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State-space formulation for simultaneous identification of both damage and input force from response sensitivity

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Abstract. A new method for both local damage(s) identification and input excitation force identification of beam structures is presented using the dynamic response sensitivity-based finite element model updating method. The state-space approach is used to calculate both the structural dynamic responses and the responses sensitivities with respect to structural physical parameters such as elemental flexural rigidity and with respect to the force parameters as well. The sensitivities of displacement and acceleration responses with respect to structural physical parameters are calculated in time domain and compared to those by using Newmark method in the forward analysis. In the inverse analysis, both the input excitation force and the local damage are identified from only several acceleration measurements. Local damages and the input excitation force are identified in a gradient-based model updating method based on dynamic response sensitivity. Both computation simulations and the laboratory work illustrate the effectiveness and robustness of the proposed method.

Keywords: response sensitivity; time domain; damage detection; excitation force; state-space.

1. Introduction

Damage identification has been a hot research topic and has received increasing attention in different disciplines such as, mechanical, aerospace and civil engineering in the past three decades. It is important to identify damage(s) at the early stage of development in these structural systems. Early damage identification allows maintenance and repair works to be properly programmed thus minimizing the maintenance costs and also avoid the occurrence of calamity due to the structure failure. Non-destructive techniques have been developed for practical and accurate damage detection, and most of them are based on measured vibration responses. The basic idea of vibration-based non-destructive damage detection is to measure the dynamic responses or dynamic characteristics of a structure, e.g. the natural frequencies, mode shapes, transfer functions, etc. at certain stages over its life span and use the measurements as database to assess the conditions of the structure.

There are a lot of non-destructive methods for damage detection in the literature. Housner *et al.* (1997) presented an extensive summary on the state-of-the-art in control and health monitoring in civil engineering structures. Salawu (1997) discussed and reviewed the use of natural frequency as a diagnostic parameter in structural assessment procedures using vibration monitoring. Doebling *et al.* (1998) provided a comprehensive review of the damage detection methods by examining changes in

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the dynamic properties of a structure. Zou *et al.* (2000) summarized the methods on vibration-based damage detection and health monitoring for composite structures, especially in delamination modeling techniques and delamination detection.

Damage detection usually requires a mathematical model on the structure in conjunction with experimental modal parameters of the structure. The identification approaches are mainly based on the change in the natural frequencies (Cawley and Adams 1979, Narkis 1994), mode shapes (Pandey *et al*. 1991, Ratcliffe 1997, Rizos *et al*. 1990) or measured dynamic flexibility (Pandey and Biswas 1994, Doebling *et al.* 1996, Lim 1991, Wu and Law 2004). The natural frequencies are easy to measure and with a high level of accuracy, and they are the most common dynamic parameters for damage detection. However, problems may arise in some structures if only natural frequencies are used, since the symmetry of the structures would lead to non-uniqueness in the solution in the inverse analysis of damage detection. This problem can be overcome by incorporating the mode shape data in the analysis. Finite element model (FEM) updating method is the most popular tool for damage detection making use of these modal parameters. A large number of gradient-based finite element model updating methods have been discussed by Friswell and Mottershead (1995). All of them have been used for damage detection (Ricles and Kosmatka 1992, Frahat and Hemez 1993, Hemez and Frahat 1995, Sinha and Friswell 2002, Jones and Turcotte 2002, Gordis 1999). The major difficulty in using finite element model updating method lies in the differentiation between the local damages and modeling errors in the structure (Wu and Law 2004). A two-stage method has been proposed to overcome this problem (Friswell and Motershead 2001). The finite element model of the undamaged structure is firstly updated to remove most of the model errors to have a more accurate model. Then the differences in the modal parameters between the damaged and the intact structures are used to estimate the changes in the system parameters.

Some literatures on damage detection in time domain using structural dynamic response have been published recently. Cattarius and Inman (1997) used the time histories of vibration response of the structure to identify damage in smart structures. Majumder and Manohar (2003, 2004) proposed a time domain approach for damage detection in beam structures using vibration data. The vibration induced by a vehicle moving on the bridge was taken to be the excitation force. Chen and Li (2004) presented a method to identify structural parameters and input time history simultaneously from output-only measurements. The structural parameters and the input time history are obtained in an iterative manner. Law and Zhu (2007) proposed an approach for damage detection in a concrete bridge structure in time domain. Both the damage and moving vehicular loads are identified successfully. Lu and Law (2007) presented a dynamic response based model updating method for identifying structural local damages in time domain. A time series analysis method was used to detect damage in plates by Trendafilova and Manoach (2007).

Several of methods are available to calculate the dynamic response of systems. Each of these methods has different level of accuracy, stability and computation efficiency (Bathe and Wilson 1976, Ralston and Wilf 1960, Goudreau and Taylor 1973). It is well known that Newmark method is most popular in engineering for dynamic analysis of either linear or non-linear systems. The state-space approach can also be used to calculate the dynamic response of structures. The advantage of state-space method and the difference with Newmark method have been discussed by the authors (Law and Lu 2004) and other researcher (Wang *et al.* 2001). The major difference between the two methods lies in: Newmark method is based on the approximation of derivatives of the second order differential equation; the state-space method is based on piecewise interpolation of the discrete force functions so that convolution integral can be carried out. And the state-space method make no assumption on the response functions, the distortion of the dynamic characteristics of the structures is smaller in comparison with the Newmark method. Generally speaking, the state-space method is comparably effective in the context of numerical stability and accuracy.

