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Exact solution of a thick walled functionally graded piezoelectric cylinder under mechanical, thermal and electrical loads in the magnetic field

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Abstract. The present paper deals with the analytical solution of a functionally graded piezoelectric (*FGP*) cylinder in the magnetic field under mechanical, thermal and electrical loads. All mechanical, thermal and electrical properties except Poisson ratio can be varied continuously and gradually along the thickness direction of the cylinder based on a power function. The cylinder is assumed to be axisymmetric. Steady state heat transfer equation is solved by considering the appropriate boundary conditions. Using Maxwell electro dynamic equation and assumed magnetic field along the axis of the cylinder, Lorentz's force due to magnetic field is evaluated for non homogenous state. This force can be employed as a body force in the equilibrium equation. Equilibrium and Maxwell equations are two fundamental equations for analysis of the problem. Comprehensive solution of Maxwell equation is considered in the present paper for general states of non homogeneity. Solution of governing equations may be obtained using solution of the characteristic equation of the system. Achieved results indicate that with increasing the non homogenous index, different mechanical and electrical components present different behaviors along the thickness direction. *FGP* can control the distribution of the mechanical and electrical components in various structures with good precision. For intelligent properties of functionally graded piezoelectric materials, these materials can be used as an actuator, sensor or a component of piezo motor in electromechanical systems.

Keywords: functionally graded piezoelectric; magnetic field; cylinder; electric potential; non homogenous

1. Introduction

Piezoelectrics are new group of materials which can be used as a sensor or actuator in electromechanical systems. These materials can exchange the mechanical deformations into electric potential and conversely, the electric potential can be exchanged to mechanical deformation. Piezoelectric sensors or actuators may be designed as many structural elements such as beam, plate or cylindrical shell. Some applications of piezoelectric structures in sensor or actuator application, while the structure is subjected to magnetic field, is important for designer and engineer. Therefore it is necessary to analyze the functionally graded piezoelectric cylinders under magnetic field subjected to mechanical, thermal and electrical loads.

Lim and He (2001) proposed an exact solution for a compositionally graded piezoelectric layer

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under combined loads. Liu *et al*. (2002) investigated an analytical model for vibration analysis of a circular piezoelectric plate. Mindlin's plate and shear deformation theories and finite element approach were employed for modeling the plate deformation. The distribution of electric potential was simulated using a sinusoidal function along the thickness direction. By considering three equations of equilibrium and Maxwell's equations, four differential equations were derived in order to obtain four unknown components (displacement, electric potential and two rotation components). Dynamic analyses of a magneto piezo electric hollow cylinder were investigated by Hou and Leung (2004). They analyzed cylinder in the general form with entire components of stress. This analysis can offer a simple and accurate tool for the prediction, identification and study of the complex dynamic characteristics of coupling mechanical and electromagnetic fields.

Transient response of a piezoelectric hollow cylinder subjected to complex loading was investigated by Dai and Wang (2006). The cylinder was loaded under an axial magnetic field, thermal shock, mechanical and transient electric excitation. Only normal stresses have been considered in the equilibrium equation.

Dai *et al.* (2006) investigated the exact solution of a functionally graded (*FG*) pressure vessel under a uniform magnetic field. The mechanical and magnetic properties were considered with a simple power low along the thickness direction. The obtained results in terms of the non homogenous index were compared with those of the homogenous case.

Dai and Fu (2007) studied the magneto-thermo-elastic interactions in hollow structures of functionally graded material and subjected to mechanical loads. The structure was subjected to a uniform temperature rising. Pietrzakowski (2008) investigated the vibration analysis of active rectangular plates. The plates were considered composites containing piezoelectric sensor/actuator layers, which operate in a velocity feedback control to achieve transverse vibration suppression. Babaei and Chen (2008) presented exact solution of infinitely long magneto elastic hollow cylinder and solid rotating cylinder that polarized and magnetized radially. The cylinder was considered orthotropic. They investigated the effect of angular velocity on the hoop and radial stresses. Khoshgoftar *et al.* (2009) presented an exact solution for a thick walled *FGP* cylinder. They supposed, all mechanical, thermal and electrical properties except Poisson ratio were non homogenous. Dai *et al.* (2010) proposed an analytical solution for a thick walled piezoelectric cylinder under magnetic field.

