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Differential quadrature method for frequency analysis of membranes having irregular domains using an eight-node curvilinear element

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1. Introduction

Membranes are widely used in various engineering such as acoustic and mechanical applications. Free vibration of plates and membranes has been presented (Irie, *et al.* 1981, Laura, *et al.* 1997, Buchanan and Peddieson 1999, Leung, *et al.* 2003, Quek and Tua 2008, Singh, *et al.* 2008). The non-dimensional differential equation for free vibration is (Kang, *et al.* 1999)

$$\frac{\partial^2 W(x,y)}{\partial X^2} + \lambda^2 \frac{\partial^2 W(x,y)}{\partial Y^2} + \Omega^2 W(x,y) = 0, \tag{1}$$

where W is the transverse deflection, ρ is the mass per unit area, ω is the circular frequency, and T is the tension per unit length. The density of the membrane is the linear function of the x. The boundary conditions is W=0 at edges. Employing the transformation rule (Bert and Malik 1996) the governing Eq. (1) becomes,

$$[J_{22}]^{-1}\sum_{i=0}^{N} B_{ij}W_{ik} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\sum_{i=0}^{N} A_{ij}W_{ik}$$

$$+\lambda^2\left[[J_{22}]^{-1}\sum_{i=0}^{N} B_{ij}W_{jk} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\sum_{i=-M}^{N} B_{ij}W_{jk}\right] + \Omega^2 W_{ij} = 0 \tag{2}$$

Combining the discretized governing Eq. (2) at each discrete point in the physical domain and the boundary condition Eq. (2) at each boundary point, we obtain.

$$\{[K] - \Omega^2[M]\}\{W\} = 0 \tag{3}$$

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2. Numerical results

Firstly, to check the validity of the HDQ (Civalek 2004, Civalek and Ülker 2004, Özpolat 2009) solution and the proposed formulation, the results for frequencies of square membrane are obtained (Table 1). It is seen from this table that when the grid point numbers reaches N=11 the present method gives accurate predictions for the first vibration modes. For second and third modes of vibration, however, the accurate results are obtained for N=13. Free vibration analysis of trapezoidal membrane (Fig. 1) is considered. The results (Table 2) obtained by the present method are compared with the finite element solution (Kang and Le 2004). The results are good agreement with the results given by Kang and Lee (2004). Another convergence study is presented in Table 3 for the first three modes of vibration for the rectangular (b/a=0.75) membrane. For the using 15 grids, the rate of convergence remains reasonable and the agreement with the results from references (Kang, *et al*. 1999, Wu, *et al.* 2007) is excellent. First three frequency values of the skew membrane (Fig. 2) are listed in Table 4 for four different skew angles. Finally, first three frequency values of the skew membrane with different aspect ratio are listed in Table 5 for $\theta = 45^o$. In general, it is concluded that in these two tables, the frequency parameter generally decreases as aspect ratio and skew angle increase, respectively.

Table 1 Comparison of first three frequency values of the square membrane

Mode number	Methods			
	Exact	HDQ N=11	HDQ N=13	HDQ N=15
1	4.4429	4.4429	4.4429	4.4429
2	7.0248	7.0251	7.0249	7.0248
3	8.8858	8.8860	8.8859	8.8859

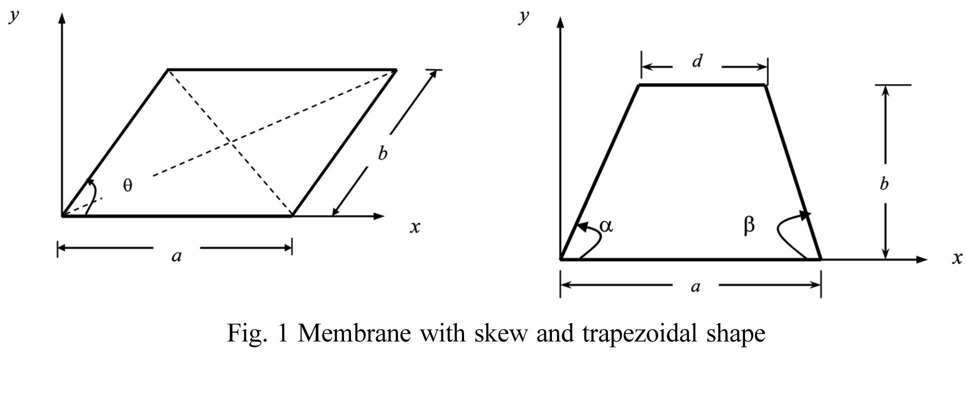

Fig. 1 Membrane with skew and trapezoidal shape

Table 2 Comparison of frequencies of the trapezoidal membrane (a/b=2.0, β=70°, α=60°)

Mode number	Methods			
	Kang and Lee (2004)	HDQ N=9	HDQ N=11	HDQ N=15
1	3.81	3.82	3.81	3.81
2	5.29	5.30	5.28	5.28
3	6.58	6.63	6.60	6.58

Table 3 Comparison of frequency values of the rectangular membrane (b/a=0.75)

Mode numbers	Methods				
	Exact Kang, *et al*. (1999)	Wu, *et al.* (2007)	Kang, *et al.* (1999)	Present HDQ (N=13)	Present HDQ (N=15)
1	4.3633	4.3633	4.3651	4.3634	4.3633
2	6.2929	6.2929	6.3006	6.2930	6.2930

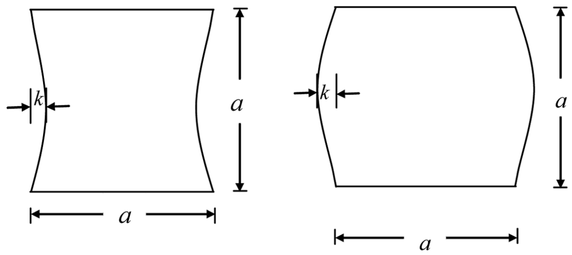

Fig. 2 Membrane with two opposite concave and two opposite convex edges

Table 4 First three frequency values of the skew membrane (*b*/*a*=3)

Mode number	Skew angles			
	θ=75°	θ=70°	θ=60°	θ=45°
1	3.4193	3.5082	3.8008	4.5882
2	3.8806	3.9649	4.2405	4.9980
3	4.5402	4.6190	4.9001	5.5948

Table 5 First three frequency values of the skew membrane with different aspect ratio (θ=45°)

Mode number	*b*/*a*			
	1	1.5	2	3
1	5.9021	5.0671	4.7904	4.5882
2	8.1478	6.4925	5.6730	4.9980
3	10.0263	8.0694	6.8155	5.5948

Table 6 Fundamental frequency values of the membrane with different geometries

Membrane Shape	*k*/*a*			
	0.1	0.15	0.20	0.25
Membrane with two opposite concave edges	4.586	4.623	4.713	4.794
Membrane with two opposite convex edges	4.402	4.385	4.299	4.283

The fundamental frequency values are listed in Table 6 for different curvilinear shaped membrane (Fig. 2). It is found that the frequency parameters of membranes with two-opposite concave edges show larger than those of two-opposite convex edges membrane. It is also concluded that, as the parameter k increase and the shape of membranes become more irregular, the frequency values monotonically increase due to the decrease of the area for two-opposite concave edges. For convex edges, however, the frequency values monotonically decrease with the increasing value of k.

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