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Real-time hybrid testing using model-based delay compensation

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Abstract. Real-time hybrid testing is an attractive method to evaluate the response of structures under earthquake loads. The method is a variation of the pseudodynamic testing technique in which the experiment is executed in real time, thus allowing investigation of structural systems with time-dependent components. Real-time hybrid testing is challenging because it requires performance of all calculations, application of displacements, and acquisition of measured forces, within a very small increment of time. Furthermore, unless appropriate compensation for time delays and actuator time lag is implemented, stability problems are likely to occur during the experiment. This paper presents an approach for real-time hybrid testing in which time delay/lag compensation is implemented using model-based response prediction. The efficacy of the proposed strategy is verified by conducting substructure real-time hybrid testing of a steel frame under earthquake loads. For the initial set of experiments, a specimen with linear-elastic behavior is used. Experimental results agree well with the analytical solution and show that the proposed approach and testing system are capable of achieving a time-scale expansion factor of one (i.e., real time). Additionally, the proposed method allows accurate testing of structures with larger frequencies than when using conventional time delay compensation methods, thus extending the capabilities of the real-time hybrid testing technique. The method is then used to test a structure with a rate-dependent energy dissipation device, a magnetorheological damper. Results show good agreement with the predicted responses, demonstrating the effectiveness of the method to test rate-dependent components.

Keywords: real-time hybrid testing; delay compensation; substructuring; MR damper.

1. Introduction

Experimental testing is an essential tool for understanding how structures respond to extreme events such as earthquakes. Hybrid (or pseudodynamic) testing provides an attractive alternative for dynamic testing of structural systems by combining physical testing with numerical simulation (Hakuno, *et al*. 1969, Takanashi, *et al*. 1975, Mahin and Shing 1985, Takanashi and Nakashima 1987, Mahin, *et al*. 1989, Shing, *et al*. 1996). The basis of the method is that the dynamic response of the structure due to a

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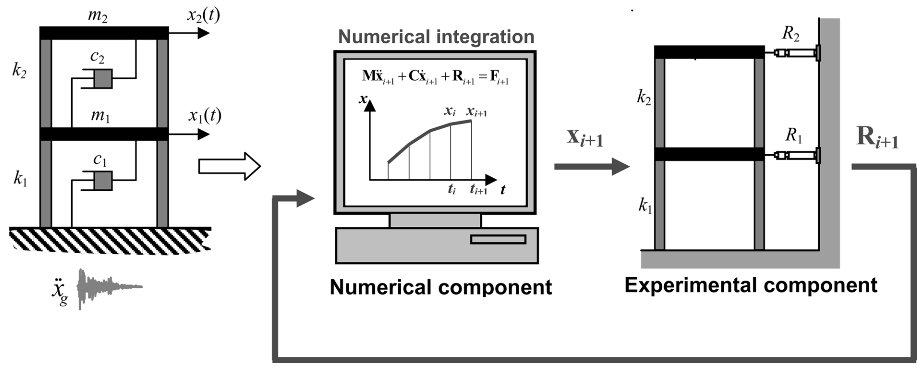

Fig. 1 Schematic of the pseudodynamic test method (hybrid testing)

particular loading (e.g. earthquake) is calculated numerically on a computer and then the restoring forces from the structure are obtained by applying the calculated displacements to the test specimen. The structure to be tested is divided into a physical component and a numerical model. In its basic form, the numerical model includes the mass on the structure (lumped at discrete locations), and the structural damping. During the test, the response of the structure is calculated using time step-integration of the equation of motion. The displacements obtained are then imposed to the physical structure using actuators at the discrete locations where lumped masses are assumed. The forces required to produce these displacements are measured and fed back to the computer to calculate the displacements corresponding to the next time step (see Fig. 1).

Because damage is intrinsically a local phenomenon, frequently it is not required to test the entire structural system. The substructure technique allows physical testing of only the parts of the structure of interest (e.g. where damage is expected, or components that are difficult to model numerically), while the rest of the structure can be modeled numerically. In conventional hybrid testing, the displacements are imposed on an extended time-scale which typically ranges from 100 to 1000 times the actual earthquake duration. Therefore, when the restoring forces depend also on the velocity (i.e. rate-dependent), the method is not applicable. An alternative approach for seismic testing is the shaking-table test method, where the entire structure is subjected to a ground acceleration history applied by the shaking-table. Because the test is conducted in real time, dynamic effects and rate-dependent behavior are completely modeled. However, the applicability of the shaking-table test method is often limited by economic considerations (because the test specimen is the entire structure), as well as limitations on the size and payload capacity of the shaking-table.

Real-time hybrid testing is a variation of the pseudodynamic test method in which the imposed displacements and response analysis are executed in a common time scale (i.e. real time), thus allowing testing of systems with rate-dependent components (Nakashima, *et al.* 1992, Horiuchi, *et al.* 1996, Horiuchi, *et al.* 1999, Nakashima and Masaoka 1999, Darby, *et al.* 2001, Shing, *et al.* 2004, Carrion and Spencer 2006, 2007, Ahmadizadeh, *et al.* 2008). Real-time hybrid testing makes it possible to test the large category of structural components associated with vibration control including passive, semi-active, and active control devices (e.g. base isolation and dampers), which are typically nonlinear and rate-dependent. Recently, special attention has been directed toward the application of this technique to evaluate the response of structures with MR dampers. Real-time hybrid experiments of structures with MR dampers have already been conducted by Emmons and Christenson (2006; http://

nees.colorado.edu/MRDamperFHT.php), Wu, *et al*. (2006), Carrion and Spencer (2006, 2007) and Spencer and Carrion (2007).

Real-time hybrid testing is challenging because it is necessary to perform all the calculations, apply the displacements, and measure and feedback the forces within a single time step (typically less than 10 ms). Because the test is conducted in real time, the dynamics of the testing system and specimen become important. Furthermore, when hydraulic actuators are used to apply forces to the test specimen, a time lag exists between when the displacement is commanded and when the actuator actually reaches the commanded position. There are also some inevitable time delays associated with the numerical calculations and the communication between the computer and data acquisition systems. Because of these time delays and lags, the force measured and fed back from the experiment does not correspond to the desired position (it is measured before the actuator has reached its target position). The effect of this error is to introduce additional energy into the system which may even cause the test to go unstable (Horiuchi, *et al*. 1996).

This paper presents an approach for real-time hybrid testing which implements model-based time delay/lag compensation. The efficacy of the proposed strategy is verified by conducting substructure real-time hybrid testing of a steel frame under earthquake loads in which the physical substructure is a small-scale specimen representing one of the columns. The performance of the testing system and compensation technique is investigated by considering structures with different fundamental frequencies. The proposed time delay/lag compensation technique is also compared to the traditionally used polynomial extrapolation method. The proposed method is then used to conduct real-time hybrid tests of a structure with a supplementary energy dissipation device, a magnetorheological damper (MR damper), which is tested experimentally. This experiment demonstrates the effectiveness of the proposed real-time hybrid testing algorithm.