Arefi and Rahimi (2011, 2012) published some linear and nonlinear studies about functionally graded piezoelectric and functionally graded piezomagnetic structures.

Arefi *et al.* (2011) investigated thermo magneto piezoelectric analysis of a functionally graded piezo magnetic cylinder. They have shown some advantageous applications of functional materials in optimization and control of distribution of mechanical, electrical and magnetic parameters.

Due to very applicable instances of sensors and actuators in industry and technical application under magnetic effects, the present paper proposes analytical solution of a functionally graded piezoelectric cylinder subjected to magnetic field. All mechanical, thermal and electrical properties are assumed non homogenous. The effect of non homogenous index is evaluated on the distribution of mechanical, electrical and magnetic components along the wall of the cylinder.

2. Fundamental equations

In the present section, the governing equations of a piezoelectric thick walled cylinder in the magnetic field can be derived. Stress and electric displacement equations can be obtained as follows

(Khoshgoftar *et al.* 2009, Rahimi *et al.* 2011, Arefi *et al.* 2011)

$$\sigma_r = c_{11}\frac{\partial u}{\partial r}+c_{12}\frac{u}{r}+e_{11}\frac{\partial \varphi}{\partial r}-\lambda_1 T$$

$$\sigma_\theta = c_{12}\frac{\partial u}{\partial r}+c_{22}\frac{u}{r}+e_{12}\frac{\partial \varphi}{\partial r}-\lambda_2 T$$

$$\sigma_z = c_{13}\frac{\partial u}{\partial r}+c_{23}\frac{u}{r}+e_{13}\frac{\partial \varphi}{\partial r}-\lambda_3 T$$

$$D_r = e_{11}\frac{\partial u}{\partial r}+e_{12}\frac{u}{r}-g_{11}\frac{\partial \varphi}{\partial r}+p_{11}T \tag{1}$$

where, $\frac{\partial u}{\partial r}, \frac{u}{r}$ are linear radial and circumferential components of strain (Arefi and Rahimi 2012). p, g, e, c are pyroelectric, dielectric, piezoelectric and elastic constants of the system, respectively. It assumed that all variable material properties obey a power function. Therefore based on the power function distribution of the mechanical, thermal and electrical material properties, the non-homogenous properties can be expressed as (Khoshgoftar *et al.* 2009, Arefi and Rahimi 2011, Arefi *et al.* 2011, Rahimi *et al.* 2011, Arefi and Rahimi 2012)

$$e_{ri} = e_{ri0}r^l \quad C_{ij} = C_{ij0}r^l \quad \alpha = \alpha_0 r^b \quad g = g_{110}r^l$$

$$p_{11} = p_{110}r^{b+1} \quad \lambda_i = \lambda_{i0}r^{b+l} \quad \mu = \mu_0 r \tag{2}$$

Thermal constants (λ_i) may be defined in terms of elastic and heat expansion coefficients as follows (Khoshgoftar *et al*. 2009)

$$\lambda_1 = c_{11}\alpha_1+c_{12}\alpha_2+c_{13}\alpha_3$$

$$\lambda_2 = c_{12}\alpha_1+c_{22}\alpha_2+c_{23}\alpha_3$$

$$\lambda_3 = c_{13}\alpha_1+c_{23}\alpha_2+c_{33}\alpha_3 \tag{3}$$

Maxwell electro dynamic equations for an elastic solid are (Dai *et al.* 2006)

$$\vec{J} = \nabla\times\vec{h}$$

$$\nabla\times e = -\mu(r)\frac{\partial\vec{h}}{\partial t}$$

$$div\,\vec{h} = 0$$

$$\vec{e} = -\mu(r)\left(\frac{\partial\vec{U}}{\partial t}\times\vec{H}\right)$$

$$\vec{h} = \nabla\times(\vec{U}\times\vec{H}) \tag{4}$$

As mentioned in the abstract, the magnetic field is assumed to be along the axis of the cylinder as $\vec{H} = (0,0,H_z)$. Fig. 1 shows the schematic figure of a thick-walled cylinder under a magnetic field along the axial direction.

