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Determination of optimal accelerometer locations using modal sensitivity for identifying a structure

Soon-Jung Kwon*

Korail Research Institute, Korea Railroad Cooperation,
Daejeon Government Complex, 139 Seonsa-ro, Seo-gu, Daejeon City, 302-701 Korea

Sungkwon Woo‡ and Soobong Shin‡†

Dept. of Civil Eng., Inha University, 253 Yonghyun-dong, Nam-gu, Incheon 402-751, Korea

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Abstract. A new algorithm is proposed to determine optimal accelerometer locations (OAL) when a structure is identified by frequency domain system identification (SI) method. As a result, a guideline is presented for selecting OAL which can reflect modal response of a structure properly. The guideline is to provide a minimum number of necessary accelerometers with the variation in the number of measurable target modes. To determine OAL for SI applications effectively, the modal sensitivity effective independence distribution vector (MS-EIDV) is developed with the likelihood function of measurements. By maximizing the likelihood of the occurrence of the measurements relative to the predictions, Fisher Information Matrix (FIM) is derived as a function of mode shape sensitivity. This paper also proposes a statistical approach in determining the structural parameters with a presumed parameter error which reflects the epistemic paradox between the determination of OAL and the application of a SI scheme. Numerical simulations have been carried out to examine the proposed OAL algorithm. A two-span multi-girder bridge and a two-span truss bridge were used for the simulation studies. To overcome a rank deficiency frequently occurred in inverting a FIM, the singular value decomposition scheme has been applied.

Keywords: OAL; SI; MS-EIDV; FIM; mode shape sensitivity.

1. Introduction

Civil structures are large in size and complex so that analytical models require a large number of degrees of freedom (DOF). Therefore, it has been acknowledged that the selection of measurement locations may influence on the estimation of structural parameters. To estimate structural parameters by using measured structural dynamic responses, system identification (SI) methods have been widely developed and applied in the frequency-domain (EI-Borgi, *et al*. 2005, Hjelmstad and Shin 1996, Jang, *et al*. 2002, Vestouni, *et al*. 2000) and in the time-domain (Ge and Soong 1998, Hjelmstad and Banan 1995, Huang 2001, Kang, *et al*. 2005). Since it is hard to measure rotational DOF and responses around the supports in actual applications, the number of measurable DOF is generally limited. Therefore, it is

*Deputy General Manager, E-mail: sjkwon21@hanmail.net
‡Assistant Professor
‡†Associate Professor, Corresponding author, E-mail: sbshin@inha.ac.kr

essential to select optimal measuring DOF to introduce better parameter estimation by applying system identification (SI) method. Especially when measurement noise is considered in the estimation, the correctness of SI results may be more dependent on the selection of measuring DOF. In spite that the influence of selecting optimal accelerometer locations (OAL) on the final parameter estimation cannot be ignored, the problem of optimal sensor locations (OSL) has not been considered seriously so far.

To measure structural dynamic responses, various types of sensors are used in field tests. Among them, accelerometers are mainly used to sense the global dynamic response of a structure. For an ambient vibration test, the locations of accelerometers can be moved sequentially with a fixed reference one to identify modal parameters of a whole bridge (Jung, *et al*. 2002). However, even in this case, the number of measured DOF is still limited compared with the total DOF of an analytical model for a bridge. Also, the sequential moving of the accelerometers does require working hours to install and connect the equipments. For a long-term health monitoring of a bridge, durability of accelerometers should be considered so that the number of permanently installed accelerometers is usually limited as well. Conventionally measurement locations have been selected based on engineers' instinct or structural symmetry rather than based on a mechanical formulation for the structural behaviors. Even when the number of accelerometers is limited, the priority of measurement locations could not be determined through any theoretical background. Some available algorithms for selecting optimal locations include 'optimum sensor location problem' (Kirkegaard and Brincker 1994, Udwadia 1994), 'optimal sensor locations' (Fadale, *et al*. 1995, Li and Yam 2001), 'optimum measurement locations' (Penny, *et al*. 1994) and 'optimal sensor placement' (Cherng 2003, Kammer 1996, Meo and Zumpano 2005, Tongco and Meldrum 1996).

Even though an OAL scheme is well-prepared, its application for SI is blocked by a paradox that an OAL is determined by pre-defined structural parameters while a SI is to estimate unknown structural parameters. These conflicting objectives cannot be resolved without additional information on the structure. The current paper proposes a statistical approach using the designed values for the structure. It determines OAL by assuming an error bound between the actual and known designed values of parameters and by applying Monte Carlo iterations with random errors within the bound.

The paper proposes a new algorithm for determining OAL and examines it through simulation studies. A frequency domain SI method has been utilized to verify its usefulness. A two-span multi-girder bridge and a two-span truss bridge were used for the simulation studies.