2. Real-time hybrid testing

The equation of motion of the structural system to be tested is given by the following second-order differential equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{R}(\mathbf{x}, \dot{\mathbf{x}}, t) = \mathbf{F}(t) \tag{1}$$

in which $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ the damping matrix (for linear damping), $\mathbf{R}$ the resisting force vector, $\mathbf{F}$ the vector of externally applied forces, and $\mathbf{x}$ the displacement vector. The dots denote differentiation with respect to time. When the structure is subjected to a ground acceleration $\ddot{\mathbf{x}}_g$, the external force vector is $\mathbf{F} = -\mathbf{M}\,\boldsymbol{\Gamma}\,\ddot{\mathbf{x}}_g$, where $\boldsymbol{\Gamma}$ is a mass influence vector. The equation of motion is discretized by dividing the time interval at a uniform spacing of Δt, and then solved using a step-by-step time integration algorithm. Amongst all the numerical algorithms proposed for hybrid testing, the central difference method (CDM) and the alpha operator-splitting method α-OS (Nakashima, *et al*. 1990, Combescure and Pegon 1997) are the most widely used and are the ones considered herein. The expression to calculate displacements using the CDM is given by

$$\mathbf{x}_{i+1} = \left[\frac{1}{\Delta t^2}\mathbf{M} + \frac{1}{2\Delta t}\mathbf{C}\right]^{-1}\left[\frac{2}{\Delta t^2}\mathbf{M}\mathbf{x}_i - \left(\frac{1}{\Delta t^2}\mathbf{M} - \frac{1}{2\Delta t}\mathbf{C}\right)\mathbf{x}_{i-1} - \mathbf{R}_i + \mathbf{F}_i\right] \tag{2}$$

in which the subscript i denotes the value of the variable at time $t_i = i\Delta t$. The advantage of CDM is that it is an explicit method; therefore it requires only information from the current and previous steps to compute the response at the next time step, making its application to hybrid testing straightforward. However, the method is conditionally stable (with stability criterion given by $\omega\Delta t \leq 2$, where ω is the highest natural frequency of the system). For structures in which the stability criterion of CDM becomes difficult to satisfy, the α-OS-method, which is unconditionally stable for systems of softening type, can be used.

3.1. Actuator dynamics

Displacements in hybrid testing are generally imposed on the structure using hydraulic actuators. When displacements are applied at fast rates, the dynamics of the actuator and attached specimen become important. Dyke, *et al*. (1995) analyzed the effects of control-structure-interaction (CSI) and showed that the dynamics of the actuator and structure are intrinsically coupled through a natural velocity feedback link. This work demonstrated that neglecting phase differences between the command input and the resulting force (i.e. neglecting the CSI) resulted in an apparent time delay associated in the literature with generation of the control forces. In structural testing, this effect is not significant when the tests are conducted at slow speeds. However, when performing fast and real-time hybrid tests, this dynamic coupling and the associated finite response time of the hydraulic actuators become particularly important, resulting in a time lag between the commanded displacement and the realization of this command by the actuator. Although the time lag is caused by both actuator dynamics and the attached test specimen, the contribution from the former is dominant (Zhao, *et al*. 2003). Typically the natural frequency of the test specimen is large compared to the bandwidth of interest and in the pseudo-static region (low frequency region), the phase of the actuator transfer function can be approximated as linear and modeled as a pure time delay. Actuator time lags can be several times larger than the typical time-step used for seismic testing. Typical values reported in the literature range from 8 to 30 msec (Horiuchi, *et al*. 1999, Darby, *et al*. 2001, Shing, *et al*. 2004, Nakashima and Masaoka 1999), however values as high as 80 msec have also been reported (Wu, *et al*. 2006).

In real-time hybrid testing, the effect of time delays (denoted here as δt_d) is that the output or imposed displacement lags the commanded displacement by an amount of time equal to the delay. Because of this delay, the force measured and fed back from the experiment does not correspond to the commanded displacement (it is measured before the actuator has reached its target position), however the algorithm assumes that the measured force corresponds to the commanded displacement. For a linear-elastic system, the resulting response, as seen by the algorithm, is a counter-clockwise hysteretic loop, instead of the straight line corresponding to the linear behavior. The effect of this counter-clockwise loop is to introduce additional energy into the simulation. Horiuchi, *et al*. (1996) demonstrated that for a linear-elastic SDOF system, the increase in the total system energy caused by the delay/lag is equivalent to introducing negative damping into the system (given by $c_{eq} = -k\delta t_d$, where k is the stiffness of the system). This artificial negative damping becomes large when either the stiffness of the system or the time delay/lag is large. When this negative damping is larger than the structural damping, the response will diverge (become unstable), and the experiment must be halted. Instability almost invariably occurs in practice due to the low levels of damping associated with structural frames and the large time delays/lags associated with large hydraulic actuators (Darby, *et al*. 2001). Therefore, introduction of compensation for time delays/lags is essential when conducting fast hybrid experiments.