In the present study, the state-space method is used in the computation of dynamic response of a prestressed Euler-Bernoulli beam. And then the sensitivities of dynamic responses with respect to the elemental flexural rigidity and the input excitation force are calculated by state-space method. In the inverse analysis, both the damage of the structure and the excitation force are identified simultaneously. Only several dynamic response measurements of the structure are needed in the inverse analysis. A prestressed single-span and a multi-span continuous beam are used as numerical examples to illustrate the effectiveness and robustness of the proposed method. Computation simulation shows the proposed method is not sensitive to measurement noise. The proposed method is further verified by a laboratory work.

2. State-space formulation for dynamic response and response sensitivity

2.1 State-space formulation for dynamic response

Fig. 1 shows a simply-supported, rectangular prestressed concrete beam with a straight concentric tendon under study. Equation of motion of a prestressed beam by finite element representation can be written as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [\bar{K}]\{x\} = [B]\{F\} \tag{1}$$

where x is a vector the displacement of the model, $\dot{x}$ is the first derivative of x with respect to time t, $\ddot{x}$ is the second derivative of x with respect to time t, M is the consistant mass matrix, C is the Rayleigh damping matrix, i.e., $C = \alpha M + \beta K$, α and β, are two associated damping constants, $\bar{K}$ is the global stiffness matrix of the prestressed beam, K is the global stiffness matrix without prestress force, $\bar{K} = K - K_g$, $\{F\}$ is a vector of the input excitation forces and $[B]$ maps these forces to the associated degrees-of-freedom of the structure. The *jth* input force is assume to be the following form

$$F^j(t) = F^j \sin \omega^j t \quad (j = 1,2,..., N_f) \tag{2}$$

Here, F^j, ω^j are parameters of the *jth* force, N_f is the total number of excitation force, if these parameters are known, then the force is determined. So they are taken as the unknown parameters

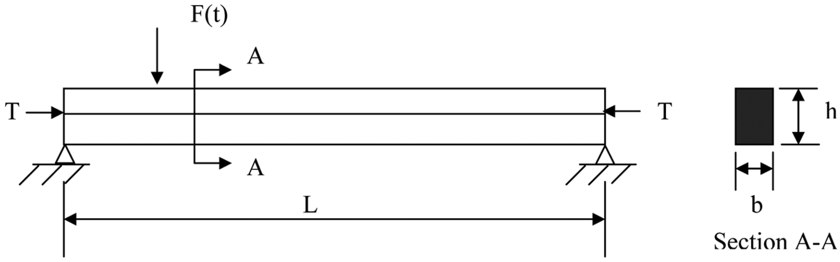

Fig. 1 A single-span prestressed beam

to be identified in the inverse problem. K_g is the global geometrical stiffness matrix and can be expressed as follows

$$[K_g] = \sum_{i=1}^{N} [k_g]_e^i \tag{3}$$

where N is the total number of element, k_g The additional geometrical stiffness matrix (Geradin and Rixen 1994) of each element can be written as

$$[k_g]^i = \frac{T^i}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix}, \ (i = 1, 2, ..., N) \tag{4}$$

Where T is the axially prestress force, it is applied at the anchoring edges of the beam, and the prestress force is assumed to be uniform along the length of the beam and not varying with time. l is the length of the element.

For an isotropic elastic material, the elemental stiffness matrix is proportional to the flexural rigidity EI and the geometric coefficient, which are usually taken as the unknown parameters to be identified in the inverse problem. The stiffness matrix of the structure is expressed as the summation of the elemental stiffness matrices as

$$[K] = \sum_{i=1}^{N} [k]_i^e \tag{5}$$

where N is the number of the elements. Damage in the ith element is modeled as a reduction in the average flexural rigidity $(EI)^i$. $[k]_i^e$ is the elemental stiffness matrix.

Substituting Eqs. (2) and (5) into Eq. (1), we have

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + (\sum_{i=1}^{N} [k]_i^e - [K_g])\{d\} = [B]\{F^j \sin \omega^j t\} \tag{6}$$

Using the state-space formulation, Eq. (1) can be written as the first order differential equation

$$\dot{X} = K^* X + \bar{F} \tag{7}$$

where, $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}_{2n \times 1}$, $K^* = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{2n \times 2n}$, $\bar{F} = \begin{bmatrix} 0 \\ M^{-1}[B]F \end{bmatrix}$, X represents a vector of state variables with a length $2n$ containing the displacements and velocities of the nodes, n is the total degree-of-freedom of the finite element model. These differential equations can then be converted to discrete equations using exponential matrix representation.

$$X_{k+1} = AX_k + \bar{D}_{k+1}\bar{F}_k \tag{8}$$

$$A = e^{K^* \Delta t} \tag{9}$$

$$\bar{D} = K^{*-1}(A - I) \tag{10}$$

where A is the exponential matrix, $(k+1)$ denotes the value at the $(k+1)$th time step of computation, the time step Δt represents the time step in the computation. I is the unit matrix. The dynamic response of the system can be obtained from Eq. (8). Once the vector X is obtained, substituting X into Eq. (7), then the acceleration response can be obtained.

