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Damage identification of substructure for local health monitoring

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Abstract. A challenging problem in structural damage detection based on vibration data is the requirement of a large number of sensors and the numerical difficulty in obtaining reasonably accurate results when the system is large. To address this issue, the substructure identification approach may be used. Due to practical limitations, the response data are not available at all degrees of freedom of the structure and the external excitations may not be measured (or available). In this paper, an adaptive damage tracking technique, referred to as the sequential nonlinear least-square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) and the substructure approach are used to identify damages at critical locations (hot spots) of the complex structure. In our approach, only a limited number of response data are needed and the external excitations may not be measured, thus significantly reducing the number of sensors required and the corresponding computational efforts. The accuracy of the proposed approach is illustrated using a long-span truss with finite-element formulation and an 8-story nonlinear base-isolated building. Simulation results demonstrate that the proposed approach is capable of tracking the local structural damages without the global information of the entire structure, and it is suitable for local structural health monitoring.

Keywords: local damage identification; local structural health monitoring; substructure technique; adaptive damage tracking.

1. Introduction

The development of a health monitoring system to ensure the reliability and safety of structures has received considerable attention recently. In particular, the ability to detect structural damages, based on measured vibration data, is of practical importance. Various analysis methodologies for structural damage identifications have been proposed (e.g., Bernal & Beck 2004, Lin, *et al.* 2005, Zhou & Yan 2006). However, most of the methodologies available in the literature (e.g., Bernal & Beck 2004) deal with linear structures and require both the reference data (the data without damage) and the data after damage. In practice, however, the reference data may not be available or difficult to establish, since the reference data are affected by the environments, such as temperature. After a severe event, such as a

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strong earthquake, it may not be feasible to conduct vibration tests to obtain meaningful data for damage identifications. It would be desirable for a data analysis method to be capable of detecting the structural damage based solely on the vibration data measured during a severe event, such as a strong earthquake, without a prior knowledge of the undamaged structure. In this connection, several time-domain damage identification methodologies have been developed recently, including the least-square estimation (LSE) (e.g., Lin, *et al.* 2001, Yang and Lin 2004, 2005, 2007a), the extended Kalman filter (EKF) (e.g., Yang, *et al.* 2006a, 2007b), the sequential nonlinear least-square estimation (Yang, *et al.* 2006b, 2007c), and others. Recently, a new technique, referred to as the sequential nonlinear least squares estimation with unknown inputs (excitations) and unknown outputs (responses) (SNLSE-UI-UO), has been developed (Yang and Huang 2006c, 2007c). In this approach, external excitations and some acceleration responses are not needed, so that the number of sensors required in the health monitoring system can be reduced. This technique is capable of tracking the structural damages on-line or almost on-line.

In practical applications, the modeling of engineering structures often involves a large number of degrees of freedom (DOFs), leading to not only numerical and computational difficulties for an accurate damage detection, but also the requirement of excessive number of sensors. It is highly desirable to reduce the required number of sensors as much as possible due to economic considerations and data management. Further, for a complex structure, there may only be a limited number of hot spots or critical areas where damages may likely to occur, and hence the health monitoring can be restricted to such critical areas, referred to as the local health monitoring. This will allow for a significant reduction of the number of required sensors. Consequently, structures can be decomposed into smaller subsystems for the purpose of local damage identification. In this connection, the so-called substructure identification (SSI) approach (e.g. Koh, *et al.* 1991, 2003) can be used.

In this paper, we present an approach using the SNLSE-UI-UO method and the sub-structure technique to identify damages at critical locations of complex structures based on a limited number of sensors and finite-element formulation. Our purpose is to demonstrate the feasibility of the local health monitoring for critical areas without the global information of the structure, thus reducing the required number of sensors and the burden of data management, such as the data transmission and analyses. Simulation results using a long-span truss with finite-element formulation and an 8-story nonlinear base-isolated building will be presented to demonstrate the accuracy of the proposed approach in tracking the local damages without the global information of the entire structure.

2. Sequential nonlinear LSE with unknown inputs and unknown outputs

Let $\mathbf{x} = [x_1(t), x_2(t), ..., x_m(t)]^T$ and $\dot{\mathbf{x}} = [\dot{x}_1(t), \dot{x}_2(t), ..., \dot{x}_m(t)]^T$ be the displacement and velocity vectors, respectively, of a m-DOF nonlinear structure to be considered. The acceleration vector $\ddot{\mathbf{x}}(t) = [\ddot{x}_1(t), \ddot{x}_2(t), ..., \ddot{x}_m(t)]^T$ is divided into two vectors, denoted by $\ddot{\mathbf{x}}^*(t) = [\ddot{x}_1^*(t), \ddot{x}_2^*(t), ..., \ddot{x}_s^*(t)]^T$ and $\bar{\ddot{\mathbf{x}}}(t) = [\bar{\ddot{x}}_1(t), \bar{\ddot{x}}_2(t), ..., \bar{\ddot{x}}_{m-s}(t)]^T$, in which $\ddot{x}_i^*(t)$ $(i = 1, 2, \ldots, s)$ and $\bar{\ddot{x}}_i(t)(i = 1, 2, \ldots, m\text{-}s)$ are unknown (unmeasured) and known (measured) acceleration responses (outputs), respectively. In a similar manner, the external excitations are divided into two vectors, $\mathbf{f}^*(t) = [f_1^*(t), f_2^*(t), \ldots, f_r^*(t)]^T$ and $\mathbf{f}(t) = [f_1(t), f_2(t), ..., f_{\bar{m}}(t)]^T$, where $f_i^*(t)(i = 1, 2, \ldots, r)$ and $f_i(t)$ $(i = 1, 2, \ldots, \bar{m})$ are unknown (unmeasured) and known (measured) excitations (inputs), respectively. The equation of motion of the m-DOF nonlinear structure can be expressed as

$$\overline{\mathbf{M}}\ddot{\mathbf{x}}(t) + \mathbf{F}_c[\dot{\mathbf{x}}(t), \theta] + \mathbf{F}_s[\mathbf{x}(t), \theta] = \eta^*\mathbf{f}^*(t) + \eta\mathbf{f}(t) - \mathbf{M}^*\ddot{\mathbf{x}}^*(t) \tag{1}$$