Only nonzero component of displacement is radial displacement $u = u(r)$. Therefore, we have

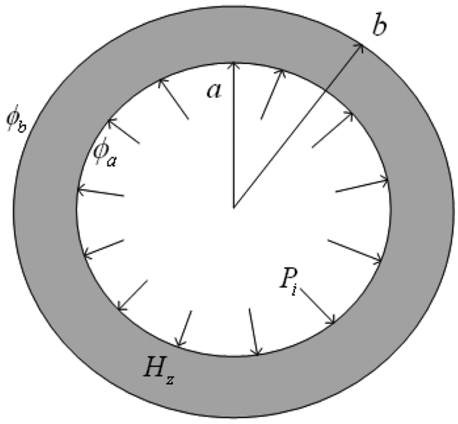

Fig. 1 The schematic figure of a thick walled cylinder under a magnetic field along the axial direction

$$\vec{U} = (u, 0, 0)$$

$$\vec{e} = -\mu(r)\left(0, H_z\frac{\partial u}{\partial t}, 0\right)$$

$$\vec{h} = (0, 0, h_z)$$

$$\vec{J} = \left(0, -\frac{\partial h_z}{\partial r}, 0\right)$$

$$h_z = -H_z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{5}$$

Equilibrium equation with considering the body force subjected to the magnetic field is (Lai *et al.* 1999, Boresi 1993, Arefi *et al.* 2011)

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + f_z = 0 \tag{6}$$

where, f_z is the magnetic force and can be defined as follows

$$f_z = \mu_0 H_z^2 \frac{\partial}{\partial r}\left(r^l \frac{\partial u}{\partial r} + r^l \frac{u}{r}\right) \tag{7}$$

Maxwell equation for electro static equilibrium is

$$\frac{\partial D_r}{\partial r} + \frac{D_r}{r} = 0 \tag{8}$$

Eqs. (6) and (8) can be solved after defining the distribution of the temperature.

2.1 Heat transfer equation

The heat transfer equation for steady state one-dimensional distribution of temperature is (Arefi *et al.* 2011, Rahimi *et al.* 2011, Khoshgoftar *et al*. 2009, Jabbari *et al.* 2002, Incropera 1999)

$$\frac{1}{r}(rk_T(r)T'(r))' = 0 \quad a \le r \le b \tag{9}$$

Heat conduction coefficient $k_T(r)$ may be introduced using power function as follows

$$k_T(r) = k_0 r^k \tag{10}$$

After two times integration of Eq. (9) with respect to r, the radial distribution of the temperature with considering the appropriate boundary conditions can be obtained as follows (Arefi *et al.* 2011, Rahimi *et al.* 2011, Khoshgoftar *et al.* 2009, Incropera 1999)

$$T(r) = \frac{-C_{T1}}{k} r^{-k} + C_{T2} \tag{11}$$

where C_{T1}, C_{T2} are the appropriate constants. These constants depend on the assumed boundary conditions. The assumed boundary conditions and derived constants are presented in Appendix A. The radial distribution of the temperature T(r) can be substituted into Eq. (1) and consequently into (Eqs. (6) and (8))

3. Solution of maxwell and equilibrium equations

By considering the non-homogenous properties of material as a power function, the solution of differential equations (Eqs. (6) and (8)) may be analytically obtained.

Substitution of variable properties (Eq. (2)) and temperature distribution (Eq. (11)) into Eq. (1) presents the stresses and electric displacement equations as follows

$$\begin{aligned}
\sigma_r &= c_{110} r^l \frac{\partial u}{\partial r} + c_{120} r^l \frac{u}{r} + e_{110} r^l \frac{\partial \varphi}{\partial r} - \lambda_1 r^{l+b} T \\
\sigma_\theta &= c_{120} r^l \frac{\partial u}{\partial r} + c_{220} r^l \frac{u}{r} + e_{120} r^l \frac{\partial \varphi}{\partial r} - \lambda_2 r^{l+b} T \\
\sigma_z &= c_{130} r^l \frac{\partial u}{\partial r} + c_{230} r^l \frac{u}{r} + e_{130} r^l \frac{\partial \varphi}{\partial r} - \lambda_3 r^{l+b} T \\
D_r &= e_{110} r^l \frac{\partial u}{\partial r} + e_{120} r^l \frac{u}{r} - g_{110} r^l \frac{\partial \varphi}{\partial r} + p_{110} r^{l+b} T
\end{aligned} \tag{12}$$