3.2. Review of previous real-time and fast hybrid experiments

The idea of conducting a real-time hybrid experiment was introduced by Hakuno, *et al*. (1969) who investigated the response of a single-degree-of-freedom using an analog computer to solve the equation of motion and to control the actuator. The existence of phase lag was identified as affecting the accuracy of the simulation. Nakashima, *et al*. (1992) conducted real-time hybrid experiments using a modified version of the Central Difference Method (CDM), called staggered CDM, together with a digital control mechanism. This control mechanism generated command signals using linear interpolation between the displacement at the previous step and the new target displacement (allowing continuous actuator motion) and corrected the signals using the difference between target and measured displacements at each substep. The system was only capable of testing systems with responses up to about 1 Hz. The effect of the dynamic response of hydraulic actuators on real-time hybrid experiments was initially considered by Horiuchi, *et al*. (1996) who identified this effect as a "response delay" and introduced a compensation technique. The method employed response prediction using an *n*-th order polynomial extrapolation based on the previous and current displacements. Nakashima and Masaoka (1999) proposed a system for continuous real-time hybrid testing. Continuous motion of the actuators was achieved by generating intermediate command displacements for the actuator using polynomial extrapolation and interpolation based on information from the previous displacements. To compensate for the actuator lag, the method introduced by Horiuchi, *et al*. (1996) was used. While accurate results were obtained, response frequencies beyond 3 Hz were not stable, primarily because of limitations in the extrapolation and interpolation procedures. Darby, *et al*. (1999) conducted real-time hybrid testing of a simple portal frame. Actuator lag compensation was performed using the method proposed by Horiuchi, *et al*. (1996) with a fourth-order polynomial. The fundamental natural frequency of the system was 5 Hz. Note that a large value of damping (10%) had to be used, because the negative damping introduced by the actuator delay was not fully cancelled by the delay compensation. Horiuchi and Konno (2001) proposed a prediction algorithm in which the acceleration was linearly predicted and then used to calculate the desired predicted displacement (which also resulted in a third-order displacement approximation). Magonette (2001) conducted fast continuous pseudodynamic testing using an adaptive Minimal Control Synthesis (MCS) algorithm with a supplementary feedforward circuit to minimize tracking phase lag. To allow testing of structures with strain-rate dependent components using the continuous pseudodynamic method (at non-real-time speeds), a simple constant correction factor was used to compensate for the difference between the forces at real time-speed and the forces obtained at the speed at which the pseudodynamic test was conducted. Darby, *et al.* (2001) conducted experiments using the simple portal frame tested by Darby, *et al.* (1999) with an improved algorithm. Actuator lag compensation was performed using the method proposed by Horiuchi, *et al*. (1996) with a fourth-order polynomial. The fundamental frequency of the system was 5.13 Hz, and a damping ratio of 5% was assumed. Blakeborough, *et al.* (2001) perform hybrid experiments on a similar frame as the one tested by Darby, *et al*. (1999), however with a physical substructure consisting of a column with two-degrees-of-freedom (lateral displacement and rotation at the top). Results for a 2.5 Hz sinusoidal excitation were satisfactory, however large errors where observed on the rotation response. The results for an actual earthquake record were less satisfactory, especially in the 6 to 7 Hz range. A high value of damping (5% critical damping) had to be used because the tests were highly prone to instabilities due to the physical coupling between the two actuators. Mosqueda, *et al.* (2004) conducted continuous hybrid experiments (with a time scale factor of 120) using distributed experimental sites connected through the internet. The algorithm employed was an event-driven version of the polynomial extrapolation/interpolation method proposed by Nakashima and Masaoka (1999). The method developed by Horiuchi, *et al*. (1996) was used to compensate for the time lag in the actuators. The test structure (two-degree-of-freedom system) had natural frequencies of 1.6 and 4.2 Hz, respectively and stiffness proportional damping with 5% on the first mode was used. Shing, *et al*. (2004) developed an algorithm for fast hybrid testing based on the unconditionally stable α-method (Hilber, *et al*. 1977) and a modified Newton technique using the initial stiffness of the structure. To account for the inherent delay due to digital control (a few milli-seconds), the PID control was supplemented with a feed-forward control implemented in the digital servo-controller (by MTS). Additionally the system included an ad hoc correction for the time lag in the actuator response caused by the dynamics of the servo-hydraulic actuator by overshooting the displacement commands in the next correction if there is a time lag in the previous correction (Shing, *et al*. 2002). The system was used to test a two-degree-of-freedom system with 2.1 and 11.8 Hz natural frequencies, and 4% critical damping in the two modes. The results between the test and theoretical solution were close, with some amplitude errors in the response. Reinhorn, *et al*. (2004) and Shao, *et al*. (2006) used a Smith predictor for delay compensation during force-controlled hybrid experiments. On these experiments, the calculated force is sent to the actuator, and the resulting displacement is measured and fed back to the simulation. The Smith predictor method, however, required an accurate model of the system to effectively compensate for the delay (Ahmadizadeh, *et al*. 2006). Using an approach similar to Horiuchi and Konno (2001), Ahmadizadeh, *et al*. (2006, 2008) used kinematics expressions assuming constant acceleration to predict the command displacements. Experiments were conducted using single degree-of-freedom system with 2 Hz natural frequency and 5% damping. The authors also presented a method for online estimation of system time delays with improved characteristics over existing methods (*e.g*., Darby, *et al*. 2002). Carrion and Spencer (2007) and Spencer and Carrion (2007) presented a method for modeling and compensating actuator dynamics for cases in which the characteristics of the test specimen (and therefore actuator dynamics) change rapidly and significantly. The compensation was implemented using a model-based feedforward-feedback compensator and the method was used to evaluate the response of a semi-actively controlled structure employing an MR damper. Ahmadizadeh (2007) and Ahmadizadeh, *et al*. (2008) presented a method to compensate for the actuator tracking errors by performing compensation on the force measurement signal; however these studies demonstrated that displacement modification procedures outperform force correction methods especially for large delays. The force correction method can also be used in conjunction with displacement modification as suggested by Mosqueda and Ahmadizadeh (2007).

Fast and real-time hybrid experiments have been conducted to date using different techniques; however, most of the experiments have been limited to structures with relatively low natural frequencies. These limitations arise due to the stability problems introduced by the time delays and lags. To solve this problem, some researchers have increased the structural damping in the model. Although solving the numerical stability problem, the additional damping reduces the accuracy of the test results.

3.3. Delay compensation

Several techniques have been proposed to compensate for time delays and actuator lags, of which the response prediction method (Horiuchi, *et al*. 1996) has been the most widely used. In this approach, instead of using the calculated displacement as the command signal to the actuator, the displacement of the actuator after the delay δt_d is predicted and used as the command signal to the actuator. Because of the time delay, the resulting displacement imposed by the actuator approximates the calculated one. This approach is illustrated in Fig. 2(a), where $x(t_{i+1})$ is the calculated displacement, $\hat{x}(t_{i+1}+\delta t_d)$ is the

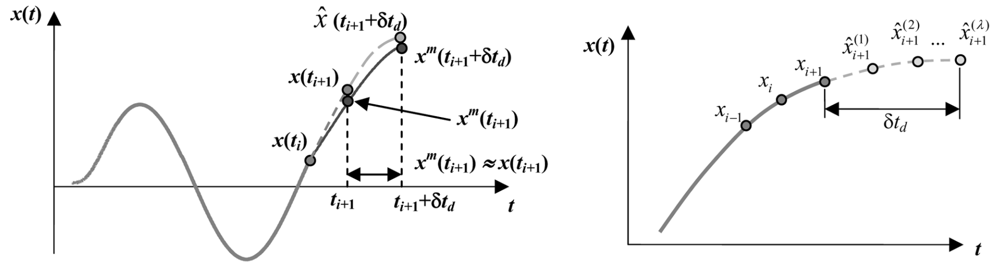

Fig. 2 (a) Delay compensation using response prediction, (b) Calculation of predicted displacements using model-based response prediction

predicted (compensated) displacement, and $x^m(t_{i+1})$ is the actual displacement imposed by the actuator.

The method used for the prediction by Horiuchi, *et al*. (1996) was an *n*-th order polynomial extrapolation based on the displacements at the present and previous time steps. The accuracy and stability of the method were investigated using a linear single-degree-of-freedom (SDOF) system. The prediction method was found to cause variations in both stiffness and damping. Furthermore, damping becomes negative and the simulation diverges when the non-dimensional parameter $\omega\delta t_d$ (where ω is the natural frequency of the system) is larger than a critical value that depends on the order of the polynomial used for the extrapolation. For multi-degree-of-freedom systems, the critical value should be based on the highest natural frequency of the structure (Horiuchi, *et al*. 1999). A third-order polynomial extrapolation was recommended because it requires little calculation time and gives a relatively large critical value $\omega\delta t_d = 1.571$. The limitation imposed by the critical value for stability becomes more important for stiff structures or multi-degree-of-freedom systems which have higher modal frequencies, or when the delays/lags are large compared to the smallest period of the structure. More rational methods for evaluating and compensating delay and lag effects are needed to extend the capabilities of the real-time hybrid test method.

4. Model-based response prediction

The polynomial extrapolation method for response prediction uses only information from the displacements at a few previous time steps. For small prediction times, the response of a structure (which is nearly harmonic) can be well represented by polynomials, but when longer prediction times are required, or for systems with larger natural frequencies (i.e. shorter natural periods), more precise prediction methods are necessary. A model-based response prediction method estimates the response using known information and physical characteristics of the system. Although complete characteristics of the system are not known in advance, there is some information about the system that is known prior to testing or can be calculated at the beginning of the experiment. This information includes: the mass matrix $\mathbf{M}$, the damping matrix $\mathbf{C}$, the external excitation $\mathbf{F}$ (e.g. ground acceleration $\ddot{\mathbf{x}}_g$ for seismic loading), and the initial elastic stiffness of the structure $\mathbf{K}_e$. By incorporating known information about the system and the excitation, a more accurate prediction of the response can be achieved, allowing larger prediction horizons, stability improvements, and testing of structures with higher natural frequencies or with more degrees-of-freedom.