in which $\overline{\mathbf{M}} = [m\times(m-s)]$ mass matrix corresponding to the (m-s)-known (measured) acceleration response vector $\ddot{\mathbf{x}}(t)$; $\mathbf{M}^* = (m\times s)$ mass matrix corresponding to the s-unknown (unmeasured) acceleration response vector $\ddot{\mathbf{x}}^*(t)$ (or unknown outputs); $\mathbf{F}_c[\dot{\mathbf{x}}(t), \theta]$ = m-damping force vector; $\mathbf{F}_s[\mathbf{x}(t), \theta]$ = m-stiffness force vector; $\eta = m\times\overline{m}$ excitation influence matrix corresponding to measured excitation vector **f**(t); $\eta^* = (m\times r)$ excitation influence matrix corresponding to unmeasured excitation vector $\mathbf{f}^*(t)$; and $\theta = [\theta_1, \theta_2, ..., \theta_n]^T$ is an n-unknown parametric vector with unknown parameters θ_i ($i = 1, 2, \ldots, n$), such as stiffness, damping and nonlinear parameters. For simplicity of presentation, the argument t of all quantities above will be dropped in the following. Further, the bold-face letter represents either a vector or a matrix.

The unknown quantities to be identified are the unknown excitation (input) vector $\mathbf{f}^*$, the unmeasured acceleration response (output) vector $\ddot{\mathbf{x}}^*$, the state vector $\mathbf{X} = [\mathbf{x}^T, \dot{\mathbf{x}}^T]^T$, including the displacement and velocity vectors, and the parametric vector θ. Our objective is to determine not only all the unknown quantities above but also the variation of the parametric vector θ due to structural damages, such as the degradation of stiffness, etc.

The observation equation associated with the equation of motion, Eq. (1), can be written as

$$\varphi(\mathbf{X})\theta + \varepsilon = \bar{\eta}\bar{\mathbf{f}} + y \tag{2}$$

where $\mathbf{X}$ is the state vector defined above, $\varphi(\mathbf{X})$ is the observation matrix that is a function of $\mathbf{X}$, $\mathbf{y} = \eta\mathbf{f} - \overline{\mathbf{M}}\ddot{\mathbf{x}}$ is known, $\bar{\mathbf{f}} = [\mathbf{f}^{*T}\ \ddot{\mathbf{x}}^{*T}]^T$ is an unknown vector consisting of unknown inputs $\mathbf{f}^*$ and unknown outputs $\ddot{\mathbf{x}}^*$, $\bar{\eta} = [\eta^* - \mathbf{M}^*]$, and $\varepsilon(t)$ is the model noise. Eq.(2) can be discretized at $t = t_k = k\Delta t$ as

$$\varphi(\mathbf{X}_k)\theta_k + \varepsilon_k - \bar{\eta}\,\bar{\mathbf{f}}_k = \mathbf{y}_k \tag{3}$$

in which $\mathbf{X}_k = \mathbf{X}(t_k)$, $\varphi(\mathbf{X}_k) = \varphi[\mathbf{X}(t_k)]$, $\theta_k = \theta(t_k)$, $\varepsilon_k = \varepsilon(t_k)$, $\bar{\mathbf{f}}_k = \bar{\mathbf{f}}(t_k)$ and $\mathbf{y}_k = \mathbf{y}(t_k)$. Note that $\mathbf{X}_k$, θ_k and $\bar{\mathbf{f}}_k$ in Eq. (3) are unknown quantities to be estimated. Hence, Eq. (3) is a nonlinear vector equation for unknowns $\mathbf{X}_k$, θ_k and $\bar{\mathbf{f}}_k$.

Instead of solving $\mathbf{X}_k$, θ_k and $\bar{\mathbf{f}}_k$ simultaneously by forming an extended composite unknown vector, $\mathbf{X}_k$, θ_k and $\bar{\mathbf{f}}_k$ are solved in two steps. The first step is to determine θ_k and $\bar{\mathbf{f}}_k$ by assuming (or under the condition) that $\mathbf{X}_k$ is given, and the second step is to determine $\mathbf{X}_k$ through a nonlinear LSE approach, referred to as the sequential nonlinear least square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) (Yang and Huang 2007c).

Let $\hat{\mathbf{X}}_{k+1|k+1}$, $\hat{\theta}_{k+1} = \hat{\theta}_{k+1}(\hat{\mathbf{X}}_{k+1|k+1})$ and $\hat{\bar{\mathbf{f}}}_{k+1|k+1}$ be the estimations of $\mathbf{X}_{k+1}$, θ_{k+1} and $\bar{\mathbf{f}}_{k+1}$, respectively, estimated at $t = (k+1)\Delta t$. Further, let $\hat{\mathbf{X}}_{k+1|k}$ and $\hat{\theta}_{k+1}(\hat{\mathbf{X}}_{k+1|k+1})$ be the estimations of $\mathbf{X}_{k+1}$ and θ_{k+1} estimated at $t = k\Delta t$, respectively. Based on the adaptive SNLSE-UI-UO (Yang and Huang 2007c), if the number of DOFs, m, of the structure is greater than the total number, $s+r$, of unknown inputs and unknown outputs, the recursive solution for $\hat{\theta}_{k+1} = \hat{\theta}_{k+1}(\hat{\mathbf{X}}_{k+1|k+1})$ and $\hat{\bar{\mathbf{f}}}_{k+1|k+1}$ are given by

