Smart Structures and Systems, Vol. 5, No. 3 (2009) 223-240
DOI: http://dx.doi.org/10.12989/sss.2009.5.3.223

Health Monitoring System (HMS) for structural assessment

José Campos e Matos*[1], Oscar Garcia‡[2], António Abel Henriques‡†[1], Joan Ramon Casas‡‡[3] and Josep Vehí‡‡†[2]

[1]*University of Porto, Faculty of Engineering, Rua Dr. Roberto Frias s/n, Civil Engineering Department, 4200-465 Porto, Portugal*
[2]*University of Girona, Campus Montilivi, Edificio P-IV, Automatic, Informatics and Electronic Department, 17071 Girona, Spain*
[3]*Technical University of Catalonia, School of Civil Engineering. C/Jordi Girona 1-3, Campus Norte, Edificio C1, 08034 Barcelona, Spain*

(Received February 25, 2008, Accepted September 22, 2008)

Abstract. As in any engineering application, the problem of structural assessment should face the different uncertainties present in real world. The main source of uncertainty in Health Monitoring System (HMS) applications are those related to the sensor accuracy, the theoretical models and the variability in structural parameters and applied loads. In present work, two methodologies have been developed to deal with these uncertainties in order to adopt reliable decisions related to the presence of damage. A simple example, a steel beam analysis, is considered in order to establish a liable comparison between them. Also, such methodologies are used with a developed structural assessment algorithm that consists in a direct and consistent comparison between sensor data and numerical model results, both affected by uncertainty. Such algorithm is applied to a simple concrete laboratory beam, tested till rupture, to show it feasibility and operational process. From these applications several conclusions are derived with a high value, regarding the final objective of the work, which is the implementation of this algorithm within a HMS, developed and applied into a prototype structure.

Keywords: uncertainty; Modal Interval Analysis (MIA); perturbation method; structural assessment; Health Monitoring System (HMS).

1. Introduction

Development of structural Health Monitoring Systems (HMS) has been a subject of increasing activity in recent years. One of the main problems to face by HMS is the treatment of uncertainty, mainly present in numerical model, physical and geometrical parameters such as loading, Young modulus, inertia, etc., and in measured variables such as displacements, strains and rotations. One of the main issues derived from the various sources of uncertainty is how to define objective and reliable criteria to distinguish between an abnormal behavior (differences between measured values and those

*Corresponding Author, E-mail: joseantonio@camposematos.com
‡E-mail: odgarcia@eia.udg.es
‡†E-mail: abel.henriques@fe.up.pt
‡‡E-mail: joan.ramon.casas@upc.edu
‡‡†E-mail: vehi@eia.udg.es

predicted by the model) due to the presence of damage, and the differences between measured and calculated results just because of the uncertainty and randomness present in the experimental data, models and physical parameters. When performing structural assessment, methodologies that take into account these uncertainties should be implemented in an efficient, fast and user friendly way. In this paper two methodologies that consider uncertainty both in the model and in recorded data are presented. One of them, Modal Interval Analysis (MIA) is a modification of the classic interval analysis theory developed by the authors. Also, it is presented an algorithm for structural assessment and one simple application of it. The final aim is to apply the developed algorithm within a Health Monitoring System (HMS), developed and installed in a real prototype structure.

2. Treatment of uncertainty

There exist different techniques for consideration of uncertainties in numerical models and measured variables. Methods based on simulation techniques have high computational cost in large structures (Mahadevan and Raghothamachar 2000, Olsson, *et al*. 2003, Schueller 2001). Reliability methods like FORM and SORM may be used, however, the evaluation of limit state functions derivatives and the implementation in a Finite Element framework is rather complex and requires several analyses (Frangopol, *et al*. 1996, Liu and Der Kiureghian 1991). Neural network and fuzzy set theory have been applied as an alternative (Ayyub and Gupta 1997, Biondini, *et al*. 2004, Hurtado 2002, Mullen and Muhanna 1999). Probability theory (Oberkampf, *et al*. 2002) and rough set theory (Fetz, *et al*. 2000) have been used recently. Perturbation based methods are also being applied to obtain results efficiently (Altus, *et al*. 2005, Reh, *et al*. 2006, Zhang, *et al*. 1996). Recently, interval analysis has been applied to evaluate uncertainties in structures (Garcia, *et al*. 2004, Gardenyes, *et al*. 2001, Jaulin, *et al*. 2001). Modal Interval Analysis (MIA) and Perturbation Method are presented here in the following for their application to the management of uncertainties in Health Monitoring Systems (HMS).

2.1. Modal Interval Analysis (MIA)

The initial idea of interval analysis (Moore 1966, Neumaier 1990) is to enclose real numbers in intervals and real vectors in boxes as a method of considering the imprecision of representing real numbers by finite digits in numerical computers. The variables are not deterministic, but taking any value between a lower and an upper limit of an interval. The variables are represented by a uniform variation and the probability distribution function is not necessary. Interval analysis has become a fundamental nonlinear numerical tool for representing uncertainties or errors, proving properties of sets, solving sets of equations or inequalities and optimizing globally via interval arithmetic (Hansen 1992, Jaulin, *et al*. 2001).

A Classic Interval number is a closed set that include the possible range of an unknown real number. Thus, instead of considering a fixed value a, the following representation is adopted:

$$A' = [\underline{a}, \bar{a}] := \{x \in R \mid \underline{a} \le x \le \bar{a}\} \tag{1}$$

where $\underline{a}$ is the minimum and $\bar{a}$ the maximum of the interval. The four elementary arithmetic operations (+, –, ×, ÷) are extended to intervals. If op denotes an arithmetic operation for real numbers, the corresponding interval arithmetic operation is:

$$C = AopB = \{aopb \mid a \in A, b \in B\} \tag{2}$$

Modal Interval Analysis (MIA) is a natural extension of Classical Interval Analysis, where the concept of interval is widened by a set of predicates that are fulfilled by the real numbers (Gardenyes, *et al*. 2001, SIGLA/X 1999).

A modal interval X is defined as a couple $X = (X', \forall)$ or $X = (X', \exists)$, where X' is its classical interval domain and the quantifiers $\forall$ (universal) and $\exists$ (existential) are a modality selection. Modal intervals of type $X = (X', \exists)$ are defined as proper intervals, while intervals of type $X = (X', \forall)$ are designated by improper intervals. A modal interval can be represented using its canonical coordinates in the form:

$$X = [a, b] = \begin{cases} ([a, b]', \exists) & \text{if } a \le b \\ ([a, b]', \forall) & \text{if } a \ge b \end{cases} \tag{3}$$

For example, the interval [2, 5] is equal to ([2, 5]′, ∃) and the interval [8, 4] is equal to ([4, 8]′, ∀).

