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Sliding mode control for structures based on the frequency content of the earthquake loading

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Abstract. A control algorithm for seismic protection of building structures based on the theory of variable structural control or sliding mode control is presented. The paper focus in the design of sliding surface. A method for determining the sliding surface by pole assignment algorithm where the poles of the system in the sliding surface are obtained on-line, based on the frequency content of the incoming earthquake signal applied to the structure, is proposed. The proposed algorithm consists of the following steps: (i) On-line FFT process is applied to the incoming part of the signal and its frequency content is recognized. (ii) A transformation of the frequency content to the complex plane is performed and the desired location of poles of the controlled structure on the sliding surface is estimated. (iii) Based on the estimated poles the sliding surface is obtained. (iv) Then, the control force which will drive the response trajectory into the estimated sliding surface and force it to stay there all the subsequent time is obtained using Lyapunov stability theory. The above steps are repeated continuously for the entire duration of the incoming earthquake. The potential applications and the effectiveness of the improved control algorithm are demonstrated by numerical examples. The simulation results indicate that the response of a structure is reduced significantly compared to the response of the uncontrolled structure, while the required control demand is achievable.

Keywords: sliding mode control; FFT; pole assignment; earthquake engineering; structural dynamics.

1. Introduction

The main purpose in structural control theory is to determine a control strategy that uses the measured structural response and the excitation signal to calculate appropriate control forces that will enhance the structural safety and serviceability against dynamic excitation like wind or earthquake. Over the past few decades various control algorithms and control devices have been developed, modified and investigated by various groups of researchers. The work of Yao (1972), Housner, *et al*. (1994), Kobori, *et al*. (1998), Spenser, *et al*. (1997), Yang (1975), Soong (1990), Casciati and Yao (1994) and Connor and Klink 1996 is the most representative. Control is applicable not only to new or existing buildings but also to monuments (Casciati and Osman 2005, Syrmakesis 2006). While many of these structural control strategies have been successfully applied, challenges as cost, reliance on external power and mechanical intricacy have delayed their widespread use.

Among various control algorithms sliding mode control (SMC) has demonstrated its potential of

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becoming a dominant control algorithm for civil engineering structures. In the work of Utkin (1992), SMC was developed for robust control of uncertain nonlinear systems. Continuous SMC without chattering effect for linear and non-linear structures, combination of SMC and base isolated building, applications of SMC to active variable damper and stiffness systems, SMC with full state or static output feedback with observer, are a variety of subjects for SMC that are investigated by Yang, *et al.* (1995). For SMC and actuator saturation Yang, *et al*. (1995) suggest the maximum control force to be a percentage of the building weight, while Cai, *et al*. (1997) give this force as a portion of the seismic force. In the work of Matheu, *et al*. (1998), the sliding-mode-control approach focuses (i) on a general scheme to achieve the regular form of the equations of motion required to uncouple the control actions from the sliding motion description, (ii) on a treatment of control redundancy, and (iii) on a method to improve sliding surface design by incorporating auxiliary dynamic systems. In this work both full-state-feedback and output-feedback cases were considered. This work provides very useful insight into sliding mode control and especially in the design of the sliding surface, which is the area of focus for the present paper. Lee, *et al*. (2004) improve the design of SMC determining the upper limit of control force based on the design spectrum. Experiments on a full-scale building using modified SMC are presented by Wu (1997). Wang and Lee (2002) proposes fuzzy sliding mode control for buildings based on genetic algorithms, while Cai, *et al.* (1997) investigates the combination of SMC and bang-bang control. Moon, *et al*. (2002) apply SMC algorithm to control a cable-stayed bridge using magnetoreological dampers. Non-linear SMC for response reduction is investigated by Sarbjeet and Datta (2000). All above studies show the wide range of interest for SMC.

In SMC the critical issues are, first the determination of the sliding surface where the system trajectory remains stable, and secondly the determination of the control force, usually applying Lyapunov stability theory, which drives the trajectory to the sliding surface and forces it to stay there. To find the sliding surface, the pole assignment or the linear quadratic regulator (LQR) methods can be used. One drawback of the pole assignment method is that it requires pre-specified values of the poles of the system on the sliding surface. To deal with this limitation, in this paper the design of the sliding surface is performed by pole assignment method, where the poles of the system on the sliding surface are calculated based on the frequency content of the incoming earthquake signal and should not be predefined by the designer.

2. Improved sliding mode control algorithm

Sliding mode control or variable structure strategies were developed specifically for robust control of uncertain nonlinear systems (Utkin 1992). The equation of motion of a structural system with n degrees of freedom controlled by m forces and subjected to an earthquake excitation a_g is:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = -\mathbf{M}\mathbf{E}a_g + \mathbf{E}_f\,\mathbf{F} \tag{1}$$

where **M**, **C**, **K** denote the mass, damping and stiffness matrices of the structure, respectively, **F** is the control force matrix and **E**, $\mathbf{E}_f$ are the location matrices for the earthquake and the control forces on the structure, respectively. In the state space approach the above Eq. (1) can be written as follows.