2. Algorithm for determining OAL

2.1. Problem definition by maximum likelihood estimation

The maximum likelihood estimation method was introduced by Fisher (O'Connor and Robertson 2003) and has been widely applied to provide statistical backgrounds in solving engineering problems. The main idea of the maximum likelihood estimation method is to obtain the best estimates of structural parameters by maximizing the likelihood of the occurrence of the measurements relative to the predictions. If noise of sensors are mutually independent and have a normal distribution with zero mean and variance of σ_n^2, the optimal values of parameter $\underset{\sim}{\mathbf{x}}$ can be correctly estimated by maximizing the probability density function of measurements $f(\tilde{\mathbf{\Phi}}_m|\mathbf{x})$ where $\tilde{\mathbf{\Phi}}_m$ is modal displacement matrix defined by Eq.(1) with limited *nmm* modes measured at limited DOF (Fadale, *et al*. 1995).

$$\tilde{\Phi}_m = \begin{bmatrix} \tilde{\phi}_{11} & \cdots & \tilde{\phi}_{1nmm} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \tilde{\phi}_{Nm1} & \cdots & \tilde{\phi}_{Nm\,nmm} \end{bmatrix} \tag{1}$$

where N_m = the number of measured DOF and nmm = the number of measured modes.

If statistical parameters are defined, the probability density function of measurements $f(\tilde{\Phi}_m|\mathbf{x})$ can be defined by Eq. (2) as a likelihood function.

$$f(\tilde{\Phi}_m|\mathbf{x}) = \left[(2\pi)^{N_m \times nmm} \prod_{i=1}^{nmm} \mathrm{Det}(\mathbf{C}_i)\right]^{-1/2} \times \exp\left[-\frac{1}{2}\sum_{i=1}^{nmm} (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi})^T \mathbf{C}_i^{-1} (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi})\right] \tag{2}$$

where $\mathbf{x}_{(N_p \times 1)}$ = unknown structural parameter vector, $\tilde{\boldsymbol{\phi}}_{ai(N_m \times 1)}(\mathbf{x})$ = the computed i-th mode shape vector at the limited N_m DOF, $\tilde{\boldsymbol{\phi}}_{mi(N_m \times 1)}$ = measured i-th mode shape vector at N_m DOF, $\mathrm{C}_{i(N_m \times N_m)}$ = the covariance matrix of the measurements of the i-th mode, and N_p = number of unknown parameters, respectively.

If Eq. (2) is modified in the log scale, the following expression of Eq. (3) can be obtained.

$$L^* = -2\ln f(\tilde{\Phi}_m|\mathbf{x}) = N_m + nmm\ln(2\pi) + \sum_{i=1}^{nmm} \ln\mathrm{Det}(\mathbf{C}_i) + (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi})^T \mathbf{C}_i^{-1} (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi}) \tag{3}$$

Since the first and the second terms of the right-hand-side of Eq.(3) are independent of the parameters **x** and can be constant if noise in measurements can be considered as mutually independent, Eq. (3) can be simplified to Eq. (4).

$$L(\mathbf{x}) = \sum_{i=1}^{nmm} (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi})^T \mathbf{C}_i^{-1} (\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi}) \tag{4}$$

Also, if measurement noise and structural parameters **x** are mutually independent and the variance σ_n^2 is identical regardless of the measurement DOF, the covariance matrix **C** can be expressed by $\mathbf{C} = \sigma_n^2\mathbf{I}$ so that Eq. (5) can be formulated as the final minimization problem in estimating the structural parameters **x** instead of the maximization of the probability density function of Eq. (2).

$$\underset{\mathbf{x} \in \Re^{N_p}}{\text{Minimized}}\ L = \sigma_n^{-2} \sum_{i=1}^{nmm} \left\|\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi}\right\|^2 \tag{5}$$

In solving the structural parameters **x** through the minimization of the least-squared error, $nmm \times N_m \geq N_p$ should be satisfied as an identifiability criterion to escape from multiple solutions with under-determined systems of equations.

2.2. Formulation of Fisher information matrix

To estimate unknown structural parameters, the information provided by experiments should be maximized. Since the test plans are determined by considering allowable test environments, it is necessary to set up the basis in comparing the quality of different experiments. To define such an optimal basis, most available methods are usually estimating structural parameters through the minimization of estimation error. Among those methods, by applying the Cramer-Rao inequality, the estimation error has a lower bound of $\mathbf{F}^{-1}$, where $\mathbf{F}$ = Fisher information matrix (FIM) defined by Eq. (6) (Goodwin and Payne 1977).

$$\mathbf{F}(\tilde{\mathbf{\Phi}}_m,\mathbf{x}) = E_{\tilde{\mathbf{\Phi}}_m|\mathbf{x}}\left\{\left[\frac{\partial \log f(\tilde{\mathbf{\Phi}}_m|\mathbf{x})}{\partial \mathbf{x}}\right]\left[\frac{\partial \log f(\tilde{\mathbf{\Phi}}_m|\mathbf{x})}{\partial \mathbf{x}}\right]^t\right\} \tag{6}$$

