | 47.562 | 47.559 | 57.557 |
| | 0.8 | 4.1655 | 4.1653 | 4.1650 | 4.3674 | 4.3672 | 4.3669 | 4.4987 | 4.4984 | 4.4981 |
| $h$ ($i$=8) | 0.4 | 0.0222 | 0.0221 | 0.0218 | 0.0185 | 0.0183 | 0.0181 | 0.0158 | 0.0154 | 0.0149 |
| | 0.8 | 0.0311 | 0.0310 | 0.0307 | 0.0296 | 0.0295 | 0.0295 | 0.0219 | 0.0217 | 0.0114 |

therefore required for high reliability of piezoelectric laminated composite shell panel. It can also be observed that sensitivity of the shell panel with random change in $E_{11}$, $E_{22}$, $G_{12}$ and $h$ decreases with increasing the environmental conditions. However, sensitivity of shell panel subjected to random change in $G_{13}$, $G_{23}$, $\alpha_{11}$ and $\alpha_{22}$ increases with increases the environmental conditions.

Table 9 shows the effect of support conditions namely *SSSS* (*S*1), *CCCC* and *CSCS*, amplitude ratio and applied voltage with environmental conditions on the mean and dispersion of the post buckling load of piezoelectric laminated [P/0/90/0/90] square cylindrical shell panel in hygrothermal environment for *a*/*h=15* and $R_1/b$=50. It is observed that for same environmental condition, amplitude ratio and applied voltage, *CCCC* shell panel shows highest dimensionless mean post buckling load while *SSSS* plate shows the least. It is due to increased effect of boundary constrained which significantly increases the stiffness of the shell panel. It is also observed that panel is more sensitive to *SSSS* support condition compared to other support conditions. It is also observed that the shell panel in most sensitive to random change in all material properties varying simultaneously while least for all random thermal expansion coefficients.

Table 10 shows the effect of thickness ratio, amplitude ratios, applied voltage and random system variables with environmental conditions on the dimensionless mean and dispersion of the post buckling load of simply supported [P/0/90] piezoelectric laminated square cylindrical shell

Table 9 Effect of support boundary conditions, amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, $\{b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10$\}$ of the post buckling load of cross-ply [P/0/90/0/90] piezolaminated square cylindrical shell panel in with $a/h$=15 and $R_1/b$=50