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}_g a_g + \mathbf{B}_f\mathbf{F} \tag{2}$$

The matrixes **X**, **A**, $\mathbf{B}_g$, $\mathbf{B}_f$ are given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix}_{2nx1}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2nx2n}, \quad \mathbf{B}_{\mathrm{g}} = \begin{bmatrix} \mathbf{O} \\ -\mathbf{E} \end{bmatrix}_{2nx1}, \quad \mathbf{B}_{\mathrm{f}} = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{E}_{\mathrm{f}} \end{bmatrix}_{2nxm} \tag{3}$$

The fundamental idea of SMC is to design a controller to drive the state trajectory on the sliding surface (or switching surface), whereas the motion on the sliding surface is stable, remains there all the subsequent time and moves toward the equilibrium position.

The first step in SMC is to design the sliding surface on which the response is stable and the second step is the determination of the control demand which will drive the response trajectory into the sliding surface and force it to stay there all the subsequent time.

The general control strategy consists of the following stages: (i) the monitoring of the incoming signal, (ii) its FFT or wavelet analysis for recognition of its dynamic characteristics, (iii) the calculation of the desired poles of the controlled system and the estimation of the sliding surface, (iv) the application of sliding mode control algorithm for the calculation of the required actions, and finally, (v) accounting for the limitations of the devices that are used, the application of these actions, considering saturation effects and time delay. A flow chart of this integrated control strategy is shown in Fig. 1. The above steps are explained in detail next.

2.1. Design of sliding surface

In most studies the sliding surface is defined as a linear combination of the state vector. In the work of Slotin (1991) a more general approach is proposed. Let $\mathbf{U}_{\mathrm{d}}$ be the desired response of the system. Usually, in the control of buildings this quantity is zero or, if more relaxed criteria are preferred, $\mathbf{U}_{\mathrm{d}}$

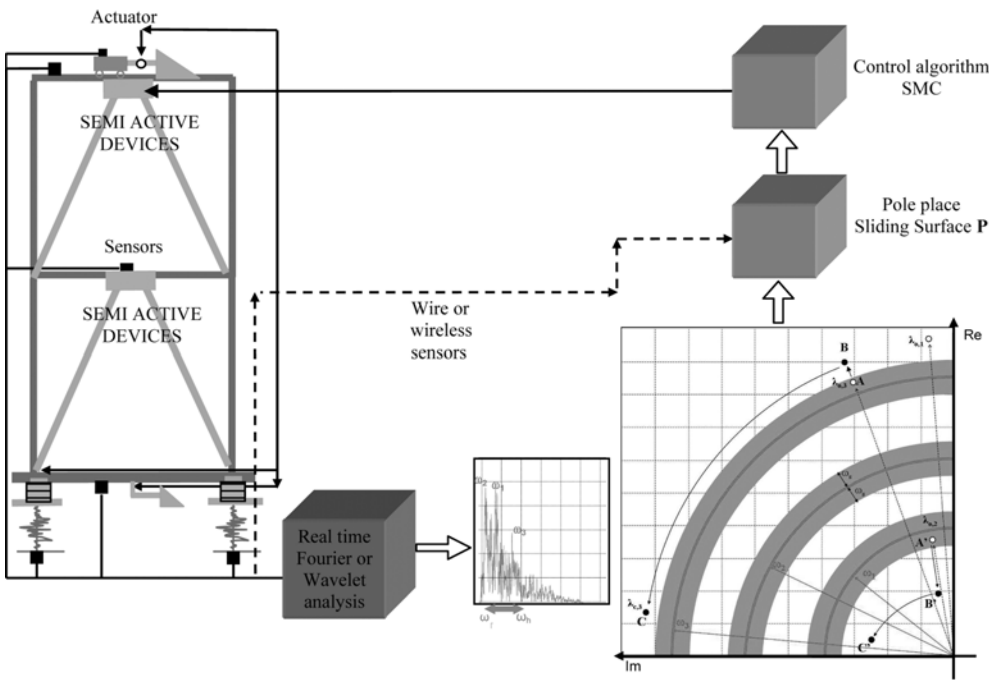

Fig. 1 The general flow chart of the proposed control strategy

could be the response of the system just before yielding of some structural members. $\mathbf{U}_d$ could also be the response of a reference model, the behavior of which we would like our systems to follow. Let $\bar{\mathbf{U}}$ be the error defined as:

$$\bar{\mathbf{U}} = \mathbf{U} - \mathbf{U}_d \tag{4}$$

The sliding, time varying, surface is defined as:

$$\mathbf{s}(\bar{\mathbf{U}}, t) = 0, \quad \text{where } \mathbf{s}(\bar{\mathbf{U}}, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \bar{\mathbf{U}} \tag{5}$$

where λ is a symmetric positive defined matrix or, more generally λ is a Hurwitz matrix. If n=2 then **s** is the weighted sum of the position and velocity error:

$$\mathbf{s} = \dot{\bar{\mathbf{U}}} + \lambda\bar{\mathbf{U}} = [\lambda\ \mathbf{I}] \begin{bmatrix} \bar{\mathbf{U}} \\ \dot{\bar{\mathbf{U}}} \end{bmatrix} = \mathbf{PX} \tag{6}$$