2.1.1. Semantic extension

Comparative to the interval function $f(\mathbf{X}') = [\min_{\mathbf{x} \in \mathbf{X}'} f(\mathbf{x}), \max_{\mathbf{x} \in \mathbf{X}'} f(\mathbf{x})]'$, $\mathbf{X}' \in \mathrm{I}(R^n)$ defined in classical interval analysis, the modal interval function $f^*(X): \mathrm{I}^*(R^n) \to \mathrm{I}^*(R)$ is defined to be (Gardenyes, *et al*. 2001).

$$f^*(\mathbf{X}) := \left[\min_{\mathbf{x}_p \in \mathbf{X}_p'} \max_{\mathbf{x}_i \in \mathbf{X}_i'} f(\mathbf{x}_p, \mathbf{x}_i), \max_{\mathbf{x}_p \in \mathbf{X}_p'} \min_{\mathbf{x}_i \in \mathbf{X}_i'} f(\mathbf{x}_p, \mathbf{x}_i) \right]. \tag{4}$$

Its counterpart modal interval function $f^{**}(X)$ is defined to be

$$f^{**}(\mathbf{X}) := \left[\max_{\mathbf{x}_i \in \mathbf{X}_i'} \min_{\mathbf{x}_p \in \mathbf{X}_p'} f(\mathbf{x}_p, \mathbf{x}_i), \min_{\mathbf{x}_i \in \mathbf{X}_i'} \max_{\mathbf{x}_p \in \mathbf{X}_p'} f(\mathbf{x}_p, \mathbf{x}_i) \right]. \tag{5}$$

Naturally, $f^*(\mathbf{X})$ is degenerated to $[\min_{\mathbf{x}_p \in \mathbf{X}_p'} f(\mathbf{x}_p), \max_{\mathbf{x}_p \in \mathbf{X}_p'} f(\mathbf{x}_p)]$ when all components of $\mathbf{X}$ are proper intervals, which is the case of $f(\mathbf{X}')$ in classical interval analysis. Furthermore, $f^*(\mathbf{X}) = f^{**}(\mathbf{X})$ when all components of $\mathbf{X}$ have the same modality $\exists$ or $\forall$.

The semantic statement of $\forall x_1 \in [a_1, b_1]' \ldots \forall x_n \in [a_n, b_n]' \exists z \in F'(\mathbf{X}') z = f(x_1, \ldots, x_n)$ for $f(\mathbf{X}') \subseteq F'(\mathbf{X}')$ in classical interval analysis can be extended accordingly for $f^*(\mathbf{X}') \subseteq F(\mathbf{X})$ in modal interval analysis, i.e., $f^*(\mathbf{X}) \subseteq F(\mathbf{X})$ is equal to:

$$\forall \mathbf{x}_p \in \mathbf{X}_p' \, Q_{F(\mathbf{x})} z \in F'(\mathbf{X}) \exists \mathbf{x}_i \in \mathbf{X}_i' \;\; z = f(\mathbf{x}_p, \mathbf{x}_i). \tag{6}$$

This is fundamental semantic theorem in modal interval analysis since it provides a semantic and physical interpretation for the modal interval function $f^*(\mathbf{X})$ and its inclusion function $F(\mathbf{X})$ (Gardenyes, *et al*. 2001). It can also be seen that the semantic statement of Eq. (6) includes both the universal quantifier statement $\forall \mathbf{x}_p \in \mathbf{X}_p'$ and the existential quantifier statement $\exists \mathbf{x}_i \in \mathbf{X}_i'$ for various function variables while the semantic statement in classical interval analysis only contains the universal quantifier statement $\forall \mathbf{x} \in \mathbf{X}'$ for all function variables. Such semantic extension is essential and complete in theory since many physical problem descriptions include both $\forall$ and $\exists$ for function variables.

2.1.2. Computational implementation

In this study, the uncertainty is included in the mathematical model of the structure by replacing the uncertain parameters by intervals. The uncertain parameter can take any value within the limits of the interval. The result of this process is an interval value, obtained once the structural equations have been analyzed with theorems of Modal Interval Analysis (MIA).

In interval arithmetic, overestimation is one of the main drawbacks because the range of uncertain is much larger than the range introduced by round off error. Overestimation is due to dependency and failure of some algebraic laws that are valid in real arithmetic. Such overestimation produces extremely and sometimes meaningless results.

Interval implementation of Finite Element Method presents a sharp bound on possible nodal displacement for treatment of uncertainty (Garcia, *et al*. 2004). The system of interval equations can be written as:

$$Kq = p \tag{7}$$

where K is the global interval matrix of the structure, p the applied interval load vector and q the unknown interval displacement vector.

When the quantities involved in the simulation take values inside intervals of variation, the set of trajectories determine a plane band bounded by two envelopes. At each step of the simulation, the envelopes, i.e. the possible maximum and minimum values of the variable, have to be determined. The function whose parameters have to be determined is defined by the interval model of the system and the parameter is determined by the interval values of the parameters. The simulation of an interval model provides intervals (ranges) which can be computed by means of interval arithmetic.

Interval arithmetic (Moore 1966, Jaulin, *et al*. 2001) considers the whole range of possible instances represented by an interval model. The computations of the natural extension of a real function are done by substituting real numbers by intervals and real operations by their interval extensions. An important property is monotonic inclusion: given f, a real function, and $f_R(X)$ its natural extension to interval X, then $x \in X$ implies $f(x) \in f_R(X)$.

In consequence, the natural extension is very useful to compute the range of a function because it guarantees the result. Unfortunately, it does not provide the exact estimate in a general case. This comes from the multi-incidence problem: interval arithmetic considers each instance of a variable in the syntax tree of a function as being independent of each other, leading to an overestimation of the actual range.

In this paper, the interval model is studied using Modal Interval Analysis (MIA), allowing computing a tight (sometimes exact) enclosure of the envelope that includes all the possible behaviours of the system.

Using modal intervals, each interval function to be evaluated is automatically analyzed and put, if possible, in its optimal form (the expression is rewritten in such a way that the exact range is obtained). Then, the Modal Interval Library (IvalDB) computes the exact range. In case that the optimal result cannot be reached, the f^* algorithm (Herrero, *et al*. 2005) is launched. This algorithm takes benefit of many optimality and coercion theorems from Modal Interval Theory to compute tight approximations of the range by using branch-and-bound techniques.

The results showed in this paper have been obtained using the improved version of the f^* algorithm, which is not detailed in this paper.

A complete introduction to modal interval analysis, including several examples can be found in

SIGLA/X (1999). Efficient tools to compute the range of modal interval functions as well as many other tools can be found in http://mice.udg.es/fstar.

2.2. Perturbation Method

2.2.1 Formulation

The Perturbation Method can be applied in a finite element framework following the steps of a deterministic analysis (Spanos and Ghanem 1989, Thomos and Trezos 2006, Reh, *et al.* 2006). The method is based on Taylor series expansion of the governing equations. The structural behavior is characterized by taking into account terms around the mean values of the basic random variables. Mean and variance of the response can be found in terms of mean and variance of the basic random variables, thus, distribution information is not required (Contreras 1980, Zhang and Ellingwood 1996, Altus, *et al.* 2005). When a perturbation δ is applied to the balanced system, the equilibrium can be defined by the following equation:

$$(K_0 + \delta K) \cdot (U_0 + \delta U) = F_0 + \delta F \tag{8}$$

where K is the stiffness matrix, defined as a function of nodal displacements U, F is the applied load vector, the variables with indexes 0 represent the central values of their probabilistic distribution (generally the mean values) and variables with δ-sign stand for perturbations around central values. Developing Eq. (8), taking into account that $K_0 \cdot U_0 = F_0$, and neglecting second order terms, the structural response dispersion can be evaluated by the relation:

$$K_0 \cdot \delta U = -\delta K \cdot U_0 + \delta F \tag{9}$$

The non deterministic nature of the structural parameters is defined by random variables denoted by X. The covariance matrix of displacements, C_u, is calculated by:

$$C_u = \frac{\partial U}{\partial X} \cdot \delta X \cdot C_\rho \cdot \delta X^T \cdot \left(\frac{\partial U}{\partial X}\right)^T \tag{10}$$

where $\partial U / \partial X$ stands for the partial derivatives of displacements U with respect to the random variables X, C_p is the correlation matrix and δX a matrix containing the standard deviation of random variables. Taking into consideration that the deviation of structural stiffness and forces, defined in Eq. (8), result from the random variables dispersion, the structural response deviation can be defined by:

$$K_0 \cdot \delta U = -\frac{\partial K}{\partial X} \cdot U_0 \cdot (\delta X) + \frac{\partial F}{\partial X} \cdot \delta X \tag{11}$$