The proposed model-based response predictor uses the information available at the beginning of each time step to predict the displacement response at the end of the prediction horizon (i.e. the time delay).

The prediction is performed using a model of the system with the uncompensated target displacement x_{i+1} as the initial condition. The response of the predictor model is then computed for the displacements $\hat{x}_{i+1}^{(k)}$ for $k = 1, 2, ..., \lambda$, where λ is the ratio between the time delay and the time step (i.e. $\lambda = \delta t_d/\Delta t$), as shown on Fig. 2(b) (the time step size Δt is set so that the time delay is an integer multiple of Δt for simplicity). The predicted displacement at the end of the prediction horizon ($\hat{x}_{i+1}^{(\lambda)}$) is then sent as a command to the actuator, providing compensation for the time delay.

To predict the response of structural systems with a small number of degrees-of-freedom, a complete model of the system is used to generate the predictions. By considering the solution of the equation of motion (e.g. Eq. (2) when using the Central Difference Method), the only unknown parameter is the restoring force vector **R**, while the mass matrix, damping matrix, and external force vector are known. An approximation of the restoring force vector can be obtained using the previous measured restoring force vector and a certain stiffness matrix **K** as given by

$$\mathbf{R}_i \approx \mathbf{R}_{i+1} + \mathbf{K}(\mathbf{x} - \mathbf{x}_{i-1}) \tag{3}$$

When the stiffness of the system does not change significantly during the experiment (*e.g.*, linear and moderately nonlinear response), using the initial elastic stiffness of the system on the approximation (i.e., $\mathbf{K} = \mathbf{K_e}$) yields good estimates of the restoring force (a similar approximation is used in the Operator-Splitting method by Nakashima, *et al.* 1990 in which the nonlinear part of the response is made explicit and therefore does not require iteration for the solution). When the response of the structure is limited to the elastic range, this approximation and the proposed model-based compensation method yields the exact prediction; theoretically the experiment should be stable for any value of the parameter $\omega\delta t_d$.

For cases when the stiffness of the system changes significantly during the experiment, an estimate of the tangent stiffness based on the last increment of the measured restoring force is used. A simple estimate of the tangent stiffness can be obtained using the Broyden formula (1965), which corresponds to a generalization of the one-dimensional secant approximation to the tangent stiffness. The tangent stiffness $\mathbf{K}_i$ is obtained by making the least change to $\mathbf{K}_{i-1}$ that satisfies the equation $\mathbf{K}_i\Delta\mathbf{x}_i = \Delta\mathbf{r}_i$, where $\Delta\mathbf{x}_i = \mathbf{x}_i - \mathbf{x}_{i-1}$ and $\Delta\mathbf{r}_i = \mathbf{R}_i - \mathbf{R}_{i-1}$ are the displacement and force increments, respectively. The equation for the stiffness using the Broyden formula is

$$\mathbf{K}_i = \mathbf{K}_{i-1} + \frac{(\Delta\mathbf{r}_i - \mathbf{K}_{i-1}\Delta\mathbf{x}_i)\Delta\mathbf{x}_i^T}{\Delta\mathbf{x}_i^T\Delta\mathbf{x}_i} \tag{4}$$

Because the main effect of actuator dynamics is to produce a time lag on the response, compensation for actuator dynamics is in essence a tracking control problem. The compensation algorithm can be viewed an additional controller whose main objective is to cause the measured displacement to more accurately track the desired displacement.

When the controller does not use a measurement of the system being controlled in computing the control action, the control system is called open-loop (Franklin, *et al.* 2002). Feedforward is a type of open-loop control that processes the reference signal directly by calculating a best guess or prediction to produce the ideal response of the plant (Ellis 2000). Feedforward control allows for the combination of knowledge of the command and the system being controlled (i.e. plant) to improve the system response (Ellis 2000). Feedforward control has the advantage that it avoids additional lags associated with feedback control, therefore, allowing much faster responses. Because the output of the system is not used in calculating the control signal (i.e. does not form a closed loop), feedforward control does not have the potential stability problems associated with feedback control, provided that the plant and controller are stable (Ellis 2000). The proposed model-based response prediction method is a type of feedforward control in which the compensated displacement is determined using information about the command (i.e. displacement response) and the plant (i.e. testing system). The information associated with the command displacement includes the dynamic properties of the test structure (i.e. mass, damping, and stiffness) and the excitation (e.g. ground acceleration); information about the plant includes the magnitude of the actuator time lag. When the initial elastic stiffness of the system (or a constant stiffness) is used on the approximation of the restoring force for the compensation algorithm (i.e. Eq. (3)), the output of the system (i.e. measured displacement and restoring force) is not included on the control, and therefore the controller is open-loop. When the tangent stiffness is used (i.e. Eq. (4)), the measurements from the system are also used for the compensation, and therefore the controller includes both a feedforward and also a feedback link. The effect of the system parameters on this feedback loop is currently under investigation.

For systems with a large number of degrees-of-freedom, using a complete model of the system to conduct the response prediction is not efficient. Considering that the earthquake response of typical structures is dominated by a few lower modes, the mode superposition technique (based on the elastic mode shapes) is used to predict the response of larger systems.

The above formulation provides a simple and efficient method for model-based response prediction which gives good predictions of the response even for systems with inelastic response. The next section describes the experimental setup used to validate the proposed approach.

5. Experimental setup

A system that combines fast hardware, for high-speed computations and communication, with high performance hydraulic components has been employed to experimentally validate the proposed real-time hybrid testing approach (see Fig. 3). The system consists of a hydraulic actuator controlled by a digital servo-controller, a displacement transducer which measures the displacements imposed by the actuator and provides position feedback, a load cell to measure the force imposed by the actuator

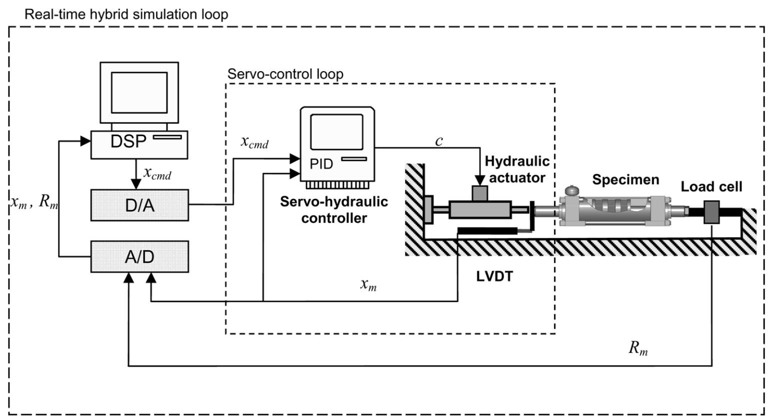

Fig. 3 Schematic of experimental setup (x_m = measured displacement, R_m = measured restoring force, x_{cmd} = command displacement, and c = command to the actuator)

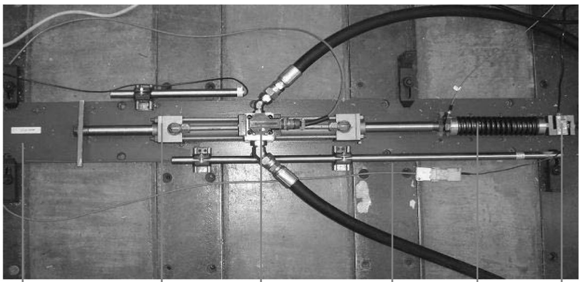

Fig. 4 Experimental setup

(which corresponds to the restoring force from the specimen), a computer with a board for real-time control (to solve the equation of motion and generate the target displacements), and digital to analog and analog to digital converters.