If n=3 the surface becomes as follows:

$$\mathbf{s} = \ddot{\bar{\mathbf{U}}} + 2\lambda\dot{\bar{\mathbf{U}}} + \lambda^2\bar{\mathbf{U}} \tag{7}$$

Thus, the problem of having for our system the same response as the response of the ideal reference model or zero response is equivalent to that of the system remaining on the surface **s**.

In the case where the surface **s** is a linear combination of the states the matrix **P** is defined by pole assignment method. Successful application of the method requires judicious placement of the poles of the system on the sliding surface. In the work of Pnevmatikos and Gantes (2007a) the feedback matrix is estimated by pole placement algorithm and a selection of the poles of the controlled system is based on the frequency content of the incoming earthquake. In this paper the sliding surface, or in other words the matrix **P**, is estimated by pole placement algorithm and the methodology is described briefly next.

The estimation of poles of the controlled system in the sliding surface is based on an on-line process, where the signal is measured, its dynamic characteristics are recognized, and the decision is taken. The strategy for the selection of the poles of the controlled system is to transform both the structure and the

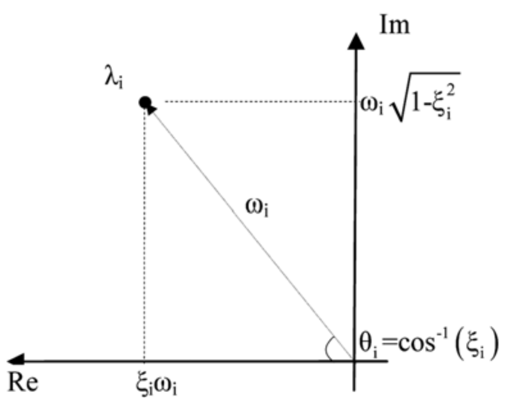

Fig. 2 Representation of the poles of the structure in the complex plane

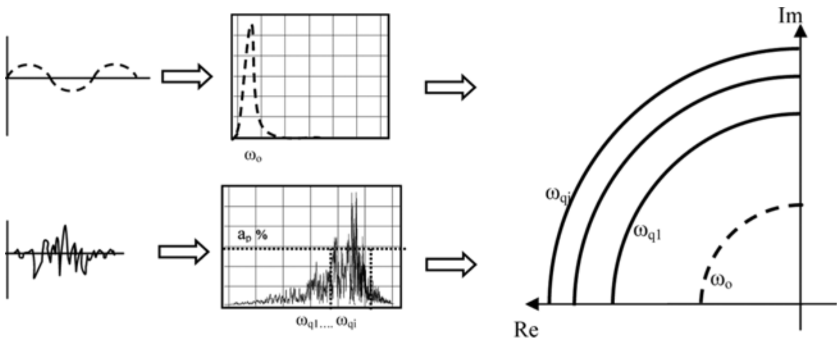

Fig. 3 Representation of frequencies of two types of loading, sinusoidal and earthquake, by cycles in the complex plane

loading into the complex plane and there, depending on their relationship, take appropriate decisions. The graphical representation of structural eigenvalues (poles) into the complex plane is shown in Fig. 2.

Then, the initial part of the incoming signal is analyzed and its frequency characteristics are recognized by means of FFT or Wavelet analysis. The frequencies that should be avoided are chosen based on their percentage of participation to the maximum value of spectrum a_p, and to their percentage of the power to the total power of the corresponding part of the incoming signal I_p. Then, cycles with radii equal to those frequencies are drawn in the complex plane. The graphical representation of the frequencies of two types of loading, sinusoidal and seismic, by cycles in the complex plane is shown in Fig. 3. All points on or near those cycles should be avoided as possible pole locations, in order to avoid resonance.

Next, zones with semi-bandwidth ω_{si} are defined around each of those cycles, where near-resonance conditions are expected, therefore poles should preferably not be located there. Then, the position of poles of the uncontrolled structure is compared to the unsafe zones, and a decision whether to move the poles of the controlled system and where to put them is taken.

