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Optimized finite element model updating method for damage detection using limited sensor information

L. Cheng and H. C. Xie*

College of Civil Engineering, Shenzhen University,
Shenzhen Durability Center for Civil Engineering, Shenzhen, 518060, Guangdong, China

B. F. Spencer, Jr. and R. K. Giles

Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign,
Urbana, IL, USA

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Abstract. Limited, noisy data in vibration testing is a hindrance to the development of structural damage detection. This paper presents a method for optimizing sensor placement and performing damage detection using finite element model updating. Sensitivity analysis of the modal flexibility matrix determines the optimal sensor locations for collecting information on structural damage. The optimal sensor locations require the instrumentation of only a limited number of degrees of freedom. Using noisy modal data from only these limited sensor locations, a method based on model updating and changes in the flexibility matrix successfully determines the location and severity of the imposed damage in numerical simulations. In addition, a steel cantilever beam experiment performed in the laboratory that considered the effects of model error and noise tested the validity of the method. The results show that the proposed approach effectively and robustly detects structural damage using limited, optimal sensor information.

Keywords: modal analysis; damage detection; flexibility matrix; PSO; FE model updating.

1. Introduction

All structures, from buildings and bridges to aircraft and offshore platforms, accumulate damage continuously throughout their service life. The ability to detect damage and evaluate the condition of a structure is of great value in ensuring public safety and protecting the significant investment made in the infrastructure. Scholars have presented many damage detection and diagnostic methods based on indentifying changes between the undamaged and damaged sates of a structure's physical properties such as natural frequencies and mode shapes. Among the many methods, modal testing has been regarded as an effective way to perform damage detection. Using measured vibration data to detect changes in structural systems due to damage has been gaining increased attention (Doebling 1998, Gao, *et al*. 2004). Doebling (1998) have given a detailed overview of vibration based damage detection methods.

*Corresponding Author, E-mail: huicaixie@hotmail.com

Many modal-based methods use changes in modal parameters, such as the natural frequency, mode shape, or modal damping to detect damage. Cawley and Adams (1979) have developed a simple method, with low computational costs, that uses changes in the ratio of modal frequencies to determine the presence of damage but not its magnitude. However, Salawu (1997) has pointed out that the inability of changes in natural frequency to provide spatial information about structural damage could limit the practical application of vibration based structural health monitoring. An additional subgroup of modal-based damage detection algorithms is one that analyzes changes in a structure's mode shapes. Changes in mode shapes tend to be more sensitive to damage than changes in natural frequencies. Another class of damage detection algorithms utilizes the dynamically measured modal flexibility matrix. Pandey and Biswas (1994, 1995) presented a damage detection and localization method based on the measured structural modal flexibility matrix. They applied this method to several numerical examples and a spliced beam where the damage was linear in nature. Their results showed that estimates using only the first two measured modes could locate the damage and determine its condition. Gao and Spencer (2002) introduced, and tested under ambient vibration conditions, a damage localization method that also uses changes in a structure's flexibility. All this research has demonstrated that changes in the modal flexibility is more sensitive to damage than changes in either natural frequencies or mode shapes. As a result, increased numbers of researchers have focused on the flexibility approach for structural damage detection (Mottershead and Friswell 1993, Zhao and DeWolf 1999, Yan and Golinval 2005, Stutz, *et al*. 2005, Duan, *et al*. 2005, Yin, *et al*. 2007).

Finite element (FE) model updating is another method for identifying structural damage and assessing a structure's condition. By modifying mass, stiffness, and damping parameters, FE model updating attempts to obtain better agreement between the model's numerical analysis and test data from the actual structure. A number of model updating methods for structural dynamics have been proposed (Friswell and Mottershead 1995, Link 1999). The iterative parameter updating method uses the parameter sensitivity to determine appropriate changes (Link 1999). The advantage of sensitivity-based parameter updating is that it identifies parameters that directly affect the structure's dynamic characteristics. Non-iterative methods that directly update the elements of the stiffness and mass matrices are one-step procedures (Berman and Nagy 1983). The results of FE model updating are mass and stiffness matrices that more closely reflect the measured modal properties of the structure. However, the updated matrices do not always maintain structural connectivity, neither are they always physically meaningful in their representation. In addition, the mathematical model used in model updating is often ill posed and special care needs to be taken in order to obtain an accurate solution.

All the previously mentioned vibration based methods are hampered in their effectiveness by the incompleteness of the available data caused by nonideal test conditions. For example, measurements can often only determine a limited number of modes; likewise, only a few sensors, a markedly insufficient number for large civil infrastructure, are available for installation. Therefore, practical application of many of the aforementioned methods is difficult at best (Law, *et al.* 1998). This paper presents a method of finite element model updating that determines optimal sensor placement and performs damage detection. Modal flexibility sensitivity analysis determines the optimal sensor locations for collecting information on structural damage. Using this method, only a limited number of degrees of freedom need to be instrumented to obtain sufficient information on the structural condition. Using the data obtained from the optimally placed sensors, the paper develops a damage detection algorithm based on changes in the flexibility matrix that can determine both the location and severity of damage. Finally, a numerical example and a simple experiment show that this approach is both effective and robust for detecting structural damage using limited sensor information.

2. Theoretical background

2.1. Damage severity index

Structural damage often occurs in one or more structural members resulting in a stiffness reduction while the mass remains unchanged. Therefore, this paper neglects the effect of damage on the mass of the structure. $\boldsymbol{K_u}$ and $\boldsymbol{K_d}$ are the undamaged and damaged system stiffness matrices respectively. $\boldsymbol{K_d}$ can be defined in terms of $\boldsymbol{K_u}$ as:

$$\mathbf{K}_d = \mathbf{K}_u + \sum_{e=1}^{m} \Delta\mathbf{K}_e = \mathbf{K}_u + \sum_{e=1}^{m} \alpha_e \mathbf{K}_e \tag{1}$$

$$\Delta\mathbf{k}_{eij} = \alpha_e \mathbf{k}_{eij} \tag{2}$$

where $\boldsymbol{K}_e$ and $\Delta\boldsymbol{K}_e$ are the global coordinate representation of the e^{th} element stiffness matrix and the change in stiffness matrix respectively; $\boldsymbol{k}_{eij}$ and $\Delta\boldsymbol{k}_{eij}$ are the elements in the stiffness matrix and the change in stiffness matrix of the e^{th} structural element, respectively. Here α_e (e=1,2… m), where m is the total number of elements in the structure, is a coefficient defining a fractional reduction in the e^{th} element stiffness matrix that represents the damage severity index. We can define

$$\alpha_e = \frac{\mathbf{k}_e^u - \mathbf{k}_e^d}{\mathbf{k}_e^u}, e = 1, 2, \ldots, m \tag{3}$$

where $\boldsymbol{k}_e^u$ and $\boldsymbol{k}_e^d$ are the stiffness matrix, in local coordinates, of the e^{th} element for the undamaged and damaged structures respectively. In this formulation, a positive value where $0 \le \alpha_e \le 1$ indicates a loss in element stiffness. The e^{th} element is undamaged when α_e=0, and the stiffness capacity of the e^{th} element has completely failed when α_e=1.