Considering Eq. (11), the covariance matrix of displacements, C_u, defined in Eq. (10), is computed as:

$$C_u = \delta U \cdot C_\rho \cdot \delta U^T \tag{12}$$

As follows, the dispersion of structural response, defined by matrix C_u, is evaluated taking into account mean values X_i, standard deviation δX_i and the correlations $\rho_{ij}(i, j = 1,2,...,m)$.

2.2.2. Computational implementation

The application of this method to current finite element computational programs requires the addition of new modulus to evaluate the covariance matrix, defined in Eq. (12), taking into account the structural uncertainty according to Eq. (11).

This implementation is performed by incorporating new instructions to compute the partial derivatives of the stiffness matrix, $\partial K/\partial X$, the partial derivatives of the nodal forces vector, $\partial F/\partial X$, and the system of equations defined in Eq. (10) to evaluate δU vector. Generally, the stiffness matrix K and the nodal forces F are obtained by integration, being the analytical calculation of the partial derivatives not practicable in most of the cases. In these circumstances, numerical evaluation is performed (e.g.: finite differentiation). An algorithm coupled to a finite element framework can be divided in the following steps:

1. Read data of structural problem, particularly, finite element mesh (geometry), material properties and applied actions. Read data to characterize random variables, i.e., their mean values, standard deviation and correlation coefficients;
2. Evaluate nodal forces F corresponding to the actions defined previously;
3. Calculate stiffness matrix K_0;
4. Compute the displacement U_0 by solving the system of equations: $K_0 \cdot U_0 = F_0$;
5. Calculate partial derivatives $\partial K/\partial X$ of stiffness matrix and partial derivatives $\partial F/\partial X$ of forces, with respect to random variables;
6. Compute displacements covariance matrix, C_u, according to Eqs. (11) and (12);
7. Calculate standard deviation of structural response from its variance that is defined by the diagonal elements of covariance matrix, corresponding to the degree of freedom which identifies the response.

3. Application

3.1. Steel beam

The random response of a steel beam, due to random fluctuation in material properties and applied loads, is studied here. To show their advantages and disadvantages, both Modal Interval Analysis (MIA) and Perturbation Method were used. The beam, clamped at the left side and simply supported at the right one, is modeled with 1D Euler-Bernoulli beam elements. The structure is subjected to a uniform distributed load (Fig. 1). Due to the low level load, the steel beam response follows a linear elastic regime. The steel modulus of elasticity, E, and the uniform load level, p, are considered as the uncertain parameters of the model and are defined by random variables which are not correlated between them. Other mechanical parameters are deterministic.

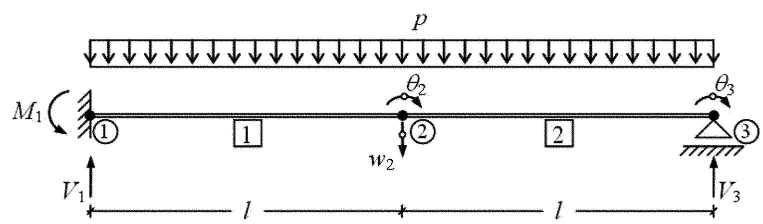

Fig. 1 Finite element model

The following values are considered: total span length $2l = 4$ m, moment of inertia $I = 2\times10^{-5}$ m^4, modulus of elasticity (E) with a mean of 200 GPa and a coefficient of variation of 5%, applied load (p) with a mean of 10 kN/m and a coefficient of variation of 15%.

The Perturbation Method, based on a finite element procedure, will be applied to obtain node 2 and node 3 displacements (θ_2, w_2 and θ_3). Such output will be quantified in terms of the mean value and dispersion (defined by standard deviation) considering an uniform distribution for all random variables. Taking into account the supports of this structure, the displacements θ_1, w_1 and w_3 are already known, being equal zero, and the corresponding forces (reactions) M_1, V_1 and V_3 are unknown. Therefore, the displacements θ_1, w_1 and w_3, originally placed in the unknown variables vector, are replaced by M_1, V_1 and V_3 and the system of equations corresponding to the evaluations of the static equilibrium becomes:

$$\begin{bmatrix} 1 & 0 & \frac{2EI}{l} & -\frac{6EI}{l^2} & 0 & 0 \\ 0 & 1 & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 & 0 \\ 0 & 0 & \frac{8EI}{l} & 0 & \frac{2EI}{l} & 0 \\ 0 & 0 & 0 & \frac{24EI}{l^3} & \frac{6EI}{l^2} & 0 \\ 0 & 0 & \frac{2EI}{l} & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 \\ 0 & 0 & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & 1 \end{bmatrix} \cdot \begin{Bmatrix} M_1 \\ V_1 \\ \theta_2 \\ w_2 \\ \theta_3 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} \frac{pl^2}{12} \\ \frac{pl}{2} \\ 0 \\ pl \\ -\frac{pl^2}{12} \\ \frac{pl}{2} \end{Bmatrix} \tag{13}$$

The mean values of the unknowns are calculated by solving Eq. (13) and taking the mean values of the parameters:

$$U_0 = \begin{Bmatrix} M_1 \\ V_1 \\ \theta_2 \\ w_2 \\ \theta_3 \\ V_3 \end{Bmatrix}_0 = \begin{bmatrix} 20 \text{ kN.m} \\ 25 \text{ kN} \\ 0.00083(3)\text{rad} \\ 0.0033(3)\text{m} \\ -0.0033(3)\text{rad} \\ 15 \text{ kN} \end{bmatrix} \tag{14}$$

The application of Eqs. (11) and (12) for this example, and having into account that C_p is an identity matrix, results in the following covariance matrix of structural response:

$$C_u = \begin{bmatrix} 9 & 11.25 & 3.75 \times 10^{-4} & 1.5 \times 10^{-3} & -1.5 \times 10^{-3} & 6.75 \\ 11.25 & 14.06 & 4.69 \times 10^{-4} & 1.88 \times 10^{-3} & -1.88 \times 10^{-3} & 8.44 \\ 3.75 \times 10^{-4} & 4.69 \times 10^{-4} & 1.736 \times 10^{-8} & 6.94 \times 10^{-8} & -6.94 \times 10^{-8} & 2.81 \times 10^{-4} \\ 1.5 \times 10^{-3} & 1.88 \times 10^{-3} & 6.94 \times 10^{-8} & 2.778 \times 10^{-7} & -2.78 \times 10^{-7} & 1.12 \times 10^{-3} \\ -1.5 \times 10^{-3} & -1.88 \times 10^{-3} & -6.94 \times 10^{-8} & -2.78 \times 10^{-7} & 2.778 \times 10^{-7} & -1.12 \times 10^{-3} \\ 6.75 & 8.44 & 2.81 \times 10^{-4} & 1.12 \times 10^{-3} & -1.12 \times 10^{-3} & 5.06 \end{bmatrix} \tag{15}$$