| *BCs* | $W_{max}/h$ | $V_k$ | $\Delta T$=0ºC, $\Delta C$=0% | | | | $\Delta T$=200ºC, $\Delta C$=3% | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ | | | Mean, $\lambda_{cr}$ | *COV*, $N_{crnl}$ $b_i$ | | |
| | | | | ($i$=1,..,7) | ($i$=6,7) | ($i$=8) | | ($i$=1,...,7) | ($i$=6.7) | ($i$=8) |
| *SSSS S*1 | 0.4 | +100 | 20.3683 | 0.0864 | 0.0095 | 0.0351 | 13.7567 | 0.1144 | 0.0353 | 0.0348 |
| | | 0 | 19.8671 | 0.0861 | 0.0092 | 0.0349 | 13.1328 | 0.1140 | 0.0351 | 0.0346 |
| | | -100 | 19.3557 | 0.0853 | 0.0090 | 0.0347 | 12.9052 | 0.1138 | 0.0346 | 0.0342 |
| | 0.8 | +100 | 23.9373 | 0.0885 | 0.0091 | 0.0359 | 16.8112 | 0.1175 | 0.0341 | 0.0355 |
| | | 0 | 22.6509 | 0.0883 | 0.0089 | 0.0356 | 16.2807 | 0.1173 | 0.0339 | 0.0351 |
| | | -100 | 22.2588 | 0.0881 | 0.0088 | 0.0352 | 15.9203 | 0.1167 | 0.0339 | 0.0350 |
| *CCCC* | 0.4 | +100 | 33.9386 | 0.0831 | 0.0061 | 0.0317 | 28.0447 | 0.0792 | 0.0216 | 0.0309 |
| | | 0 | 33.4859 | 0.0762 | 0.0053 | 0.0291 | 27.9202 | 0.0790 | 0.0214 | 0.0308 |
| | | -100 | 33.0505 | 0.0697 | 0.0050 | 0.0284 | 27.3732 | 0.0788 | 0.0210 | 0.0306 |
| | 0.8 | +100 | 35.1462 | 0.0838 | 0.0060 | 0.0314 | 31.6875 | 0.0796 | 0.0215 | 0.0315 |
| | | 0 | 34.8826 | 0.0837 | 0.0060 | 0.0311 | 31.1374 | 0.0794 | 0.0215 | 0.0314 |
| | | -100 | 34.3266 | 0.0836 | 0.0059 | 0.0308 | 30.7750 | 0.0791 | 0.0213 | 0.0311 |
| *CSCS* | 0.4 | +100 | 26.9696 | 0.0832 | 0.0071 | 0.0329 | 22.1426 | 0.0834 | 0.0252 | 0.0317 |
| | | 0 | 26.3378 | 0.0828 | 0.0069 | 0.0325 | 21.9698 | 0.0831 | 0.0249 | 0.0315 |
| | | -100 | 25.2126 | 0.0825 | 0.0067 | 0.0322 | 21.3282 | 0.0827 | 0.0248 | 0.0313 |
| | 0.8 | +100 | 29.4202 | 0.0829 | 0.0069 | 0.0334 | 24.3549 | 0.0830 | 0.0256 | 0.0320 |
| | | 0 | 28.6263 | 0.0827 | 0.0064 | 0.0329 | 23.6565 | 0.0829 | 0.0253 | 0.0318 |
| | | -100 | 28.0402 | 0.0821 | 0.0061 | 0.0327 | 23.0248 | 0.0823 | 0.0251 | 0.0314 |

panel. For the same amplitude ratio, applied voltage and environmental condition, it is observed that the dimensionless mean and corresponding *COV* of post buckling load increases with increase the *a/h* ratio subjected to random change in system properties. However, for the thin shell panel effect of amplitude ratio, applied voltage and environmental condition on the dimensionless mean and *COV* of post buckling load are important than thick shell panel. It is concluded that thin shell panel is more sensitive to random change in all material properties than thick shell panel.

The effect of aspect ratio, amplitude ratio, applied voltage and environmental conditions with random material properties on the dimensionless mean and dispersion of the hygrothermoelectromechanical post buckling of simply supported piezoelectric laminated [P/0/90/90/0] shell panel is examined in Table 11. It is observed that for the same amplitude ratio, applied voltage and environmental conditions increasing the aspect ratio, the dimensionless mean post buckling load decreases and corresponding *COV* increases subjected to random change in thermo mechanical properties, however, *COV* decreases with the random change in *h*. It is also observed that increase the environmental

Table 10 Effects of plate thickness ratios ($a/h$), amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, $\{b_i$, ($i$=(1,..., 7), (6,7) and (8)=0.10$\}$ of the post buckling load of cross ply [P/0/90] simply supported piezolaminated square cylindrical shell in hygrothermal environment with $R_1/b$=50