By substituting Eq. (2) into Eq. (6), the following equation of Eq. (7) can be derived.

$$\mathbf{F}(\tilde{\mathbf{\Phi}}_m,\mathbf{x}) = E_{\tilde{\mathbf{\Phi}}_m|\mathbf{x}}[\{\nabla \ln(\tilde{\mathbf{\Phi}}_m|\mathbf{x})\}\{\nabla \ln(\tilde{\mathbf{\Phi}}_m|\mathbf{x})\}^T]$$

$$\mathbf{F}(\tilde{\mathbf{\Phi}}_m,\mathbf{x}) = \sum_{i=1}^{nmm}[\mathbf{\Theta}_i^T\mathbf{C}_i^{-1}\mathbf{\Theta}_i + \Psi_i] \quad (N_p \times N_p) \tag{7}$$

where $\mathbf{\Theta}_i$ is defined by Eq.(8) as a sensitivity matrix for mode i each column of which is the mode shape sensitivity with respect to each unknown parameter and $\mathbf{\Psi}_i$ is a function defined by Eq. (9).

$$\mathbf{\Theta}_i = \left(\frac{\partial \tilde{\phi}_k}{\partial x_l}\right)_i \quad k = 1, 2,\ldots, N_m \; l = 1, 2,\ldots, N_p \; i = 1, 2,\ldots, nmm \tag{8}$$

$$(\mathbf{\Psi}_i)_{lm} = \frac{1}{2}tr\left[\mathbf{C}_i^{-1}\frac{\partial \mathbf{C}_i}{\partial x_l}\mathbf{C}_i^{-1}\frac{\partial \mathbf{C}_i}{\partial x_m}\right] \quad i = 1, 2,\ldots, nmm \; l, m = 1, 2, \ldots, N_p \tag{9}$$

If we can use the same assumption on the covariance matrix $\mathbf{C}$ of $\mathbf{C} = \sigma_n^2\mathbf{I}$, $\partial \mathbf{C}_i/\partial x_j = 0$ can be concluded. Also, if the sensors are exposed to the same type of measurement noise as assumed, the measurement noise is independent of the structural parameters $\mathbf{x}$. Then the FIM can be simplified to Eq. (10).

$$\mathrm{F}(\tilde{\mathbf{\Phi}}_m,\mathbf{x}) = \sum_{i=1}^{nmm}[\mathbf{\Theta}_i^T\mathbf{\Theta}_i] \qquad (N_p \times N_p) \tag{10}$$

2.3. Computation of mode shape sensitivity

The mode shape sensitivity of Eq. (8) can be computed directly by solving Eq. (11) when only distinct eigenvalues are considered. For a case of multiple eigenvalues, a similar direct computation is also proposed by Lee and Jung (1997).

$$\begin{bmatrix} \mathbf{K} - \lambda_i\mathbf{M} & -\mathbf{M}\phi_i \\ -\phi_i^T\mathbf{M} & 0 \end{bmatrix}\begin{Bmatrix} \partial \phi_i/\partial x_j \\ \partial \lambda_i/\partial x_j \end{Bmatrix} = \begin{Bmatrix} -\mathbf{G}_j \\ 0.5\phi_i^T\mathbf{M} \end{Bmatrix}\phi_i \tag{11}$$

where $\mathbf{K}_{(N\times N)}$, $\mathbf{M}_{(N\times N)}$ = stiffness and mass matrix, l_i = eigenvalue of mode i, x_j = the j-th structural parameter, and $\mathbf{G}_{j(N\times N)} = \partial \mathbf{K}(\mathbf{x})/\partial x_j$ = the j-th kernel matrix composed of constant coefficients.

In case $\mathbf{M}$ can be assumed as known and $\mathbf{K}$ and ϕ_i can be considered as given with estimated parameters $\mathbf{x}$ at an iteration step during the estimation process, the mode shape sensitivity can be expressed by

$$(\tilde{\mathbf{S}}_i)_j = \partial \tilde{\phi}_i/\partial x_j = D_j^{(i)}\tilde{\phi}_i \tag{12}$$

where $\mathbf{D}^{(i)}_{j(N_m \times N_m)}$ = a constant coefficient matrix as a function of (λ_i, $\mathbf{x}$, $\boldsymbol{\phi}_i$, $\mathbf{K}(\mathbf{x})$, $\mathbf{M}$, $\mathbf{G}_j$).