This equipment is located at the Smart Structures Technology Laboratory (SSTL) at the University of Illinois at Urbana-Champaign (http://sstl.cee.uiuc.edu). The load frame is shown in Fig. 4. A double-ended hydraulic actuator manufactured by Nopak is used. The actuator has a ±152 mm (6 in) stroke and is equipped with low-friction Teflon seals to reduce friction effects. The actuator has a cylinder diameter of 38.1 mm (1.5 in) and a rod diameter of 25.4 mm (1.0 in) which results in an effective piston area of 633 mm^2 (0.98 in^2). A Schenck-Pegasus 132A two-stage servo-valve rated for 10 gpm at 1,000 psi pressure drop is used to control the actuator. A pressure accumulator provides supplemental oil flow for high-speed testing. The system is connected to the main hydraulic power supply line of the Newmark Structural Engineering Laboratory operating at 3,000 psi pressure. The actuator is controlled by a Schenck-Pegasus 5910 digital servo-hydraulic controller in displacement feedback mode. The displacement of the actuator is measured using a Lucas-Schaevitz 10,000 DC-EC linear variable differential transformer (LVDT) having a range of 254 mm (10 in). An Omega load cell with a range of ± 4,540 N (1.0 Kip) is used to measure the applied force. A computer with a dSPACE DS1003 parallel processing DSP board based on Texas Instrument TMS320C40 processor is used to solve the equations of motion and provide real time control. A dSPACE DS2102 High-Resolution 6-channel D/A Board and a dSPACE DS2002/DS2003 32-channel A/D Board are employed to convert the signals from digital to analog, and analog to digital, respectively, both having a resolution of 16 bits. The algorithms (numerical integration and compensation techniques) are implemented in SIMULINK/MATLAB and then downloaded to the dSpace processor using the Real-time Workshop.

The dynamics of the actuator and the resulting time-lag of the actuator are critical parameters in real-time hybrid testing. To understand the behavior of the system over a wide range of frequencies, the frequency response function (FRF) from the commanded displacement to the measured displacement was determined using a random excitation as input. The excitation signal was a bandlimited white noise with a bandwidth of 50 Hz and an RMS of 0.01 in. A PC-based SigLab spectrum analyzer manufactured by DSP Technology was employed for data acquisition and analysis. The data was captured using a sampling rate of 128 Hz. The FRF was calculated using 2048 FFT points, resulting in a frequency resolution of 0.0625 Hz, a Hanning windowing with 50% overlap, and 10 averages. Eight-pole elliptic anti-aliasing (AA) filters were used during the data acquisition. Fig. 5 shows the magnitude and phase of the transfer function. After about 15 Hz, the magnitude starts to roll-off and assuming a cutoff

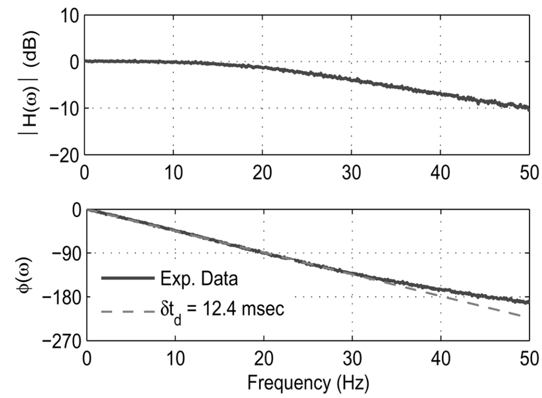

Fig. 5 Transfer function from command displacement to measured displacement

frequency of -3dB, the bandwidth of the system is determined to be 28.7 Hz. The phase plot shows that within the bandwidth, the phase of the system is nearly linear. From this slope, the time lag of the actuator is approximated as a time delay of 12.4 msec.

6. Verification experiments

Real-time hybrid testing of a simple structural system under earthquake loads was conducted to verify the efficacy of the testing system and compensation strategy. The test structure consisted of a single-story, one-bay steel frame assumed to have a rigid floor beam (i.e. shear frame) and is shown schematically in Fig. 6. The structure was divided into two substructures. The left column is tested experimentally (physical substructure), while the right column is modeled numerically (numerical substructure). The columns have stiffnesses of 3.61 KN/mm (20.6 Kip/in) and 6.65 KN/mm (38.0 Kip/in) for the physical and numerical substructures, respectively. The force-displacement relationship corresponding to the numerical substructure is modeled using a bilinear hysteretic model with a yield displacement of 1 mm (0.04 in) and a ratio of post- to pre-yield stiffness of 0.02. Because the purpose of this experiment is to verify the testing system and methodology, instead of using an actual steel cantilever column as the physical substructure, an alternative small-scale specimen was used. The specimen was designed to minimize backlash effects and deformation of the loading frame while fitting into the experimental setup. The specimen is composed of a steel compression spring having a length of 203 mm (8 in) and an outside diameter of 49.2 mm (1.9375 in), see Fig. 7(a). The spring has a stiffness of 41.0 N/mm (234 lb/

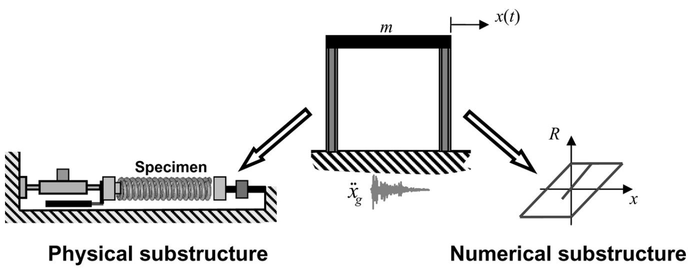

Fig. 6 Structural model and substructures for hybrid experiment

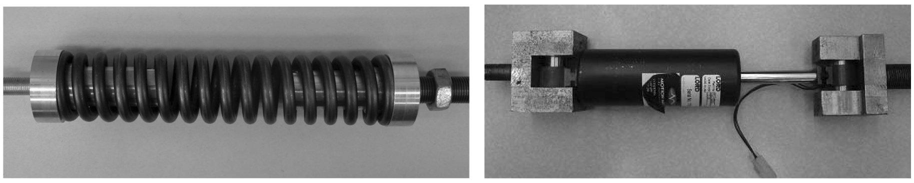

Fig. 7 Test specimens: (a) Compression spring, (b) Small scale MR damper

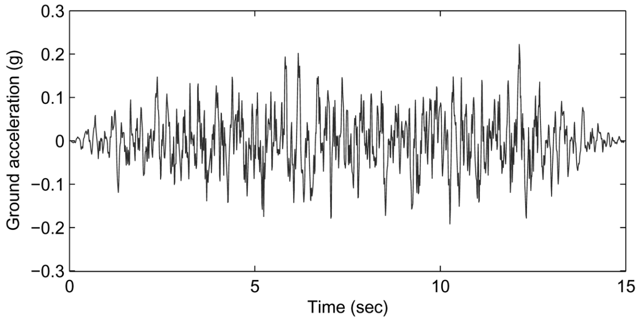

Fig. 8 Input ground acceleration

in); therefore a force scaling factor of 88.0 is used. The linear elastic behavior of the spring specimen allows repeatable test results which can be compared against the theoretical expected response, providing an assessment of the effectiveness of the proposed approach.