2.2. Structural flexibility and sensitivity analysis

2.2.1. Modal flexibility matrix

The undamped free vibration of a dynamic structural system can be described by the second order differential equation as

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = \mathbf{0} \tag{4}$$

where **M** and **K** are the mass and stiffness matrices, respectively, and **Y** is the displacement vector. The eigenequation can then be expressed as follows

$$\mathbf{K}\mathbf{\Psi} - \mathbf{M}\mathbf{\Psi}\mathbf{\Omega} = \mathbf{0} \tag{5}$$

where Ω is the eigenvalue matrix and $\Omega = diag[\omega_1^2, \omega_2^2, \ldots \omega_n^2]$ where ω_i is the i^{th} squared natural frequency, and $\Psi = [\psi_1, \psi_2, \ldots \psi_n]$ are the mode shapes normalized to unit mass such that

$$\mathbf{\Psi}^{\mathbf{T}}\mathbf{K}\mathbf{\Psi} = \mathbf{\Omega} \tag{6}$$

$$\mathbf{\Psi}^{\mathbf{T}}\mathbf{M}\mathbf{\Psi} = \mathbf{I} \tag{7}$$

Solving Eqs. (6-7) for the stiffness and flexibility matrices, approximations in modal form using only the first few modes are obtained as follows

$$\mathbf{K} = \mathbf{M}\boldsymbol{\Psi}\boldsymbol{\Omega}\boldsymbol{\Psi}^{\mathbf{T}}\mathbf{M} = \mathbf{M}\left(\sum_{i=1}^{n}\omega_i^2\psi_i\psi_i^T\right)\mathbf{M} \tag{8}$$

$$\mathbf{F} = \boldsymbol{\Psi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}^{\mathbf{T}} = \sum_{i=1}^{n}\frac{\psi_i\psi_i^T}{\omega_i^2} \tag{9}$$

Eqs. (8) and (9) indicate the influences of the various frequency modes on the stiffness and flexibility matrices, respectively. The influence of the i^{th} mode on the stiffness matrix **K** increases with the square of the modal frequency ω_i^2; whereas for the flexibility matrix **F**, the influence decreases with ω_i^{-2}. This implies that nearly all of the modes are required to obtain a reasonably accurate representation of the stiffness matrix. However, because obtaining the higher mode shapes of a structure through experimentation is challenging, stiffness matrix based damage detection strategies may be difficult to implement in practice. Contrarily, only a few modes are required to achieve good accuracy in estimating the flexibility matrix. By extrapolation, only the first few modes can also accurately estimate the truncated flexibility matrix at the measured degree of freedoms (DOFs)(Oi, *et al.* 2001). This result is promising because the practical restrictions of experimentation only allow measurements at a limited number of DOFs and can only obtain a few modes. Partitioning the full DOF set into measured, *m*, and non-measured DOFs and using only the first *s* lower mode shapes and frequencies, Eq. (9) can be rewritten as

$$\mathbf{F}_{mm} = \boldsymbol{\Psi}_{\mathbf{s}_m}\boldsymbol{\Omega}_s^{-1}\boldsymbol{\Psi}_{\mathbf{s}_m}^{\mathbf{T}} = \sum_{i=1}^{s}\frac{\psi_{i_m}\psi_{i_m}^T}{\omega_i^2} \tag{10}$$

where $\mathbf{F}_{mm}$ is the measured flexibility matrix, $\psi_{\mathbf{S}_m}$ are the mode shapes of the structure at the measured *m* DOFs, and $\boldsymbol{\Omega}_s$ correspond to the eigenvalues of the measured modes.

2.2.2. Sensitivity analysis of the flexibility matrix

The flexibility matrix can be rewritten as the columns $\mathbf{F}=[\mathbf{F}_1, \mathbf{F}_2,\dots \mathbf{F}_m]$, and we have

$$F_{lk} = \sum_{p=1}^{s}\frac{\psi_{lp}\psi_{kp}}{\omega_p^2},\ (1, k = 1, 2 \dots m) \tag{11}$$

Structural flexibility, just like mass, stiffness or damping, is a natural system property. Therefore, the flexibility matrix can be defined as a function of the structure's physical parameters. Here, the stiffness of element $\boldsymbol{k}_{eij}$ is selected as the parameter to use and the partial derivative of the flexibility may be written as

$$\frac{\partial F_{lk}}{\partial k_{eij}} = \frac{\partial}{\partial k_{eij}}\left(\sum_{p=1}^{s}\frac{\phi_{lp}\phi_{kp}}{\omega_p^2}\right) = \sum_{p=1}^{s}\frac{\partial}{\partial k_{eij}}\left(\frac{\phi_{lp}\phi_{kp}}{\omega_p^2}\right) = \sum_{p=1}^{s}\left[-\frac{2}{\omega_p^3}\frac{\partial\omega_p}{\partial k_{ij}}\psi_{lp}\psi_{kp} + \frac{1}{\omega_p^2}\left(\frac{\partial\psi_{lp}}{\partial k_{eij}}\psi_{kp} + \frac{\partial\psi_{kp}}{\partial k_{eij}}\psi_{lp}\right)\right] \tag{12}$$

where $\partial F_{lk}/\partial k_{eij}$ is calculated by taking the partial derivative of Eq. (11) with respect to k_{eij} and is called the flexibility sensitive index. In Eq. (12), $\partial\omega_p/\partial k_{eij}$ and $\partial\varphi_{ip}/\partial k_{eij}$ represent the eigenpair sensitivity. Zhang and Wei (1999) derive the following:

$$\frac{\partial\omega_p}{\partial k_{eij}} = \begin{cases} \dfrac{\psi_{ip}\psi_{jp}}{\omega_p}, i \neq j \\ \dfrac{\psi_{ip}^2}{2\omega_p}, i = j \end{cases}, \text{ and } \frac{\partial\omega_{lp}}{\partial k_{eij}} = \sum_{k=1}^{n} \beta_k \psi_{lk}$$

$$\text{If } p \neq k,\ \beta_k = \begin{cases} \dfrac{1}{w_k^2 - w_p^2}(\psi_{ik}\psi_{jp} + \psi_{jk}\psi_{ip}), i \neq j \\ \dfrac{1}{w_k^2 - w_p^2}\psi_{ik}\psi_{jp}, i = j \end{cases}, \text{ otherwise if } p = k,\ \beta_k = 0$$