| $a/h$ | $W_{max}/h$ | $V_k$ | $\Delta T$=0°C, $\Delta C$=0% | | | | $\Delta T$=100°C, $\Delta C$=1% | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,...,7) | *COV*, $N_{crnl}$ $b_i$ ($i$=6,7) | *COV*, $N_{crnl}$ $b_i$ ($i$=8) | Mean, $\lambda_{cr}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,..,7) | *COV*, $N_{crnl}$ $b_i$ ($i$=6.7) | *COV*, $N_{crnl}$ $b_i$ ($i$=8) |
| 5 | 0.4 | +100 | 3.9108 | 0.0790 | 0.0048 | 0.0236 | 3.3917 | 0.0892 | 0.0426 | 0.0234 |
| | | 0 | 3.8662 | 0.0786 | 0.0044 | 0.0234 | 3.1763 | 0.0890 | 0.0423 | 0.0231 |
| | | -100 | 3.5598 | 0.0781 | 0.0041 | 0.0231 | 2.9746 | 0.0889 | 0.0421 | 0.0229 |
| | 0.8 | +100 | 5.9586 | 0.0797 | 0.0042 | 0.0259 | 4.9992 | 0.0889 | 0.0414 | 0.0248 |
| | | 0 | 5.4687 | 0.0795 | 0.0039 | 0.0257 | 4.4889 | 0.0887 | 0.0411 | 0.0246 |
| | | -100 | 5.2674 | 0.0789 | 0.0037 | 0.0353 | 4.0875 | 0.0883 | 0.0409 | 0.0242 |
| 10 | 0.4 | +100 | 10.5670 | 0.0823 | 0.0106 | 0.0259 | 8.8058 | 0.1002 | 0.0839 | 0.0288 |
| | | 0 | 9.9802 | 0.0822 | 0.0105 | 0.0258 | 8.4682 | 0.0999 | 0.0835 | 0.0286 |
| | | -100 | 9.4697 | 0.0820 | 0.0103 | 0.0255 | 8.0979 | 0.0995 | 0.0831 | 0.0281 |
| | 0.8 | +100 | 12.1927 | 0.0836 | 0.0096 | 0.0277 | 11.0842 | 0.0990 | 0.0829 | 0.0271 |
| | | 0 | 11.7253 | 0.0834 | 0.0093 | 0.0273 | 10.6593 | 0.0987 | 0.0827 | 0.0268 |
| | | -100 | 11.2122 | 0.0831 | 0.0090 | 0.0273 | 10.1929 | 0.0986 | 0.0825 | 0.0268 |
| 20 | 0.4 | +100 | 20.5671 | 0.0866 | 0.0183 | 0.0287 | 19.6064 | 0.1229 | 0.1136 | 0.0314 |
| | | 0 | 19.9814 | 0.0863 | 0.0182 | 0.0286 | 18.8926 | 0.1225 | 0.1135 | 0.0312 |
| | | -100 | 19.3915 | 0.0760 | 0.0181 | 0.0286 | 18.2650 | 0.1223 | 0.1131 | 0.0311 |
| | 0.8 | +100 | 23.5487 | 0.0876 | 0.0172 | 0.0299 | 22.6587 | 0.1103 | 0.1113 | 0.0332 |
| | | 0 | 22.6354 | 0.0873 | 0.0170 | 0.0296 | 22.1674 | 0.1101 | 0.1112 | 0.0331 |
| | | -100 | 22.0646 | 0.0871 | 0.0168 | 0.0295 | 21.9428 | 0.1099 | 0.1107 | 0.0328 |

conditions the sensitivity of the shell increases.

Table 12 shows the effect of side to curvature ratio ($R/b$) mean amplitude ratios, applied voltage and environmental conditions with the random system properties on the dimensionless mean and dispersion of the hygrothermoelectromechanical post buckling load and simply supported piezoelectric laminated [P/0/90/90/0] square cylindrical shell panel $a/h$=15. From the table it is observed that with the change in the environmental conditions the sensitivity of the shell increases subjected to random change in material properties and thickness. It is observed that for the same amplitude ratio, applied voltage and environmental condition, the dimensionless mean post buckling load decreases, while, *COV* increases with the increase with the curvature to length ratio and mean post buckling load and *COV* is higher for flat plate.

Table 13 shows the effect of piezoelectric laminated layup sequence and layers, amplitude ratio and applied voltage with environmental conditions on dimensionless mean and *COV*, $\{b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10$\}$ of post buckling load of simply supported piezoelectric laminated cylindrical shell panel having $a/h$=30 and $R_1/b$=60. From the table it is observed that the post buckling load of