The variances of displacement components, θ_2, w_2 and θ_3, correspond to the values obtained at the diagonal of matrix C_u, respectively, at lines (or columns) 3, 4 and 5. Hence, standard deviation of θ_2, w_2 and θ_3 are:

$$\delta\theta_2 = \sqrt{1.736 \times 10^{-8}} = 0.000132 rad$$

$$\delta w_2 = \sqrt{2.778 \times 10^{-7}} = 0.000527 m$$

$$\delta\theta_3 = \sqrt{2.778 \times 10^{-7}} = 0.000527 rad$$

In Interval Analysis, only uniform distributions can be considered. So the modulus of elasticity E = 200 GPa with 5% coefficient of variation is represented by the interval, E = [190, 210] GPa, where the lower and upper values of the interval correspond to the mean minus/plus one standard deviation. Analogously, the applied load p = 10 kN/m. with 15% coefficient of variation is defined by the interval P = [8.5, 11.5] kN/m. Using Eq. (13) as the system of interval equations and the previous defined input interval values, node 2 and node 3 displacements (θ_2, w_2 and θ_3) (Fig. 1) are obtained. Such output is also an interval value.

Table 1 presents the main results obtained with both methods. Regarding Interval Analysis this structure was analyzed with the Modal Interval Analysis (MIA), obtaining a lower overestimation. The

Table 1 Beam results - comparison of methods

Solution	$\underline{w}_2$ (m)	$\overline{w}_2$ (m)	$\underline{\theta}_2$ (rad)	$\overline{\theta}_2$ (rad)	$\underline{\theta}_3$ (rad)	$\overline{\theta}_3$ (rad)
Modal interval	3.651×10^{-3}	2.982×10^{-3}	9.127×10^{-4}	7.456×10^{-4}	-2.982×10^{-3}	-3.651×10^{-3}
Perturbation method	3.860×10^{-3}	2.806×10^{-3}	9.653×10^{-4}	7.013×10^{-4}	-2.806×10^{-3}	-3.860×10^{-3}

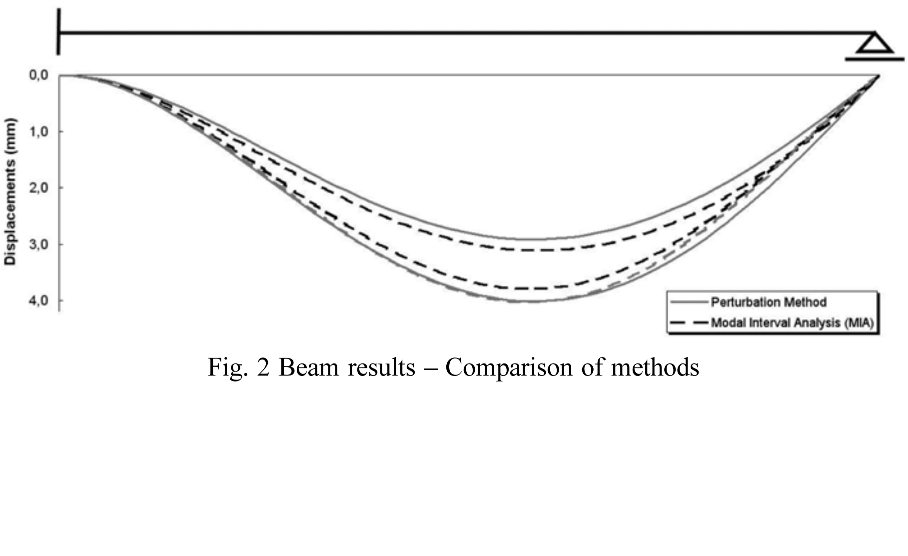

Fig. 2 Beam results – Comparison of methods

nomenclature used here is the same of Fig. 1 being the under scored variables the minimum value and the upper scored the maximum one. In Fig. 2 is presented the beam deformed shape obtained for all methodologies.

Results in Fig. 2 show the range of possible values of the response taking into account the randomness of the modulus of elasticity and loading. A response outside the range can be interpreted as a malfunction or damage of the structure.

3.2. Concrete beam

The aim of this example is to present a simple algorithm for structural assessment, based in a direct and consistent comparison between obtained data and numerical results, both affected by uncertainty. In order to obtain the numerical results, the previously presented methodologies for uncertainty treatment were considered.

A reinforced concrete beam with a square section of 0.15 (b) × 0.15 (h) m^2 and a total length of 2.10 m was tested in the laboratory up to failure (Fig. 3). The beam is loaded by two punctual forces F applied at third parts of the span. Such forces are applied by an actuator. The error in the measurement of applied load can be modelled as a random variable with a mean equivalent to the load value and a standard deviation equal to 5% of it. Compressive tests were executed in some samples during the pouring of the beam to determine the concrete elasticity modulus. From the samples it is estimated a mean value of 32.6 GPa and standard deviation of 1.94 GPa (E_{cm} = 32.6 GPa; δE_c = 1.94 GPa).

A finite element model of this beam was considered in order to analyze its behaviour (Fig. 4). The model is constituted by 1D Euler Bernoulli beam elements, two displacements by each node. In both

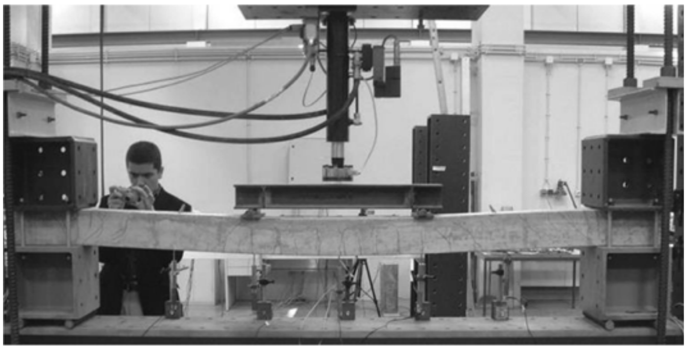

Fig. 3 Laboratory test of concrete beam

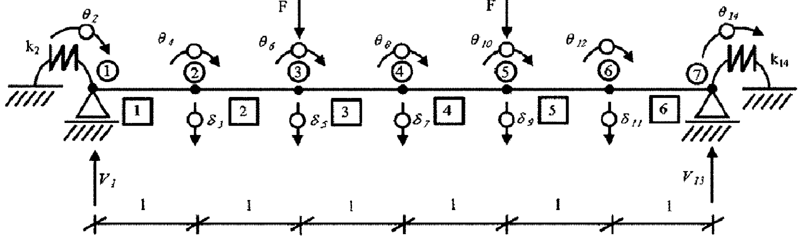

Fig. 4 Finite element model

ends the beam is simply supported, being the rotational displacements also partially restrained due to the test set-up at the supports (Fig. 3). To take into account such restraint, a rotational spring was considered in the model (k = 6000 kN.m/rad; δk = 60 kN.m/rad).