2.2.3. Optimal sensor placement

Using a Taylor series expansion, the partial derivative of the flexibility with respect to a physical parameter **X** is:

$$\mathbf{F}(\mathbf{X}) = \mathbf{F}(\mathbf{X}_0) + \frac{\partial\mathbf{F}(\mathbf{X})}{\partial\mathbf{X}}\Delta\mathbf{X} \tag{13}$$

$$\Delta\mathbf{F} = \mathbf{F}(\mathbf{X}) - \mathbf{F}(\mathbf{X}_0) = \frac{\partial\mathbf{F}(\mathbf{X})}{\partial\mathbf{X}}\Delta\mathbf{X} = \sum_{i=1}^{m}\frac{\partial\mathbf{F}(x_i)}{\partial x_i}\Delta x_i \tag{14}$$

When x_i is the element stiffness $\boldsymbol{k}_{eij}$, Eq. (14) can be rewritten as

$$\Delta F_{lk} = \sum_{e=1}^{m}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial F_{lk}(k_{eij})}{\partial k_{eij}}\Delta k_{eij}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial F_{lk}(k_{eij})}{\partial k_{eij}}\sum_{i=1}^{m}\alpha_e k_{eij}\right) \tag{15}$$

Written as a matrix, Eq. (15) is

$$\Delta F_{lk} = \mathbf{S}_k \bullet \mathbf{A} \tag{16}$$

where $\mathbf{A}=[\alpha_1, \alpha_2, \ldots \alpha_m]^T$ is a vector called the damage index of the element defined in the previous section. $\mathbf{S}_k$ is therefore

$$\mathbf{S}_k = \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial F_{lk}(k_{1ij})}{\partial k_{1ij}}k_{1ij},\ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial F_{lk}(k_{2ij})}{\partial k_{2ij}}k_{2ij}, \ldots, \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial F_{lk}(k_{mij})}{\partial k_{mij}}k_{mij}\right]$$

Theoretically, the damage index vector **A** can then be derived from Eq. (16) when the change in flexibility determined by measurement is known as follows

$$\mathbf{A}^* = (\mathbf{S}_k^T\mathbf{S}_k)^{-1}\mathbf{S}_k^T\Delta F_{lk} \tag{17}$$

In practice, noise and ill-conditioned matrices make obtaining the results of Eq. (16) difficult. To help resolve these difficulties, the Fisher Information Matrix (FIM) is defined as $\mathbf{Q}_k = \mathbf{S}_k^T \cdot \mathbf{S}_k$ (Fedorov 1972, Kammer 1991, Shi 2002). According to research by Fedorov (1972) and Kammer (1991), the determinant of the FIM must have a maximum value if the damage index vector has an unprejudiced estimation. The FIM can be rewritten as the sum of all DOFs' contribution sub-matrices

$$\mathbf{Q}_k = \sum_{i=1}^{n} (\mathbf{S}_k^i)^T \mathbf{S}_k^i = \sum_{i=1}^{n} \mathbf{Q}_k^i \tag{18}$$

where $\mathbf{S}_k^i$ is the i^{th} row of matrix $\mathbf{S}_k$ that represents the structural damage information given by the i^{th} DOF. The contribution of each DOF to the determinant of the FIM is different as some may contain more information than others. The DOFs that contribute most to the FIM determinant are the optimal locations to place sensors. Placing sensors at these locations will get the most information out of a given project's limited sensor resources and perform damage detection effectively. Eq. (18) shows that building the FIM requires only one column of the change in flexibility matrix. Therefore, the FIM can be derived from the average of all the columns in the flexibility matrix.

2.2.4. Number and performance estimate for selected sensor locations

Theoretically, structural modes are orthogonal vectors. However, noise and measurement error make it difficult to guarantee the modes obtained by estimated methods are orthogonal. In extreme cases, the geometric angle between two vectors is so small that some modes hide other important modes. Therefore, when choosing DOFs to instrument, attention needs to be paid to selecting those that have a large geometric angle between the set. Ideally, the mode shape vectors should be linearly independent of each other.

From mathematics, the Modal Assurance Criterion (MAC) is a method of comparing two vectors with an equal number of components (Carbe and Dohrmann 1995). Modal analysis uses the MAC for matching and validating the model with test data. The MAC matrix is defined as:

$$\mathrm{MAC}_{ij} = \frac{(\psi_i^T \psi_j)^2}{(\psi_i^T \psi_i)(\psi_j^T \psi_j)}, \; i,j = 1,2,\ldots,S \tag{19}$$

where ψ_i and ψ_j are the i^{th} and j^{th} mode shape vectors and S is the total number of targeted mode shapes. The value of MAC_{ij}, that always varies between 0 and 1, is a quantitative correlation factor between two vectors. If $\mathrm{MAC}_{ij}(i \neq j)$ is unity then the i^{th} and j^{th} mode shape vectors are identical within a scale factor; therefore, these two vectors cannot be distinguished one from the other. If $\mathrm{MAC}_{ij}(i \neq j)$ is zero then the two vectors are orthogonal vectors and are easily distinguishable. In order to have two reasonably orthogonal modes, the off-diagonal MAC values should be less than 0.05.

For the i^{th} DOF of the FE model and the j^{th} mode shape, the Modal Kinetic Energy (MKE) is defined as

$$\mathrm{MKE}_{ij} = \psi_{ij} \sum_{k}^{m} M_{ik} \psi_{kj} \tag{20}$$

where MKE_{ij} represents the modal kinetic energy of the i^{th} DOF for the j^{th} mode shape and m is the total number of measured DOFs. Eq. (20) shows that if the j^{th} mode ψ_{ij} includes all DOFs and is mass normalized, the sum of MKE_{ij} will be unity. The DOFs, i.e. sensor locations, that make the corresponding MKE_{ij} largest should be used. To ensure the sensors detect and measure sufficient modes, the sum of the MKE_{ij} should be a minimum of 0.9 but come as close to unity as limited sensor resources allow.

Both of the aforementioned criteria should be used together to estimate the number of sensors needed and where to place them.

2.3. Structural damage detection

The observation that damage causes changes to the dynamic characteristics of a structure is the basis of vibration based damage detection algorithms. Accordingly, the correlation between parameters obtained through dynamic testing and FE model analysis can be determined. In this paper, the damage detection problem is transformed into an optimization problem under certain constraints. A healthy FE model is used as the baseline model and the flexibility difference matrix is the objective function that is solved for using the Particle Swarm Optimization (PSO) algorithm (Parsopoulos and Vrahatis 2002, Kennedy and Eberhart 1995, Shi and Eberhart 1999, Hou and Lu 2003).