Several tests were conducted by changing the fundamental frequency of the test structure. During each experiment the mass of the structure was adjusted accordingly to obtain the desired natural frequency. Modal damping was assumed to be 2% of critical damping. The structure was subjected to the artificial earthquake record shown in Fig. 8 (which was used for the MOST experiment, Spencer, *et al*. 2004). The Central Difference Method was used for integration of the equation of motion with a time step $\Delta t = 0.0062$ sec which corresponds to half of the time delay (i.e. $\delta t_d = 2\Delta t$). This time step was adequate to accurately integrate the equation of motion for all the natural frequencies considered in the experiments (0.5 to 15 Hz). The stability criterion for the CDM was easily satisfied with this time step.

For the initial set of experiments, the structure was assumed to be linear–elastic; therefore the ratio of post- to pre-yield stiffness in the numerical substructure was set to 1.0. Tests corresponding to natural frequencies of the test structure of 0.5, 1, 5, 6, 7, 8, 10, 13, and 15 Hz were conducted. The amplitude of the ground acceleration was scaled so that the maximum displacement (calculated using the analytical model) was about 5.08 mm (0.2 in). For each natural frequency, tests were conducted using (a) no delay compensation, (b) proposed model-based compensation, and (c) third-order polynomial extrapolation. To evaluate the accuracy of the tests results, the analytical response was calculated using a numerical model of the test structure with the experimentally measured stiffness of the test specimen. Fig. 9 shows the experimental results for the structure with 1.0 Hz natural frequency. The duration of the earthquake record used is 15 sec, therefore the additional 5 sec shown in the figure correspond to zeros padded at the end of the input record to observe the effect of the compensation methods on the free vibration response of the system. As can be seen, excellent agreement is obtained between the test results and the analytical solution for both compensation methods (polynomial extrapolation and model-based). The test without delay compensation results in a larger response than the exact analytical

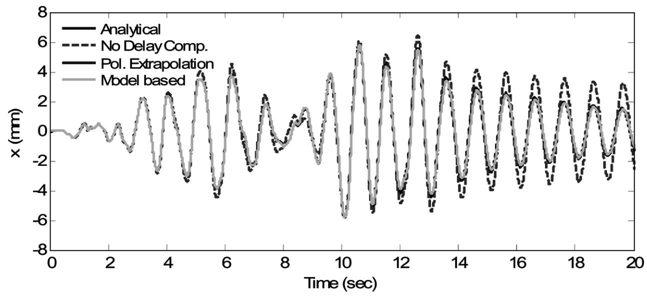

Fig. 9 Test results for 1 Hz natural frequency

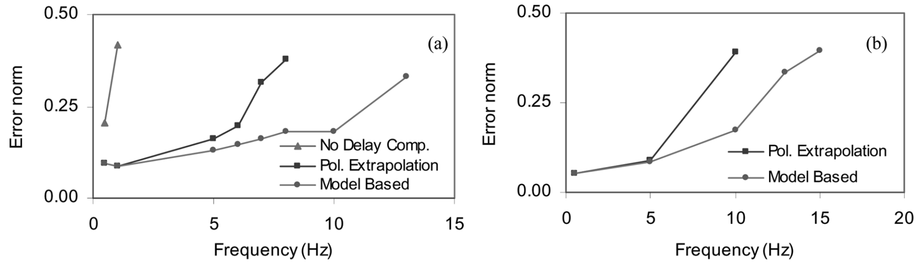

Fig. 10 Test results using different delay compensation methods: (a) Linear case, (b) Inelastic case

solution due to the effect of the negative damping introduced by the uncompensated phase lag from the actuator. To measure the error between analytical and experimental results and allow comparison between the different compensation strategies, the RMS of the error normalized by the RMS of the analytical solution is used. Fig. 10(a) shows the results for the tests conducted considering different natural frequencies of the test structure and using the different compensation strategies. For each delay compensation method, the highest natural frequency plotted corresponds to the highest natural frequency of the test structure that was tested without the experiment becoming unstable.

As observed, when no delay compensation is used the experiment becomes unstable even for relatively low frequencies of the structure. The theoretical stability limit corresponds to the frequency where the negative damping introduced by the actuator lag becomes equal to the inherent structural damping (1.46 Hz for the test structure considered). Using the proposed model-based compensation method, experiments with natural frequencies as high as 13 Hz were successfully conducted. The maximum frequency that was effectively tested using the polynomial extrapolation was 8 Hz. Fig. 10 (a) shows that the error is small and therefore the tests are accurate for frequencies of about 5-6 Hz for the polynomial extrapolation and up to 10 Hz for the model-based delay compensation method. Fig. 11 shows the results for the test corresponding to a natural frequency of 10 Hz using model-based delay compensation; results are seen to match well with the analytical solution.

The maximum natural frequencies for which the experiment was stable using both delay compensation methods are smaller than theoretically predicted. Possible causes are: the effect of experimental errors present in the hybrid experiment, which tend to exacerbate responses at higher frequencies; differences in the estimated actuator time delay and the actual phase lag; the fact that the phase lag is close, but no exactly, a constant time delay; and the effect of the roll-off in the magnitude of the actuator transfer function (e.g. at a frequency of 15 Hz the magnitude is 0.94, causing an undershoot error which adds energy to the system and results in a negative damping which can led to system instability, Mahin, *et al.* 1989).

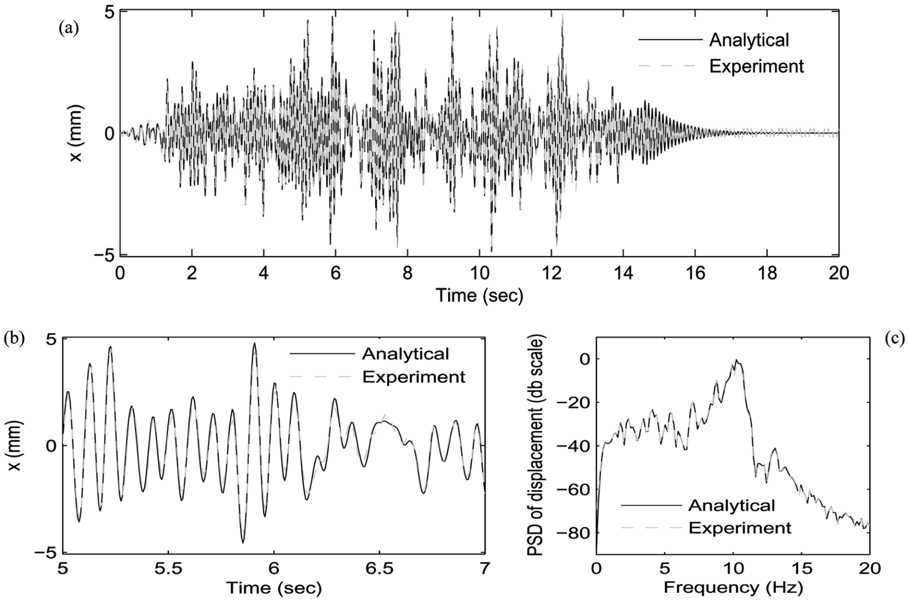

Fig. 11 Test results for 10 Hz natural frequency using model-based compensation: (a) Displacement response, (b) Displacement response (close up view), (c) Power spectrum of displacement