Depending on the computed $\mathbf{G}_j$, the linear independence of the column vectors in $\tilde{\mathbf{S}}_i$ may not be guaranteed so that a rank deficiency problem can occur. For a critical case that $\mathbf{G}$ is constant for all the members, rank($\tilde{\mathbf{S}}_i$)= 1 can be possibly occurred. To resolve such a rank deficiency problem involved in the sensitivity matrix, the singular value decomposition scheme has been applied. The sensitivity matrix can be decomposed with singular values as

$$\tilde{\mathbf{S}}_i = \mathbf{U}_i \mathbf{W}_i \mathbf{V}_i^T \tag{13}$$

where $\mathbf{U}_{i(N_m \times N_p)}$, $\mathbf{W}_{i(N_p \times N_p)}$, $\mathbf{V}_{i(N_p \times N_p)}$= column orthogonal matrix, diagonal matrix, and row orthogonal matrix of mode i, respectively, where $\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I}_{(N_p \times N_p)}$. The diagonal matrix $\mathbf{W}_i$ is defined with singular values at the diagonal terms as

$$\mathbf{W}_i = \lfloor diag(\omega_j^{(i)}) \rfloor \; j = 1, \ldots, N_p \tag{14}$$

where $\omega_j^{(i)}$ = positive or zero singular value of $\tilde{\mathbf{S}}_i$ with $\omega_1 \geq \omega_2 \geq \ldots \geq \omega_{Np} \geq 0$ (Press, *et al.* 1989).

The sensitivity matrix $\tilde{\mathbf{S}}_i$ is singular if the condition number is infinite and is ill-conditioned if the condition number is too large. In determining OAL, only limited number of DOF are selected with relatively large singular values in computing the sensitivity matrix. Therefore, the selection of OAL is not so much influenced by truncating the terms with singular values less than a pre-defined tolerance.

2.4. Modal sensitivity effective independence distribution vector (MS-EIDV)

A usual approach to determine OAL is to maximize characteristic properties of FIM of Eq. (10). One of the methods proved as efficient is the scheme of effective independence distribution vector (EIDV) proposed by Penny *et al.* (1994). The original idea of EIDV was presented with FIM of mode shapes without a mathematical background. Since the current approach is to determine locations of accelerometers based on the computed mode shape sensitivities rather than the mode shape themselves, the method is called modal sensitivity EIDV (MS-EIDV) in the paper to discern from EIDV. The distribution vector $\mathbf{e}_d$ of MS-EIDV is computed with the contributions from all the participating modes as Eq. (15). In the computed distribution vector, we can eliminate components with relatively small values one-by-one which contribute less to the modal behavior.

$$\mathbf{e}_d = \sum_{i=1}^{nmm} \mathbf{e}_{di} \text{ where } \mathbf{e}_{di} = diag[\mathbf{E}_i] \text{ with } \mathbf{E}_i = \tilde{\mathbf{S}}_i[\tilde{\mathbf{S}}_i^T\tilde{\mathbf{S}}_i]^{-1}\tilde{\mathbf{S}}_i^T \tag{15}$$

where $\mathbf{E}_{i(N_m \times N_m)}$, $\mathbf{e}_{di(N_m \times 1)}$ = idempotent matrix and the distribution vector of mode i, respectively. A typical characteristic of an idempotent matrix $\mathbf{E}$ is that its rank is the same as the sum of the diagonal terms of the matrix $\mathbf{E}$. If $rank(\tilde{\mathbf{S}}_i)$ is full, $\mathbf{E}_i = \mathbf{I}$ so that $rank(\mathbf{E}_i) = N_p$ will be satisfied.

2.5. Consideration of parameter error bound in determining OAL

To determine OAL by the proposed algorithm, values of structural parameters should be provided. However, in identifying a structure by a SI scheme, structural parameters are the unknowns to be estimated. Design properties may be used as the values for structural parameters in determining OAL but they may not agree with the actual properties of a structure.

To accommodate this conflict between OAL and SI, a statistical approach is proposed in the paper by considering a parameter error bound around the initial design values for the structure. Monte Carlo iterations are applied to determine a proper set of OAL with random errors within the parameter error bound assumed based on an engineering judgment.

2.6. Application of frequency domain SI algorithm

A frequency domain SI algorithm modified from the algorithm proposed by Jang, *et al.* (2002) has been applied to investigate the effectiveness of the proposed algorithm for selecting OAL. The applied SI algorithm has demonstrated its usefulness in identifying structural parameters even with noisy and sparse measured data. As a similar process to the minimization of Eq. (5), a constrained optimization problem of Eq. (16) is formulated to solve for an optimal stiffness parameter vector **x**.

$$\underset{\mathbf{x}\in\Re^{N_p}}{\text{Minimized}}\quad J(\mathbf{x}) = J_o(\mathbf{x}) + J_r(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{nmm}\|\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi}\|^2 + \frac{\beta}{2}\|\Delta\mathbf{K}(\mathbf{x})\|_F^2 \tag{16}$$

where $\mathbf{x}_{lo}$, $\mathbf{x}_{up(N_p\times 1)}$ = lower and upper bounds of the parameter vector **x** and β = regularization factor, respectively. The primary error vector $\mathbf{e}_{(N_m\times 1)}(\mathbf{x})$ is defined by the error in the equality condition as Eq. (17).