2.2 State-space formulation for response sensitivity

The sensitivity of the dynamic response with respect to the physical parameters and the force parameters can be derived as follows.

Performing differentiations on both sides of Eq. (7) with respect to flexural rigidity $(EI)^i$ of each element, we have

$$\frac{\partial \dot{X}}{\partial EI^i} = K^* \frac{\partial X}{\partial EI^i} + \frac{\partial K^*}{\partial EI^i} X = K^* \frac{\partial X}{\partial EI^i} + \begin{bmatrix} -M^{-1} \dfrac{\partial K}{\partial EI^i} x \\ -M^{-1} \beta \dfrac{\partial K}{\partial EI^i} \dot{x} \end{bmatrix} \quad (i = 1, 2, \ldots, N) \tag{11}$$

Let, $Y = \dfrac{\partial X}{\partial EI^i}$, $\overline{\overline{P}} = \begin{bmatrix} -M^{-1} \dfrac{\partial K}{\partial EI^i} x \\ -M^{-1} \beta \dfrac{\partial K}{\partial EI^i} \dot{x} \end{bmatrix}$, Eq. (11) can be rewritten as

$$\dot{Y} = K^* Y + \overline{\overline{P}} \tag{12}$$

Performing differentiations on both sides of Eq. (7) with respect to the force parameter F^j and ω^j, we have

$$\frac{\partial \dot{X}}{\partial F^j} = K^* \frac{\partial X}{\partial F^j} + \begin{bmatrix} 0 \\ -M^{-1}[B] \sin \omega^j t \end{bmatrix} \tag{13}$$

$$\frac{\partial \dot{X}}{\partial \omega^j} = K^* \frac{\partial X}{\partial \omega^j} + \begin{bmatrix} 0 \\ -M^{-1}[B] F^j t \cos \omega^j t \end{bmatrix} \tag{14}$$

Let, $Z_1 = \dfrac{\partial X}{\partial F^j}$, $\overline{\overline{P}}_1 = \begin{bmatrix} 0 \\ -M^{-1}[B] \sin \omega^j t \end{bmatrix}$, $Z_2 = \dfrac{\partial X}{\partial \omega^j}$, $\overline{\overline{P}}_2 = \begin{bmatrix} 0 \\ -M^{-1}[B] F_j t \cos \omega^j t \end{bmatrix}$

Eqs. (13) and (14) can be rewritten as

$$\dot{Z}_1 = K^* Z_1 + \overline{\overline{P}}_1 \tag{15}$$

$$\dot{Z}_2 = K^* Z_2 + \overline{\overline{P}}_2 \tag{16}$$

Note that Eqs. (12), (15) and (16) have the same form of Eq. (7), similarly, these sensitivities can also be obtained.

3. Inverse problem

In the forward problem, the dynamic responses and their sensitivities with respect to the average flexural rigidity of each element and parameters of each force can be obtained from for a given set of parameters EI^i, F^j, ω^j (i=1,2…N, j=1,2…N_f). In the inverse problem, these parameters are required to be identified from the measured responses. In other words, these parameters are chosen to best fit the measurement data. There are in general two ways to fit the data: one is simply using the least-squares method which minimizes the square error sum; the other is the sensitivity-based analysis method which has different formulation for different problems, and it is often obtained approximately by neglecting the higher order of the formulation. The latter approach is adopted in this study. The identification problem can be expressed as follows: to find the vector of the force parameters $\{P_F\} = [F^1, \omega^1, F^1, \omega^1,..., F^{Nf}, \omega^{Nf}]^T$ and the vector of average flexural rigidity $\{P_{EI}\} = [EI^1, EI^1,..., EI^N]^T$ such that the calculated acceleration or displacement best matches the measured response, i.e.

$$[Q]\{\ddot{d}\} = \{\hat{\ddot{d}}\} \tag{17a}$$

or

$$[Q]\{d\} = \{\hat{d}\} \tag{17b}$$

where the selection matrix $[Q]$ is a constant matrix with elements of zeros or ones, matching the degrees-of-freedom corresponding to the measured acceleration or displacement components.

Let

$$\{\delta z\} = \{\hat{\ddot{d}}\} - [Q]\{\ddot{d}_{cal}\} \tag{18a}$$

or

$$\{\delta z\} = \{\hat{d}\} - [Q]\{d_{cal}\} \tag{18b}$$

The identification problem can be accomplished from the following two subsections.

3.1 Excitation force identification from measured dynamic response

In the penalty function method (Friswell and Mottershead 1995), we have

$$\{\delta z\} = [S_F]\{\delta P_F\} \tag{19}$$

where $\{\delta z\}$ is the error in the measured output, the flexural rigidity of intact structure is used for we do not know the true flexural rigidity of the damage structure. And $\{\delta P_F\}$ is the perturbation in the force parameters, $[S_F]$ is the two-dimensional sensitivity matrix, which is the acceleration (or displacement) response with respect to the force parameters in time domain. When writing in full, the left hand side of Eq. (18) can be written as