2.3.1. Objective function

An objective function f reflects the deviation between the analytical prediction and the real behavior of a structure. FE model updating can be posed as a minimization problem to find the x^* design set such that

$$f(x^*) \le f(x), \forall x$$
$$x_i \in [x_{li}, x_{ui}], i = 1, 2, 3, \ldots, S \tag{21}$$

where $f(x)$ represents the objective function, x is the design variable and $\boldsymbol{x}^* = \{x_1^*, x_2^*, \ldots, x_s^*\}$ is a column matrix representing the optimal value, and S represents the number of design variables. The optimization problem uses an iterative process to determine the best-fit values for the objective function. The general objective function is formulated in terms of the difference between the FE and experimental quantities. The modal flexibility error is given by the expression

$$[\Delta \mathbf{F}_{mm}] = [\mathbf{F}_{mm}]_t - [\mathbf{F}_{mm}]_a \tag{22}$$

where $[\mathbf{F}_{mm}]_t$ is the experimental modal flexibility matrix obtained at the measured DOFs, $[\mathbf{F}_{mm}]_a$ is the analytical flexibility matrix corresponding to the measured DOFs, and $[\Delta \mathbf{F}_{mm}]$ is the modal flexibility error residual.

As shown in sections 2.1 and 2.2, the modal flexibility error can be expressed as a function of the element stiffness $\boldsymbol{k}_e^d$ if the structural damage only reduces the element stiffness. Instead of using the damaged stiffness value of each element, $\boldsymbol{k}_e^d$, as the updating parameter, α_e, its difference from the undamaged stiffness value $\boldsymbol{k}_e^u$ is chosen. Substituting Eq. (3) into Eq. (22) yields

$$\Delta \mathbf{F}(\alpha_e) = [\mathbf{F}_{mm}]_t - [\mathbf{F}_{mm}]_a \tag{23}$$

As is known, if α_e equals 1, then the eigenequation may be meaningless; therefore, an upper limit on α_e is set at 0.6 for the numerical analysis.

Eq. (23) is the function, in matrix form, to be minimized. To carry out the least squares minimization, the norm of a matrix called the Frobenius Norm is used. Therefore, the minimization problem using the Frobenius Norm is

$$\|\Delta\mathbf{F}(\alpha_e)\|_F^2 = \sum_{j=1}^{m}\sum_{k=1}^{m}(\Delta\mathbf{F}(\alpha_e))_{jk}^2 \tag{24}$$

Finally, the minimization problem mathematically is

$$\begin{gathered} f(\alpha) = \|\Delta\mathbf{F}(\alpha)\|_F^2, \ \forall\alpha \\ f(\alpha^*) \le f(\alpha) \\ \alpha_e \in [0, 0.6], i = 1, 2, 3, \ldots, m \end{gathered} \tag{25}$$

2.3.2. Optimization algorithm

Particle Swarm Optimization is a population intelligence based stochastic zero-order optimization algorithm developed by Kenney and Eberhart (1995). The social behavior of animals, such as birds flocking or fish schooling, inspired the algorithm. PSO shares many similarities with other evolutionary computational techniques. For example, it does not require analytic information on the derivative of the objective function, but only its numerical values. This property is important for those functions whose gradients are either unavailable or computationally expensive.

The PSO optimization procedure starts with a population of random candidate solutions named "particles". Each particle has its own position, velocity, and a fitness value assigned by the evaluation function. According to a few simple rules, the population adaptively updates their positions and velocities to travel around the solution space searching iteratively for the optimal solution. When a particle calculates its new position, it uses two prior values: the best position it has achieved so far (called "*pbest*"), and the best global position the population has obtained so far (called "*gbest*"). Each iteration step changes the particle velocity using the independent randomly weighted "pbest" and "gbest" information and then updates the particle's position as follows.

$$\begin{aligned} v_{ij}^{k+1} &= w \times v_{ij}^{k} + c_1 \times r_1 \times (P_{ij} - x_{ij}^{k}) + c_2 \times r_2 \times (G_j - x_{ij}^{k}) \\ x_{ij}^{k+1} &= x_{ij}^{k} + v_{ij}^{k+1} \end{aligned} \tag{26}$$

where $\boldsymbol{x}_i=\{x_{i1}, x_{i2},...x_{id}\}$, i=1,2,…, p represents the position of the i^{th} particle, d is the number of optimized parameters, $\mathbf{v}_i=\{v_{i1}, v_{i2},...v_{id}\}$, i=1,2,…, p is the velocity of the moving particles, p is the size of the particle population, c_1 and c_2 are learning factors that are usually equal to 2, and r_1 and r_2 are random numbers uniformly distributed in the interval (0,1). The index w is defined as follows

$$w = w_{\max} - k \times \frac{w_{\max} - w_{\min}}{k_{\max}} \tag{27}$$

where, typically, $w_{\max}$=0.9 and $w_{\min}$=0.4 (Shi and Eberhart 1999), $k_{\max}$ is the maximum number of iterations, P_{ij} is the best position x_i has achieved so far, and G_j is the best global position of the population. In addition, the position x_i must satisfy the constraint of [$x_{\min}$, $x_{\max}$] and the velocity v_i must be bound by a predetermined maximum value v_{max}. This parameter is crucial to the success of the algorithm and is usually set to be less than or equal to x_{max}. Eq. (26) defines the global version of PSO, there are also other formulations for different purposes.

The PSO optimization algorithm used to minimize the objective function stated in Eq. (25), has been programmed in the MATLAB environment in order to implement the proposed damage detection algorithm.