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \mathbf{K}_{\theta,k+1}[\mathbf{y}_{k+1} - \varphi(\hat{\mathbf{X}}_{k+1|k+1})\hat{\theta}_k + \bar{\eta}\,\hat{\bar{\mathbf{f}}}_{k+1|k+1}] \tag{4}$$

$$\hat{\bar{\mathbf{f}}}_{k+1|k+1} = -\mathbf{S}_{k+1}\bar{\eta}^T[\mathbf{I} - \varphi(\hat{\mathbf{X}}_{k+1|k+1})\mathbf{K}_{\theta,k+1}][y_{k+1} - \varphi(\hat{\mathbf{X}}_{k+1|k+1})\hat{\theta}_k] \tag{5}$$

in which

$$\mathbf{K}_{\theta,k+1} = (\Lambda_{k+1}\mathbf{P}_{\theta,k}\Lambda_{k+1}^T)\varphi^T(\hat{\mathbf{X}}_{k+1|k+1})[\mathbf{I} + \varphi(\hat{\mathbf{X}}_{k+1|k+1})(\Lambda_{k+1}\mathbf{P}_{\theta,k}\Lambda_{k+1}^T)\varphi^T(\hat{\mathbf{X}}_{k+1|k+1})]^{-1} \quad (6)$$

$$\mathbf{S}_{k+1} = \{\bar{\eta}^T[[\mathbf{I} - \varphi(\hat{\mathbf{X}}_{k+1|k+1})\mathbf{K}_{\theta,k+1}]]\bar{\eta}\}^{-1} \quad (7)$$

$$\mathbf{P}_{\theta,k} = [\mathbf{I} + \mathbf{K}_{\theta,k}\bar{\eta}\mathbf{S}_k\bar{\eta}^T\varphi(\hat{\mathbf{X}}_{k|k})][\mathbf{I} - \mathbf{K}_{\theta,k}\varphi(\hat{\mathbf{X}}_{k|k})](\Lambda_{k+1}\mathbf{P}_{\theta,k-1}\Lambda_k^T),\ k = 1, 2, \ldots \quad (8)$$

In equations above, Λ_{k+1} is a diagonal matrix, referred to as the adaptive factor matrix, with diagonal elements $\lambda_1(k+1)$, $\lambda_2(k+1)$,…, $\lambda_n(k+1)$, where $\lambda_j(k+1)$ is referred to as the adaptive factor associated with the jth unknown parameter of θ_{k+1} at $t_{k+1} = (k+1)\Delta t$. The determination of the adaptive factor matrix Λ_{k+1} has been described in Yang & Lin (2005), Yang and Huang (2007c). Further, the estimation $\hat{\mathbf{X}}_{k+1|k+1}$ is needed for the computation of $\varphi\hat{\mathbf{X}}_{k+1|k+1}$ in Eqs. (4)-(8). The recursive solution for $\hat{\mathbf{X}}_{k+1|k+1}$ is given by

$$\hat{\mathbf{X}}_{k+1|k+1} = \hat{\mathbf{X}}_{k+1|k} + \bar{\mathbf{K}}_{k+1}[\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k})] \quad (9)$$

in which

$$\hat{\mathbf{X}}_{k+1|k} = \mathbf{\Phi}_{k+1,k}\hat{\mathbf{X}}_{k|k} + \mathbf{B}_1\ddot{\mathbf{x}}_k + \mathbf{B}_2\ddot{\mathbf{x}}_{k+1} \quad (10)$$

$$\hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k}) = \varphi(\hat{\mathbf{X}}_{k+1|k})\hat{\theta}_{k+1}(\hat{\mathbf{X}}_{k+1|k}) - \bar{\eta}\hat{\bar{\mathbf{f}}}_{k|k} \quad (11)$$

$$\bar{\mathbf{K}}_{k+1} = \bar{\mathbf{P}}_{k+1|k}\mathbf{\Psi}_{k+1,k+1}^T[\mathbf{I} + \mathbf{\Psi}_{k+1,k+1}\bar{\mathbf{P}}_{k+1|k}\mathbf{\Psi}_{k+1,k+1}^T]^{-1} \quad (12)$$

$$\bar{\mathbf{P}}_{k+1|k} = \mathbf{\Phi}_{k+1,k}\bar{\mathbf{P}}_{k|k}\mathbf{\Phi}_{k+1,k}^T \quad (13)$$

$$\bar{\mathbf{P}}_{k|k} = \bar{\mathbf{P}}_{k|k-1} - \bar{\mathbf{K}}_k\mathbf{\Psi}_{k,k}\bar{\mathbf{P}}_{k|k-1} = (\mathbf{I} - \bar{\mathbf{K}}_k\mathbf{\Psi}_{k,k})\bar{\mathbf{P}}_{k|k-1} \quad (14)$$

In Eq. (10), $\mathbf{\Phi}_{k+1,k}$ is the transition matrix for the state vector from k to k+1, and $\mathbf{B}_1$ and $\mathbf{B}_2$ are given by

$$\mathbf{\Phi}_{k+1,k} = \begin{bmatrix} \mathbf{I}_m & (\Delta t)\mathbf{I}_m \\ 0 & \mathbf{I}_m \end{bmatrix};\ \mathbf{B}_1 = \begin{bmatrix} (0.5-\beta)(\Delta t)^2\mathbf{I}_m \\ (1-\gamma)(\Delta t)\mathbf{I}_m \end{bmatrix};\ \mathbf{B}_2 = \begin{bmatrix} \beta(\Delta t)^2\mathbf{I}_m \\ \gamma(\Delta t)\mathbf{I}_m \end{bmatrix} \quad (15)$$

in which $\beta = 0.25$ and $\gamma = 0.5$ are the Newmark-Beta numerical integration coefficients, and $\mathbf{I}_m$ is a (m×m) unit matrix. For those acceleration responses that are not measured (unknown outputs), $\ddot{\mathbf{x}}^*$, we set $\gamma = 0$ and $\beta = 0$ for approximation. In Eq. (11), $\hat{\theta}_{k+1}(\hat{\mathbf{X}}_{k+1|k})$ is obtained from Eqs. (4)-(8) in which on the right hand sides are replaced by $\varphi(\hat{\mathbf{X}}_{k+1|k})$, and $\mathbf{\Psi}_{k+1,k+1} = [\partial\hat{\mathbf{y}}_{k+1}(\mathbf{X}_{k+1})/\partial\mathbf{X}_{k+1}]_{\mathbf{X}_{k+1} = \hat{\mathbf{X}}_{k+1|k}}$ can be expressed analytically as a function of the partial derivative of the data matrix for computation (Huang 2006). Thus, all the unknown quantities can be estimated from the recursive solutions above.