The uncertainty is considered in the elasticity modulus (E_c), applied load (F) and in the spring stiffness (k). Uniform distributions were considered for all uncertain variables. Other variables are considered to be deterministic (Length l = 0.35 m; Inertia I = 4.21875×10^{-5} m^4). During the test, displacement transducers were placed to measure the beam deflection, namely δ_3, δ_5, δ_7, δ_9 and δ_{11}.

Perturbation Method was applied having into account the defined uncertainty. The obtained results were compared with those from the displacement transducers affected by their sensitivities. If the transducer gives a value of δ_m, we may say that the real vertical displacement is within the interval $[\delta_m - \Delta\delta; \delta_m + \Delta\delta]$, being $\Delta\delta$ the transducer accuracy. Such comparison was made for each level of load applied in the test. When there is no intersection between numerical data and experimental results intervals, it is possible to state that the beam presents a different behaviour from the elastic one. This, in turn, can be considered as a potential damage. In this case, the damage is the cracking in the beam and therefore the prediction of the moment when the cracking appears ("Cracking Load") is the main point of interest. Fig. 5 presents the results for the control variable δ_7. From this comparison it was obtained a "Cracking Load" of 16.90 kN for a 1.55 mm vertical displacement.

In Fig. 6 it is present the results obtained with Modal Interval Analysis (MIA) for the control variable δ_7. In order to determine the "Cracking Load" a comparison was made between numerical and obtained data intervals for each load step. It was determined a "Cracking Load" of 16.40 kN for a 1.45 mm vertical displacement.

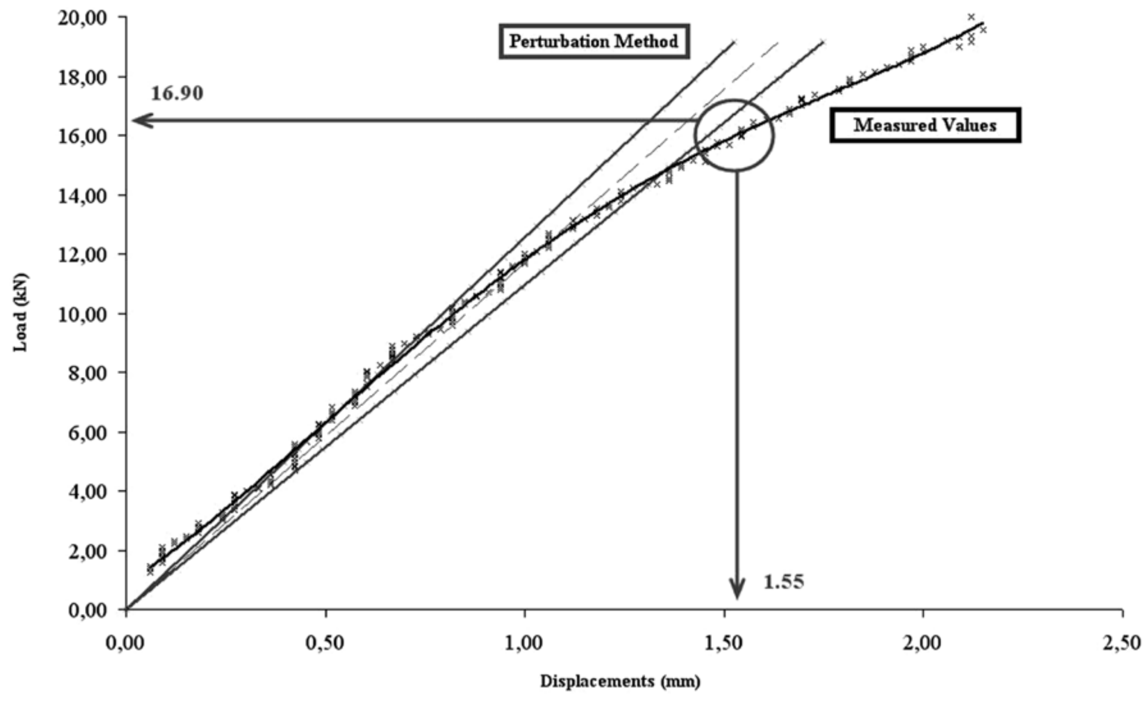

Fig. 5 Obtained results – Perturbation method

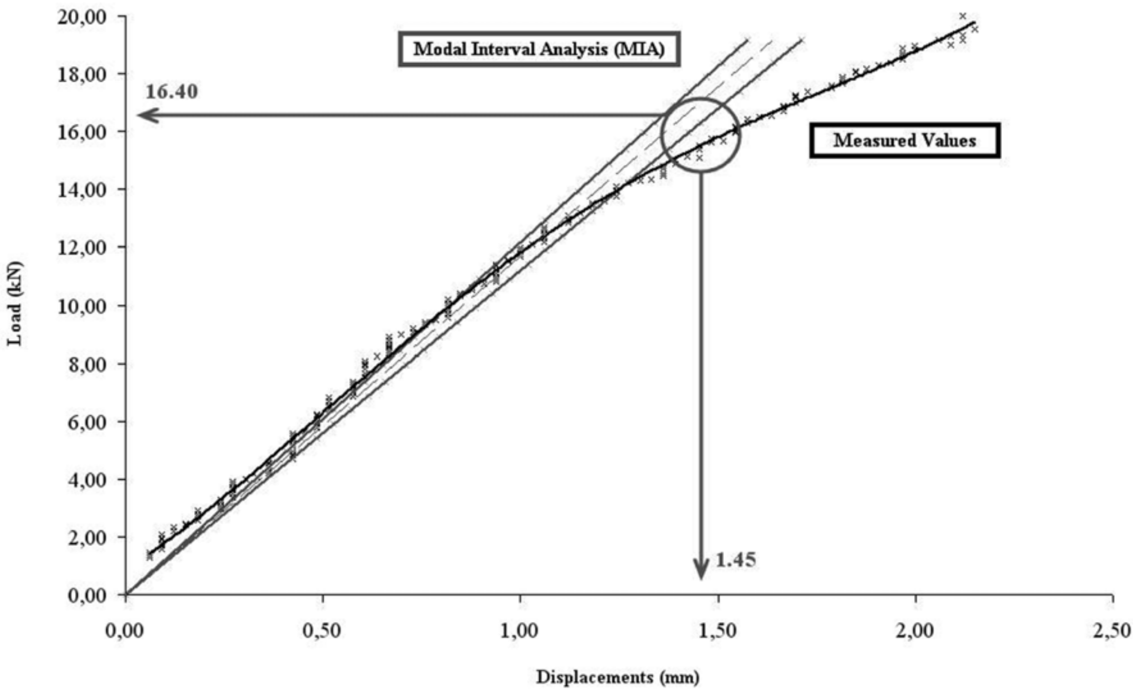

Fig. 6 Obtained results – modal interval analysis (MIA)

Table 2 Beam results - Comparison of methods

Solution	“Cracking Load” (kN)	vertical displacement (mm)
Modal interval	16.90	1.55
Perturbation method	16.40	1.45
Measured value	17.20	1.45

Table 2 presents a comparison between measured values and the results obtained with Perturbation Method and Modal Interval Analysis (MIA). The measured value of “Cracking Load”, and the respective vertical displacement, was determined by a direct analysis of obtained data. It is considered to be the point where the graphic Load (kN) – Displacements (mm) changes it linearity.