$$\{\delta z\} = \begin{Bmatrix} \hat{\ddot{d}}(t_1) \\ \hat{\ddot{d}}(t_2) \\ \vdots \\ \hat{\ddot{d}}(t_l) \end{Bmatrix} - [Q]\begin{Bmatrix} \ddot{d}_{cal}(t_1) \\ \ddot{d}_{cal}(t_2) \\ \vdots \\ \ddot{d}_{cal}(t_l) \end{Bmatrix} \tag{20a}$$

$$\{\delta z\} = \begin{Bmatrix} \hat{d}(t_1) \\ \hat{d}(t_2) \\ \vdots \\ \hat{d}(t_l) \end{Bmatrix} - [Q]\begin{Bmatrix} d_{cal}(t_1) \\ d_{cal}(t_2) \\ \vdots \\ d_{cal}(t_l) \end{Bmatrix} \tag{20b}$$

where l is the number of identification equation, and l should be greater than the unknown force parameters to make sure that the set of equation is over-determined. Eq. (19) can be solved by the standard simple least-squares methods as follows

$$\{\delta P_F\} = [(S_F)^T(S_F)]^{-1}(S_F)^T\delta z \tag{21}$$

or

$$P_{F\,j+1} = F_{F\,j} + [(S_F)_j^T(S_F)_j]^{-1}(S_F)_j^T\delta z_j \tag{22}$$

The subscript j indicates the iteration number at which the sensitivity matrix is computed.

Like many other inverse problems, Eq. (19) is an ill-conditioned problem. In order to provide bounds to the solution, the damped least-squares method (DLS) (Tikhonov 1963) is used and singular-value decomposition is used in the pseudo-inverse calculation. Eq. (19) can be written in the following form in the DLS method

$$\{\delta P_F\} = ((S_F)^T(S_F) + \lambda I)^{-1}(S_F)^T\delta z \tag{23}$$

where λ is the non-negative damping coefficient governing the participation of least-squares error in the solution. The solution of Eq. (23) is equivalent to minimizing the function

$$J(\{\delta P_F\}, \lambda) = \|(S_F)\delta P_F - \delta z\|^2 + \lambda\|\delta P_F\|^2 \tag{24}$$

with the second term in Eq. (24) provides upper and lower bounds to the solution. When the parameter λ approaches zero, the estimated vector $\{\delta P_F\}$ approaches to the solution obtained from the simple least-squares method. In this study, the well-known L-curve method (Hansen 1998) is adopted to determine the appropriate regularization parameters λ. The L-curve is a plot $\|\delta P_v\|$ of the norm of the regularized solution versus the corresponding residual norm $\|[S_v]\delta P_v - \delta R\|$ for all valid regularization parameters. The vertical part of the L-curve corresponds to perturbations of the regularized solution resulting from contamination errors, and the horizontal part represents small changes of the regularized solution caused by regularization errors (Hansen 1998). Therefore, the optimal regularization parameter is a point on this curve that is at the "corner" of the vertical piece.

3.2 Damage Identification from measured dynamic response

Once the force has been obtained from the above, now we move to damage identification. Again by using the penalty function method, we have

$$\{\delta z\} = [S_{EI}]\{\delta P_{EI}\} \tag{25}$$

where $\{\delta z\}$ is the error in the measured output, since we do not know the true elemental flexural rigidity of the damage structure, the flexural rigidity of intact structure is used. And $\{\delta P_{EI}\}$ is the perturbation in the elemental flexural rigidity, $[S_{EI}]$ is the two-dimensional sensitivity matrix, which is the acceleration (or displacement) response with respect to the elemental flexural rigidity in time domain.

Similarly, the elemental flexural rigidity can be obtained from

$$\{\delta P_{EI}\} = ((S_{EI})^T(S_{EI}) + \lambda I)^{-1}(S_{EI})^T\delta z \tag{26}$$

or

$$(P_{EI})_{j+1} = (P_{EI})_j + ((S_{EI})_j^T(S_{EI})_j + \lambda I)^{-1}(S_{EI})_j^T\delta z_j \tag{27}$$

The subscript j indicates the iteration number.

3.3 Algorithm of iteration

As both the excitation force and the damaged structure are unknown, the following iterative algorithm is used to solve the problem.

(1) Iteration of the excitation force parameters

Starting with an initial guess $\{(P_F)_0\}$ for the unknown force parameter vector $\{P_F\}$ and the intact elemental flexural rigidity, the procedure of iteration is given as:

Step 1: Solve Eq. (8) at $j=k+1$ iteration step with known $\{(P_F)_k\}$ for acceleration $\{\ddot{d}\}$ or displacement $\{d\}$ and compute the value $\{\delta z_k\}$

Step 2: Solve Eqs. (15) and (16) at $j=k+1$ iteration step with known $\{(P_F)_k\}$ for $\left\{\dfrac{\partial \ddot{d}}{\partial (P_F)_k}\right\}$ or $\left\{\dfrac{\partial d}{\partial (P_F)_k}\right\}$ to get the sensitivity matrix.

Step 3: Find $\{(P_F)_{k+1}\}$ from Eq. (22) or Eq. (23)

Step 4: Repeat Steps 1 to 3 until $\left\|\dfrac{\{(P_F)_{k+1}\} - \{(P_F)_k\}}{\{(P_F)_{k+1}\}}\right\| \le tolerance$ 1. The tolerance equals 1.0×10^{-5} in this study.