By substitution of stress, electric displacement and Lorentz's force relations into the equilibrium equation (Eq. (6)) and Maxwell equation (Eq. (8)), the governing differential equation of the system may be obtained as follows

$$\begin{aligned}
&[C_{110} + \mu_0 H_z^2] r^2 \frac{\partial^2 u}{\partial r^2} + [(C_{110} + \mu_0 H_z^2)(l+1)] r \frac{\partial u}{\partial r} + [C_{120} l - C_{220} + \mu_0 H_z^2 (l-1)] u \\
&+ [e_{110}] r^2 \frac{\partial^2 \varphi}{\partial r^2} + [e_{110}(l+1) - e_{120}] r \frac{\partial \varphi}{\partial r} - C_{T1}[\lambda_{10}(1 - \frac{1}{k}(b+l+1)) + \frac{1}{k}\lambda_{20}] r^{b-k+1} + C_{T2}[-\lambda_{10}(b+l+1) + \lambda_{20}] r^{b+1} = 0 \\
&[e_{110}] r^2 \frac{\partial^2 u}{\partial r^2} + [e_{110}(l+1) + e_{120}] r \frac{\partial u}{\partial r} + [e_{120} l] u - [g_{110}] r^2 \frac{\partial^2 \varphi}{\partial r^2} - [g_{110}(l+1)] r \frac{\partial \varphi}{\partial r} \\
&+ p_{110} C_{T1} [\frac{1}{k}(b+l+1) - 1] r^{b-k+1} - p_{110} C_{T2} [b+l+1] r^{b+1} = 0
\end{aligned} \tag{13}$$

Eq. (13) includes two unknown functions u, φ. The solution of Eq. (13) is composed of the homogenous and particular solutions. Eq. (14) indicates these solutions

$$u = u_h + u_p$$
$$\varphi = \varphi_h + \varphi_p \tag{14}$$

where, subscript h represents the homogenous solution and subscript p represents the particular solution of the problem. The homogenous solution can be obtained by eliminating the non-homogenous terms of differential equation (Eq. (13)). Changing the variable as $r = e^s$ presents the homogenized differential equation as follows

$$\begin{aligned}
&[(C_{110} + \mu_0 H_z^2)\frac{d^2}{ds^2} + (C_{110} + \mu_0 H_z^2) l \frac{d}{ds} + (C_{120} l - C_{220} + (l-1)\mu_0 H_z^2)]u \\
&\quad + [e_{110}\frac{d^2}{ds^2} + (e_{110} l - e_{120})\frac{d}{ds}]\varphi = 0 \\
&[e_{110}\frac{d^2}{ds^2} + (e_{110} l + e_{120})\frac{d}{ds} + l e_{120}]u + [-g_{110}\frac{d^2}{ds^2} - g_{110} l \frac{d}{ds}]\varphi = 0
\end{aligned} \tag{15}$$

Characteristic equation for Eq. (15) may be obtained using the determinant of Eq. (16) as follows (Khoshgoftar *et al.* 2009, Arefi *et al.* 2011)

$$A_1\lambda^4 + A_2\lambda^3 + A_3\lambda^2 + A_4\lambda = 0 \tag{16}$$

where, $\lambda: = \frac{d}{ds}, \lambda^2 = \frac{d^2}{ds^2}$ and A_i are the constants depend on the properties of materials as follows

$$\begin{aligned}
A_1 &= (C_{110} + \mu_0 H_z^2) g_{110} + e_{110}^2 \\
A_2 &= 2l(C_{110} g_{110} + e_{110}^2 + g_{110}\mu_0 H_z^2) \\
A_3 &= \mu_0 H_z^2 g_{110}(l^2 + l - 1) + g_{110}(C_{120} l - C_{220} + l^2 C_{110}) + e_{110}^2 l^2 + l e_{110} e_{120} - e_{120}^2 \\
A_4 &= g_{110} l (C_{120} l - C_{220} + (l-1)\mu_0 H_z^2) + l^2 e_{110} e_{120} - l e_{120}^2
\end{aligned} \tag{17}$$