3. Identification of sub-structure

Consider a complex structure, such as the one shown in Fig. 1(a), and suppose we are interested in

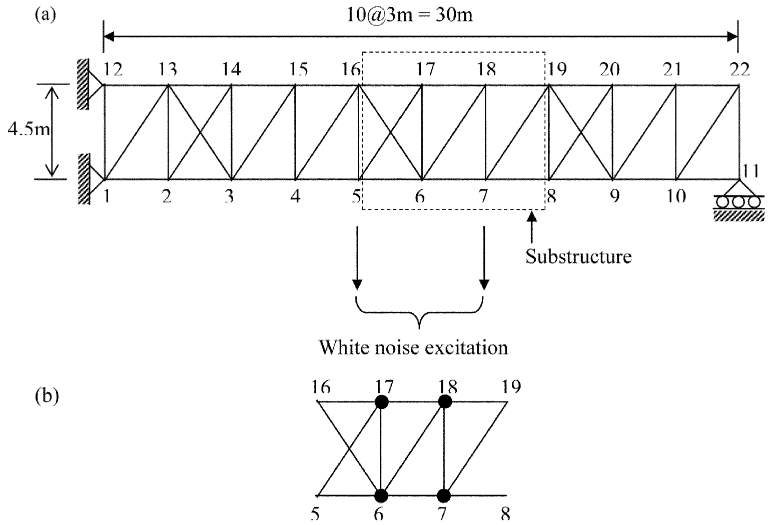

Fig. 1 Long-span truss: (a) full structure with white noise excitation; and (b) substructure

monitoring some critical areas where damages may occur. For simplicity of presentation, let us consider only one critical area, consisting of 12 members as shown in Fig. 1(a) by dashed lines, for the monitoring purpose. This critical area is referred to as the sub-structure as shown in Fig. 1(b). From Fig. 1(b), the sub-structure formed by these 12 critical members consists of 4 masses at nodes 6, 7, 17 and 18, referred to as the internal nodes, and 4 interface nodes at nodes 5, 8, 16 and 19. Let $\mathbf{u}_r(t)$ be the displacement vector of the internal nodes, and $\mathbf{u}_s(t)$ be the displacement vector of the interface nodes. Then, the equation of motion of the sub-structure can be expressed as

$$[\mathbf{M}_{rs} \;\; \mathbf{M}_{rr}]\begin{bmatrix}\ddot{\mathbf{u}}_s(t)\\ \ddot{\mathbf{u}}_r(t)\end{bmatrix} + [\mathbf{C}_{rs} \;\; \mathbf{C}_{rr}]\begin{bmatrix}\dot{\mathbf{u}}_s(t)\\ \dot{\mathbf{u}}_r(t)\end{bmatrix} + [\mathbf{K}_{rs} \;\; \mathbf{K}_{rr}]\begin{bmatrix}\mathbf{u}_s(t)\\ \mathbf{u}_r(t)\end{bmatrix} = [\mathbf{f}_r(t)] \tag{16}$$

in which $\mathbf{f}_r(t)$ is the external excitations to the sub-structure at the internal nodes, and the entire structure has been assumed to be linear elastic for simplicity of presentation.

The interaction effects at the interface nodes can be considered as the inputs (excitations) and the above equation can be expressed as

$$\mathbf{M}_{rr}\ddot{\mathbf{u}}_r(t) + \mathbf{C}_{rr}\dot{\mathbf{u}}_r(t) + \mathbf{K}_{rr}\mathbf{u}_r(t) = \mathbf{f}_r(t) - \mathbf{M}_{rs}\ddot{\mathbf{u}}_s(t) - \mathbf{C}_{rs}\dot{\mathbf{u}}_s(t) - \mathbf{K}_{rs}\mathbf{u}_s(t) \tag{17}$$

Now, some of the acceleration responses of the internal nodes may not be measured, referred to as the unknown outputs $\ddot{\mathbf{x}}^*$ in Eq. (1), and some of the accelerations at the interface nodes may not be measured, referred to as unknown inputs (excitations) $\mathbf{f}^*$ in Eq. (1). When some acceleration responses at the interface nodes are measured, their corresponding velocity and displacement responses are obtained by the Newmark-β integration method and these terms on the right hand side of Eq. (17) should be moved to the left hand side. Thus Eq. (17) can be cast appropriately into the form of Eq. (1), and the SNLSE-UI-UO solution described previously can be used.

4. Simulation results

To demonstrate the accuracy of the sub-structure technique using adaptive SNLSE-UI-UO for parametric identifications and damage detections at critical locations, a long-span truss with the finite element formulation and an 8-story non-linear base-isolated building will be considered. For both examples, the sampling frequency is 500 Hz for all measured responses.

4.1. Long-span truss with finite-element model

A planar long-span truss, consisting of 44 members (or elements) and a total of 41 DOFs as shown in Fig. 1(a), will be considered (Bernal 2002, Gao and Spencer 2002). As observed from Fig. 1(a), the truss is statically indeterminate. Now, only the substructure shown in Fig. 1(b) will be identified and monitored. The finite-element substructure model consists of 12 members with uniform cross-section, 4 internal nodes, and 4 interface nodes, where each node has 2 DOFs (horizontal and vertical). Twelve critical members (or elements) to be monitored in Fig. 1(b) are denoted as follows: member 1 (nodes 5-6), member 2 (nodes 6-7), member 3 (nodes 7-8), member 4 (nodes 16-17), member 5 (nodes 17-18), member 6 (nodes 18-19), member 7 (nodes 5-17), member 8 (nodes 6-18), member 9 (nodes 7-19), member 10 (nodes 6-17), member 11 (nodes 7-18), member 12 (nodes 6-16).