The graphical representation of selection of poles with the above procedure is illustrated in Fig. 4. If the poles are outside of the unsafe zones, then they are left provisionally at the same position, as is the case with pole $\lambda_{o,1}$ in Fig. 4. Otherwise, they are moved outside of the zone, along the radius between the initial pole and the origin of axes, (movement AB, A′B′ or A″B″ in Fig. 4). The direction of movement will be outwards if the pole is outside of the cycle zone (see pole $\lambda_{o,2}$ in Fig. 4) or inwards if the pole is inside the cycle zone (see pole $\lambda_{o,3}$ in Fig. 4). After that, based on the desired equivalent percentage of damping ζ_c, the poles can be moved along a cycle, with centre the axes origin, and radius defined by the new position, with direction towards the real axis (movement BC or B′C′ in Fig. 4). Additional questions which arise with regard to the above process and concerning the number of frequencies that must be accounted for and represented in the complex plane, the bandwidth of the unsafe zone ω_{si}, the amount of equivalent percentage of damping ζ_c, that should be added to determine the final location of the pole of the controlled structure, is answered in detail in Pnevmatikos (2007).

Based on the new position of the poles the sliding matrix **P** is obtained by pole placement algorithm, and consequently the sliding surface, **s**, is obtained from Eq. (6).

The next step is to design the controller (determine the control law of the applied control force), and check that the poles of the controlled system are not near the poles of the earthquake, or alternatively that its eigenfrequencies are outside of the main frequency window of the incoming earthquake.

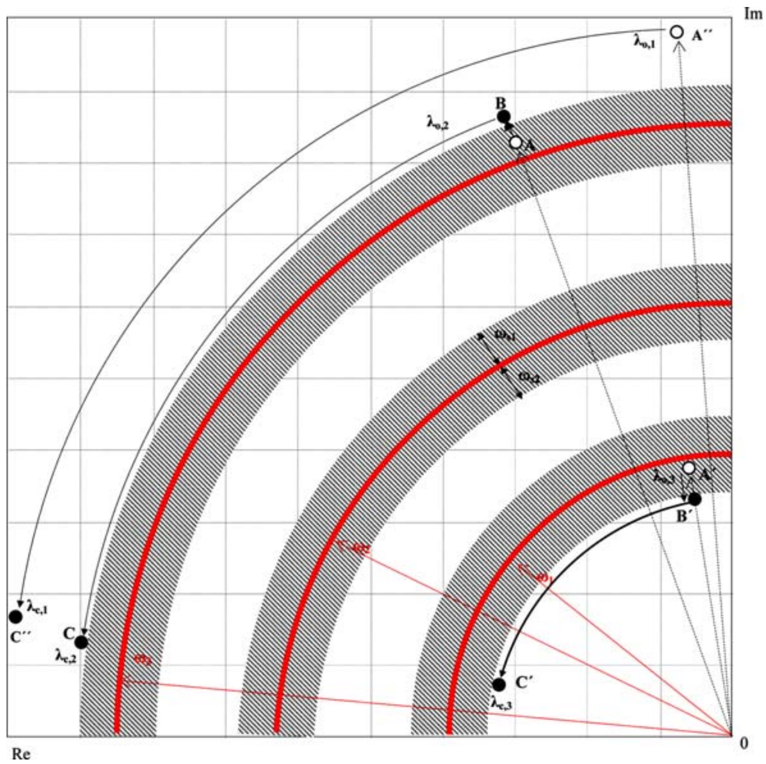

Fig. 4 The graphical representation of selection of poles of the controlled structure (●) from poles of the uncontrolled structure (○), based on the cycles of frequency of the incoming earthquake

2.2. Design of the controller

The controller is designed in order to drive the response trajectory into the sliding surface s = 0 and force it to stay there all the subsequence time, as shown in Fig. 5. To achieve this goal a Lyapunov function V is chosen and the control forces are obtained for the condition that the derivative of the function V should be negative. According to Yang, *et al*. (1995), the control forces F are given by:

$$\mathbf{F} = \mathbf{G} - \delta \cdot \lambda^{\mathrm{T}}, \quad \mathbf{G} = -(\mathbf{P}\mathbf{B}_{\mathrm{f}})^{-1}\mathbf{P}(\mathbf{A}\mathbf{X} + \mathbf{B}_{\mathrm{g}}a_{\mathrm{g}}), \quad \lambda = \mathbf{s}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{f}} \tag{8}$$

Where δ is the sliding margin matrix, a diagonal, rxr matrix, with elements $\delta_i \geq 0$.
If the control force of Eq. (8) is replaced into Eq. (2) we obtain:

$$\dot{\mathbf{X}} = -\mathbf{B}\delta\mathbf{B}^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}\mathbf{P}\mathbf{X} = -\mathbf{A}_{\mathrm{c}}\mathbf{X} \tag{9}$$

The poles of the controlled system are the eigenvalues of matrix $\mathbf{A}_c$. The last step is to check that the above eigenfrequencies of $\mathbf{A}_c$ are not located within the main frequency window of the incoming earthquake signal, or otherwise, that the poles of $\mathbf{A}_c$ are not inside the unsafe zones.

