Smart Structures and Systems, Vol. 6, No. 1 (2010) 17-27
DOI: http://dx.doi.org/10.12989/sss.2010.6.1.017

Statistical approach to a SHM benchmark problem

Sara Casciati*

Department ASTRA, University of Catania, Via delle Maestranze 99, 96100 Siracusa, Italy

(Received April 15, 2007, Accepted May 14, 2009)

Abstract. The approach to damage detection and localization adopted in this paper is based on a statistical comparison of models built from the response time histories collected at different stages during the structure lifetime. Some of these time histories are known to have been recorded when the structural system was undamaged. The consistency of the models associated to two different stages, both undamaged, is first recognized. By contrast, the method detects the discrepancies between the models from measurements collected for a damaged situation and for the undamaged reference situation. The damage detection and localization is pursued by a comparison of the *SSE* (sum of the squared errors) histograms. The validity of the proposed approach is tested by applying it to the analytical benchmark problem developed by the ASCE Task Group on Structural Health Monitoring (SHM). In the paper, the results of the benchmark studies are presented and the performance of the method is discussed.

Keywords: damage detection; damage localization; regression analysis; structural health monitoring; sum of the squared errors.

1. Introduction

The research in the field of Structural Health Monitoring (SHM) led to the development of several algorithms for damage detection and localization which fulfil the requirements of effectiveness, simplicity, reliability and low amount of data storage (Casciati and Casciati 2006, Casciati 2008, Masri, *et al*. 2000). Within this framework, a method exploiting the response surface approximation theory (Breitung and Faravelli 1996, Draper and Smith 1981) was formulated by the author in (Casciati 2005, 2009). Empirical models are built from two different sets of data: a reference one which is associated to the undamaged case and a second set of data, subsequently collected, which is *a priori* unknown since it could either represent an undamaged or damaged case. A solution to the problem of damage detection and localization is then pursued by comparing the *SSE* (sum of the squared errors) histograms associated with these models.

In view of the implementation of this algorithm into intelligent devices, its validation on a benchmark problem is preliminarily performed. For this purpose, the benchmark problem proposed by the ASCE SHM Task Group (Johnson, *et al*. 2000, 2004) is suitable and is considered as a case study in this paper.

2. The statistical approach

The approach adopted in this paper assumes that the measurement data from an undamaged baseline

*E-mail: sara.casciati@uict.it

structure are available, and that another data set (marked with the suffix "$_{\mathrm{new}}$" in the following) is subsequently collected from the same structure whose state in the meantime has become unknown (either damaged or undamaged). Given this pair of data sets, the goal is then to conclude whether or not the new state is associated with damage.

Let N_S the number of sensors placed across a structure. Each recorded time history is broken up into several segments. Each segment is approached by regression analysis, so that the results from all the segments can be used for a statistical analysis. Let n_{set} be the number of segments, each of p points, with $p = \alpha N_S$ and α a positive integer.

A regression analysis is repeatedly applied by assuming in turn the measurements taken at each sensor as the response variable, vector $\mathbf{y}$. At each instant of time, the corresponding measurements from the remaining channels, matrix $\mathbf{X}$, are considered as the predictors in the regression scheme. In other words, for each sensor j and for each segment i, one can write:

$$\mathbf{y}(j, i) = f(\mathbf{X}(j, i)) + \eta_L(j, i) + \eta(j, i), \quad j = 1, \ldots, N_s, \quad i = 1, \ldots, nset \tag{1}$$

where $\mathbf{y}$ is the $p \times 1$ vector of the response variables, $\mathbf{X}$ is the $p \times (N_S\text{-}1)$ matrix of the predictors, η_L and η are the $p \times 1$ vectors of the lack of fit errors and of the pure errors, respectively. The values contained in these matrices vary depending on the sensor j whose measurements are assumed as responses and on the considered segment i along the corresponding time history. For simplicity of notation, this dependence will be omitted in the following.

In Eq. (1), one distinguishes the systematic lack of fit errors, η_L, due to the selected form of $f(.)$, and the random errors, or pure errors, η. The latter errors (Breitung and Faravelli 1996) arise from the remark that, if the same experiment is run m times for the same values of $\mathbf{X}$, the resulting outcomes $\mathbf{y}_1, \ldots, \mathbf{y}_m$ may result different due to the randomness of the variables. It is generally assumed that the total error terms, $\varepsilon=\eta_L+\eta$, are normally distributed uncorrelated random variables, with zero mean and variance σ^2, i.e., $\varepsilon \sim N(0, \sigma)$. This assumption allows the analyst to perform the statistical tests, such as the t- or F-tests. Since the aim of the proposed algorithm is only to perform regression analyses, no hypothesis on the error distribution needs to be introduced (Draper and Smith 1981).