Let $\mathbf{M}_i$ and $\mathbf{K}_i$ be the local mass matrix and the local stiffness matrix, respectively, of the ith element (member) with an uniform cross-section in the local coordinate system,

$$\mathbf{M}_i = \frac{\overline{m}_i L_i}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}; \quad \mathbf{K}_i = k_i\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

in which L_i and $\overline{m}_i$ are the length and the mass per unit length of the ith element (or member) of the sub-structure, respectively, and $k_i = E_i A_i / L_i$ is the equivalent stiffness parameter, where E_i and A_i are the Young's modulus and cross-sectional area of the ith element (or member), respectively. The local element mass and element stiffness matrices $\mathbf{M}_i$ and $\mathbf{K}_i$ are transformed into $\overline{\mathbf{M}}_i$ and $\overline{\mathbf{K}}_i$, which are the element matrices in the global coordinate system of the sub-structure, using the transformation matrix $\mathbf{T}$, i.e.,

$$\overline{\mathbf{M}}_i = \mathbf{T}^T\mathbf{M}_i\mathbf{T}; \quad \overline{\mathbf{K}}_i = \mathbf{T}^T\mathbf{K}_i\mathbf{T} \tag{19}$$

in which $\mathbf{T}$ is a (4×4) matrix with its (i, j) element, $\mathbf{T}_{ij}$, as: $\mathbf{T}_{11}=\mathbf{T}_{22}=\mathbf{T}_{33}=\mathbf{T}_{44}=\cos\phi$, $\mathbf{T}_{12}=\mathbf{T}_{34}=\sin\phi$, $\mathbf{T}_{21}=\mathbf{T}_{43}=-\sin\phi$, and $\mathbf{T}_{ij}=0$ for other i and j, where ϕ = the angle between the local and global coordinates. Finally, the element mass and stiffness matrices $\overline{\mathbf{M}}_i$ and $\overline{\mathbf{K}}_i$ are expanded to (m×m) matrices denoted by $\tilde{\mathbf{M}}_i$ and $\tilde{\mathbf{K}}_i$, and the global mass and stiffness matrices $\mathbf{M}$ and $\mathbf{K}$ of the sub-structure, Fig.1(b), are obtained by summing up $\tilde{\mathbf{M}}_i$ and $\tilde{\mathbf{K}}_i$ for all the elements, i.e.

$$\mathbf{M} = \sum_{i=1}^{p}\tilde{\mathbf{M}}_i; \quad \mathbf{K} = \sum_{i=1}^{p}\tilde{\mathbf{M}}_i = \sum_{i=1}^{p}k_i\mathbf{S}_i \tag{20}$$

in which for simplicity of presentation $\tilde{\mathbf{K}}_i$ is expressed in terms of $k_i\mathbf{S}_i$, where $k_i = E_iA_i/L_i$ is the equivalent stiffness parameter and $\mathbf{S}_i$ is a (m×m) matrix of the ith element. In Eq. (20), p is the total number of elements (members).

In the literature, the Rayleigh damping is usually assumed and the damping matrix **C** is expressed as:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{21}$$

in which α and β are the mass-proportional and the stiffness-proportional damping coefficients. All the truss members are made of steel (with $E = 200$ Gpa) with an area of 64.5 cm^2. For simplicity of computation, the element mass matrix $\mathbf{M}_i$ is approximated by a diagonal matrix with diagonal element 1.75×10^5 kg (Bernal 2002), and hence the global mass matrix **M** is diagonal after transformation. The structural parameters are: $k_i = 430$ MN/m ($i = 1,2,...,6$), $k_i = 238.52$ MN/m ($i = 7,8,9, 12$), $k_i = 286.67$ MN/m ($i = 10, 11$), $\alpha = 0.1064$ s^{-1} and $\beta = 3.4\times10^{-3}$ s. With the structural properties above, the first three natural frequencies and the corresponding modal damping ratios are: $\omega_i = 0.64$, 1.19 and 1.54 Hz, and $\zeta_i = 2\%$, 1.99% and 2.2%.

Suppose the truss shown in Fig. 1(a) is subject to two vertical white noise excitations applied vertically at nodes 5 and 7. The measured responses include: (i) the horizontal and vertical accelerations at all interface nodes and internal node 18, (ii) the horizontal acceleration of internal node 6, and (iii) the vertical accelerations of internal nodes 7 and 17. Note that the horizontal accelerations of internal nodes 7 and 17, the vertical acceleration of internal node 6, and all external white noise excitations $\mathbf{f}^*(t)$ at nodes 5 and 7 are not measured (unknown). All the measured quantities are simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. The sampling frequency is 500Hz. The unknown quantities to be identified include: α, β, k_i ($i = 1, 2,\ldots, 12$), the state vector, and the unknown white noise excitation $\mathrm{f}^*(t)$ at node 7.

The initial guesses for α, β and k_i are $\alpha_0 = 0.2$ s^{-1}, $\beta_0 = 6\times10^{-3}$ s, $k_{i,0} = 300$ MN/m ($i = 1, 2, \ldots, 12$), respectively. The initial values for the state vector and the unknown excitation are zero, and the initial gain matrices $\mathbf{P}_{\theta,0}$ and $\bar{\mathbf{P}}_{0|0}$ are taken to be $\mathbf{P}_{\theta,0} = 10^{-2}\,\mathbf{I}_{25}$ and $\bar{\mathbf{P}}_{0|0} = 10^6\mathbf{I}_{16}$.

Two damage patters are considered. For damage pattern 1, a damage occurs at $t = 5$ sec, at which time the equivalent stiffness k_2 of member 2 in Fig. 1(b) is reduced linearly from 430 MN/m to 344 MN/m (20% reduction) within 2 seconds. Based on the adaptive SNLSE-UI-UO and sub-structure techniques, the identified structural parameters are presented in Fig. 2 as solid curves. Also shown in Fig. 2 as dashed curves are the theoretical results for comparison.