Table 11 Effects of aspect ratio ($a/b$), amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, $\{b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10$\}$ of the post buckling load of cross ply [P/0/90/90/0] simply supported piezolaminated cylindrical shell panel in hygrothermal environment with $a/h$=10 and $R_1/b$=60

| $a/b$ | $W_{max}/h$ | $V_k$ | $\Delta T$=0°C, $\Delta C$=0% | | | | $\Delta T$=100C, $\Delta C$=1% | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ | | |
| | | | | ($i$=1,...,7) | ($i$=6,7) | ($i$=8) | | ($i$=1,...,7) | ($i$=6,7) | ($i$=8) |
| 0.5 | 0.4 | +100 | 17.9407 | 0.0809 | 0.0061 | 0.0369 | 16.0612 | 0.0997 | 0.0632 | 0.0303 |
| | | 0 | 16.7984 | 0.0806 | 0.0059 | 0.0368 | 14.3484 | 0.0994 | 0.0631 | 0.0301 |
| | | -100 | 15.8269 | 0.0804 | 0.0057 | 0.0366 | 13.9604 | 0.0993 | 0.0628 | 0.0299 |
| | 0.8 | +100 | 18.9891 | 0.0813 | 0.0055 | 0.0383 | 17.0673 | 0.0979 | 0.0611 | 0.0312 |
| | | 0 | 18.1584 | 0.0811 | 0.0052 | 0.0380 | 16.5479 | 0.0976 | 0.0610 | 0.0310 |
| | | -100 | 17.8426 | 0.0810 | 0.0050 | 0.0379 | 16.1305 | 0.0973 | 0.0607 | 0.0308 |
| 1.0 | 0.4 | +100 | 12.2385 | 0.0878 | 0.0093 | 0.0321 | 11.0256 | 0.1117 | 0.0896 | 0.0319 |
| | | 0 | 11.6860 | 0.0873 | 0.0087 | 0.0317 | 10.2678 | 0.1113 | 0.0894 | 0.0317 |
| | | -100 | 11.0043 | 0.0869 | 0.0083 | 0.0315 | 10.1349 | 0.1111 | 0.0893 | 0.0314 |
| | 0.8 | +100 | 14.3578 | 0.0871 | 0.0095 | 0.0332 | 13.6885 | 0.1102 | 0.0872 | 0.0328 |
| | | 0 | 13.6587 | 0.0868 | 0.0091 | 0.0329 | 13.0009 | 0.1099 | 0.0870 | 0.0326 |
| | | -100 | 13.0786 | 0.0864 | 0.0088 | 0.0328 | 12.9754 | 0.1098 | 0.0866 | 0.0325 |
| 1.5 | 0.4 | +100 | 6.9829 | 0.0973 | 0.0449 | 0.0299 | 5.7445 | 0.2157 | 0.1963 | 0.0297 |
| | | 0 | 6.5580 | 0.0971 | 0.0444 | 0.0298 | 5.1869 | 0.2153 | 0.1961 | 0.0295 |
| | | -100 | 6.3185 | 0.0970 | 0.0442 | 0.0296 | 4.9968 | 0.2151 | 0.1959 | 0.0294 |
| | 0.8 | +100 | 7.2616 | 0.0975 | 0.0436 | 0.0312 | 6.4891 | 0.2102 | 0.1892 | 0.0316 |
| | | 0 | 6.9905 | 0.0973 | 0.0435 | 0.0311 | 5.8903 | 0.2101 | 0.1888 | 0.0315 |
| | | -100 | 6.6004 | 0.0972 | 0.0433 | 0.0310 | 5.6468 | 0.2098 | 0.1887 | 0.0313 |

angle ply [45/P/-45/45/-45] shell panel is highest while *COV* is lowest compared to other lamination layup sequence subjected to random material properties. However, *COV* is highest for shell panel subjected to random change in lamina panel thickness. It is also observed that increases the amplitude ratio increases the nonlinear stiffness matrix which increases the stiffness of the shell panel. It is also expected that environmental conditions lowers the dimensionless post buckling load while increases the *COV*.