To evaluate the performance of the delay compensation methods for nonlinear structures, tests were conducted considering a structure with inelastic response (numerical substructure with bilinear force-displacement relationship). Tests corresponding to natural frequencies of the structure of 0.5, 5, 10, 13, and 15 Hz were performed using both delay compensation methods. For the model-based compensation method, the initial elastic stiffness was used as the approximation of the restoring force (Eq. 3). The ratio between the maximum displacement and yield displacement (i.e. displacement ductility) was about 4.0. The error norms for the test results are shown on Fig. 10(b). As can be seen, the trends are similar to the linear case, however the maximum frequencies that were achieved are slightly larger (10 Hz for the polynomial extrapolation and 15 Hz for model-based compensation). This greater stability is due to the damping introduced by the energy dissipated during inelastic deformation.

7. Structure with MR damper

The use of magnetorheological dampers (MR dampers) as supplemental damping devices for reducing the response of civil engineering structures under severe earthquakes and winds is becoming increasingly accepted (Spencer and Nagarajaiah 2003). MR dampers can be used as semiactive control devices, offering the reliability of passive devices, yet maintaining the versatility and adaptability of fully active systems. Recent developments also suggest the use of MR dampers as smart passive control devices (Jung, *et al.* 2007). Some of the attractive features of MR dampers include very low power requirements (allowing operation under battery power), ability to provide a readily controllable damping force, large achievable force capacity, and low sensitivity to temperature changes (Spencer, *et al.* 1997). The second application considered in this study consists of a building structure with an MR damper used as a supplementary energy dissipation device.

The damper considered is a RD-1005 MR fluid damper manufactured by Lord Corporation (Fig. 7(b)). The damper is 216 mm (8.5 in) long in its extended position, and the main cylinder is 38.1 mm (1.5 in) diameter. The damper has a stroke of ± 25.4 mm (1.0 in) and can generate forces up to about 3,000 N (0.67 kip). Input current commands are supplied to the damper using a RD-1002 Wonder Box™ Device Controller from Lord. By selecting the input current, the characteristics of the damper may be changed in real-time to vary the forces exerted by the damper.

7.1. Characterization of MR damper

A model that represents the behavior of the MR damper is necessary to calculate the expected response of the test structure and therefore allow comparison with the results from the real-time hybrid experiment. A series of simple tests were conducted to measure the response of the damper and provide the necessary data to characterize/model its behavior. The MR damper (with 0 V input voltage) was subjected to sinusoidal displacement excitations of 5.08 mm (0.2 in) and 10.16 mm (0.4 in) amplitude with frequencies of 0.637 Hz, 1.273 Hz, and 2 Hz. Fig. 12 shows the response of the MR damper for the tests corresponding to 10.16 mm (0.4 in) amplitude. As observed the response of the damper is rate-dependent and nonlinear.

A simple model that can be used to represent the behavior of MR dampers (and ER dampers) is based on the Bingham viscoplastic model. The model can be idealized using mechanical elements, e.g. a viscous damper and a Coulomb friction element placed in parallel (Stanway, *et al*. 1987, Spencer, *et al*. 1997, Maiti, *et al*. 2006). For the data obtained from the tests conducted in this study, it was found that a better fit can be obtained by adding a stiffness term to the model. The model is represented schematically in Fig. 13, the force produced by the damper (F_D) is given by

$$F_D = f_c \operatorname{sgn}(\dot{x}) + c\dot{x} + kx + f_o \tag{5}$$

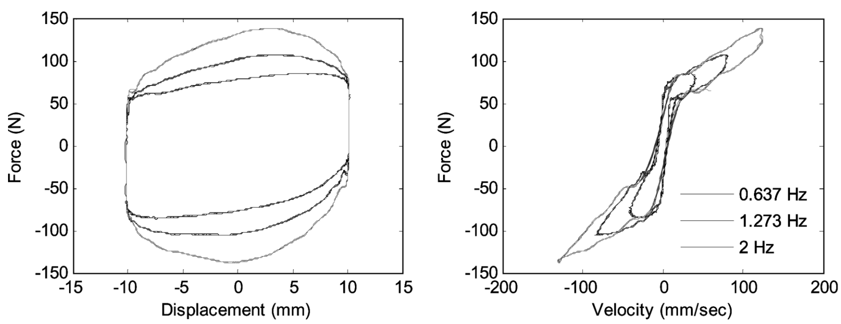

Fig. 12 Response of MR damper for test with 10.16 mm amplitude sinusoidal displacement

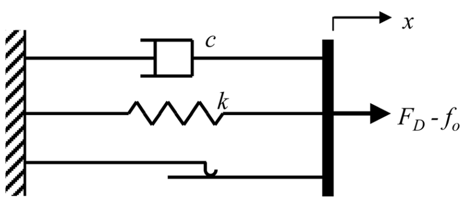

Fig. 13 Model for MR damper

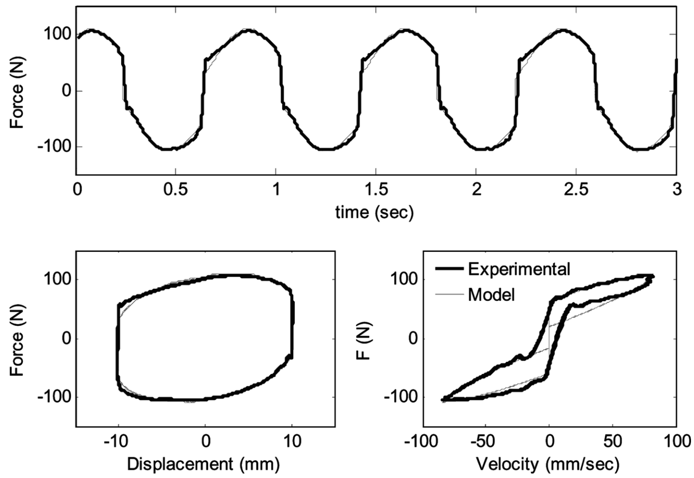

Fig. 14 Comparison between experimentally measured damper force and predicted by model for 1.273 Hz frequency test with 10.16 mm (0.4 in) amplitude

where f_c = frictional force, c = damping coefficient, k = stiffness coefficient, and f_o = offset in force. The parameters for the model where determined to fit the data using non-linear least squares parameter estimation, which yielded the following results: f_c = 39.8 N (8.94 lb), c = 0.801 N-s/mm (4.57 lb-s/in), f_o = 1.49 N (0.336 lb), and k = 2.14 N/mm (12.2 lb/in). Fig. 14 shows a comparison between the response of the damper measured experimentally and that predicted by the model for the test corresponding to a frequency of 1.273 Hz and an amplitude of 10.16 mm (0.4 in) (similar results were obtained for the other tests.). As observed, except for the response at velocities near zero, the behavior of the damper is well represented by the model.