By solution of Characteristic equation of the system (Eq. (16)), the roots of characteristic equation may be obtained as three types (real, complex and double roots) (Arefi *et al.* 2011, Khoshgoftar *et al.* 2009)

$$u_c = c r^{\lambda}$$
$$u_c = (c_1 + c_2 \ln(r)) r^{\lambda}$$
$$u_c = r^p (c_1 \cos(q \ln(r)) + c_2 \sin(q \ln(r))) \tag{18}$$

where q, p are the imaginary and real parts of the complex root. c_1, c_2 are the constants of integration and can be obtained using the applied boundary conditions. By substitution of the radial displacement in one of the Eq. (13), the other component (electric potential) may be analytically obtained.

Non-homogenous term of solution of differential equation (Eq. (13)) indicates that the particular solution of the system may be supposed as

$$u_p = X_1 r^{b-k+1} + X_2 r^{b+1}$$

$$\varphi_p = X_3 r^{b-k+1} + X_4 r^{b+1} \tag{19}$$

By considering the Eq. (16), we see that one root is zero and the other roots are real integers. From Eq. (15), the homogenous solution of the problem can be analytically obtained as follows

$$u_h = C_1 r^{m_1} + C_2 r^{m_2} + C_3 r^{m_3}$$

$$\varphi_h = C_1' + C_2' r^{m_1} + C_3' r^{m_2} + C_4' r^{m_3} \tag{20}$$

where, m_1, m_2, m_3 are the non zero roots of characteristic equation (Eq. (16)) and C_i, C_i' are dependent constants that must be evaluated using Eq. (13) and boundary conditions.

Selection of u as mentioned in Eq. (20) is due to one zero root of characteristic equation (Eq. (16)). Due to existence of only derivatives of φ such as $\frac{\partial \varphi}{\partial r}, \frac{\partial^2 \varphi}{\partial r^2}$ and inexistence of φ in Eq. (15), we see that C_1' appears in Eq. (20).

By disregarding the thermal effect, the electric displacement can be obtained using substitution of solution presented in Eq. (20) into last equation of Eq. (12) as follows

$$\begin{aligned}
D &= e_{110} r^l \frac{\partial u}{\partial r} + e_{120} r^l \frac{u}{r} - g_{110} r^l \frac{\partial \varphi}{\partial r} \\
D &= e_{110} r^l \left[C_1 m_1 r^{m_1-1} + C_2 m_2 r^{m_2-1} + C_3 m_3 r^{m_3-1} \right] + e_{120} r^l \left[C_1 r^{m_1-1} + C_2 r^{m_2-1} + C_3 r^{m_3-1} \right] \\
&- g_{110} r^l \left[C_2' m_1 r^{m_1-1} + C_3' m_2 r^{m_2-1} + C_4' m_3 r^{m_3-1} \right] = \\
& r^{l+m_1-1}[e_{110} C_1 m_1 + e_{120} C_1 - g_{110} C_2'] + r^{l+m_2-1}[e_{110} C_2 m_2 + e_{120} C_2 - g_{110} C_3' m_2] + \\
& r^{l+m_3-1}[e_{110} C_3 m_3 + e_{120} C_3 - g_{110} C_4' m_3]
\end{aligned} \tag{21}$$

Second differential equation of the problem can be satisfied by substitution of Eq. (21) into Eq. (13).

4. Results and discussion

In the present section, by devoting the numerical values, the behavior of *FGP* cylinder can be analytically studied. The present section investigates the effect of non-homogenous parameters on the mechanical, electrical and thermal behavior of *FGP* cylinder under a magnetic field. Solution procedure can be continued with equaling the value of *k*,*b*,*l* and considering the properties as follows (Khoshgoftar *et al.* 2009)

$$r_i = 0.6(m), r_o = 1(m)$$

$$c_{11} = c_{33} = 111\ Gpa,\ c_{22} = 220\ Gpa$$

$$c_{13} = c_{23} = 115Gpa,\ c_{12} = 77.8Gpa$$
$$e_{11} = 15.1(\mathrm{VmN}^{-1}), e_{12} = e_{13} = -5.1(\mathrm{VmN}^{-1})$$
$$\alpha_2 = 1\times 10^{-4}(1/K), \alpha_1 = \alpha_3 = 1\times 10^{-5}(1/K)$$
$$g_{11} = 5.62\times 10^{-9}(\mathrm{C}^2/\mathrm{Nm}^2)$$
$$p_{11} = -2.5\times 10^{-4}(\mathrm{C}/\mathrm{Nm}^2)$$
$$\mu_0 = 4\pi\times 10^{-7}(\mathrm{H/m}), H_z = 2.23\times 10^{9}(\mathrm{A/m}) \tag{22}$$