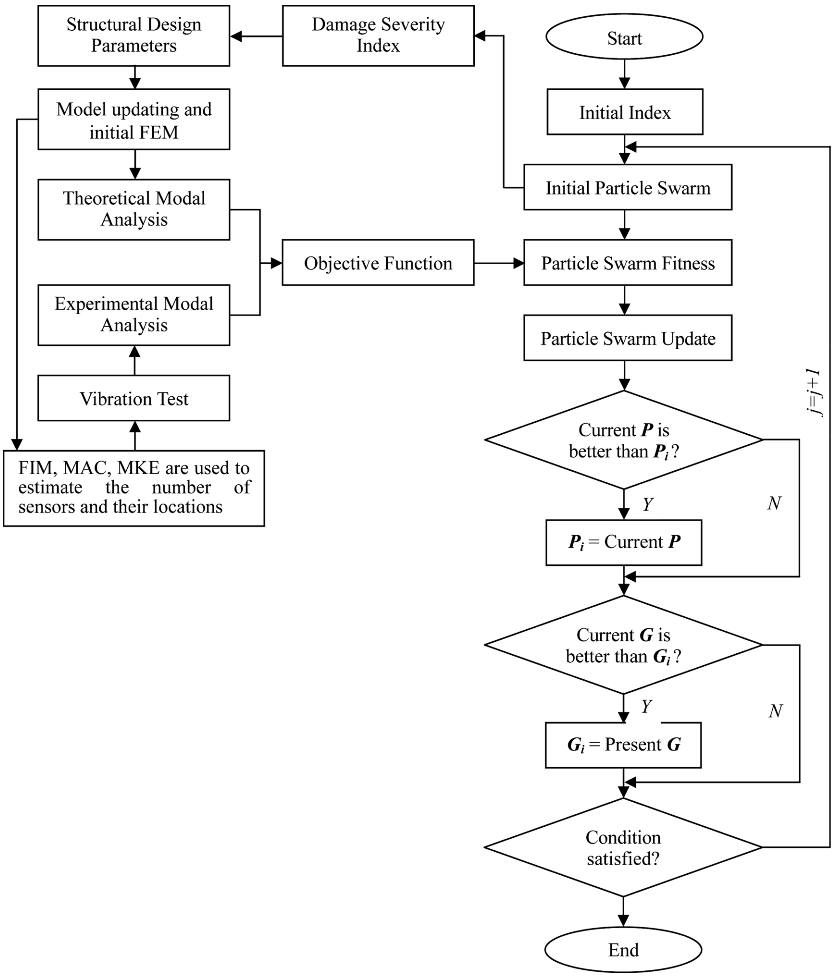

Fig. 1 Damage detection algorithm flowchart

2.3.3. Damage detection program

Fig. 1 shows a flowchart for the proposed damage detection algorithm.

3. Case studies

3.1. Numerical example and results

A numerical example with a simply supported, 6-bay planar-truss illustrates the performance and robustness of the proposed method. The FE model of the structure consists of 31 two-dimensional bar elements and 14 nodes with 25 DOFs as shown in Fig. 2. The geometric and physical parameters of the structure are as follows: the elastic modulus is E=70GPa, the mass density is ρ=2.77·10^{3}kg/m^3, and the cross-sectional area of each element is A=0.001m^2. Each bay of the truss is 1.52m square. The nodes and elements are numbered as shown in Fig. 2.

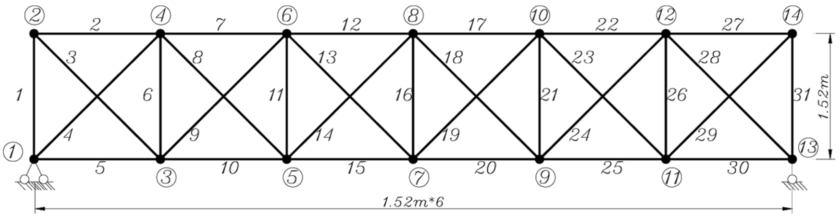

Fig. 2 Finite element model of a simply supported, 6-bay planar truss

Two damage cases are simulated in the truss structure. Case I has a 25% loss of stiffness in element 16. Case II has stiffness losses of 15% in element 7, 15% in element 12, and 20% in element 11. Damage localization and quantification were considered under noisy conditions.

Modal analysis was performed in the MATLAB environment to determine the FE natural frequencies and the mass-normalized mode shapes. The first six damaged and undamaged modes and frequencies were used to construct the flexibility matrix. The mode shapes were contaminated with 3% random noise to simulate measurement noise. The contaminated signal is represented as

$$\psi_{is}^{*} = \psi_{is}(1 + r_i \beta |\psi_{\max,s}|) \tag{28}$$

where ψ_{is}^{*} and ψ_{is} are the mode shape components of the s^{th} mode at the i^{th} DOF with and without noise, respectively, r_i is a random number with a mean equal to zero and a variance equal to 1, β is the random noise level, and $\psi_{\max,s}$ is the largest component in the s^{th} mode shape.

Table 1 shows the optimal number and placement of sensors, sorted in descending importance of

Table 1 DOFs and nodes for the sensor configuration

Sensor index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DOFs	14	12	10	8	18	16	20	23	22	19	1	6	5	4	15	9	11	13	17	24	21	7	2	3	25
Nodes	8y	7y	6y	5y	10y	9y	11y	13y	12y	11x	2x	4y	4x	3y	9x	6x	7x	8x	10x	14x	12x	5x	2y	3x	14y