For damage pattern 2, a damage occurs at $t = 5$ sec, at which time the equivalent stiffness k_2 of member 2 is reduced abruptly from 430 MN/m to 301 MN/m, then another damage occurs at $t = 7$ sec, at which time the equivalent stiffness k_6 of member 6 is reduced abruptly from 430 MN/m to 365.5 MN/m. Based on the adaptive SNLSE-UI-UO technique, the identified structural parameters are presented in Fig. 3 as solid curves, whereas the theoretical results are shown as dashed curves for comparison. The identified white noise excitation $\mathrm{f}^*(t)$ at node 7 for a segment from 2 to 2.2 seconds is presented in Fig. 4 as a solid curve, whereas the dashed curve in the same figure is the theoretical result. It is observed from Figs. 2-4 that the adaptive SNLSE-UI-UO technique tracks the substructural parameters, their variations due to damage, and the unknown excitation very well.

Simulation studies for a long-span bridge similar to that shown in Fig. 1(a) and subject to earthquake excitations have been conducted. Numerical results also indicate that the adaptive SNLSE-UI-UO along with the sub-structural technique is capable of identifying local structural damages with impressive accuracy without the information of the global structure (Yang & Huang 2006d). Due to space limitation, these simulation results are not presented herein.

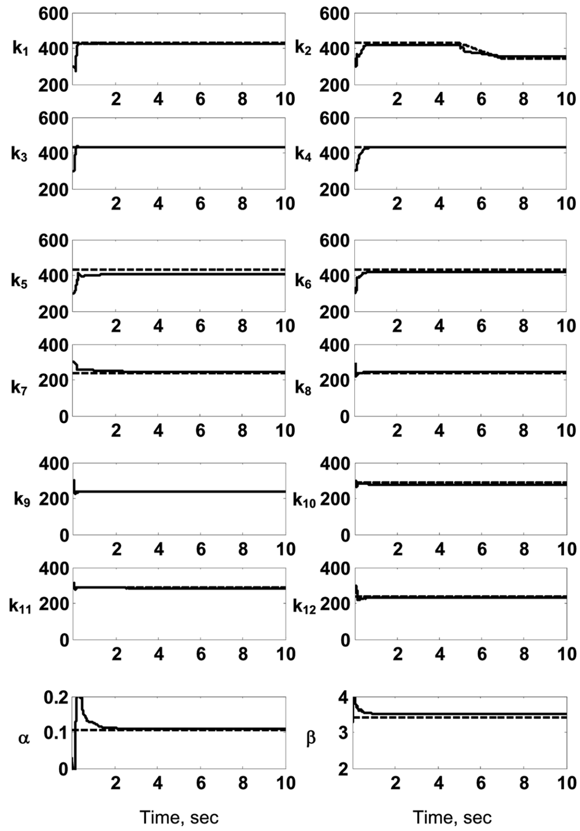

Fig. 2 Identified parameters for a substructure of a long span truss (Case 1); (damage pattern 1) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec

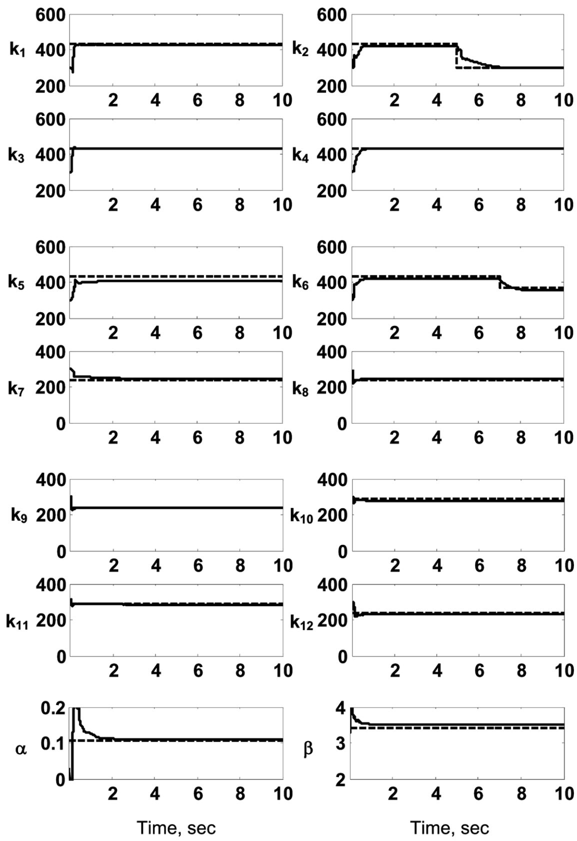

Fig. 3 Identified parameters for a substructure of a long span truss (Case 1); (damage pattern 2) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec

4.2. 8-story non-linear base-isolated building

Consider an eight-story non-linear hysteretic shear-beam building subject to an earthquake ground acceleration $\ddot{x}_0(t)$, as shown in Fig. 5(a). The properties of the building are as follows: (i) the mass of each floor is identical with m_i = 345.6 metric tons; (ii) the stiffness k_i (i = 1,2,...,8) of eight-story units are: 340.4, 325.7, 284.9, 268.6, 243, 207.3, 168.7 and 136.6 MN/m, respectively; (iii) the linear viscous damping coefficients c_i (i = 1,2,...,8) for each story unit are 490, 467, 410, 386, 348, 298, 243 and 196 kN·sec/m, respectively. A lead-core rubber bearing isolation system is used to reduce the response of the building. The stiffness restoring force of the lead-core rubber-bearing is model by the Bouc-Wen model (Wen (1989).

$$F_{sb} = \alpha_b k_b x_b + (1 - \alpha_b) k_b D_{yb} v_b \tag{22}$$

in which the subscript b stands for the base-isolation system, x_b is the drift of the isolator, k_b is the stiffness, α_b is the ratio of the post yielding stiffness to the pre-yielding stiffness, D_{yb} is the yielding deformation, and v_b is the hysteretic component. The hysteretic component, v_b, is modeled by

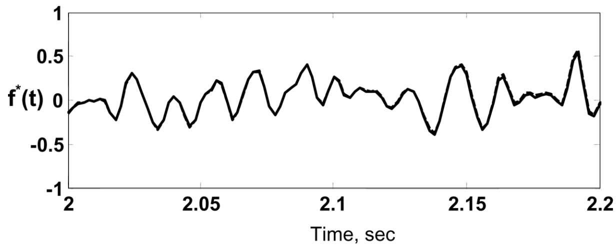

Fig. 4 Identified unknown white noise excitation for long-span truss; unit of $f^*(t)$ in 10^4 N