The solution of the problem with appropriate boundary conditions can be obtained. Mechanical and electrical boundary conditions are

$$P_a = 1\times 10^{7}(\mathrm{pa}) \quad P_b = 0$$
$$\varphi_a = 0 \quad \varphi_b = 0 \tag{23}$$

In the present section, for the analysis of the piezo magnetic behavior of a *FGP* cylinder, the non-homogenous coefficient can be varying between -1, 1 with incremental value 0.5.

Fig. 2 shows the radial distribution of the radial displacement along the thickness direction for five values of non-homogenous index 0, ±0.5, ±1. The radial displacement decreases, with increasing the value of non-homogenous index.

Shown in Fig. 3 is radial distribution of the radial stress along the thickness direction for five values of non-homogenous index 0, ±0.5, ±1. The absolute value of the radial stress decreases with increasing the value of non-homogenous index.

Fig. 4 shows the radial distribution of the circumferential stress along the thickness direction.

Fig. 5 shows the radial distribution of the axial stress along the thickness direction. Through the variation of the non-homogenous index between -1, 1, there are two regions. At the regions near the inner radius, the axial stress increases with increasing the non-homogenous index. Conversely, at the regions near the outer radius, the axial stress decreases with increasing the non-homogenous index.

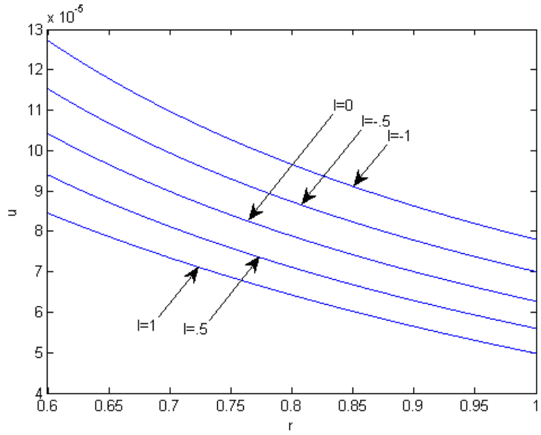

Fig. 2 The radial distribution of the radial displacement along the thickness direction

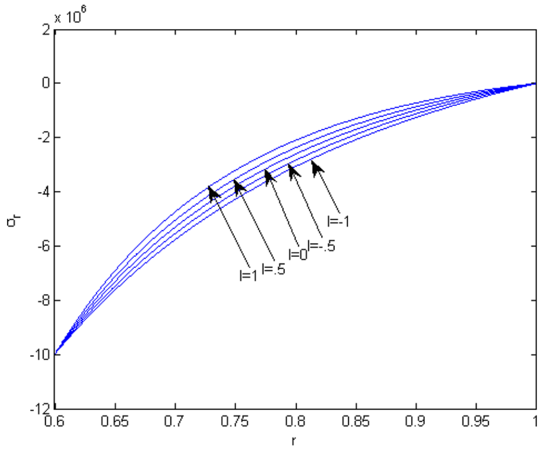

Fig. 3 The radial distribution of the radial stress along the thickness direction

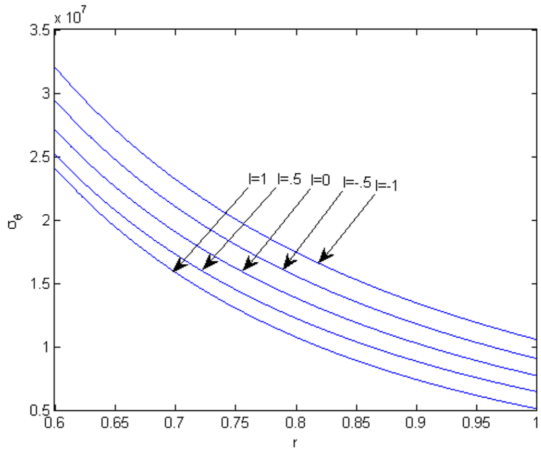

Fig. 4 The radial distribution of the circumferential stress along the thickness direction