Fig. 7 presents the beam deformed shape determined using the Modal Interval Analysis (MIA) and the Perturbation Method for two different loads. The measured results for each displacement transducer affected by its accuracy (δ_3, δ_5, δ_7, δ_9 and δ_{11}) are also displayed. For a load level F = 8.4390 kN < “Cracking Load” (top figure) all the measured data is within the deformed shape for both Modal Interval Analysis (MIA) and Perturbation Method, while for F = 19.1510 kN > “Cracking Load” (bottom figure) all transducers present results outside the respective range. When cracks start to appear the structure presents a non-linear behaviour and so the elastic model can not be applied anymore. The damage in the beam due to cracking is clearly identified by the two proposed techniques.

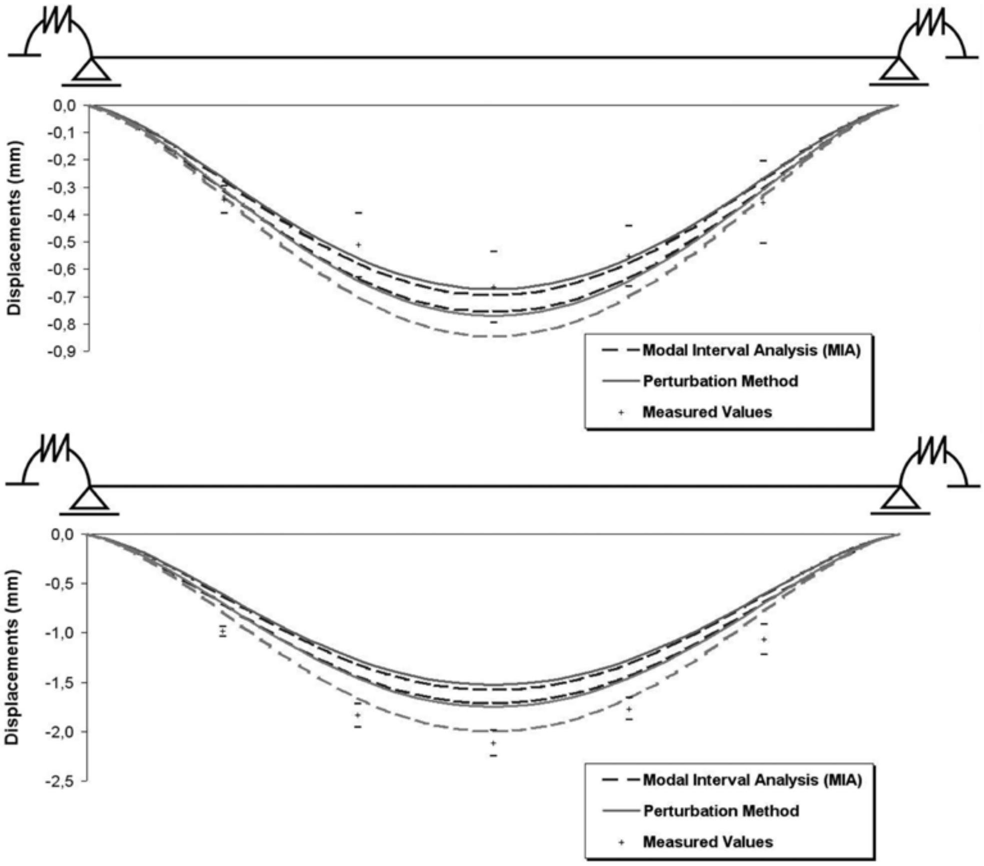

Fig. 7 Deformed shape (vertical displacements in mm) determined with different methods and measured data (Top – F = 8.4390 kN; Bottom – F = 19.1510 kN)

4. Assessment of real structures

The final objective of developing the structural assessment technique, dealing with uncertainties, is their application to real civil engineering structures. To this end, a first interesting application of previously presented algorithms is intended to be carried out, as presented below.

4.1. SMARTE research project

SMARTE Research Project, entitled as "Infrastructure remote management system based on electric and fibre optic sensors" was executed by a consortium of three companies, namely, BRISA, Portuguese Highways, FEUP, Faculty of Engineering of Porto (Civil Engineer Department), and INESC, National Institute of Computers and Systems (Optoelectronic Department). The project was financed by Adi – Portuguese Innovation Agency through the POSI program.

The aim of this project is to design, install and characterize a Health Monitoring System (HMS) able to perform an on-line and remote control of the performance of the structure during its construction and service phases (Perdigão, *et al*. 2004, 2006). The identified prototype structure, where the developed

Fig. 8 Sorraia river bridge

system was applied for the first time, is the Sorraia River Bridge located in Salvaterra de Magos (Portugal).

4.1.1. Sorraia river bridge

This structure is a pre-stressed concrete bridge, with a total length of 270 m, constructed by the balanced cantilever method (Fig. 8). The structure is divided into three spans, being the side spans 75 m and the central span 120 m long. The bridge section is of box girder type. The section height varies from 2.55 m at mid span to 6.00 m in support region. The reinforced concrete piers, based on a cap with five piles, are 7.5 m high and exhibit a hollow type rectangular section. The piles are cast “in situ” with 2.00 m diameter and 30 m long (GRID 2003).

4.2. Health Monitoring System (HMS)

The Health Monitoring System (HMS) implemented in Sorraia River Bridge is divided on the sensory, data acquisition, communication, data processing and archiving and damage detection and modeling systems (Fig. 9).

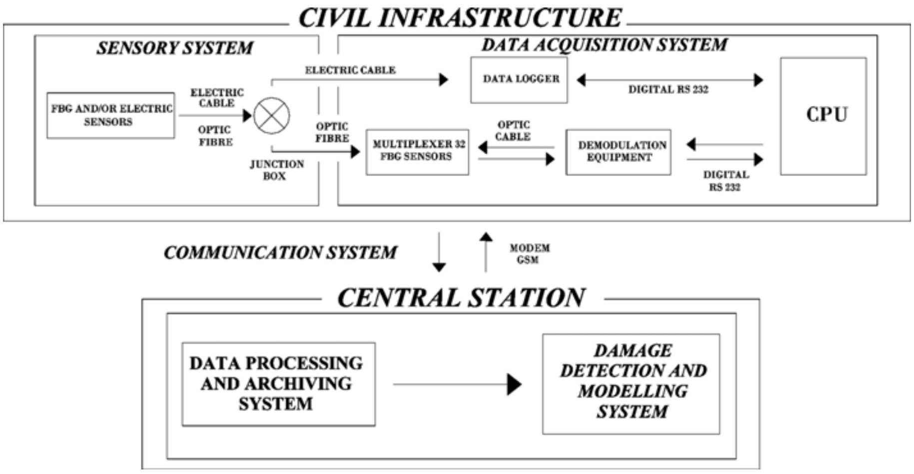

Fig. 9 Long term Health Monitoring System (HMS) (Figueiras, *et al*. 2004, Matos, *et al*. 2005a)