Typically, a polynomial form is assumed for $f(.)$. Since the structural behavior of the system is linear also after the braces removal (i.e., the occurrence of "damage"), a first-order polynomial is assumed as sufficient to model the case under consideration. Therefore, for the j-th sensor and for the i-th segment along its time history, the following linear equations can be written for the undamaged and unknown state cases, respectively

$$\mathbf{y} = \mathbf{A}\beta + \varepsilon$$
$$\mathbf{y}_{\mathrm{new}} = \mathbf{A}_{\mathrm{new}}\beta_{\mathrm{new}} + \varepsilon_{\mathrm{new}} \tag{2}$$

where $\mathbf{A}$ is a $p \times N_S$ matrix whose j-th column is a $p \times 1$ vector of 1's, and the remaining N_S - 1 columns are the $p \times 1$ vectors of the predictors which were collected in matrix $\mathbf{X}$. The problem turns to be one of linear regression analysis, and β is the $1 \times N_S$ vector of the unknown linear regression coefficients. As previously mentioned, the suffix "$_{\mathrm{new}}$" denotes that the same quantities are referred to the unknown state of the structure.

The classical linear regression analysis provides the following estimates for the β-vectors

$$\mathbf{b} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{y}$$

$$\mathbf{b}_{\text{new}} = (\mathbf{A}_{\text{new}}^T \mathbf{A}_{\text{new}})^{-1} \mathbf{A}_{\text{new}}^T \mathbf{y}_{\text{new}} \tag{3}$$

The estimates values of the response vectors $\mathbf{y}$ are:

$$\hat{\mathbf{y}} = \mathbf{A}\mathbf{b} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{y} = \mathbf{P}_A\mathbf{y}$$

$$\hat{\mathbf{y}}_{\text{new}} = \mathbf{A}_{\text{new}}\mathbf{b}_{\text{new}} = \mathbf{A}_{\text{new}}(\mathbf{A}_{\text{new}}^T\mathbf{A}_{\text{new}})^{-1}\mathbf{A}_{\text{new}}^T\mathbf{y}_{\text{new}} = \mathbf{P}_{A,\,\text{new}}\mathbf{y}_{\text{new}} \tag{4}$$

with $\mathbf{P}_A$ and $\mathbf{P}_{A,\text{new}}$ matrices of size p by p.

The further step in multivariate statistics usually consists of performing the analysis of variance. For this purpose, each of the items listed below is computed together with its own number of degrees of freedom is denoted as λ (Casciati 2009):

- the total sum of squares, SSY, with $\lambda = p$:

$$SSY = \mathbf{y}^T\mathbf{y}$$

$$SSY_{\text{new}} = \mathbf{y}_{\text{new}}^T\,\mathbf{y}_{\text{new}} \tag{5}$$

- the regression sum of squares, SSR, with $\lambda = N_S$:

$$SSR = \hat{\mathbf{y}}^T\mathbf{y} = (\mathbf{A}\mathbf{b})^T\mathbf{y}$$

$$SSR_{\text{new}} = \hat{\mathbf{y}}_{\text{new}}^T\mathbf{y}_{\text{new}} = (\mathbf{A}_{\text{new}}\mathbf{b}_{\text{new}})^T\mathbf{y}_{\text{new}} \tag{6}$$

- the residual sum of squares, SSE, with $\lambda = p\text{-}N_S$:

$$SSE = \mathbf{y}^T\mathbf{y} - (\mathbf{A}\mathbf{b})^T\mathbf{y}$$

$$SSE_{\text{new}} = \mathbf{y}_{\text{new}}^T\mathbf{y}_{\text{new}} - (\mathbf{A}_{\text{new}}\mathbf{b}_{\text{new}})^T\mathbf{y}_{\text{new}} \tag{7}$$

The first two terms are sometimes corrected by the mean, i.e. $y_m=(\Sigma_i\, y_i)/p$. In this case, $p(y_m)^2$ is subtracted to both SSY and SSR, whose degrees of freedom become p-1 and N_S -1, respectively, while the SSE stays unmodified. The SSE divided by the corresponding number of degrees of freedom, provides an estimate s^2 of the variance σ^2 of the total error term. That is, the estimate of the standard deviation of the total error is given by

$$s = \sqrt{\frac{SSE}{p - N_S}} \tag{8}$$

When replicates of a single experiment are possible, i.e., when one can associate to the same matrix $\mathbf{A}$ different response vectors $\mathbf{y}$, this subset of experiments provides an estimate of the pure error variance. The variance of the lack of fit error is then obtained by subtracting this estimate to s^2. In the particular case under investigation, the measurements are collected in a continuous monitoring. This prevents one from collecting replicates able to estimate the pure error variance and, hence, the lack of fit and the pure error cannot be distinguished one from the other.