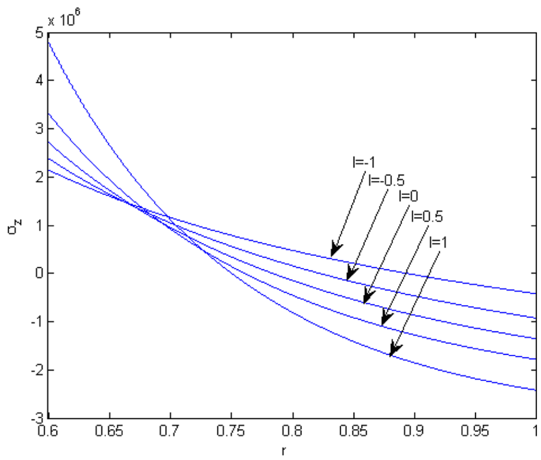

Fig. 5 The radial distribution of the axial stress along the thickness direction

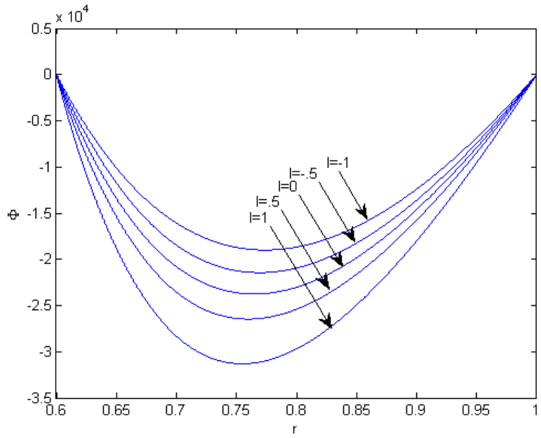

Fig. 6 The radial distribution of the electric potential along the thickness direction

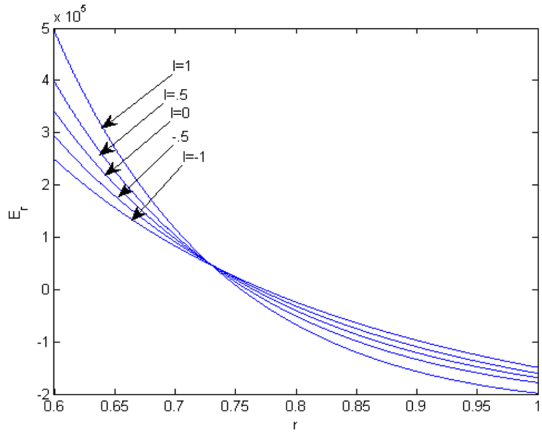

Fig. 7 The radial distribution of the electric field along the thickness direction

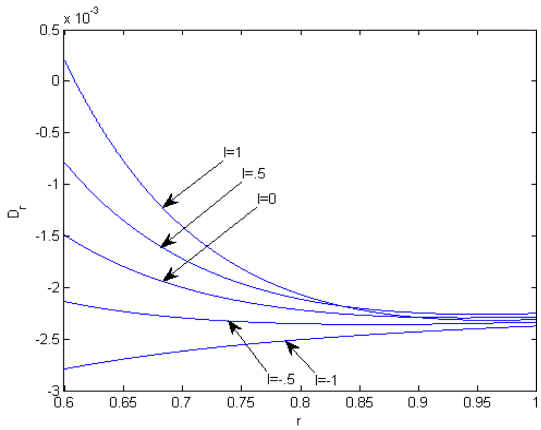

Fig. 8 The radial distribution of the electric displacement along the thickness direction