$$\tilde{\boldsymbol{\phi}}_{ai}(\mathbf{x}) - \tilde{\boldsymbol{\phi}}_{mi} = [\lambda_i\mathbf{B}\widehat{\mathbf{K}}_i^{-1}(\mathbf{x})\tilde{\mathbf{M}} - \tilde{\mathbf{I}}]\tilde{\boldsymbol{\phi}}_{mi} \tag{17}$$

where $\mathbf{B}_{(N_m\times N)}$ = a Boolean matrix to indicate the measured DOFs among the full modal displacement vector $\mathbf{f}_i$, $\mathbf{I}_{(N_m\times N_m)}$ = identity matrix with the size of measured DOFs, and $\tilde{\mathbf{M}}_{(N\times N_m)}$ = part of mass matrix corresponding to the measured DOFs, respectively.

The error vector defined by Eq. (17) is basically equal to the output error between measured and computed modal displacements. The pseudo-stiffness matrix in Eq. (17) is defined by

$$\widehat{\mathbf{K}}_i(\mathbf{x}) = \mathbf{K}(\mathbf{x}) - \lambda_i\lfloor\tilde{\mathbf{O}}\ \overline{\mathbf{M}}\rfloor \tag{18}$$

where $\tilde{\mathbf{O}}$ = part of zero matrix corresponding to the measured DOFs and $\overline{\mathbf{M}}$ = part of mass matrix corresponding to the unmeasured DOFs.

A regularization term of a Frobenius norm of Eq. (19) is added to alleviate the inherent ill-posed properties of such an inverse problem during optimization process (Park, *et al.* 2001).

$$\|\Delta\mathbf{K}(\mathbf{x})\|_F^2 = \mathbf{K}(\mathbf{x}) - \|\Delta\mathbf{K}(\mathbf{x}_o)\|_F^2 \tag{19}$$

where $\mathbf{x}_o$ = the baseline value of structural parameters.

3. Simulation study

Two simulation cases were carried out to examine the proposed OAL algorithm. A two-span multi-girder bridge and a two-span truss bridge were used for the simulation studies. Each structure was modeled with 8 parameter groups and the priority in locating the accelerometers was provided for the number of measurable mode shapes varying between 3 and 6. To verify the reliability of the selected OAL, a frequency-domain SI method was applied to estimate the structural parameters. To evaluate the optimality of the selected OAL, three different statistical error indices of ANM (average of normalized mean parameters), ARMS (average of root mean square error), and ASD (average of standard deviation) were computed as defined by Eq. (20).

$$\mathrm{ANM} = \frac{1}{N}\sum_{i=1}^{N_g} \bar{x}_{g_i} \times N_{g_i}$$

$$\mathrm{ARMS} = \frac{1}{N_e}\sum_{i=1}^{N_g} \sqrt{x_{g_i}(\bar{x}_{g_i} - x_{b_i})^2}$$

$$\mathrm{ASD} = \frac{1}{N_e}\sum_{i=1}^{N_g} \bar{s}_{g_i} \times N_{g_i} \tag{20}$$

where N_e = the total number of elements in the analytical model, N_g = the number of groups, $\bar{x}_{g_i}$ = the mean value of the estimated structural parameters through Monte Carlo iterations, $\bar{x}_{b_i}$ = the baseline value for the i-th group parameter, $\bar{s}_{g_i}$ = the standard deviation of the i-th group parameter obtained through Monte Carlo iterations, respectively.

3.1. Simulation Study on two-span non-symmetric multi-girder bridge

As the first simulation case, a two-span three-girder bridge as shown in Fig. 1 is selected. The bridge is idealized as a non-symmetric grid model with respect to the middle support to investigate the rank deficiency problem frequently occurred in inverting the FIM. The section, material and grouping information are provided in Fig. 1 and also summarized in Table 1. It was assumed that only the vertical directions could be measured for this type of structure.

Fig. 2 shows the variation of the selected OAL depending on the number of measurable target modes. The circles drawn in the figures indicate the minimal OAL to the vertical direction. Compared with the case of symmetric bridge with respect to the middle support, the arrangement of OAL for the current bridge is not symmetric and tends to locate at a longer span with a higher priority (Kwon 2006).