The flow chart of Fig. 1 summarizes the whole procedure.

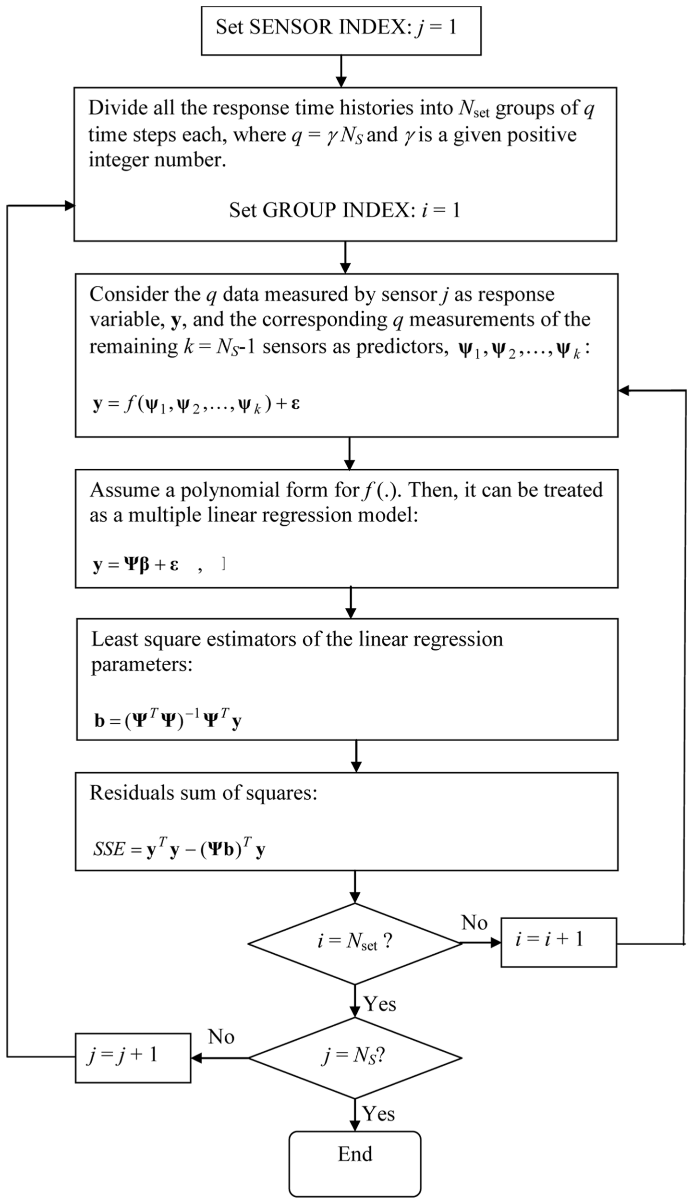

Fig. 1 Flowchart summarizing the algorithm which leads to the damage indices calculation

3. Comparing two models

In the previous section two datasets collected during different tests on the same structure were used to identify the corresponding linear regression models. In the literature, the model comparison is usually performed on a statistical basis by adopting the F- or the t-test. This procedure, however, resulted to be very sensitive to noise in the measurements (Casciati 2009) and, therefore, inadequate for SHM applications. The proposed methodology relies instead on a different approach for model comparison which is outlined by assembling together both the measurements of the undamaged structure and the ones of the unknown (new) structural state. In this case, the following matrices are used in the linear regression analysis:

$$\mathbf{y}_{\mathrm{glob}}\begin{bmatrix} \mathbf{y} \\ \mathbf{y}_{\mathrm{new}} \end{bmatrix}, \quad \mathbf{A}_{\mathrm{glob}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}_{\mathrm{new}} \end{bmatrix} \tag{9}$$

where $\mathbf{y}_{\mathrm{glob}}$ is a $2p \times 1$ vector and $\mathbf{A}_{\mathrm{glob}}$ is a $2p \times N_S$ matrix. In agreement with this notation, the result of the analysis of variance will be denoted as SSE_{glob}.

Repeating the procedure for all segments $i = 1, \ldots, n_{set}$ of the response time history of sensor j, the analyses provide n_{set} values of SSE from the undamaged dataset, and n_{set} values of SSE_{glob}, so that, for the considered sensor j, histograms of these quantities can be drawn to represent their distributions. The two histograms can be plotted together and compared after multiplying by two the SSE ordinates, which refer to the undamaged state of the structure alone (Casciati 2009). This factor of 2 is required in order to achieve a histogram of doubled size as the one obtained from the global dataset.

By permuting the time history considered as response variable so that all sensors $j = 1, \ldots, N_S$ are considered, a histograms' comparison can be built for each sensor location. Damage can be detected by simply observing the shift between the means of the two histograms.