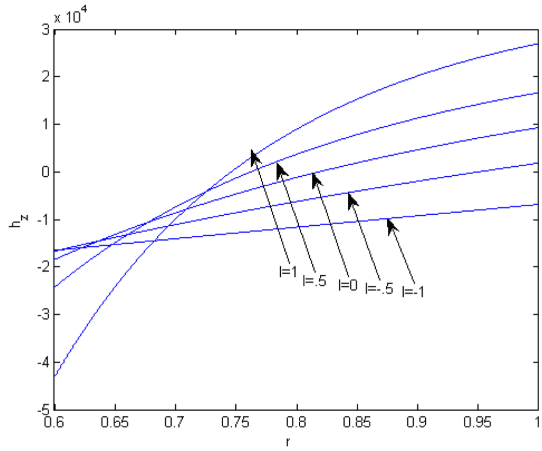

Fig. 9 The radial distribution of the magnetic field disturbance along the thickness direction

Shown in Figs. 6 and 7 are the radial distribution of the electric potential and electric field along the thickness direction, respectively. Fig. 6 indicates that the absolute value of the electric potential increases with increasing the non-homogenous index. Fig. 7 is behaviorally similar to Fig. 5.

Fig. 8 shows the radial distribution of the electric displacement along the thickness direction. This figure indicates that the absolute value of the electric displacement decreases with increasing the non-homogenous index. The variation of electric displacement for different non- homogenous index at the inner radius is significant. Conversely, this variation is not significant at the outer radius.

This difference is due to nonzero inner pressure. Therefore, we can express that the applied pressure impresses the electric displacement considerably. Fig. 9 shows the radial distribution of the magnetic field disturbance along the thickness direction.

5. Conclusions

The present paper proposed an analytical solution for the *FGP* cylinder under magnetic field. All properties were assumed to be non-homogenous except Poisson ratio. Cylinder is assumed to be made of *FGPM*. These materials can be used to control the distribution of mechanical and electrical components such as deformations, stresses and electric potential.

The achieved results indicate that the response of a *FGP* cylinder along the thickness is divided to two types. In the one group of them, at the entire of wall of the cylinder, there is no junction between the curves. Conversely, at another group of them, there is one junction between the curves. Based on this states, there are two types of responses in the analysis.

The distribution of the radial displacement, radial and circumferential stress, electric potential and electric displacement are uniformly along the thickness direction and therefore there is no junction between the figures.

The distribution of the axial stress, electric field and magnetic field disturbance are non-uniformly along the thickness direction and therefore there is one junction between the curves. For components such as axial stress and magnetic field, the effect of non homogenous index is very important for an analysis.

The other result is that the applied pressure can change the value of electric displacement, considerably.

For example at outer surface that the applied pressure is zero, the distribution of electric displacement does not impress. Conversely, at inner surface that applied pressure is considerable, distribution of the electric displacement strongly depends on the non homogenous index.

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CC

Nomenclature

C_{ij} : Elastic stiffness
D : Electric displacement
e : Piezoelectric coefficient
k : Heat conductivity coefficient
p : Pyroelectric coefficient
r : Radius
E : Electric field
T : Temperature
u : Displacement
α : Heat expansion coefficient
φ : Electric potential
g : Dielectric coefficient
$\vec{H}$: Magnetic intensity vector
$\vec{h}$: Perturbation of the magnetic intensity vector
$\vec{e}$: Perturbation of the magnetic field vector
$\vec{J}$: Electric current density vector
μ_0 : Magnetic permeability
f_z : Lorentz's force
σ : Stress
ε : Strain

Appendix A

$$D_{11}T(a)+D_{12}T'(a)=f_1$$
$$D_{21}T(b)+D_{22}T'(b)=f_2$$

$$C_{T1}=\frac{D_{21}f_1-D_{11}f_2}{D_{21}\left(D_{12}r_a^{-(k+1)}-D_{11}\dfrac{r_a^{-k}}{k}\right)-D_{11}\left(D_{22}r_b^{-(k+1)}-D_{21}\dfrac{r_b^{-k}}{k}\right)}$$

$$C_{T2}=\frac{\left(D_{12}r_a^{-(k+1)}-D_{11}\dfrac{r_a^{-k}}{k}\right)f_2-f_1\left(D_{22}r_b^{-(k+1)}-D_{21}\dfrac{r_b^{-k}}{k}\right)}{D_{21}\left(D_{12}r_a^{-(k+1)}-D_{11}\dfrac{r_a^{-k}}{k}\right)-D_{11}\left(D_{22}r_b^{-(k+1)}-D_{21}\dfrac{r_b^{-k}}{k}\right)}$$