Notes: 5x indicates the x direction of the 5th node and so forth

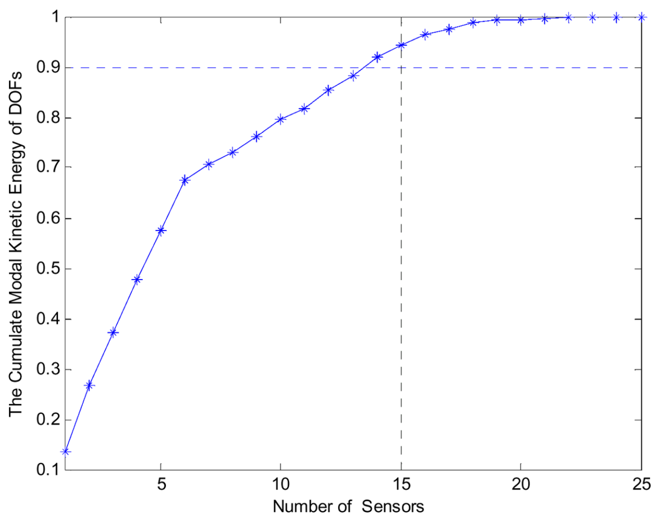

Fig. 3 The cumulate modal kinetic energy of the optimal sensors

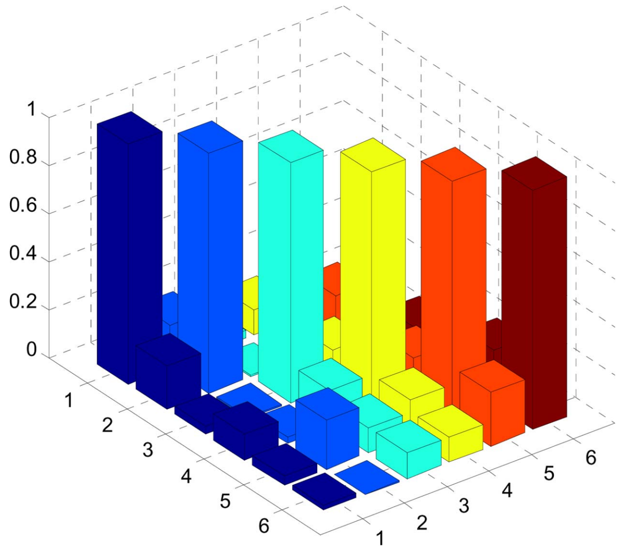

Fig. 4 MAC matrix between the first 6 mode shapes in the first optimal 10 DOFs

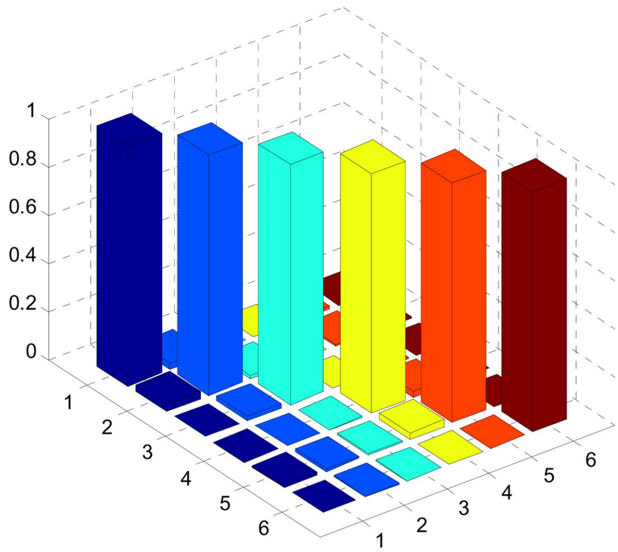

Fig. 5 MAC matrix between the first 6 mode shapes in the first optimal 15 DOFs

the DOF to the FIM, as calculated by the sensor placement optimization routine performed in MATLAB.

Fig. 3 shows the cumulative MKE listed according to the indices in Table 1. Fig. 3 shows that the cumulative MKE for the first 15 optimal sensors accounts for 94.33% of the total MKE. Figs. 4 and 5 are three-dimensional bar charts showing the MACs of the first six modes for two estimates that include different numbers of DOFs. These charts show that the maximum off-diagonal MAC value when 10 DOFs are included is 0.2250; however, when the first 15 sensor placements are included, the maximum off-diagonal MAC value is 0.0279. Therefore, enough structural information for damage detection purposes is contained in the first 15 optimal sensor locations.

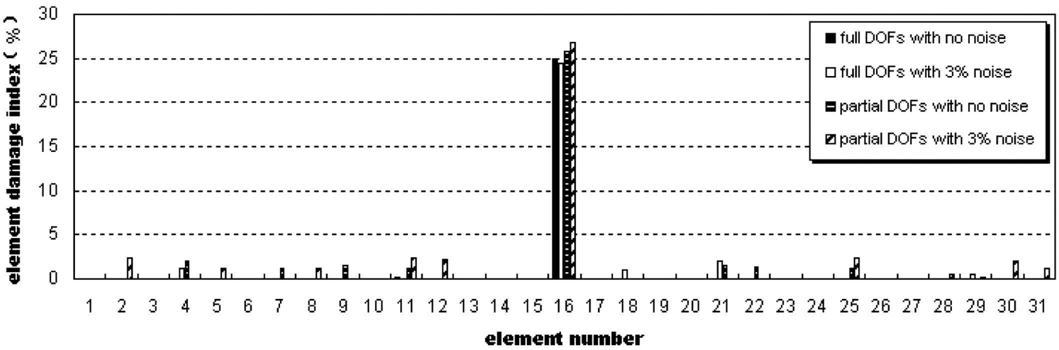

Fig. 6 Results for damage Case I

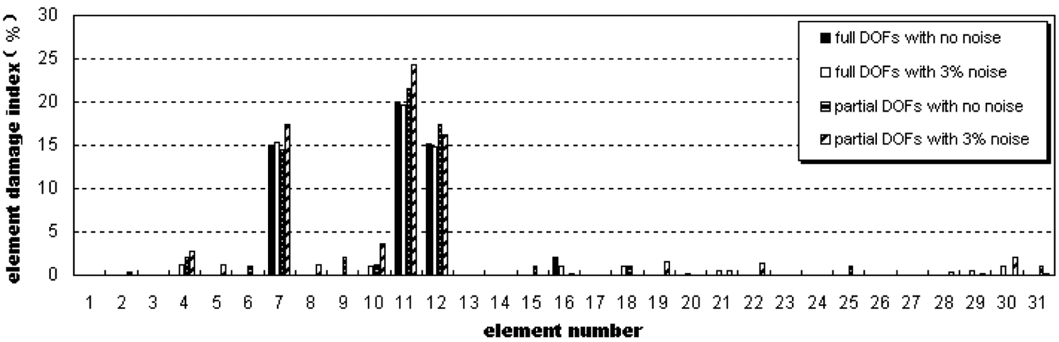

Fig. 7 Results for damage Case II

Using MATLAB, the damage detection algorithm was performed 20 times with appropriate parameters. The two damage cases considered included two additional variables: all sensors or optimal sensors only and noise simulation ignored or included. Figs. 6 and 7 show the results of the numerical simulations.

Fig. 6 shows that when all DOFs are included and the noise neglected, the algorithm correctly identifies both element 16 as the damaged element and the magnitude of the stiffness reduction as 25%. However, when all DOFs are included and the noise is added, the algorithm successfully identifies the damaged element but the magnitude of stiffness reduction is 24.37%. This result shows an error of 2.52% from the actual value. When using only the first 15 optimal DOFs as test points, the damaged element is correctly identified and the magnitude without noise is 25.77% (3.08% error), while when noise is considered, the magnitude is 26.74% (6.96% error). These results show that even with noisy data and only a limited number of sensors, accurate results are obtainable. Even though the damage index in undamaged elements is not always zero, the values present are small and therefore negligible.

Fig. 7 shows the results of Case II with three damaged elements. The algorithm always correctly identified the damaged elements. The magnitude of the stiffness reduction in those elements were as follows (citing element 7, 11, and 12 respectively): when all DOFs are used and noise is neglected, 15%, 20.02%, 15.23%; when all DOFS are used and noise is included, 15.4% (2.67% error), 19.6% (2.00% error), 14.8% (1.33% error); when only 15 DOFs are included and error is neglected, 14.5% (3.33% error), 21.6% (8.00% error), 17.5% (16.67% error); and when only 15 DOFs are included and the error is not neglected, 17.5% (16.67% error), 24.3% (21.5% error), 16.12% (7.47% error).

Therefore, when many elements are damaged simultaneously, this method works well when all DOFs are used. When using only a limited number of optimal sensor locations, the magnitude results deviate from the actual value; however, the error still does not prevent identifying the damaged elements.