Two situations are possible:

(1) if no significant shift is noticed, the unknown state of the structure is classified as still undamaged;

(2) instead, if there is a significant shift between the two histograms, then one detects that the structure underwent some damage.

Furthermore, once damage has been detected, it can also be localized by comparing the plots obtained at different sensor locations. The damage will be localized near the sensors where the larger shifts are observed.

Based on these observations, a damage index is formulated as the absolute value of the difference between the means of the two histograms and it is used in the application discussed in the next section.

4. Application to the benchmark problem

The test structure under consideration is the 2-bay by 2-bay, 4-story, scaled steel frame shown in Fig. 2(a). A data generation program ("datagen.m") was implemented by the Task Group within the Matlab software environment in order to create an analytical model of this structure (Fig. 2(b)) and to compute its response under different excitation conditions. The m-file can be downloaded from wusceel.cive.wustl.edu/asce.shm/. For the purposes of this paper, the user-defined parameters are set as their default values. In particular, a noise level of 10 is included in the analysis, the sampling time interval is 0.001 s, and the total duration of the analysis is 40 s.

Among the output of the data generation file, the acceleration-time histories computed at the measurement points are collected in a single matrix $\mathbf{a}$ of size $N_P \times N^*_S$. The number of rows is equal to the total number of points in each time history, i.e., $N_P = 40001$, while the number of columns is equal to the total number of "sensors", N^*_S=16, i.e., the number of nodes which are included in the set for which the response time histories are provided. These nodes numerically model a spatial distribution of sensors and they are the

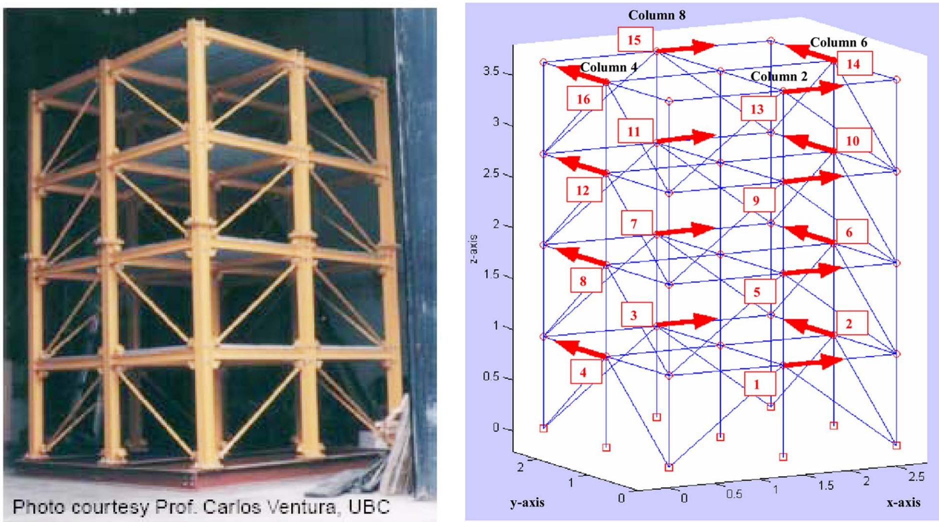

Fig. 2 The ASCE benchmark structural system: (a) real structure at UBC (University British Columbia), re-arranged from Johnson, *et al.* (2000) (left); (b) analytical model and sensors location (right)

ones indicated in Fig. 2(b)). In particular, at each floor level, the accelerations of the nodes belonging to the four middle columns are calculated for a single degree of freedom (d.o.f). The resulting total number of "sensors" associated to each monitored d.o.f. is $N_S = N_S^*/2 = 8$. Therefore, the distribution of sensors for this application is rather sparse, while the proposed damage detection method was originally developed in the context of a distributed monitoring system. Meaningful results are, however, achieved and are discussed in the present paper.

When running "datagen.m", the user is prompted to select among several options which correspond to different assumption on the numerical model and its loading conditions. Among the several cases considered by the benchmark, attention is focused on cases 1, 2 and 3. Both cases 1 and 2 see the load applied to all the floors along one of the two axis directions, but case 1 uses a numerical model with 12 degrees of freedom (d.o.f.), while case 2 uses 120 d.o.f.'s. Case 3 adopts the same model as case 1, but the load is only applied to the top and its direction is along the bisector of the *x*-*y* plane.

For all cases, two damage patterns are studied: in damage pattern 1, all the first (bottom) level of braces is removed; in damage pattern 2, also the third level of braces is removed. In next section the statistical approach is applied with α set equal to 40; therefore, $p = 320$ and $n_{set} = \text{int}(N_P/p) = 125$ (Casciati 2004).