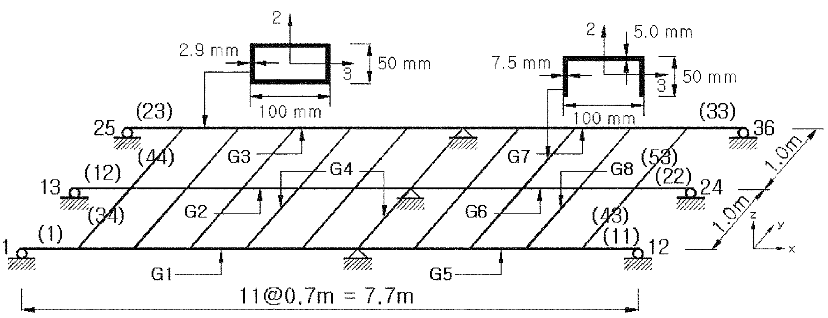

Fig. 1 Two-span non-symmetric three-girder bridge model

Table 1 Material and sectional properties of the girder bridge

Group	Area [mm^2]	E [GPa]	Density [Kg/m^3]	I_{33} [mm^4]	I_{22} [mm^4]	J [mm^4]
1, 5	836.4			3.638×10^5	1.052×10^6	8.080×10^5
2, 6	836.4	210	7850	3.638×10^5	1.052×10^6	8.080×10^5
3, 7	836.4			3.638×10^5	1.052×10^6	8.080×10^5
4, 8	1175.0			2.945×10^5	1.864×10^6	1.721×10^5

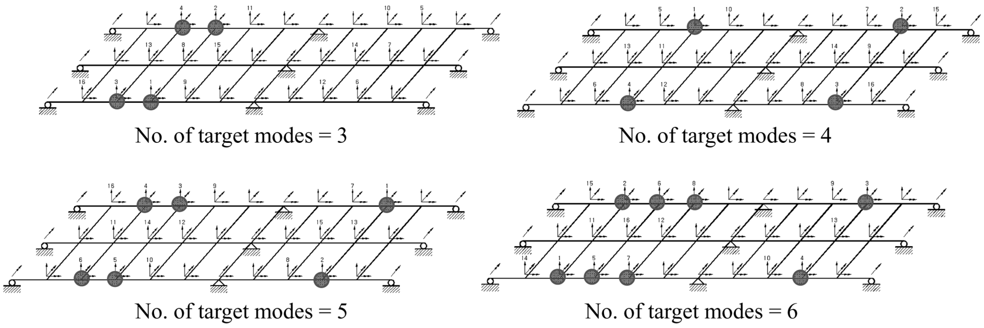

Fig. 2 Variation of OAL for the girder bridge with the number of target modes

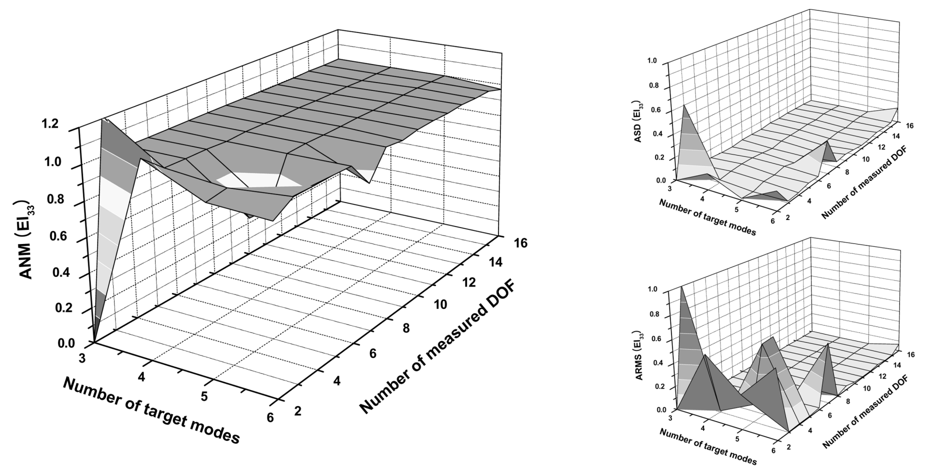

Fig. 3 Variation of statistical error indices for the girder bridge with the numbers of target modes and measured DOF

Fig. 3 shows the computed statistical error indices with the variations in the number of target modes and the number of measuring DOF. From the figure, we can observe that the use of 4 accelerometers could provide reliable SI results as far as the number of measured modes is larger than 3. As the number of target modes increases to 4, 5, and 6 modes, the minimum number of accelerometers also increases to 4, 6, and 8, respectively with the OAL as selected in Fig. 2.

Fig. 4 compares the estimated structural parameters by using the selected OAL from MS-EIDV with those from EIDV when the number of target modes was 5 and the number of used accelerometers was 7. In the figure, when the estimated parameter is the same as the exact one, the value is normalized to 1.0. Each figure shows the estimated parameters of group 1 or group 5 among all 8 groups in the