3.2. Experiment and results

To further test the proposed algorithm, an experiment using a cantilever beam was designed and executed.

3.2.1. Experimental setup

A simple steel channel cantilever beam 2.8 m long with the cross section shown in Fig. 8, served as the experimental structure. The steel cantilever had an elastic modulus of $E = 210$ GPa and a mass density of $\rho = 7864$ kg/m^3. The cantilever was damaged with a cut in the flanges, 420 mm from the support and to a depth of $h = 16$ mm with a width of $w = 3$ mm. Fig. 8 also shows the position of the damage location.

The experiments to determine the modal parameters of the cantilever were performed in the structures laboratory of Shantou University's Civil Engineering Department. PCB capacitive accelerometers (model 3701DFA20G) measured the structural response while a PCB piezoelectric load cell (model 208C02) measured the input excitation. A National Instruments SCXI data acquisition system with appropriate signal conditioners and Labview8.0 collected the experimental data.

In order to determine the quantity of damage caused by the cut in the cantilever, a finite element model of the structure first needs to be created. In this experiment, a FE model with ten elements was created as shown in Fig. 9. This model was updated with experimental measurements of the modal properties of the actual undamaged cantilever. Then, after damaging the cantilever, the model was again updated with the modal data from the damaged structure to determine the decrease in stiffness in the finite elements. The experimentation showed a decrease in the second element which corresponds to the location of the actual damage in the cantilever beam.

3.2.2. System identification

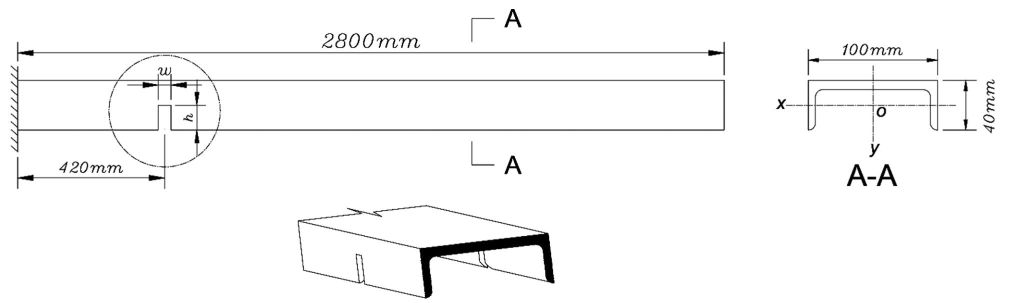

Fig. 8 Dimensions and plans of experimental cantilever beam

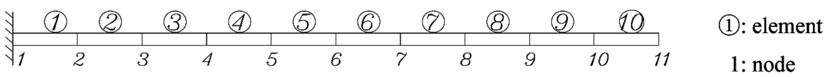

Fig. 9 FE model of cantilever beam

(a) Tested cantilever beam and PCB capacitive accelerometers

(b) SCXI data acquisition system

(c) Piezoelectric signal conditioner

(d) Capacitance signal conditioner

Fig. 10 Experimental apparatus

Table 2 The first 4 natural frequencies of the cantilever beam (Unit: Hz)

Case \ Modes	Frequency			
	f_1	f_2	f_3	f_4
Undamaged	4.5734	27.1409	73.0549	141.9115
Damaged	4.1630	26.7593	72.5458	137.9598

Vibration experiments were conducted on the damaged and undamaged cantilever and the modal parameters identified using the ERA method. Table 2 and Fig. 11 show the first four natural frequencies and mode shapes respectively for the undamaged and damaged cantilever beam.

3.2.3. Damage detection result and analysis

Analysis of the MAC and MKE values for the structure indicates the number of sensors needed and their optimum locations. Under the experiment's conditions and considering only planar vibration, ten accelerometers were used to measure the vertical response of the cantilever structure. Using the data collected from the experiments, damage detection was carried out using the computer program created for the numerical example.

Fig. 12 shows the damage index for the various elements of the FE model. Only two of the 10 elements show any significant damage index value. Element 2 shows the highest value of 39.6%. This value is more than double the 14.4% shown by Element 1. These values indicate that Element 2 is

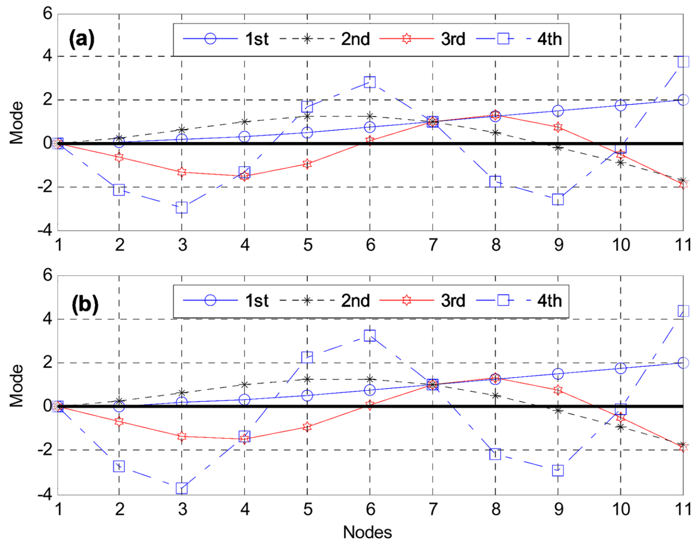

Fig. 11 The first 4 mode shapes of the undamaged (a) and damaged (b) cantilever beam

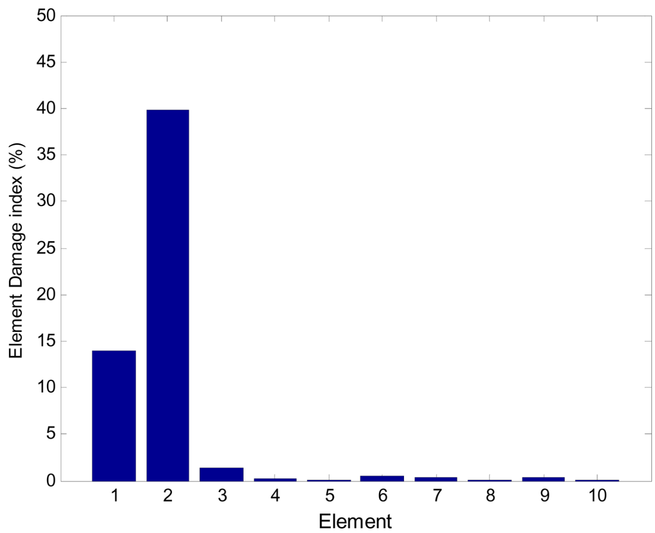

Fig. 12 Element damage index for the FE model

where the damage has occurred, but because of the nature of the cantilever and the ungraded stiffness of the finite elements, some of the damage has leaked into the stiffness of the first element. Therefore, even though the damage location is correct, the indication of damage severity lacks precision. Nevertheless, the vibration data from the structure can properly locate the local damage because the elemental stiffness of the single damaged element declines while the rest remain relatively unchanged. A deeper crack would induce a greater stiffness change and increase the nonlinear behavior at the damage location. The results of this experiment show the effectiveness of the method.