5. Results for the benchmark structure

The results for cases 1 and 2 of the benchmark problem are given in Table 1, while Table 2 refers to case 3 only. For each case, damage patterns 1 and 2 are considered (in Tables 1 and 2 the actual damage locations are denoted by an asterisk). Damage pattern 1 is correctly identified in all cases by detecting the damage at the sensors on the first floor only.

For case 2 and damage pattern 1, the histograms' comparison at each sensor location is shown in Fig. 3.

Table 1 Results for CASE 1 and CASE 2 and damage patterns 1 and 2 (a star marks the location where actually damage developed)

CASE 1, Damage 1		CASE 1, Damage 2		CASE 2, Damage 1		CASE2, Damage 2	
Damage index	Sensor	Damage index	Sensor	Damage index	Sensor	Damage index	Sensor
0.0305	10	0.0735	14	0.1040	12	0.1465	6
0.0396	12	0.0794	16	0.1056	10	0.1492	8
0.0471	6	0.4782	6	0.1326	6	0.2244	16
0.0593	8	0.4959	8	0.1344	8	0.2357	14
0.0752	16	**0.8333***	**10***	0.1481	16	**0.7142***	**2***
0.0828	14	**0.8496***	**12***	0.1537	14	**0.7151***	**4***
1.0757*	**2***	**1.6321***	**2***	**0.8721***	**2***	**0.7298***	**10***
1.1150*	**4***	**1.6671***	**4***	**0.8887***	**4***	**0.7339***	**12***

Table 2 Results for CASE 3 and damage patterns 1 and 2

CASE 3, Damage 1				CASE 3, Damage 2			
y-dof		x-dof		y-dof		x-dof	
Damage index	Sensor	Damage index	Sensor	Damage index	Sensor	Damage index	Sensor
0.0041	14	0.0116	15	0.5813	14	0.1126	13
0.0118	16	0.0148	13	0.6023	16	0.1172*	**11***
0.0353	8	0.0567	5	1.3998*	12*	0.1221	15
0.0477	6	0.0597	7	1.4100*	10*	0.1258*	**9***
0.0724	12	0.1302	11	6.6094	8	0.7058	5
0.0816	10	0.1307	9	6.6256	6	0.7070	7
1.2044*	**4***	**0.4573***	**1***	**10.8644***	**4***	**2.1328***	**1***
1.2091*	**2***	**0.4704***	**3***	**10.8765***	**2***	**2.1370***	**3***

From this figure, it is evident that a significant shift between the histograms occurs at the sensors located on the floor 1 where the damage is introduced. In this case, all the other regions do not seem to be affected by the damage (since the histograms stay close to each other).

The histograms' comparison at each sensor location for case 2 and damage pattern 2 is shown in Fig. 4. Also in this case, a significant shift between the histograms occurs only at the sensors located on the floors where the damage is introduced.

When considering case 1 and damage pattern 2 (see Table 1) a small shift of the histograms is observed in Fig. 5 also for the second floor, whose response is influenced by the removal of the braces on the first and third floors. This effect becomes more significant for case 3 and damage pattern 2 (see Table 2), where the shifts at the sensors on the second floor are larger than the ones at the sensors on the third floor. Therefore, in this case the damage could have been introduced either at the second or at the third floor. This ambiguity is mainly due to the reduced number of degrees of freedom used by the numerical models of the structure under investigation. Furthermore, the number of sensors available is the minimum to guarantee the system observability. These circumstances affect the performance of the method, which is not optimal.