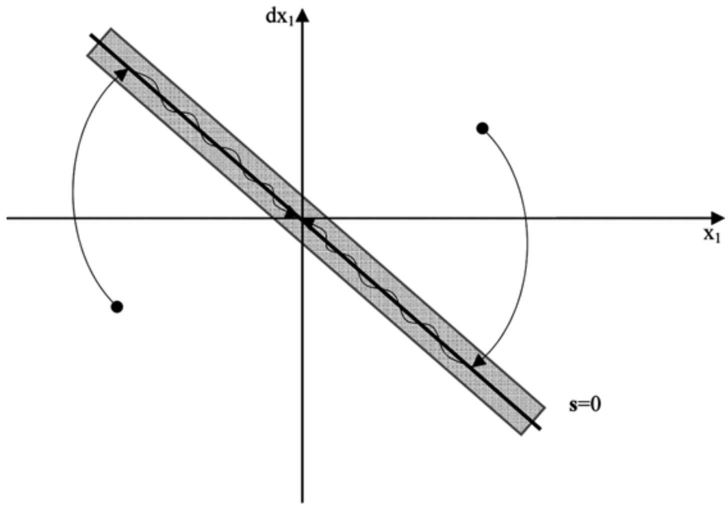

Fig. 5 The movement of the system's trajectory towards to sliding surface and next to the equilibrium point

Yang, *et al*. (2000) proved that when each degree of freedom is implemented by a controller (n = m) the external earthquake excitation can be completely compensated. However, because **G** includes the restoring, damping, inertial and seismic forces, the magnitude of **G** is very large for controlling conventional civil engineering structures. Thus, the control force should be restricted to a certain level and a saturated controller should be considered in the design of SMC. In this case, full compensation of the response cannot be achieved. If the maximum control force is bounded by $\pm\mathbf{f}_{max}$ a control force is estimated as follows:

$$\mathbf{F} = \begin{cases} \mathbf{G} - \delta \cdot \lambda^T, & \text{if } |\mathbf{G} - \delta \cdot \lambda^T| \le \mathbf{f}_{max} \\ \mathbf{f}_{max} sign(\mathbf{G} - \delta \cdot \lambda^T), & \text{otherwise} \end{cases} \tag{10}$$

The force $\mathbf{f}_{max}$ is specified by the device capacity. Usually the devices' capacity is large enough (30 tones capacity for MR dampers) and increases with time.

Another approach to obtain the control force is described by Wang and Lee (2002), where the very large seismic forces are avoided by considering an upper bound δ of disturbance a_g. Qing, *et al*. (2003) propose a control force law which is suitable for semi active control devices, such as MR dampers. Using their suggested control law the chattering problem of the sliding mode motion can be eliminated. The efficiency of the above control algorithms depends on the estimation of matrix **P.** In this paper the control law Eq. (8) suggested by Yang, *et al*. (1995) is followed and the estimation of matrix **P** is performed according to the proposed methodology described in section 2.1.

Using the above procedure for the calculation of the sliding surface by pole placement and the estimation of control forces, a program in MATLAB has been developed. The main files, their function and the simulink model are shown in Fig. 6. To examine the efficiency of the control strategy proposed above the procedure is applied to single and multi degree of freedom systems subjected to harmonic and earthquake excitations.