(2) Iteration of the elemental flexural rigidity

Starting with the modified excitation force parameter vector $\{P_F\}$ and the intact elemental flexural rigidity, the procedure of iteration is used for damage identification:

Step 1a: Solve Eq. (8) at $j=k+1$ iteration step with known $\{(P_{EI})_k\}$ for acceleration $\{\ddot{d}\}$ or displacement $\{d\}$ and compute the value $\{\delta z_k\}$

Step 2a: Solve Eqs. (11) at $j=k+1$ iteration step with known $\{(P_{EI})_k\}$ for $\left\{\dfrac{\partial \ddot{d}}{\partial (P_{EI})_k}\right\}$ or $\left\{\dfrac{\partial d}{\partial (P_{EI})_k}\right\}$ to get the sensitivity matrix.

Step 3a: Find $\{(P_{EI})_{k+1}\}$ from Eq. (27).

Step 4a: Repeat Steps 1a to 3a until $\left\|\dfrac{\{(P_{FI})_{k+1}\} - \{(P_{FI})_k\}}{\{(P_{FI})_{k+1}\}}\right\| \le tolerance$ 2. The tolerance equals 1.0×10^{-8} in this study.

The identified excitation force obtained in (1) can be further improved using the updated physical parameters obtained in (2) and repeating Steps 1 to 3. On the other hand, the vector of physical parameters can also be further improved using the modified excitation force and repeating Steps 1a to 3a. This will be illustrated in the numerical simulation.

4. Computation simulation

4.1 Example 1: A single span prestressed beam

A single span Euler-Bernoulli beam with axial prestress force is studied as shown in Fig. 1. The beam is assumed to be simply supported. The magnitude of prestress force is 1.0×10^6 N and it is assumed to be a constant along the beam. The physical parameters of the beam under study are: the mass density $\rho = 2.5\times10^3$ kg/m^3, Young's modulus $E = 3.3\times10^{10}$ N/m^2, the length, $L = 20$ m, the width $b = 0.6$ m and the height $h = 1.0$ m.

A finite element model of the prestressed beam is constructed using Euler-Bernoulli beam elements. The *FE* model has 10 elements and 22 degrees of freedom. The first five natural frequencies of the beam are: 4.07, 16.42, 37.03, 65.93and103.25Hz. The damping ratios for these five modes are all equal to 0.02. The time step Δt is taken to be 0.001 second in computation of the responses and time duration is 2 seconds.

4.1.1 Identification of one local damage and one impulsive force

A local damage is simulated by assuming a 5% reduction in average flexural rigidity at the 3rd element. The impulsive force is assumed to act at the beam at time 0.05 second and last for 0.05 second, and it is assumed to be a constant in such a small time interval. It can be expressed mathematically

$$F(t) = \begin{cases} 8000N & 0.05 \le t \le 0.1 \\ 0 & 0 \le t \le 0.05 \text{ or } t > 0.1 \end{cases} \tag{28}$$

It is applied at 6 metres from the left support. White noise is added to the calculated responses of the beam to simulate the noisy measurement data with

$$acc_{measured} = acc_{calculated} + E_p * N_{noise} * \sigma(acc_{calculated}) \tag{29}$$

where E_p is the noise level, N_{noise} is a standard normal distribution vector with zero mean value and unit standard deviation, $\sigma(acc_{calculated})$ is the standard deviation of the original acceleration response. 1%, 5% and 10% noise are added to the calculated responses to study the effect of noise level on the identified results.

Three acceleration measurements at the 3rd node, 5th node, and 7th node of the beam are used in the identification and responses of the first 2 seconds are used. Table 1 shows the iteration number and optimal regularization parameter λ_{opt} corresponding to different noise level. Fig. 2 shows the identified results of the impulsive force and the damage. From this figure, the following observations can be obtained:

(1) The identified impulsive force and the local damage are both close to the true values. This shows that the proposed method is correct and effective to identify both excitation force and damage in the beam from noisy dynamic response measurement.

(2) The identified results are similar for different noise level under study, especially, all the curves of the impulsive force are almost coincided, indicting that the present method is insensitive to the artificial measurement noise.

Table 1 Iteration number and regularization parameters corresponding to different noise level

		1% noise	5% noise	10% noise
Iteration number	Force	15	16	18
	Damage	10	11	11
λ_{opt}	Force	7.4×10^{-4}	7.4×10^{-4}	7.3×10^{-4}
	Damage	6.5×10^{-7}	1.18×10^{-6}	1.22×10^{-6}

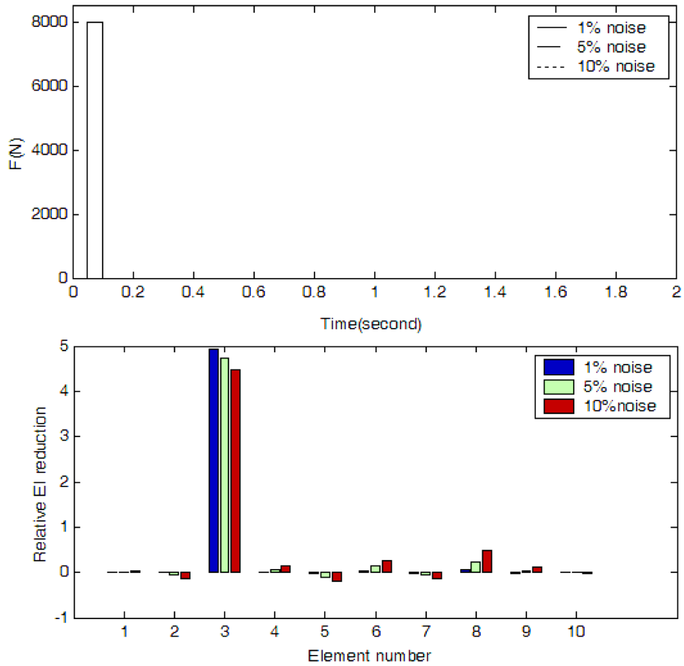

Fig. 2 Identification of an impulsive force and a local damage under different noise levels