4. Conclusions

The first few natural frequencies and modes are sufficient to accurately estimate the flexibility matrix at the measured DOFs. Sensitivity analysis of the modal flexibility matrix provides a means of determining optimal sensor placement for collecting modal data. The MAC and MKE serve to estimate the ideal number of sensors and the performance for each optimized sensor location. According to optimization theory, the damage detection problem can be transformed into an optimization problem with certain constraints. The change in the flexibility matrix constructed using incomplete modal data becomes the objective function of the optimization routine. The PSO optimization algorithm successfully minimized the objective function and performed damage detection. A numerical simulation on a planar-truss structure showed that the proposed method is effective and robust for detecting structural damage using limited sensor information. Experimentation on a cantilever beam further showed the efficacy of the proposed methods.

Though civil infrastructure instrumented with thousands of sensors is the ideal, it is far from the reality of current economic factors. This paper raises the question of how to glean more information about a structure's health using fewer optimally placed sensors. In doing so, the MAC and MKE proved essential in choosing the minimum number of sensors needed and their performance at the optimized locations. The models and experiments presented in this paper dictated that the maximum off-diagonal MAC value be limited to less than 0.05 to select modes with large geometric angles between them. Similarly, ensuring that the MKE be at least 0.9 but as close to unity as possible ensured that the sensors in this paper's models and experiments detected and measured a sufficient number of modes. Additional research and experimentation may find the limiting values for the MAC and MKE change with each structure. The next stage of study will need to address the performance of these methods on different structures with varying damage cases.

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References

Berman, A. and Nagy, E.J. (1983), "Improvements of large analytical model using test data", *AIAA J.*, **21**, 1168-1173.

Carne, T.G. and Dohrmann, C.R. (1995), "A modal test design strategy for model correlation", In: Bathel, ed. *Proc 13th Int. Modal Analysis Conf*, New York: Union College, Schenectady, 927-933.

Cawley, P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. Strain Anal.*, **14**, 49-57.

Doebling, S.W. (1998), "A summary review of vibration – based damage identification methods", *Shock Vib.,* **30**(2), 91-105.

Duan, Z.D., Yan, G.R., Ou, J.P. and Spencer, Jr. B.F. (2005), "Damage localization in ambient vibration by constructing proportional flexibility matrix", *J. Sound Vib.,* **284**, 455-466.

Fedorov, V.V. (1972), *Theory of Optimal Experiments*, New York: Academic, translated and edited by W.J. Studden and E. M. Klimko, 28.

Friswell, M.I. and Mottershead, J.E. (1995), *Finite element model updating in structural dynamics*, Kluwer Academic Publishers, Dordrecht.

Gao, W.C., Liu, W. and Zou, J.X. (2004), "Damage detection methods based on changes of vibration parameters: a summary review", *J. Vib. Shock*, **23**(4), 1-17 (in Chinese).
Gao, Y. and Spencer, Jr. B.F. (2002), "Damage localization under ambient vibration using changes in flexibility", *J. Earthq. Eng*., **1**(1), 136-144.
Hou, Z.R. and Lu, Z.S. (2003), "Particle swarm optimization with applications based on matlab", *Comput. Simulat*., **20**(10), 68-70 (in Chinese).
Kammer, D.C. (1991), "Sensor placement for on-orbit modal identification and correlation of large space structures", *J. Guid. Control Dynam.*, **14**(2), 251-259.
Kennedy, J. and Eberhart, R.C. (1995), "Particle swarm optimization", *Proc. of IEEE Conf. on Neural Networks*, Piscataway, NJ, IV: 1942-1948.
Law, S.S., Shi, Z.Y. and Zhang, L.M. (1998), "Structural damage detection from incomplete and noisy test data", *J. Eng. Mech*. ASCE, **124**(11), 1280-1288.
Link, M. (1999), "Updating of analytical models-review of numerical procedures and application aspects", *Proc. of Structural Dynamics Forum*, Los Alamos.
Mottershead, J.E. and Friswell, M.I. (1993), "Model updating in structural dynamics: a survey", *J. Sound Vib*., **167**, 347-375.
Pandey, A.K. and Biswas, M. (1994), "Damage detection in structures using changes in flexibility", *J. Sound Vib*., **169**(1), 3-17.
Pandey, A.K. and Biswas, M. (1995), "Experimental verification of flexibility difference method for locating damage in structures", *J. Sound Vib*., **184**(2), 311-328.
Parsopoulos, K.E. and Vrahatis, M.N. (2002), *Particle swarm optimization method for constrained optimization problems*, Intelligent Technologies: from theory to applications, Amsterdam: IOS Press, 214-220.
Qi, B.H., Wu, R.F., Xie, H.C. and Li, G.H. (2001), "A flexibility matrix method for damage identification of truss structure", *Comput. Mech*., **18**(1), 42-47 (in Chinese).
Salawu, O.S. (1997), "Detection of structural damage through changes in frequency: a review", *Eng. Struct.*, **19**, 718-723.
Shi, Y.H. and Eberhart, R.C. (1999), "Empirical study of particle swarm optimization", *Proc. of Congress on Evolutionary Computation*, Piscataway, NJ: IEEE Service Center, 1945-1949.
Shi, Z.Y. (2002), "Structural damage detection from limited sensor information", *J. Vib. Eng*., **15**(2), 203-206 (in Chinese).
Stutz, L.T., Castello, D.A. and Rochinha, F.A. (2995), "A flexibility-based continuum damage identification approach", *J. Sound Vib*., **279**, 641-667.
Yan, A. and Golinval, J.C. (2005), "Structural damage localization by combining flexibility and stiffness methods", *Eng. Struct*., **27**, 1752-1761.
Yin, T., Zhu, H.P. and Yu, L. (2007), "Noise analysis for sensitivity-based structural damage detection", *Appl. Math. Mech-Engl*., **28**(6), 741-750.
Zhang, D.W. and Wei, F.X. (1999), *Model updating and damage detection*, Beijing: Science Press, 1999 (in Chinese).
Zhao, J. and DeWolf, J.T. (1999), "Sensitivity study for vibrational parameters used in damage detection", *J. Struct. Eng*., **125**(4), 410-416.

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