The effect of in plane thermo-mechanical loading (biaxial and uniaxial) with variable temperature, change in moisture concentration and amplitude ratios on the dimensionalized mean and dispersion of thermo-mechanical post buckling load of cross ply [P/0/45/0/45] piezoelectric laminated composite cylindrical shell with simply supported support boundary condition with $b/h$=80, $R_1/b$=70, for moisture, temperature and electrical loading condition having *COV*, $b_i$, $\{i$=(1..7), (6,7) and (8)=0.10$\}$ is shown in Table 14. It is observed that the dimensionless mean and *COV* of post buckling load with random change in material properties for shell subjected to biaxial compression thermoelectromechanical loading is more than uniaxial compression. However, *COV* of shell panel increases with random change

Table 12 Effects of curvature to side ratios ($R/b$), amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, $\{b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10$\}$ of the post buckling load of cross ply [P/0/90/90/0] simply supported piezolaminated cylindrical shell panel having $a/h$=15

| $R_1/b$ | $W_{max}/h$ | $V_k$ | $\Delta T$=0°C, $\Delta C$=0% | | | | $\Delta T$=100°C, $\Delta C$=1% | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ | | | Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ | | |
| | | | | ($i$=1,...,7) | ($i$=6,7) | ($i$=8) | | ($i$=1,..,7) | ($i$=6.7) | ($i$=8) |
| 10 | 0.4 | +100 | 15.6534 | 0.0791 | 0.0022 | 0.0321 | 13.3053 | 0.0453 | 0.0116 | 0.0621 |
| | | 0 | 14.5187 | 0.0789 | 0.0022 | 0.0319 | 12.8408 | 0.0451 | 0.0112 | 0.0620 |
| | | -100 | 14.2395 | 0.0788 | 0.0021 | 0.0319 | 12.3735 | 0.0449 | 0.0112 | 0.0618 |
| | 0.8 | +100 | 17.3465 | 0.0796 | 0.0021 | 0.0335 | 15.9445 | 0.0487 | 0.0138 | 0.0609 |
| | | 0 | 16.9341 | 0.0795 | 0.0020 | 0.0334 | 15.5984 | 0.0486 | 0.0136 | 0.0607 |
| | | -100 | 16.2564 | 0.0795 | 0.0020 | 0.0332 | 14.0329 | 0.0484 | 0.0135 | 0.0605 |
| 50 | 0.4 | +100 | 14.8706 | 0.0794 | 0.0022 | 0.0329 | 12.6400 | 0.0466 | 0.0121 | 0.0621 |
| | | 0 | 13.7902 | 0.0793 | 0.0021 | 0.0327 | 12.4201 | 0.0463 | 0.0120 | 0.0621 |
| | | -100 | 13.4174 | 0.0792 | 0.0021 | 0.0327 | 11.9799 | 0.0462 | 0.0118 | 0.0620 |
| | 0.8 | +100 | 16.9991 | 0.0798 | 0.0023 | 0.0337 | 14.6922 | 0.0499 | 0.0139 | 0.0609 |
| | | 0 | 16.5198 | 0.0797 | 0.0023 | 0.0336 | 14.3918 | 0.0498 | 0.0138 | 0.0606 |
| | | -100 | 16.0485 | 0.0797 | 0.0022 | 0.0336 | 13.9012 | 0.0496 | 0.0135 | 0.0604 |
| ∞ | 0.4 | +100 | 34.6456 | 0.0797 | 0.0023 | 0.0337 | 29.0487 | 0.0811 | 0.0278 | 0.0317 |
| | | 0 | 33.9564 | 0.0797 | 0.0022 | 0.0334 | 28.2479 | 0.0809 | 0.0276 | 0.0316 |
| | | -100 | 33.5046 | 0.0794 | 0.0021 | 0.0334 | 27.9869 | 0.0806 | 0.0275 | 0.0313 |
| | 0.8 | +100 | 37.7623 | 0.0798 | 0.0025 | 0.0338 | 31.6979 | 0.0811 | 0.0273 | 0.0325 |
| | | 0 | 36.3655 | 0.0796 | 0.0024 | 0.0338 | 30.7606 | 0.0810 | 0.0270 | 0.0321 |
| | | -100 | 36.2646 | 0.0795 | 0.0024 | 0.0336 | 30.4249 | 0.0807 | 0.0269 | 0.0321 |

in lamina shell panel thickness subjected to biaxial compression. when the moisture concentration and temperature increases, there is very slightly decrease in mean increases in *COV* of thermo-mechanical post buckling load due to random system property.