4.2 Example 2: two-span continuous beam

A two-span continuous Euler-Bernoulli beam with axial prestress force is $1.2\times10^6 N$ is studied as shown in Fig. 3. The beam is assumed to be simply supported at both ends. The physical parameters of the beam under study are: the mass density $\rho = 2.5\times10^3$ kg/m^3, Young's modulus $E = 3.3\times10^{10}$ N/m^2, the length $L = 30$ m, the width $b = 0.6$ m and the height $h = 1.0$ m.

4.2.1 Identification of four local damages and one combined sinusoidal excitation force

A finite element model of the prestressed beam is constructed using Euler-Bernoulli beam elements. The *FE* model has 16 elements and 34 degrees of freedom. The first five natural frequencies of the intact beam are: 7.26, 11.39, 29.23, 37.00 and 65.90 Hz. The damping ratios for these five modes are all equal to 0.02. The external excitation force is assumed to be $F(t) = 16000\times(1+0.1\sin12\pi t + 0.05\sin50\pi t)$ N, and it is applied at the 3rd nodes of the finite element model. The time step Δt is taken to be 0.001 second in computation of the responses and time duration is 2 seconds. The local damages are simulated by assuming a 10%, 5%, 10% and 15% reduction in average flexural rigidity at elements 2, 3, 12 and 13, respectively. Three acceleration measurements at 5th, 11th and 14th node

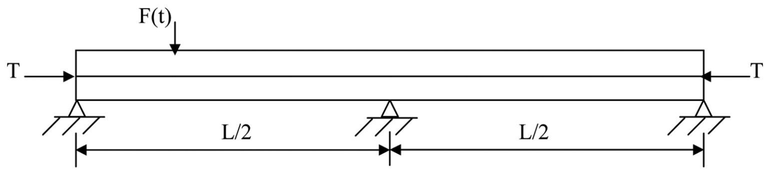

Fig. 3 A continuous prestressed beam

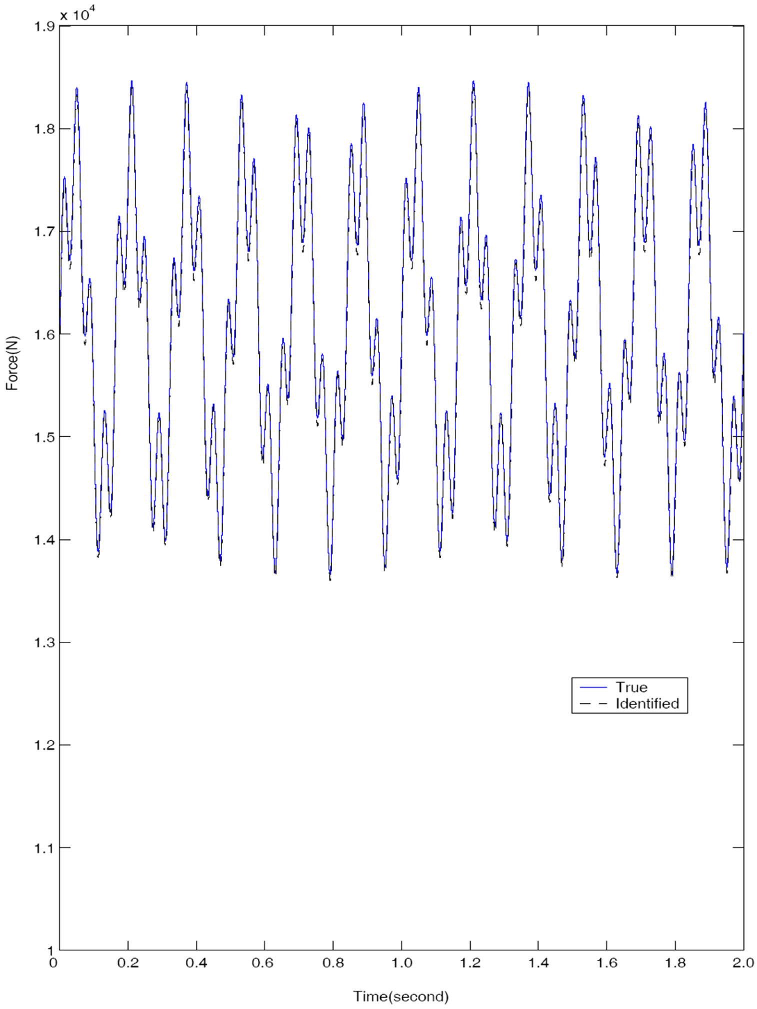

Fig. 4 Identified force time history after the second round of iteration (10% noise level)

are used and 2000 time steps are used. 10% noise is added to the calculated acceleration to simulate the measured response. Figs. 4 and 5 show the identified results of the excitation force and the damage in the first round and in the second round, respectively. The iteration number and optimal regularization parameter for force and damage identification in the first round are 20, 4.6×10^{-4} and 17, 1.5×10^{-6}, respectively. The iteration number and optimal regularization parameter for force and damage identification in the second round are 22, 1.5×10^{-6} and 18, 1.3×10^{-6}, respectively. This shows that the proposed method is effective for both multiple damages identification and excitation force identification.