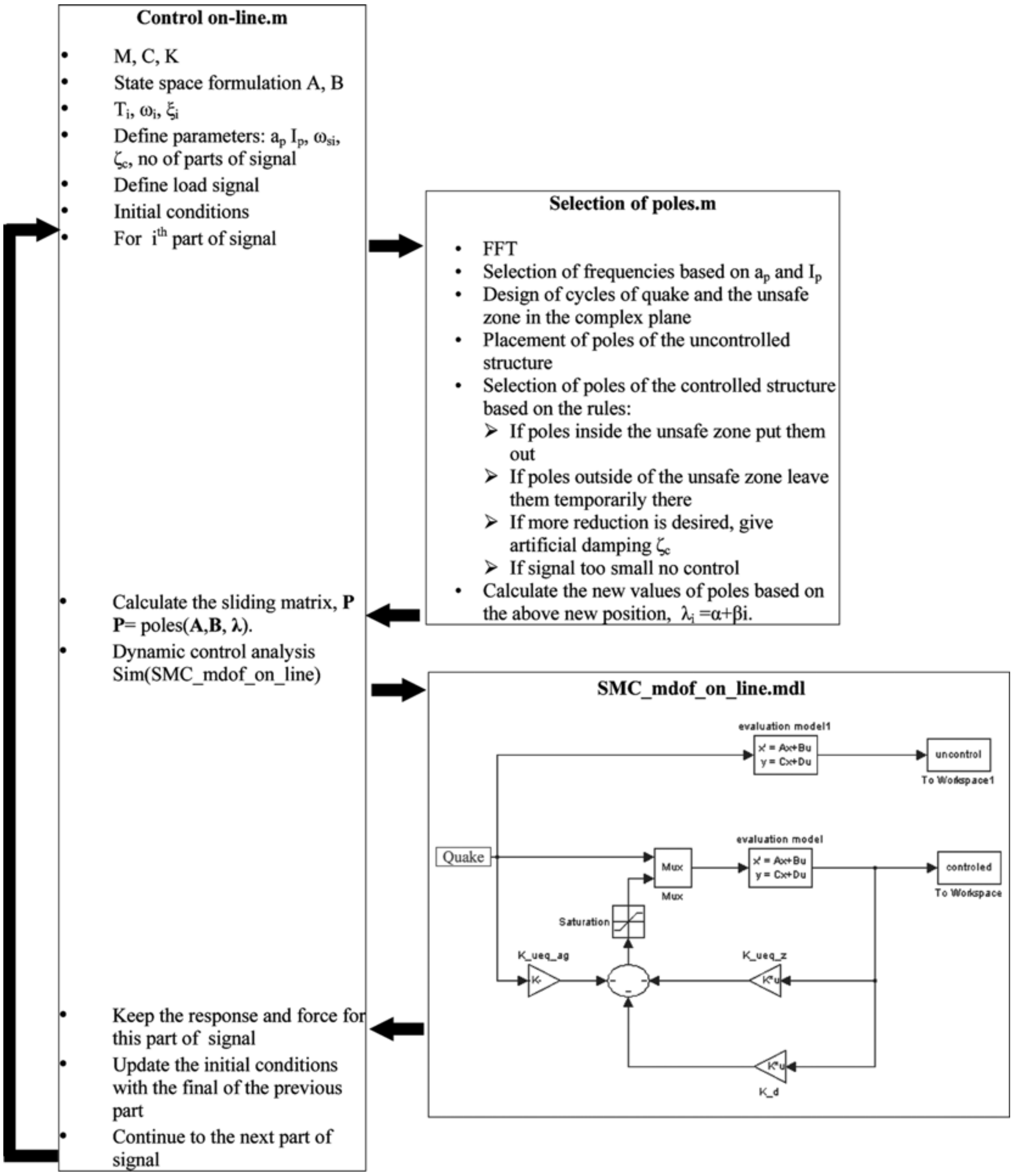

Fig. 6 The control strategy based on sliding mode control algorithm suitable for buildings subjected to earthquake actions

3. Examples and numerical experiments

The above dynamic control strategy has been applied to one single, one three and one eight degree-of-freedom system, modeling buildings with the properties shown in Fig. 7. The systems have been subjected to ten earthquake records as well as to sinusoidal and pulse dynamic excitations, all scaled at 0.3 g.