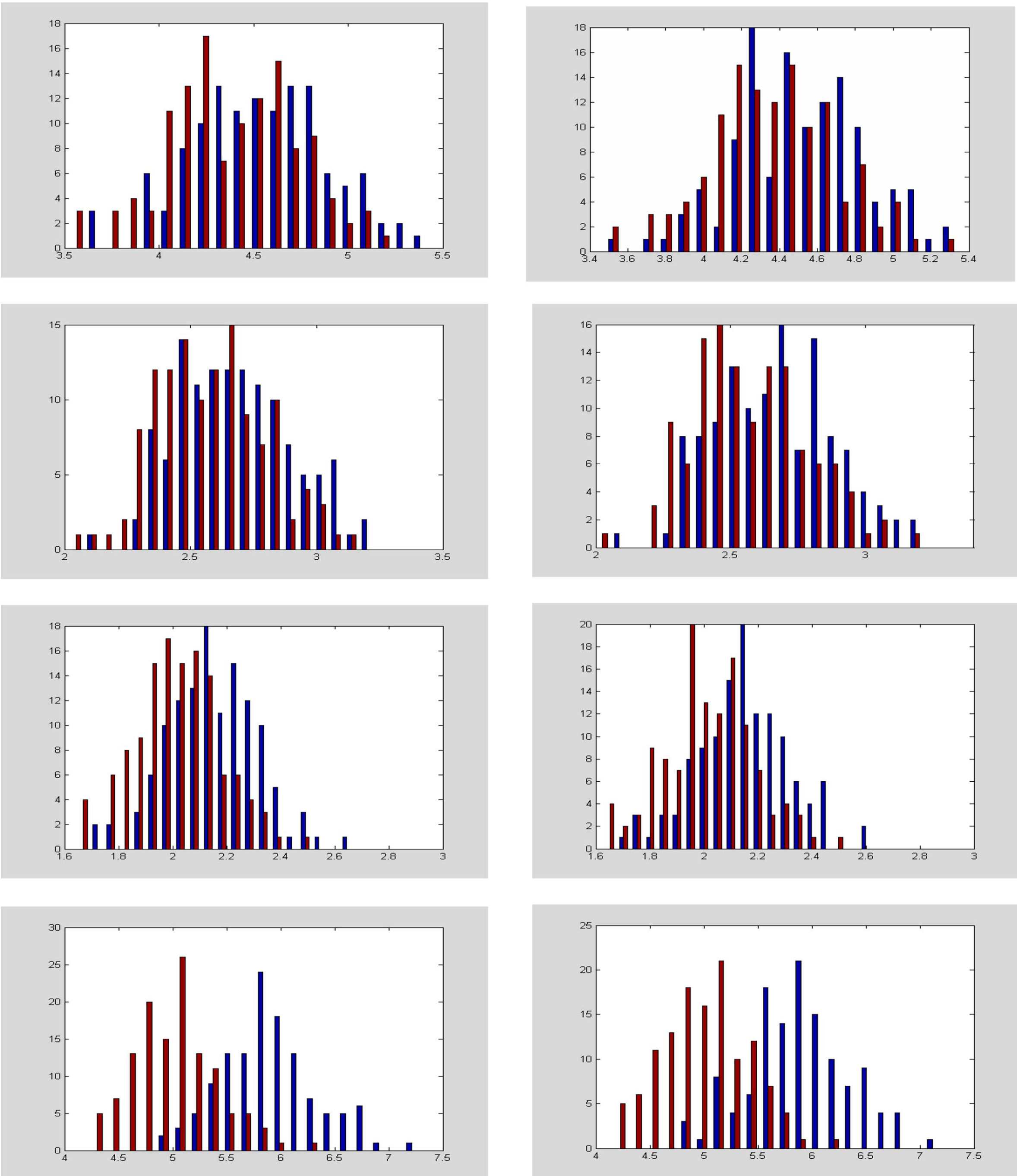

Fig. 3 Histograms comparison at different sensor locations for CASE 2, damage pattern 1. The results are displayed so that the sensors placement in Fig. 1(b) is reproduced (from the first floor (bottom) to the top). The histograms coming from the model built on of the doubled size set of response time histories from the undamaged structure are drawn in blue, and in red the ones obtained after mixing damaged and undamaged time histories