5. Laboratory work

The proposed method is further demonstrated with laboratory results from a simply supported

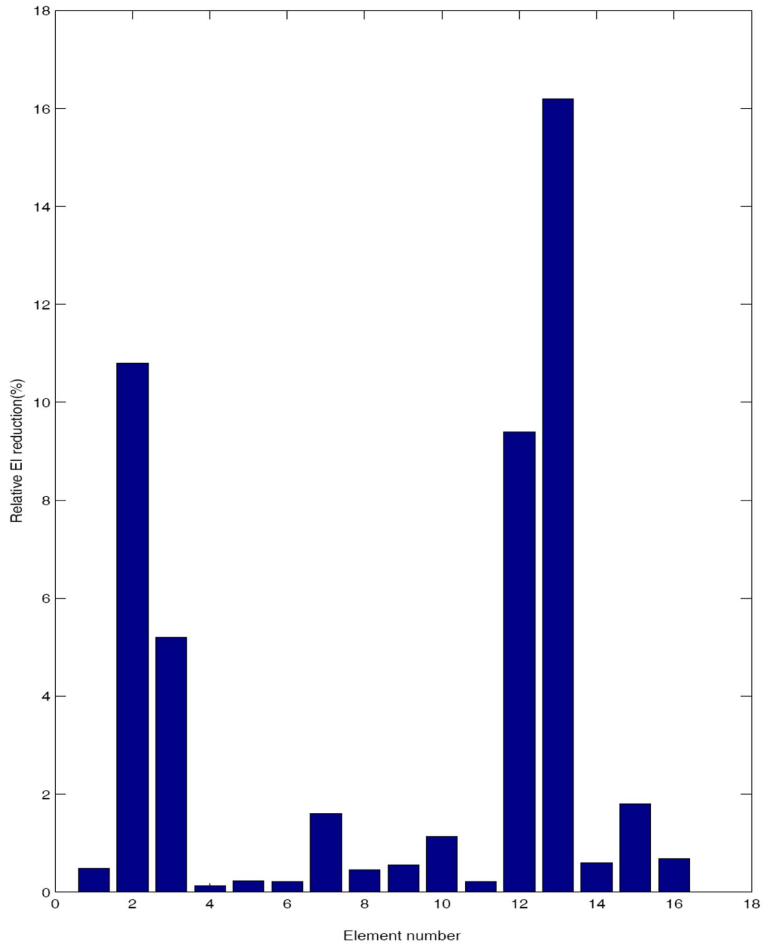

Fig. 5 Identification of multiple damages (10% noise level)

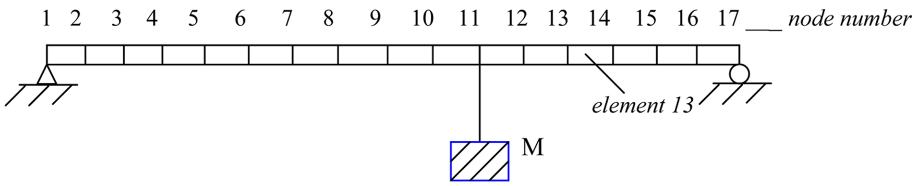

Fig. 6 Experimental set-up

steel beam as shown in Fig. 6. The parameters of the beam are: length 2.0 m, width 25 mm and height 19 mm, the elastic modulus and mass density of the material are 206.5 Gpa and 7.832×10^3 kg/m^3, respectively. It is discretized into sixteen Euler beam elements with two degrees-of-freedom at each node. A mass of 2.61 kg is hanged by a fine nylon rope at node 11 of the beam, and the excitation generated by cutting the rope will serve as the input force. The true value of the force is 25.58 N and is an "impact" acting at the initial time t=0. Mathematically, it is expressed as

$$f(t) = \begin{cases} Mg & t = 0 \\ 0 & t > 0 \end{cases} \tag{30}$$

The flexural rigidities of all the elements and the assumed impulsive force are taken as the unknowns in the inverse analysis. The initial values of the damage parameters for all the finite elements are all zero. The initial vector of the force parameters is $\{(P_F)_0\} = [0, 0, 0, 0, 0, 0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi]^T$.

The sampling frequency is 2000 Hz. Acceleration responses are collected by B&K 4370 accelerometers at nodes 7 and 9, and 2000 data of these accelerometers from 0.0 to 1.5 second were used for the identification. A commercial data logging system INV303 and the associated signal analysis package DASP2003 are used in the data acquisition. Damage is introduced by removing equal thickness of 0.5mm material from both sides of the beam over a length of 9 mm in element 13, with one edge of the damage zone starting at node 13. The equivalent reduction in the second moment of inertia of element 13 is found to be 11.3% after condensing the middle degrees-of-freedom to the two end nodes 13 and 14 by Guyan reduction. The first five natural frequencies of the undamaged and the damaged beam are shown in Table 2 along with those from the finite element model. The calculated frequencies are found very close to the measured values indicating a model which is accurate enough for the subsequent damage identification.