7.2. Real-time hybrid experiment

Because MR dampers, as most semiactive control devices, are rate-dependent (and highly nonlinear), real-time hybrid testing is required to accurately test structural systems that incorporate such devices.

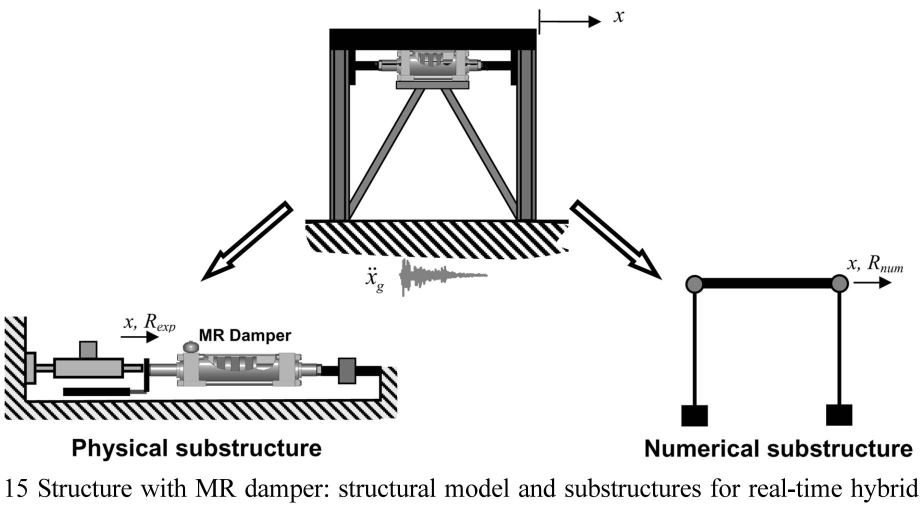

Fig. 15 Structure with MR damper: structural model and substructures for real-time hybrid test

The test structure for this example is a one-story, one-bay, steel shear frame with an installed MR damper, as shown schematically in Fig. 15. The structure is identical to the one used in the previous example, with the exception of the added MR damper. To conduct the real-time hybrid experiment, the structure was divided into two substructures: the MR damper was tested experimentally (physical substructure) while the rest of the structure was tested numerically (numerical substructure), see Fig. 15.

The stiffness of the test structure, structural damping, earthquake record, and numerical integration method were the same as in the previous example. Delay compensation was performed using the proposed model-based approach. The amplitude of the ground acceleration was scaled so that the maximum displacement was about 5.08 mm (0.2 in). Because the specimen is small-scale, a force scaling factor of 100 was used to represent the behavior of a full-scale damper (Yang, *et al*. 2002). Input voltage for the damper was set to 0 V throughout the experiment, therefore the MR damper was used as a passive energy dissipation device.

Because the force generated by the MR damper is rate dependent, it is important to impose accurately not only the displacement but also the velocity. Preliminary experiments conducted on the MR damper using sinusoidal input displacements showed that a sampling time of 0.001 sec allows both the displacement and velocity to be accurately imposed. Therefore, a time step of 0.001 sec was selected for numerical integration in the real-time hybrid experiment.

Experiments were conducted for natural frequencies of the test structure of 0.5, 1.0, and 2.0 Hz. The

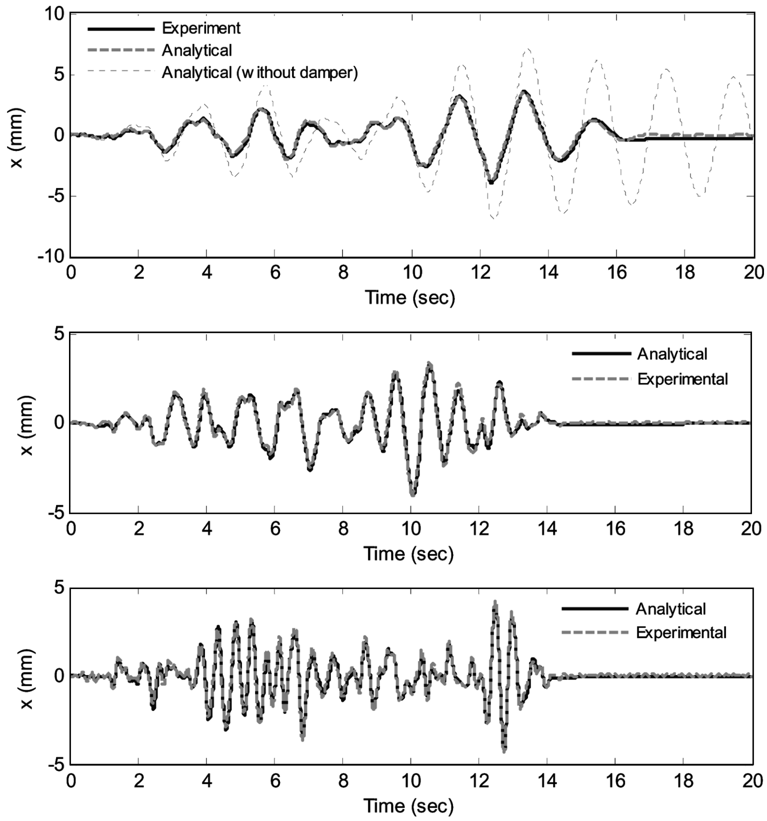

Fig. 16 Results from real-time hybrid experiment: (*top*) 0.5 Hz structure, (*middle*) 1.0 Hz structure, (*bottom*) 2.0 Hz structure

behavior of the numerical substructure (i.e. steel frame) was assumed to remain in the linear range and therefore the ratio of the post- to pre-yield stiffness in the numerical substructure was set to 1.0. Fig. 16 shows a comparison between the results from the real-time hybrid experiments as well as the analytical predicted responses for the three cases of natural frequencies considered. As observed, there is a very good agreement between the experimental results and analytical solution. Small discrepancies can be attributed to experimental errors and differences between the actual damper response and the model used to characterize its behavior. Fig. 16 also shows the analytical response of the structure without the damper for the 0.5 Hz natural frequency case; as observed here, the damper significantly reduces the response of the structure (the maximum displacement for the case with the damper is about half of the corresponding one without the damper).

8. Conclusions and future work

An approach for real-time hybrid testing has been presented in which time delay/lag compensation is implemented using model-based response prediction. Two sets of verification experiments were conducted. Experimental results from the first example demonstrate that the proposed approach performs well for both elastic and inelastic response. Model-based compensation allows testing systems with natural frequencies as high as 13 Hz for linear response and 15 Hz for inelastic response, which corresponds to a relatively large bandwidth for hybrid testing. The model-based compensation method allows accurately testing of systems with natural frequencies about twice as large as the traditionally-used polynomial extrapolation method. However, the computationally cost is somewhat larger for the model-based approach. Experimental results from the second example (structure with the MR damper) verify that the model-based approach for delay compensation and testing system presented are capable of accurately testing rate-dependent devices. Although the specimens and actuator used in this study are of small scale and load capacity, the approach is applicable to larger load capacity actuators and test specimens. The method will be used in the near future to test a more complex, multi-degree-of-freedom structure with the MR damper used as a semiactive control device (i.e. the input voltage to the damper will be modified during the test using structural control algorithms). Furthermore, substructure real-time hybrid testing will allow efficient testing of strategies for supplementary energy dissipation and vibration control (including passive, semiactive, and active control devices).

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