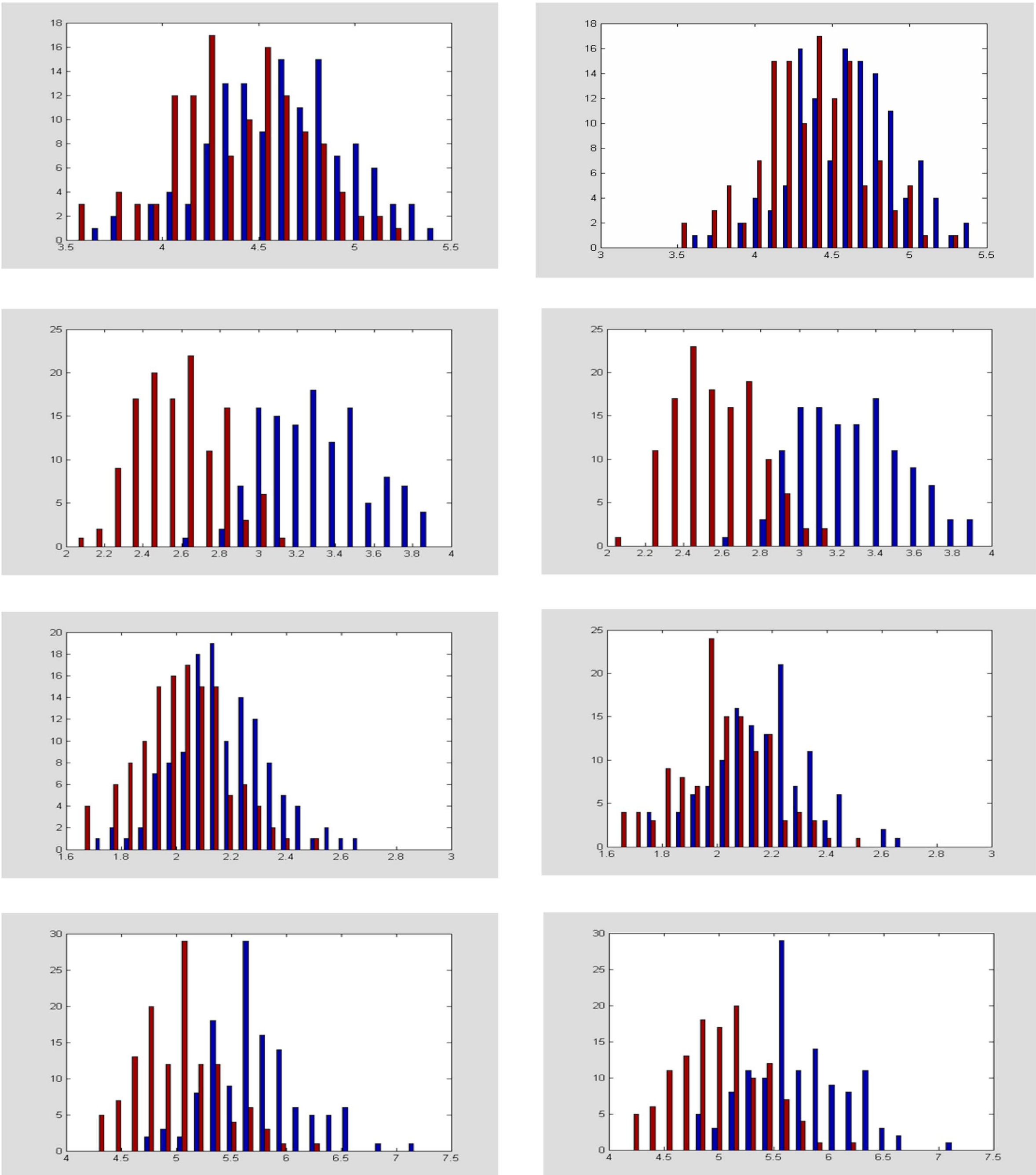

Fig. 4 Histograms comparison at different sensor locations for CASE 2, damage pattern 2. The results are displayed so that the sensors placement in Fig. 1(b) is reproduced (from the first floor (bottom) to the top). The histograms coming from the model built on of the doubled size set of response time histories from the undamaged structure are drawn in blue, and in red the ones obtained after mixing damaged and undamaged time histories