Table 2 Calculated and measured natural frequencies of the test beam (Hz)

Modal frequency		1st	2nd	3rd	4th	5th
Intact	Measured	10.523	41.316	92.615	165.353	254.625
	Calculated	10.459	41.423	93.116	165.274	256.242
Damaged	Measured	10.282	40.508	91.264	164.695	250.583
	Calculated	10.271	40.744	92.258	164.703	253.328

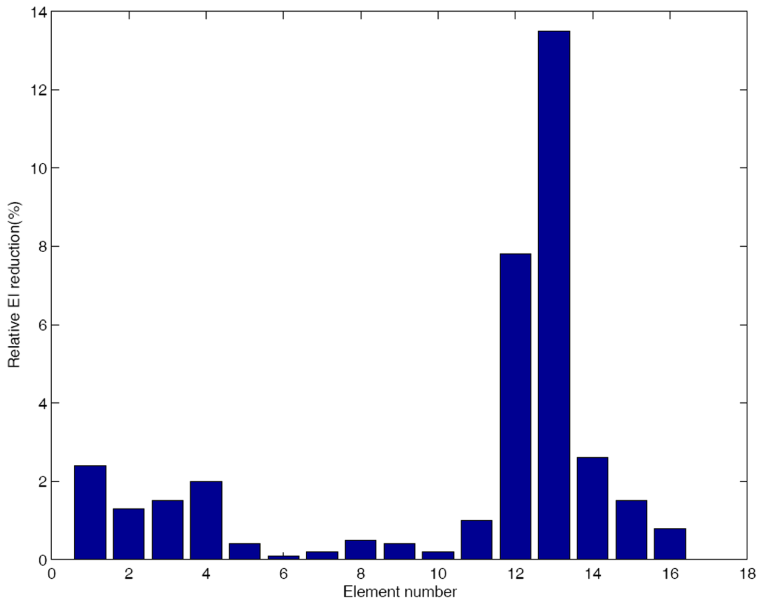

Fig. 7 Damage detection after the second round of iteration

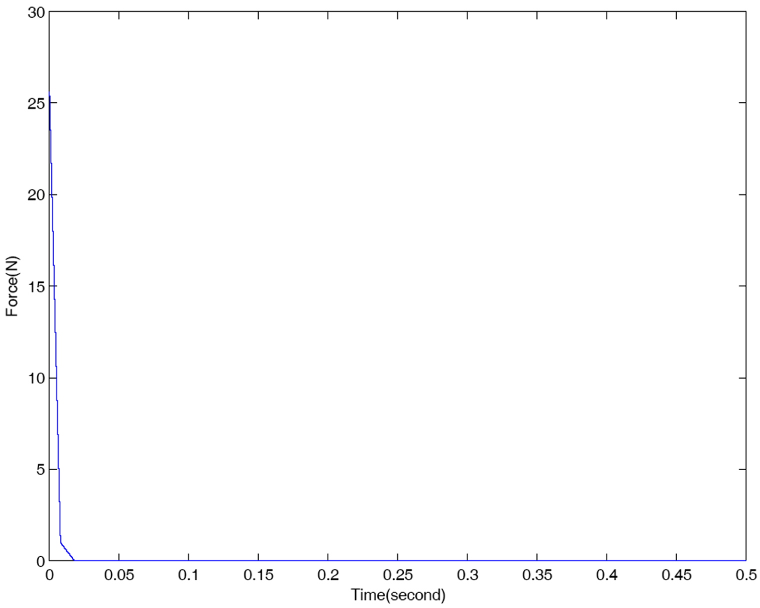

Fig. 8 The identified force time history after the second round of iteration

The iteration stops after two cycles. All the force parameters are identified simultaneously. The required number of iteration for convergence in the second cycle of iteration is 14 and 35 for the force and the damages, and the corresponding optimal regularization parameters are 0.53 and 0.87 respectively. Fig. 7 shows the identified damage, and the identified damage in element 13 is 13.5% which is close to the true value. But there is a large false identification in element 12. This observation can be explained since element 12 is in immediate adjacent to the damage and the vibration energy in the element would be much more disturbed than those in other elements as discussed by Shi and Law (2000). Fig. 8 shows the identified time history of the force with a peak of 25.6 N at $t = 0$ which is very close to the true value. The natural frequencies of the beam calculated with the identified parameters are shown in Table 2 and they are found matching the experimental values very well indicating the success of the identification.

6. Conclusions

This paper presented a method for both identifying input excitation force and local damage. The state-space approach is used to calculate the structural dynamic response and sensitivities in time domain. Comparison is made on the state-space method and Newmark method, which shows that the two methods have the same accuracy while the state-space method is more efficient than Newmark method. Local damages and the input excitation force are identified in a gradient-based model updating method based on dynamic response sensitivity. The advantage of the proposed method is only several dynamic response measurements are need in the inverse analysis. Both Numerical simulations and the laboratory work illustrated the effectiveness and robustness of the proposed method.

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