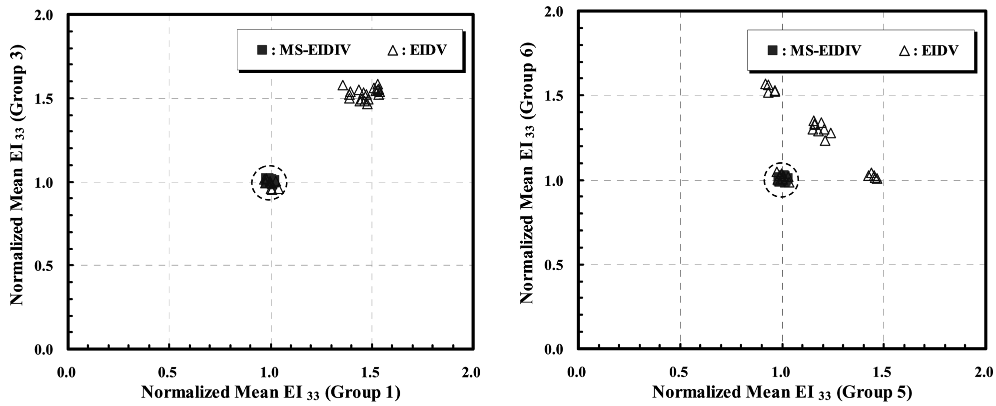

Fig. 4 Comparison of estimated parameters of the girder bridge from MS-EIDV and EIDV methods

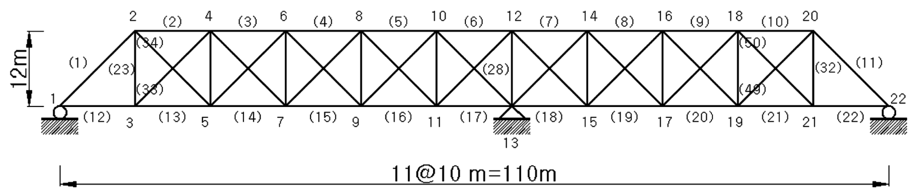

Fig. 5 Non-symmetric truss bridge model

horizontal and vertical axes. From the figure, it can be observed that the results from MS-EIDV gather more closely around 1.0 than those from EIDV.

3.2. Simulation Study on two-span non-symmetric truss bridge

As the second simulation study, a statistically indeterminate two-span truss bridge has been studied as shown in Fig. 5. At each joint of the truss, the horizontal and vertical DOF were considered as candidate OAL. Like the first simulation study, the truss was idealized as a non-symmetric one with respect to the middle support. The material and sectional properties of the truss bridge are summarized in Table 2.

Fig. 6 shows the variation of selected OAL depending on the number of measurable target modes. Differently from the case of two-span multi-girder bridge, the locations of accelerometers tend to uniformly distribute on both spans in all the cases with a different number of measurable target modes. For the case the number of target modes is equal to 4, more horizontal DOF could be selected than the other cases.

Fig. 7 shows the variation of statistical error indices with the number of target modes and the number

Table 2 Material and sectional properties of the truss bridge

Group	Member	Area [mm^2]	E [GPa]	Density [Kg/m^3]
1, 5	Left Top, Right Top	250	210	7850
2, 6	Left Bottom, Right Bottom	300		
3, 7	Left Vertical, Right Vertical	200		
4, 8	Left Diagonal, Right Diagonal	220		