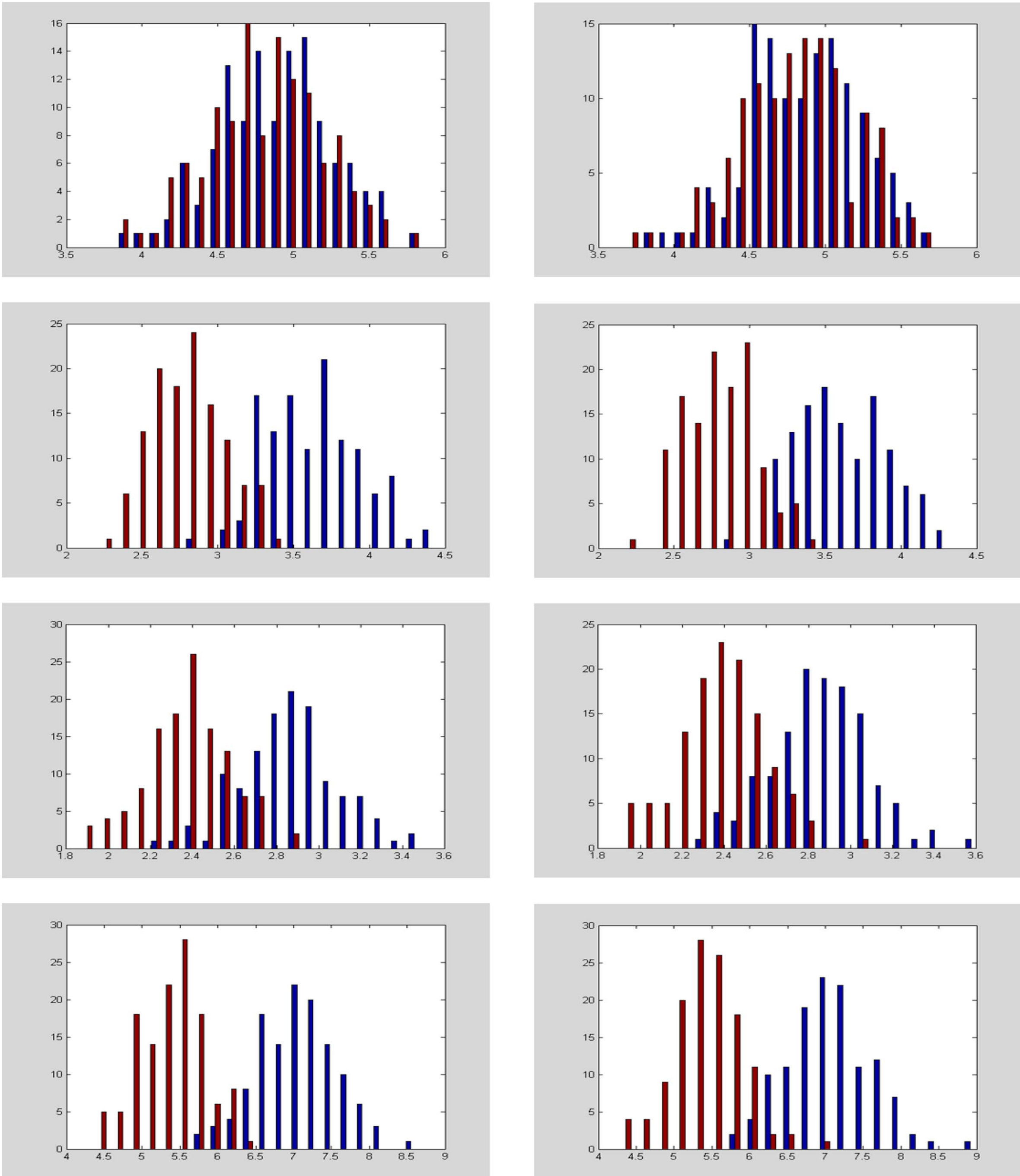

Fig. 5 Histograms comparison at different sensor locations for CASE 1, damage pattern 2. The results are displayed so that the sensors placement in Fig. 1(b) is reproduced (from the first floor (bottom) to the top). The histograms coming from the model built on of the doubled size set of response time histories from the undamaged structure are drawn in blue, and in red the ones obtained after mixing damaged and undamaged time histories

6. Conclusions

Given the response time histories collected at different times during the structure lifetime, a method based on the statistical comparison of models is employed for damage detection. The benchmark problem of the ASCE SHM Task Group (Johnson, *et al*. 2000, 2004) is considered as case study. The reference model associated to response time histories collected when the undamaged system is first constructed. A second set of data is then considered as representative of an unknown state, which could either be damaged or undamaged. The approach identifies the presence of damage and localizes it by a comparison of the sum of the squared errors (*SSE*) histograms.

The results from the benchmark study show that the method is well promising for structural health monitoring applications, provided that a sufficiently dense network of sensors is available.

Acknowledgments

This study is supported by grants from the Athenaeum Research Fund of the University of Catania.

References

Breitung, K. and Faravelli, L. (1996), *Response surface methods and asymptotic approximations*, Chapter 6 in F. Casciati and J.B. Roberts (eds.), *Mathematical Models for Structural Reliability Analysis*, CRC Press, Boca Raton, USA, 227-285.

Casciati, F. and Casciati, S. (2006), “Structural health monitoring by lyapunov exponents of non-linear time series”, *Struct. Control Health Monit.,* **13**(1), 132-146.

Casciati, S. (2004), “Statistical models comparison for damage detection using the ASCE benchmark”, *Proc. EWSHM,* DEStech Publications, Lancaster, Pennsylvania, USA, 695-702.

Casciati, S. (2005), *Data Detection and Localization in the Space of the Observed Data*, PhD Thesis, Graduate School of Civil Engineering, University of Pavia, Italy.

Casciati, S. (2008), “Stiffness identification and damage localization via differential evolution algorithms”, *Struct. Control Health Monit.,* **15**(3), 436-449.

Casciati, S. (2009), “Using response surface models to detect and localize distributed cracks in a complex continuum”, submitted for publication.

Draper, N. and Smith, H. (1981), *Applied Regression Analysis*, Wiley, New York.

Johnson, E.A., Lam, H.F., Katafygiotis, L.S. and Beck, J.L. (2000), “A benchmark problem for structural health monitoring and damage detection”, *14th Engineering Mechanics Conf.*, Austin, Texas, May 2000, ASCE.

Johnson, E.A., Lam, H.F., Katafygiotis, L.S. and Beck, J.L. (2004), “Phase I IASC-ASCE structural health monitoring benchmark using simulated data”, *J. Eng. Mech.* ASCE, **130**(1), 3-15.

Masri, S.F., Smyth, A.W., Chassiakos, A.G., Caughey, T.K. and Hunter, N.F. (2000), “Application of neural networks for detection of changing in nonlinear systems”, *J. Eng. Mech.* ASCE, **126**(1), 666-683.

FC