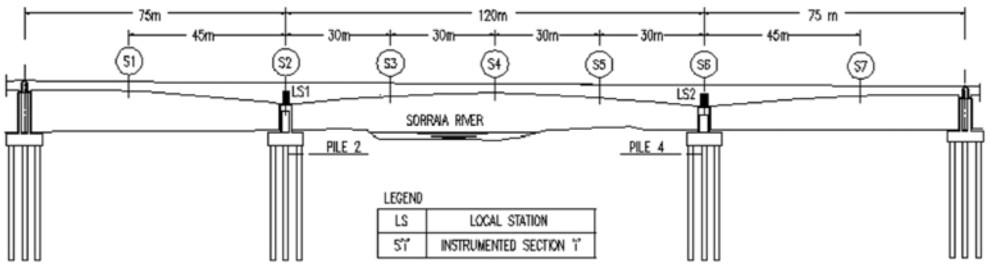

Fig. 10 Instrumentation plan

4.2.1. Sensory system

In Fig. 10 is presented the instrumentation plan of Sorraia River Bridge. In order to ease the implementation of automatic and remote monitoring, the whole sensor network will be measured from two locations (Local Station - LS). There are 42 fibre Bragg grating sensors (FBG) (temperature and strain sensors) and 42 electric strain gages placed in bridge deck (sections S1 to S7), 8 FBG and 8 electric strain gages in bridge piles (piles 2 and 4), and 2 FBG and 2 electric strain gages in concrete shrinkage proves inside and outside the deck. Additionally, there are 2 humidity and 2 external temperature electric sensors, inside and outside bridge deck (Matos, *et al*. 2005a, 2005b).

4.2.2. Data acquisition system

A Data Logger is placed in local station 1 and 2 (Fig. 10), being the demodulation equipment and multiplexer located in local station 1. In local station 1 it is also placed the CPU unity that congregates all data. The collection of data from various sensors installed in the structure should not be so scanty as to jeopardize its usefulness, nor should it be so voluminous as to overwhelm interpretation. Selection of the most appropriate data acquisition algorithm is so an important component of this system (Mufti 2001).

4.2.3. Communication system

Within the structure the communication between equipments is made by RS232 electric cables. From the bridge site, local station 1, to the central station, placed in FEUP (more than 300 km of distance), the communication is realized via modem GSM (Global System for Mobile Communication Service). A remote communication, with a wireless network hierarchy, is used (Tennyson 2001).

4.2.4. Data processing and archiving system

Raw data received by the CPU unity, placed in central station, is continuously processed and archived. After filtered, the measured strains are used to determine the curvature of each instrumented section. Having these and other restrictions into account it is possible to obtain the structure deformed shape (vertical displacements) (Matos, *et al*. 2005b).

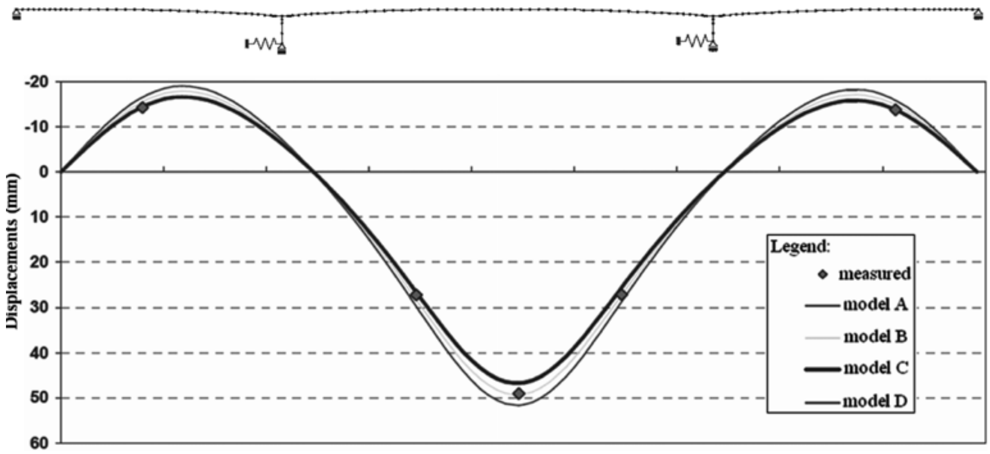

Fig. 11 Numerical and measured displacements (mm), obtained during load test (Matos, *et al*. 2005b)

4.2.5. Damage detection and modeling system

The numerical model of Sorraia River Bridge was calibrated with the results obtained during load test. Within such test an external temporary monitoring system was implemented in the bridge. Such system was constituted by displacement transducers, placed in the same monitored deck sections (S1 to S7) (Fig. 10) (Sousa, *et al*. 2005). In Fig. 11 are presented the measured vertical displacements and the numerical results with different boundary conditions at the piers for the case of vehicles placed at mid span. During model updating after load test, it was verified that the boundary conditions at supports were very important for an accurate structural modeling. As the information about their structural behavior is not complete, this uncertainty must be considered in the model. This will be carried out by means of the two methods presented in this paper. Once the calibration of the theoretical models has been carried out with the results from the load test, in the future and during the service life of the bridge, the Health Monitoring System (HMS) implemented in the bridge will be a warning tool to predict any malfunction or damage of the structure.

5. Conclusions and future developments

In this paper, two methodologies that account the uncertainties in the decision making process of structural assessment are presented, namely, the Perturbation Method and the Modal Interval Analysis (MIA). Their application to a simple case (steel beam) derives in the following conclusions:

Perturbation Method presents the following advantages: it is a very efficient method in terms of time spent in computational calculation; to take into account the uncertainties in a finite element framework, it requires additionally the calculation of partial derivatives of stiffness matrix and load vector, evaluation of δU vector (according to Eq. (12)) by a new system of equations identical (in terms of dimensions) to the system used to evaluate the nodal displacements, and some product of matrices to calculate matrix of covariance; the dispersion of structural response is available with a single structural analysis; correlation between variables could be easily taken into account; information about variables uncertainty is defined only by two parameters (mean value and standard deviation). As disadvantages, this method requires that the uncertainties are defined by statistical parameters and not by intervals; also, the implementation in a Finite Element framework requires addition of new modulus in the computer code to evaluate the partial derivatives of stiffness matrix and load vector, and the covariance matrix of structural response.

From Interval Analysis it is possible to conclude that is a methodology which considers the whole uncertainty, warranting that the real answer is within the range of output values. It was also verified that the computational cost is directly influenced by the dimension of the stiffness matrix. Higher dimension corresponds to a higher cost maintaining the output accuracy. As a conclusion the computational effort is relatively low and the calculus velocity adequate for the simple examples presented here. However, this can be a serious problem in the application to large structures with an important number of degrees of freedom.

Comparing results obtained from the different techniques, it can be observed that they give similar values, however, Modal Interval Analysis (MIA) method gives narrower intervals and Perturbation Method gives intermediate values. A sharper bound is obtained with Modal Interval Analysis (MIA) which allows to concluding about its accuracy.

Such methodologies were also applied with a structural assessment algorithm, based in a direct and consistent comparison between obtained and numerical data. A simple laboratory example, a concrete beam tested till rupture, describes the application of it presenting the respective potentialities.

In this paper, the works developed for structural assessment of a real structure, Sorraia River Bridge, are presented. Based on advantages and disadvantages of each method and having into account the results obtained in both examples, it was decided to use the Perturbation Method, together with the developed structural assessment algorithm, in the Health Monitoring System (HMS) of Sorraia River Bridge, as this method can be used with systems with all kind of complexity, presenting similar computational costs.