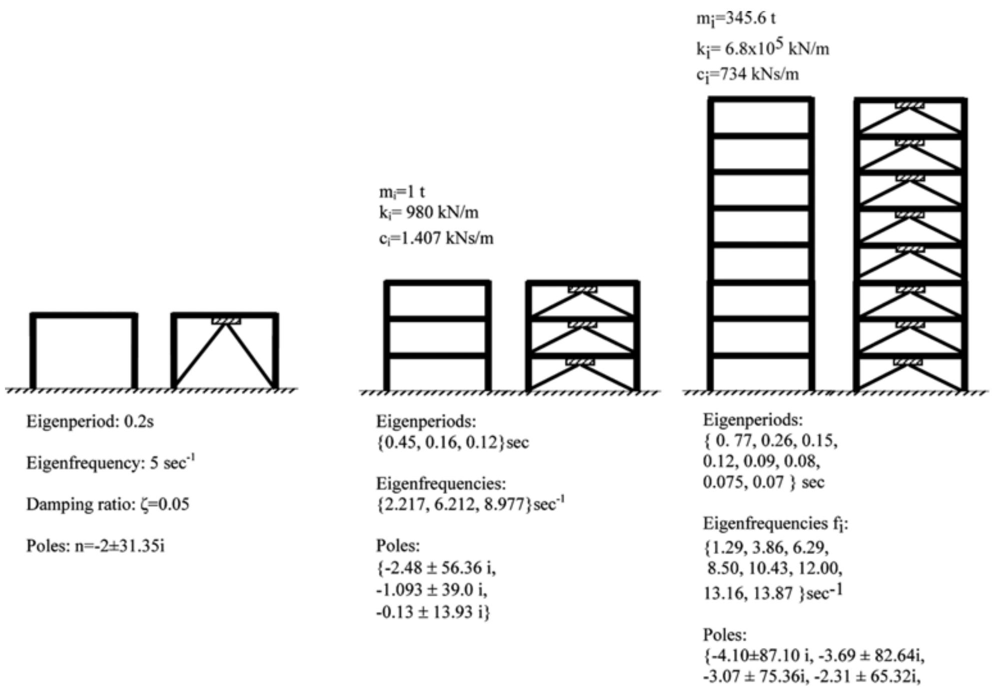

Fig. 7 The simulation models and their dynamic characteristics

Each signal is cut into 16 pieces. The significant frequencies are chosen using the values of a_p and I_p equal to 40% and 80%, respectively. The value of the bandwidth of the unsafe zone ω_s is calculated equal to 10 rad/s for the single degree-of-freedom, 50 rad/s for the three degree-of-freedom and 60 rad/s for the eight degree-of-freedom model. The value of the equivalent damping ratio, ξ_c, is calculated equal to 0.95. Finally, for the construction of δ matrix, the value of the parameter δ_i is taken equal to 5×10^{-6}. Based on the above values dynamic control simulations were performed and the results were obtained.

The representative results from the Kalamata (1986) earthquake are presented. In Figs. 8 and 9 the response of the controlled and the uncontrolled single degree-of-freedom structure, and the associated control demand are shown. The numerical results for all systems subjected to the Kalamata (1986) excitation are shown in Table 1. Similar results have been obtained for all three buildings subjected to all dynamic excitations, but they are not presented here due to limited space.

From the analysis results it is shown that the relative displacement is reduced, compared to the uncontrolled one, by 90% to 100% in the case where there are as many control devices as the number of the dynamic degrees of freedom. When the number of devices is reduced then the percentage of reduction of the displacement is decreased. Specifically, the reduction of displacement of the three story building with only one device located on the first or the third story is between 55% and 98%. For the eight story building with five control devices the reduction is 45% to 80% while for three devices the percentage of reduction of the displacements is 5% to 15%.

The total acceleration is reduced also from 30% to 95% in the case where the number of control devices is equal to the number of degrees of freedom. This percentage changes in the case of the three

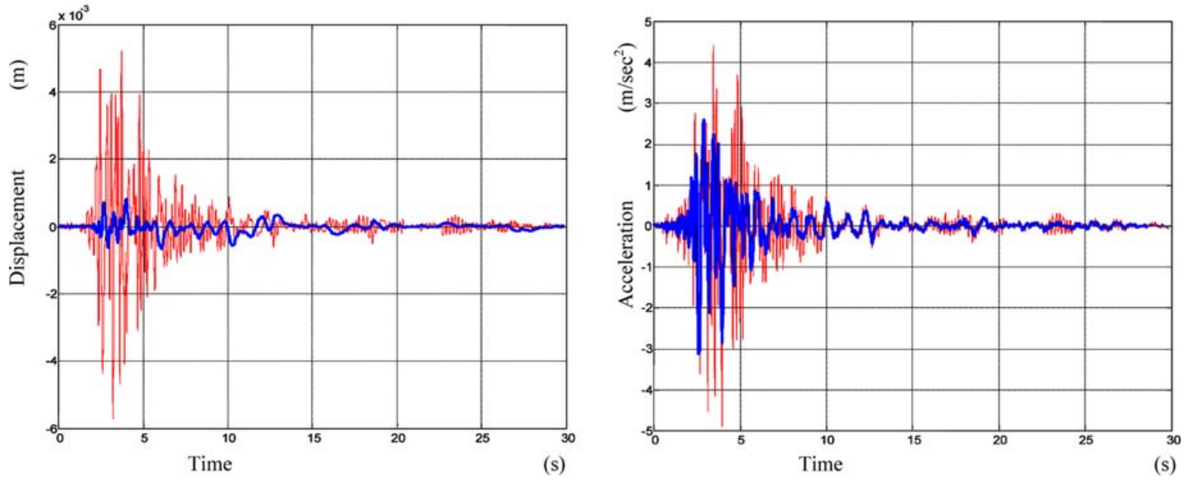

Fig. 8 Displacement and acceleration of controlled (bold line) and uncontrolled (thin line) system for single degree-of-freedom structure subjected to Kalamata (1986) earthquake