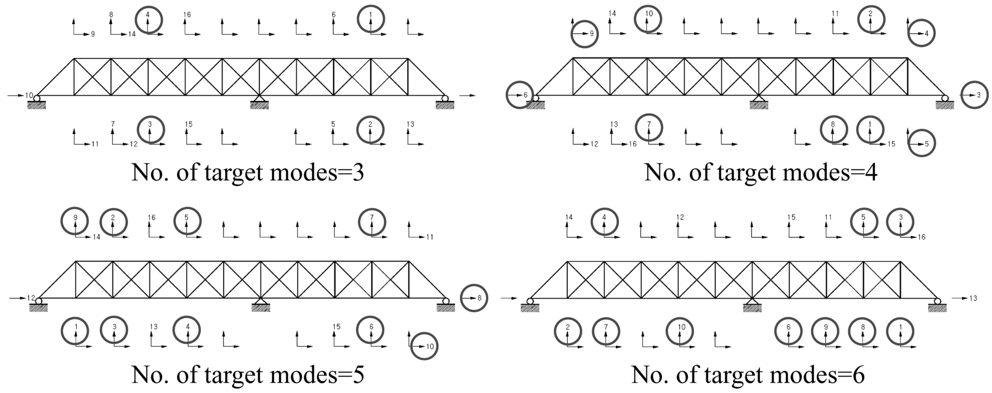

Fig. 6 Variation of OAL for the truss bridge with the number of target modes

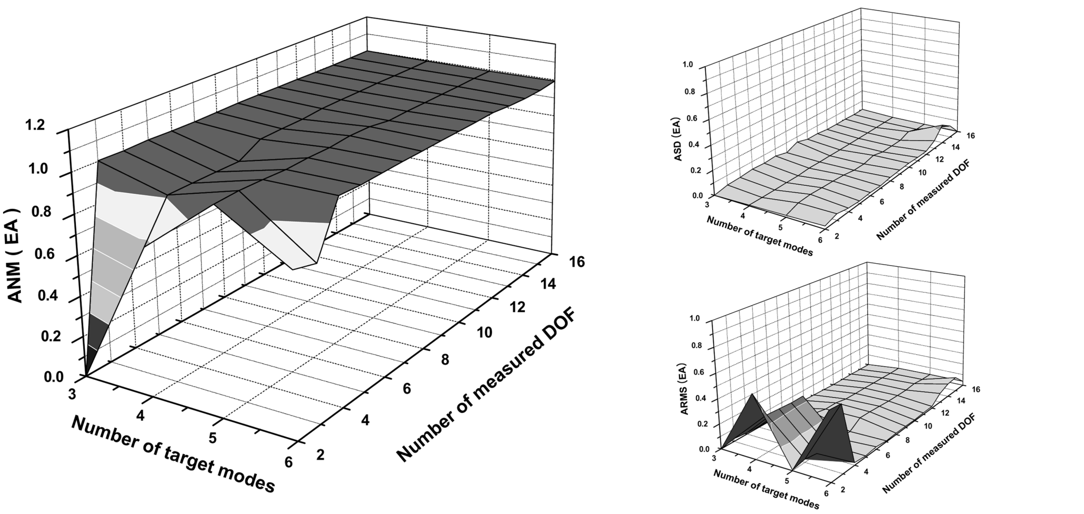

Fig. 7 Variation of statistical error indices for the truss bridge with the numbers of target modes and measured DOF

of measuring DOF. Fig. 7 indicates that at least 8 accelerometers are required to estimate structural parameters reliably when the number of measurable target modes is 4. To measure up to 5 modes, at least 10 accelerometers are also needed with stable standard deviation in the parameter estimation. Likewise, to measure up to 6 modes, 10 accelerometers are enough to obtain converged SI results of structural parameters but more than 14 accelerometers are needed to get stable standard deviation in the parameter estimation with the consideration on the measurement noise.

Fig. 8 compares the estimated structural parameters by using the selected OAL from MS-EIDV with those from EIDV when the number of target modes was 5 and the number of used accelerometers was 10. Each figure shows the estimated parameters of 2 groups among all 8 groups in the horizontal and vertical axes. Differently from the case of the girder bridge, the estimated parameters gather close from both methods. However, the values by EIDV are separated into two locations and one of them stays relatively away from 1.0.

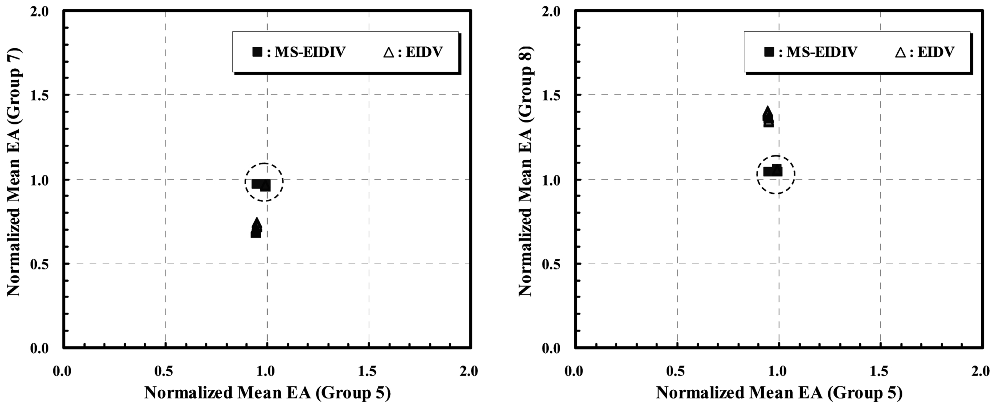

Fig. 8 Comparison of estimated parameters of the truss bridge from MS-EIDV and EIDV methods

4. Conclusions

A new algorithm to determine OAL is proposed for identifying a structure by a frequency domain SI scheme. Compared with the conventional EIDV method, the proposed algorithm of MS-EIDV is developed with the likelihood function of measurements and formulates a FIM with mode shape sensitivity by maximizing the likelihood of the occurrence of the measurements relative to the predictions. The concept of error bound on the structural parameters is introduced in the paper to accommodate the conflict between in determining OAL and in applying SI. Monte Carlo iterations with random errors within a pre-defined error bound are applied to determine more reliable OAL.

To examine the proposed algorithm, simulation studies have been carried out on a two-span multi-girder bridge and a truss structure. For each simulation case, a guideline could be presented to determine the priority of OAL by drawing three statistical error indices with variations in the number of measurable target modes and the number of measured DOF. From the figures, the required minimum number of accelerometers could be determined depending on the number of measurable target modes. The usefulness of the proposed MS-EIDV algorithm compared with EIDV could be demonstrated through the simulated examples.

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