## 5. Conclusions

A *DISFEM* probabilistic procedure is presented to study the second-order statistics (mean and *COV*) of hygrothermoelectromechanical induced post buckling load of piezoelectric laminated composite cylindrical shell panel with system randomness using *HSDT* with von-Karman nonlinearity. The effects of combination of multiple random variables varying simultaneously or individually with shell panel geometric parameters, amplitude ratios, applied voltage, boundary conditions and various modes of moisture and temperature change have been examined in present study. The following conclusion can be drawn from this study:

Table 13 Effects of lay-up sequence, amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, {$b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10} of the post buckling load of simply supported piezolaminated cylindrical shell panel in hygrothermal environment having $a/h$=30 and $R_1/b$=60

| Lay-up | $W_{max}/h$ | $V_k$ | $\Delta T$=0°C, $\Delta C$=0% Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,...,7) | ($i$=6,7) | ($i$=8) | $\Delta T$=100°C, $\Delta C$=1% Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,...,7) | ($i$=6,7) | ($i$=8) |
|---|---|---|---|---|---|---|---|---|---|---|
| P/[0/90]$_{2s}$ | 0.4 | +100 | 23.7787 | 0.0870 | 0.0375 | 0.0372 | 21.3493 | 0.2936 | 0.2364 | 0.0402 |
| | | 0 | 22.5461 | 0.0869 | 0.0375 | 0.0370 | 21.0649 | 0.2935 | 0.2361 | 0.0400 |
| | | -100 | 22.1085 | 0.0867 | 0.0373 | 0.0366 | 20.9871 | 0.2933 | 0.2360 | 0.0397 |
| | 0.8 | +100 | 26.6464 | 0.0862 | 0.0353 | 0.0381 | 24.5613 | 0.2794 | 0.2219 | 0.0416 |
| | | 0 | 26.0916 | 0.0860 | 0.0352 | 0.0380 | 24.2578 | 0.2790 | 0.2217 | 0.0415 |
| | | -100 | 25.5986 | 0.0860 | 0.0350 | 0.0378 | 23.8712 | 0.2789 | 0.2213 | 0.0412 |
| [45/P/-45 /45/-45] | 0.4 | +100 | 39.6458 | 0.0858 | 0.0213 | 0.0422 | 37.4834 | 0.1937 | 0.1162 | 0.0447 |
| | | 0 | 38.9757 | 0.0857 | 0.0210 | 0.0421 | 37.1047 | 0.1936 | 0.1161 | 0.0444 |
| | | -100 | 37.8684 | 0.0854 | 0.0205 | 0.0419 | 36.8762 | 0.1930 | 0.1158 | 0.0441 |
| | 0.8 | +100 | 42.4988 | 0.0844 | 0.0202 | 0.0426 | 41.5897 | 0.1894 | 0.1115 | 0.0459 |
| | | 0 | 41.8697 | 0.0842 | 0.0199 | 0.0423 | 41.1239 | 0.1892 | 0.1110 | 0.0458 |
| | | -100 | 41.0176 | 0.0841 | 0.0198 | 0.0422 | 40.6495 | 0.1886 | 0.1106 | 0.0454 |
| [P/30/-30/P] | 0.4 | +100 | 28.0270 | 0.0866 | 0.0280 | 0.0378 | 26.3484 | 0.2432 | 0.1990 | 0.0429 |
| | | 0 | 27.8410 | 0.0865 | 0.0277 | 0.0377 | 26.0254 | 0.2430 | 0.1986 | 0.0426 |
| | | -100 | 26.9808 | 0.0863 | 0.0272 | 0.0371 | 25.6947 | 0.2430 | 0.1985 | 0.0422 |
| | 0.8 | +100 | 32.9491 | 0.0852 | 0.0268 | 0.0399 | 29.4641 | 0.2271 | 0.1968 | 0.0434 |
| | | 0 | 31.8087 | 0.0851 | 0.0264 | 0.0393 | 29.2016 | 0.2270 | 0.1966 | 0.0433 |
| | | -100 | 30.9816 | 0.0851 | 0.0261 | 0.0392 | 29.0050 | 0.2264 | 0.1959 | 0.0430 |