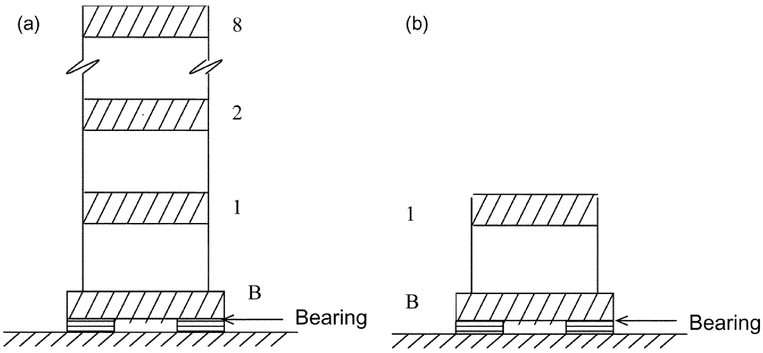

Fig. 5 An 8-story non-linear base-isolated building: (a) full structure; (b) substructure

$$\dot{v}_b = D_{yb}^{-1}[A_b\dot{x}_b + -\beta_b|\dot{x}_b||v_b|^{n_b-1}v_b - \gamma_b\dot{x}_b|v_b|^{n_b}] = f_b(x_b, v_b) \tag{23}$$

in which A_b, β_b, n_b and γ_b are parameters characterizing the hysteresis loop. Properties of the base-isolation system are: $m_b = 450$ metric tons, $k_b = 180.5$ MN/m, linear viscous damping $c_b = 26.17$ kN.sec/m, $\alpha_b = 0.6$, $D_{yb} = 4$ cm, $A_b = 1.0$, $\beta_b = 0.5$, $n_b = 3$ and $\gamma_b = 0.5$. For a small amplitude vibration (linear), the first natural frequency is $\omega_1 = 5.24$ rad/sec. The El Centro earthquake $\ddot{x}_0(t)$ with a peak ground acceleration of 0.3 g (PGA = 0.3 g) is considered as the external excitation.

A substructure, consisting of the rubber bearing and the first story as shown in Fig. 5(b), is considered as critical location for damage identification. In this example, the equation of motion is expressed in terms of the coordinate x_i representing the inter-story drift of the ith story. Two different cases will be considered.

4.2.1. Case 1: Earthquake excitation is measured

The absolute accelerations of the isolator and the first floor, $\ddot{x}_{ba}$ and $\ddot{x}_{1a}$, and the El Centro earthquake ground acceleration, $\ddot{x}_0(t)$, are measured. All measured quantities are simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. In this case, the RMS of a particular response signal is computed from the temporal average over 30 seconds. The sampling frequency is 500Hz. Parameters α_b, D_{yb}, A_b and n_b are assumed to be known constants. The unknown quantities to be identified are: c_1, k_1, c_2, k_2, c_b, k_b β_b and γ_b, as well as the state vector of the substructure.

Suppose a damage occurs at $t = 15$ sec, at which time the equivalent stiffness k_b is reduced abruptly from 180.5 MN/m to 144.4 MN/m (20% reduction). The initial guesses for c_i, k_i, c_b, k_b, β_b and γ_b are: $c_{i,0}$

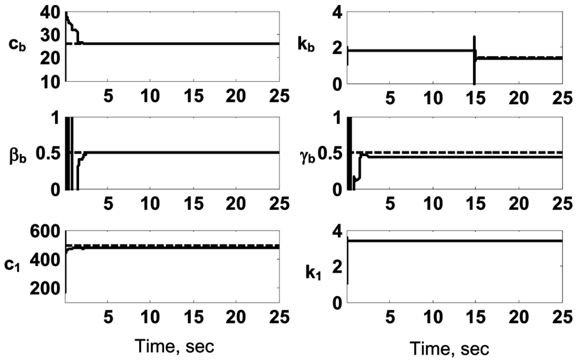
Fig. 6 Identified parameters for a substructure of a 8-story base-isolated building (Case 1); k_b in 10^4 kN/m, k_1 in 10^5 kN/m, c_b and c_1 in kN.s/m

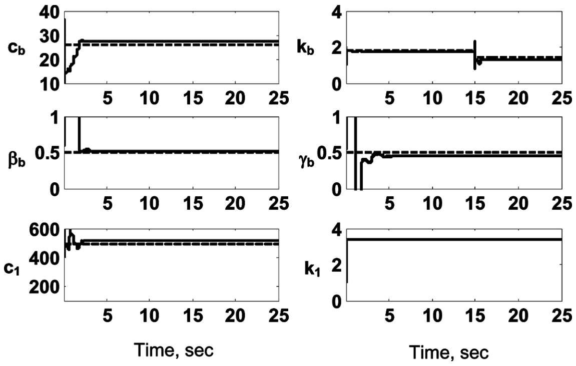
Fig. 7 Identified parameters for a substructure of a 8-story base-isolated building (Case 2); k_b in 10^4 kN/m, k_1 in 10^5 kN/m, c_b and c_1 in kN.s/m

= 300 kN·sec/m, $k_{i,0}$ = 100 MN/m (i = 1, 2), $c_{b,0}$ = 10 kN·sec/m, $k_{b,0}$ = 10 MN/m, $\beta_{b,0}$ = 1, and $\gamma_{b,0}$ = 1, respectively. The initial values for the state variables are zero, and the initial gain matrices $\mathbf{P}_{\theta,0}$ and $\bar{\mathbf{P}}_{0|0}$ are taken to be $\mathbf{P}_{\theta,0} = 10^{10}\,\mathbf{I}_6$ and $\bar{\mathbf{P}}_{0|0} = \mathbf{I}_4$. Based on the adaptive SNLSE-UI-UO technique, the identified parameters for the sub-structure are presented in Fig. 6 as solid curves. Since the predictive results for c_2 and k_2 are similar to that for c_1 and k_1, they are not presented in the figure. Also shown in Fig. 6 as dashed curves are the theoretical results for comparison. It is observed from Fig. 6 that the proposed approach is able to track the sub-structural parameters and their variations due to damage.