Acknowledgements

The authors are indebted to LABEST-FEUP, and MICE-LAB team for their help and useful discussion. They would like to thank AdI – Innovation Agency for the financial support, through the POSI program, of the SMARTE research project, to the partners INESC and BRISA for their special contribution, and Tâmega Constructors for their useful support during sensor application. The authors want also to thank the Spanish Ministry of Education through Research Projects HP-2005-0101 and BIA2005-24136-E. Thanks to FCT – Portuguese Foundation for Science and Technology for the financial support to PhD Research of Jose Campos e Matos. Oscar Garcia would like to thank the Agència de Gestió d'Ajuts Universitaris i de Recerca (AGAUR) of the Generalitat de Catalunya for the financial support (FI Grant) awarded to assist in the completion of his PhD dissertation.

References

Altus, E., Totry, E. and Givli, S. (2005), "Optimized functional perturbation method and morphology based effective properties of randomly heterogeneous beams", *Int. J. Solids Struct.*, **42**(8), 2345-59.

Ayyub, B.M. and Gupta, M.M. (1997), *Uncertainty Analysis in Engineering and Sciences: Fuzzy Logic,*

Statistics, and Neural Network approach, Kluwer Academic Publishers.
Biondini, F., Bontempi, F. and Malerba, P.G. (2004), "Fuzzy reliability analysis of concrete structures", *Comput. Struct.,* **82**(13-14), 1033-52.
Contreras, H. (1980), "The stochastic finite element", *Comput. Struct.*, **12**, 341-8.
Fetz, T., Oberguggenberger, M. and Pittschmann, S. (2000), "Applications of possibility and evidence theory in civil engineering", *Int. J. Uncertain*, *Fuzz.*, **8**(3), 295-309.
Figueiras, J., Felix, C., Matos, J.C. and Sousa, H. (2004), "An integrated system for structural health monitoring – application to the sorraia river bridge", *IABMAS*, October 18-22, Kyoto, Japan.
Frangopol, D.M., Lee, Y.H. and William, K.J. (1996), "Nonlinear finite element reliability analysis of concrete", *J. Eng. Mech.* ASCE, **122**(12), 1174-82.
Garcia, O., Matos, J.C., Vehi, J., Sainz, M. and Casas, J.R. (2004), "Application of interval analysis to solve civil engineering problems - some examples", *Structural Health Monitoring*, July 7-9, Munich. Germany.
Gardenyes, E., Sainz, M.A., Jorba, L., Calm, R., Estela, R., Mielgo, H. and Trepat, A. (2001), "Modal Intervals", *Reliable Computing*, **7**(2), 77-111.
GRID (2003), A13 – Highway Almeirim-Marateca, Track Salvaterra de Magos – A10 – Santo Estêvão, Sorraia River Bridge, Design Project, Descriptive and Justification Memory, Lisbon.
Hansen, E. (1992), *Global Optimization Using Interval Analysis*, Marcel Dekker, New York.
Herrero, P., Sainz, M.A., Vehi, J. and Jaulin, L. (2005), "Quantified set inversion algorithm with applications to control", *Reliable Computing*, **11**, 369-382.
Hurtado, J.E. (2002), "Analysis of one-dimensional stochastic finite elements using neural networks", *Probabilist. Eng. Mech.*, **17**(1), 35-44.
Jaulin, L., Kieffer, M., Didrit, O. and Walter, E. (2001), *Applied Interval Analysis*, Springer, London.
Liu, P. and Der Kiureghian, A. (1991), "Finite element reliability of geometrically nonlinear uncertain structures", *J. Eng. Mech.* ASCE, **117**(8), 1806-25.
Mahadevan, S., Raghothamachar, P. (2000), "Adaptive simulation for system reliability analysis of large structures", *Comput. Struct.*, **77**(6), 725-34.
Matos, J.C., Casas, J.R. and Figueiras, J.A. (2005a), "A new methodology for damage assessment of bridges through instrumentation: application to the sorraia river bridge", *Struct. Infrastruct. E.*, **1**(4), 239-252.
Matos, J.C., Sousa, H., Assis, W. and Figueiras, J.A. (2005b), "Structure assessment by continuous monitoring - application to sorraia river bridge", *Fib Symposium Keep Concrete Attractive*, May 23-25, Budapest. Hungary.
Moore, R.E. (1966), *Interval Analysis*, Prentice Hall, Englewood Cliffs.
Mufti, A. (2001), *Guidelines for structural health monitoring*, Design Manual No. 2. ISIS Canada Corporation. Manitoba, Canada.
Mullen, R.L. and Muhanna, R.L. (1999), "Bounds of structural response for all possible loading combinations", *J. Struct. Eng.* ASCE, **125**(1), 98-106.
Neumaier, A. (1990), *Interval methods for systems of equations*, Cambridge University Press.
Oberkampf, W.L., DeLand, S., Rutherford, B.M., Diegert, K.V. and Alvin, K.F. (2002), "Error and uncertainty in modelling and simulation", *Reliab. Eng. System Safe.*, **75**, 333-357.
Olsson, A., Sandberg, G. and Dahlblom, O. (2003), "On Latin hypercube sampling for structural reliability analysis", *Struct. Saf.*, **25**(1), 47-68.
Perdigão, V., Barros, P., Dias, I., Pereira, D., Alves, J., Sousa, H., Matos, J.C. and Figueiras, J.A. (2004), "Structural health monitoring system applied to sorraia river bridge", *Portuguese Structural Concrete Meeting,* November 17-19, Porto, Portugal.
Perdigão, V., Barros, P., Matos, J.C., Henriques, A.A., Figueiras, J.A., Dias, I. and Pereira, D. (2006), "SMARTE - development and implementation of a long-term structural health monitoring", *Third Int. Conf. Bridge Maintenance*, Safety and Management, July 16-19, Porto, Portugal.
Reh, S., Beley, J.D., Mukherjee, S. and Khor, E.H. (2006), "Probabilistic finite element analysis using ANSYS", *Struct. Saf.*, **28**(1-2), 17-43.
Schueller, G.I. (2001), "Computational stochastic mechanics - recent advances", *Comput. Struct.*, **79**(22-25), 2225-2234.
SIGLA/X group (1999), "Modal intervals. basic tutorial. applications of interval analysis to systems and control", *Proc. of the Workshop MISC'99*, University of Girona, Spain, 157-227.

Sousa, H., Matos, J.C., Assis, W., Felix, C. and Figueiras, J.A. (2005), "Load test of sorraia river bridge in highway A13 (Almeirim-Marateca)", Technical Report, June.

Spanos, P.D. and Ghanem, R. (1989), "Stochastic finite element expansion for random media", *J. Eng. Mech.* ASCE, **115**(5), 1035-53.

Tennyson, R. (2001), *Installation, use and repair of fiber optic sensors*, Design Manual No. 1. ISIS Canada Corporation, Manitoba, Canada.

Thomos, G.C. and Trezos, C.G. (2006), "Examination of the probabilistic response of reinforced concrete structures under static non-linear analysis", *Eng. Struct.*, **28**(1), 120-33.

Zhang, J., Ellingwood, B. (1996), "SFEM for reliability with material nonlinearities", *J. Struct. Eng.* ASCE, **122**(6), 701-4.

Zhang, Y., Chen, S., Liu, Q. and Liu, T. (1996), "Stochastic perturbation finite elements", *Comput. Struct.,* **59**(13), 425-29.

FC