Table 14 Effects of in-plane thermo mechanical loading, amplitude ratios and applied voltage with environmental conditions on the dimensionless mean and *COV*, {$b_i$, ($i$=(1,...,7), (6,7) and (8)=0.10} of the post buckling load of simply supported piezolaminated cylindrical shell [P/0/45/0/45] in hygrothermal environment having, $a/h$=60 and $R_1/b$=40

| In plane loading | $W_{max}/h$ | $V_k$ | $\Delta T$=0°C, $\Delta C$=0% Mean, $\lambda_{crnl}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,...,7) | ($i$=6,7) | ($i$=8) | $\Delta T$=100°C, $\Delta C$=1% Mean, $\lambda_{cr}$ | *COV*, $N_{crnl}$ $b_i$ ($i$=1,...,7) | ($i$=6,7) | ($i$=8) |
|---|---|---|---|---|---|---|---|---|---|---|
| Biaxial | 0.4 | +100 | 47.9452 | 0.0664 | 0.0414 | 0.0227 | 45.3954 | 0.1119 | 0.0745 | 0.0189 |
| | | 0 | 47.1956 | 0.0662 | 0.0413 | 0.0225 | 44.7996 | 0.1116 | 0.0742 | 0.0186 |
| | | -100 | 46.6875 | 0.0661 | 0.0411 | 0.0223 | 44.3661 | 0.1112 | 0.0741 | 0.0183 |
| | 0.8 | +100 | 48.8223 | 0.0647 | 0.0388 | 0.0213 | 47.1132 | 0.1014 | 0.0729 | 0.0171 |
| | | 0 | 48.2178 | 0.0644 | 0.0386 | 0.0212 | 46.7348 | 0.1010 | 0.0726 | 0.0170 |
| | | -100 | 47.0468 | 0.0643 | 0.0385 | 0.0210 | 46.2265 | 0.1005 | 0.0725 | 0.0168 |
| Uniaxial | 0.4 | +100 | 69.6872 | 0.0697 | 0.0426 | 0.0206 | 68.6042 | 0.1062 | 0.0757 | 0.0176 |
| | | 0 | 69.2496 | 0.0694 | 0.0425 | 0.0204 | 68.2472 | 0.1058 | 0.0756 | 0.0175 |
| | | -100 | 68.6371 | 0.0691 | 0.0422 | 0.0203 | 67.5591 | 0.1057 | 0.0751 | 0.0172 |
| | 0.8 | +100 | 71.5761 | 0.0658 | 0.0410 | 0.0188 | 70.7862 | 0.1022 | 0.0734 | 0.0162 |
| | | 0 | 70.8495 | 0.0652 | 0.0409 | 0.0185 | 70.3325 | 0.1021 | 0.0731 | 0.0160 |
| | | -100 | 70.1582 | 0.0651 | 0.0407 | 0.0181 | 69.1354 | 0.1019 | 0.0730 | 0.0158 |