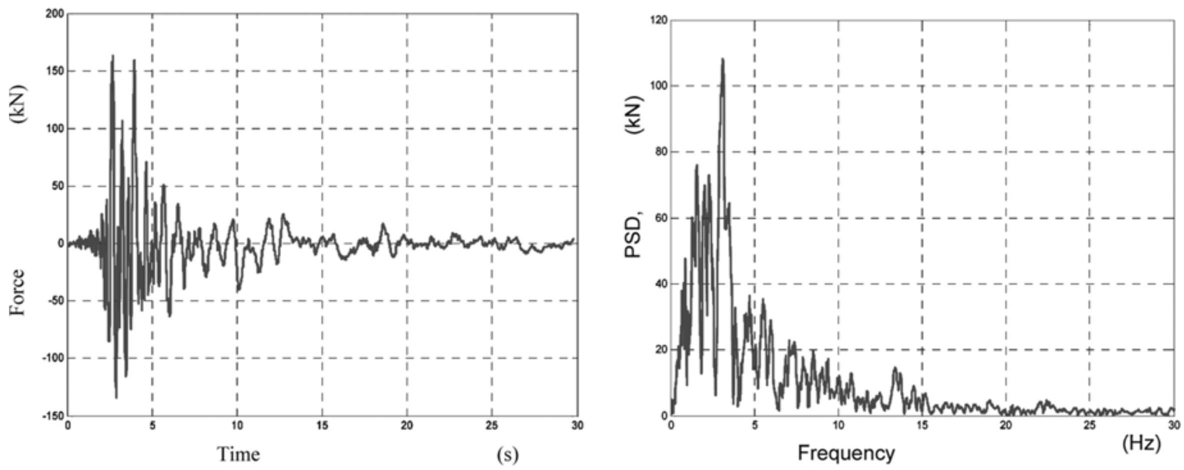

Fig. 9 The demanded force and the Power Spectral Density, PSD, of force for single degree-of-freedom structure subjected to Kalamata (1986) earthquake

story building with one control force and becomes 20% to 60%. For the eight story building with five control devices reduction between 5% and 30% is observed while for the three control devices the reduction is negligible.

In general, when the control devices are equal to the number of degrees of freedom, the structure tends to perform a rigid body motion. As the number of control devices is reduced this motion changes and relative displacements between the floors are observed.

The maximum control forces for the accomplishment of the above reduction in the response are in the order of 125 kN to 140 kN for the single story building, 3 kN to 17 kN for the three story building and reach the saturation value of 1000 kN for the eight story building. The different order of magnitude of the control force between the buildings is due to their different mass.

Table 1 Numerical results for the single, three and eight degree of freedom system subjected to Kalamata (1986) earthquake

		Uncontrolled system	Controlled system		Uncontrolled system	Controlled system		Uncontrolled system	Controlled system
u_1 (mm)	Single degree of freedom system	5.70	0.56	Three degree of freedom system	37.10	0.003	Eight degree of freedom system	29.80	0.03
$\ddot{u}_1$ (m/sec^2)		4.93	3.11		9.39	2.99		4.32	2.99
F_1 (kN)			169			3.00			1014
u_2 (mm)					66.80	0.003		58.40	0.03
$\ddot{u}_2$ (m/sec^2)					14.56	2.99		6.21	2.99
F_2 (kN)						3.00			1039
u_3 (mm)					83.40	0.003		84.70	0.03
$\ddot{u}_3$ (m/sec^2)					17.71	2.99		7.87	2.99
F_3 (kN)						3.00			1039
u_4 (mm)								108.00	0.03
$\ddot{u}_4$ (m/sec^2)								8.82	2.99
F_4 (kN)									1039
u_5 (mm)								127.40	0.03
$\ddot{u}_5$ (m/sec^2)								9.83	2.99
F_5 (kN)									1038
u_6 (mm)								142.60	0.03
$\ddot{u}_6$ (m/sec^2)								10.37	2.99
F_6 (kN)									1037
u_7 (mm)								153.00	0.03
$\ddot{u}_7$ (m/sec^2)								10.75	2.99
F_7 (kN)									1041
u_8 (mm)								158.20	0.03
$\ddot{u}_8$ (m/sec^2)								11.15	2.99
F_8(kN)									1037

4. Summary and conclusions

A modified sliding mode control algorithm where the sliding surface is calculated on-line based on the frequency content of the incoming earthquake action is proposed. The sliding surface is calculated by pole placement algorithm where the poles of the controlled structure, in the sliding surface, are obtained through the transformation of the frequency content of the incoming earthquake to the complex plane. The locations of poles on the sliding motion are not predetermined; instead they are estimated based on the dynamic characteristics of the incoming earthquake signal. Those locations are updated during the time of excitement of the structure by the earthquake, and are also different from one earthquake to another. The controlled force is designed based on classical Lyapunov stability theory.

Software in MATLAB was developed in order to simulate numerically the proposed control strategy. The success of the application of the algorithm depends on the well-chosen position of the poles of the system on the sliding surface and on checking that the eigenfrequencies of the controlled system are not located near the main frequency window of the incoming earthquake signal. The proposed control strategy is suitable for non stationary signals since the poles are calculated as the signal reaches the structures and the change in frequency content with time can be captured.

The numerical simulations verified that sufficient reduction of the response, in terms of both displacements and accelerations, can be achieved for all examined earthquakes with small amount of required equivalent control force.

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