4.2.2. Case 2: Earthquake excitation is not measured

In this case, inter-story drifts x_b, x_1 and x_2 are measured. The earthquake ground acceleration $\ddot{x}_0(t)$ is not measured and hence it is unknown. The measured drifts were simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. Finally, the inter-story accelerations $\ddot{x}_b$, $\ddot{x}_1$ and $\ddot{x}_2$ were computed by differentiations. Similar to Case 1, parameters α_b, D_{yb}, A_b and n_b are assumed to be known constants. The unknown parameters to be identified are: c_1, k_1, c_2, k_2, c_b, k_b β_b, γ_b, the unknown earthquake excitation $\ddot{x}_0(t)$, and the state vector of the substructure.

Suppose a damage occurs at t = 15 sec, at which time the equivalent stiffness k_b is reduced abruptly from 180.5 MN/m to 144.4 MN/m (20% reduction). The initial unknown excitation is zero, and the following assumed initial values and matrices are identical to that of Case 1 above: (i) initial state variables, (ii) initial parametric values c_1, k_1, c_2, k_2, c_b, k_b β_b and γ_b, and (iiii) $\mathbf{P}_{\theta,0}$ and $\bar{\mathbf{P}}_{0|0}$. Based on the adaptive SNLSE-UI-UO technique, the identified parameters are presented in Fig. 7 as solid curves. Also shown in Fig. 7 as dashed curves are the theoretical results for comparison. Again, the predicted results for c_2 and k_2 are not presented, since these results are similar to that of c_1 and k_1 shown in Fig. 7. The identified earthquake ground acceleration $\ddot{x}_0(t)$ for a segment from 2 to 5 seconds is presented in Fig. 8 as a solid curve, whereas the dashed curve is the theoretical result. It is observed from Figs. 7 and 8 that the adaptive SNLSE-UI-UO is able to track both the structural parameters and their variations due to damage, as well as the unknown earthquake excitation. Finally, the predicted hysteresis loops for the stiffness restoring force F_{sb} versus the drift x_b of the base isolator are presented in Fig. 9 as solid curves, whereas the dotted curves represent the theoretical results. As shown in Fig. 9, the predictive capability of our approach is quite reasonable.

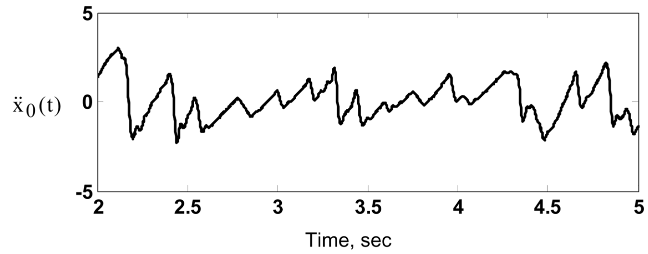

Fig. 8 Identified unknown earthquake ground acceleration for 8-story base-isolated building; unit of earthquake acceleration $\ddot{x}_0(t)$ in m/s^2

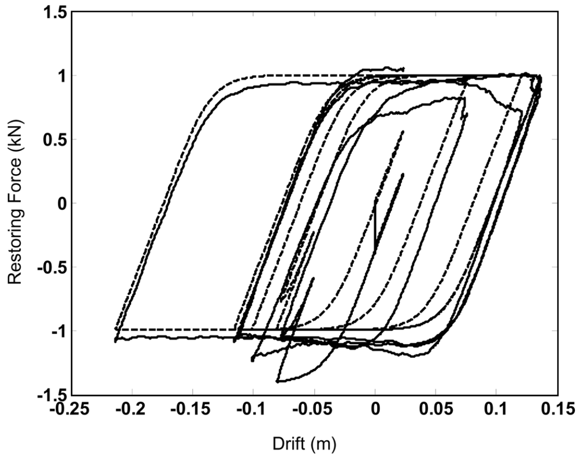

Fig. 9 Identified hysteresis loops for the rubber-bearing of 8-story base-isolated building

Similar to other time-domain analysis methods [e.g., Lin, *et al.* (2001)], the use of the adaptive SNLSE requires the initial estimates of unknown parameters and covariance matrices to initiate the recursive solution. To guarantee the convergence of the solution, these initial values should be physically reasonable, for instance the initial estimates for the damping coefficients and stiffness should be positive values. Then, as long as the initial estimates are in the same order of magnitude as the true values, the solution should converge. Further, matrices $\mathbf{P}_{\theta,0}$ and $\bar{\mathbf{P}}_{0|0}$ are covariance matrices of estimation errors, and their orders of magnitude can be estimates based on the estimated response quantities (Yang & Huang (2007c).

5. Conclusions

In this paper, the recently proposed adaptive sequential nonlinear least-square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) (Yang, *et al.* (2007c) along with the sub-structure approach have been used to identify structural damages at critical locations of a complex structure. This approach allows for the damage monitoring of critical sub-structures without the need of information for the global complex structure, thus reducing significantly the total number of sensors required. Even for the critical sub-structure, the external excitations (inputs) and some acceleration responses (outputs) are not required to be measured, again reducing the required number of sensors.

Simulation results using a long-span truss with finite-element formulation and an 8-story hysteretic base-isolated building demonstrate that the proposed approach is: (1) capable of identifying local structural damages using a limited number of sensors, and (2) suitable for local health monitoring without the global information of the complex structure.

Experimental verifications for the proposed local damage detection technique are important for practical applications. Experimental tests using a small-scaled building model have been conducted to verify the capability of the SNLSE-UI-UO approach (Yang & Huang (2007d). Currently, experimental tests are being conducted to verify the validity of the local damage tracking technique proposed herein. Finally, the sensitivity of the damage detection capability is related to the number of sensor measurements. The problem of determining the minimum number of sensors and their optimal locations for the satisfactory damage detection is a challenging problem of current research.

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