The mean dimensionless post buckling load increases with increase the amplitude ratios and applied voltage, while decreases with increases the piezoelectric layers and also increases when shifted from outer side to inner side of the shell panel from neutral axis. The shell panel is most sensitive to random change with system properties such as $E_{11}$, $E_{22}$, $G_{12}$ and $h$. The tight control of these parameters is therefore required for high reliability of piezoelectric laminated composite shell panel. The sensitivity of the shell panel with random change in $E_{11}$, $E_{22}$, $G_{12}$ and $h$ decreases with increasing the environmental conditions, however, sensitivity of shell panel subjected to random change in $G_{13}$, $G_{23}$, $\alpha_{11}$ and $\alpha_{22}$ increases with increases the environmental conditions. The *CCCC* shell panel shows highest dimensionless mean post buckling load while *SSSS* shell panel shows the least, however, *SSSS* shell panel is most sensitive compared to *CCCC* and *CSCS* shell panel. In general, thin shell panel is more sensitive than thick shell panel subjected to random change in all material properties and lamina shell panel thickness. Increasing the aspect ratio, the dimensionless mean post buckling load decreases and corresponding *COV* increases subjected to random change in thermo mechanical properties, however *COV* decreases with the random change in $h$. In general, increase the environmental conditions the sensitivity of the shell panel increases with random change in system properties acting simultaneously or individually. It is because of environmental conditions make the strength of the shell panel lower. The dimensionless mean and *COV* of post buckling load with random change in material properties for shell subjected to biaxial compression thermoelectromechancial loadings is more than uniaxial compression. The effect of randomness in the thermal expansion coefficients, the lamina shell panel thickness on *COV* post buckling load subject to hygrothermoelectromechanical loading is quite significant. It is therefore desirable to account the uncertainty in the system properties for reliable economical and safe design.

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## Notations

| | |
|---|---|
| $Aij$, $Bij$, *etc* | : Laminate stiffnesses |
| $a$, $b$ | : Plate length and breadth |
| $h$ | : Thickness of the shell |
| $\nu_{12}$ | : Poisson's ratio of cylindrical shell |
| $b_i$ | : Basic random material properties |
| $E_{11}$, $E_{22}$ | : Longitudinal and Transverse elastic moduli |
| $G_{12}$, $G_{13}$, $G_{23}$ | : Shear moduli |
| $K_l$, | : Linear bending stiffness matrix |
| $K_g$ | : Hygro-thermal-eletro-mechanical geometric stiffness matrix |
| $D$ | : Elastic stiffness matrices |
| $ne$, $n$ | : Number of elements, number of layers in the laminated plate |
| $N_{\xi 1}^T$, $N_{\xi 2}^T$, $N_{\xi 1 \xi 2}^T$ | : In-plane thermal buckling loads |
| $N_{\xi 1}^V$, $N_{\xi 2}^V$, $N_{\xi 1 \xi 2}^V$ | : In-plane electrical loads |
| $E_k$ | : The electric field vector |
| $nn$ | : Number of nodes per element |
| $N_i$ | : Shape function of ith node |
| $D_k$ | : The electric displacement vector |
| $\bar{C}_{ijkl}^p$ | : Reduced elastic material constants |
| $f$, $\{f\}^{(e)}$ | : Vector of unknown displacements, displacement vector of eth element |
| $u$, $v$, $w$ | : Displacements of a point on the mid plane of shell |
| $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ | : Displacement of a point ($\xi_1$ $\xi_2$ $\zeta$) |
| $\bar{\sigma}_{ij}$, $\bar{\varepsilon}_{ij}$ | : Stress vector, Strain vector |
| $\xi_{kl}$ | : The dielectric coefficient matrix |
| $e_{kl}$ | : The matrix of the piezoelectric coefficients |
| $\phi_1$, $\phi_2$ | : Rotations of normal to mid plane about the $\xi_1$ and $\xi_2$ axis respectively |
| $\theta_1$, $\theta_2$, $\theta_3$ | : Two slopes and angle of fiber orientation wrt $\xi_1$-axis for kth layer |
| ($\xi_1$ $\xi_2$ $\zeta$) | : Cartesian coordinates |
| $V_k$ | : Applied voltage across the kth ply |
| $\lambda$, $Var(.)$ | : Eigen value and variance |
| $N_{\mathrm{crnl}}$ | : The dimensionalized post buckling load |
| $\lambda_{crnl}$ | : The nondimensionlized critical buckling load |
| $RVs$ | : Random variables |
| $\Delta T$, $\Delta C$ | : Difference in temperatures and moistures |
| $\alpha_{11}$, $\alpha_{22}$, $\beta_{11}$, $\beta_{22}$ | : Thermal expansion and hygroscopic coefficients along $\xi_1$ and $\xi_2$ axis, respectively |