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Optimal sensor placement techniques for system identification and health monitoring of civil structures

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Abstract. Proper pretest planning is a vital component of any successful vibration test on engineering structures. The most important issue in dynamic testing of many engineering structures is arriving at the number and optimal placement of sensors. The sensors must be placed on the structure in such a way that all the important dynamic behaviour of a structural system is captured during the course of the test with sufficient accuracy so that the information can be effectively utilised for structural parameter identification or health monitoring. Several optimal sensor placement (OSP) techniques are proposed in the literature and each of these methods have been evaluated with respect to a specific problem encountered in various engineering disciplines like aerospace, civil, mechanical engineering, etc. In the present work, we propose to perform a detailed characteristic evaluation of some selective popular OSP techniques with respect to their application to practical civil engineering problems. Numerical experiments carried out in the paper on various practical civil engineering structures indicate that effective independence (EFI) method is more consistent when compared to all other sensor placement techniques.

Keywords: system identification; optimal sensor placement; effective independence; fisher information matrix; variance method; energy based methods.

1. Introduction

The problem of identification of modal parameters of a structural model using dynamic data has received much attention over the years because of its importance in structural model updating, structural health monitoring and structural control. In experimental modal testing, the quality of information that the data give on the modal parameters depends on the number and location of sensors in the structure.

The characterization of the dynamic behaviour of a real structure is possible only if an adequate minimum amount of information is available. This, in turn, implies that an adequate minimum number of sensors must be judiciously placed on the structure in order to obtain the desired information for the identification of the structural behaviour.

The set of measured degrees of freedom for large scale civil structures, usually the displacements of low frequency modes, must give sufficient information to describe the dynamic behaviour of a structural system with sufficient accuracy in order to make use of this information for effective structural parameter identification or health monitoring. Hence, the fundamental problem is how many and which degrees of freedom should be taken in the structural identification process. In solving this

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problem, due consideration should be given to the economic factors which may require that a limited number of sensors are placed at accessible locations on the real structure. In view of this, it is essential that these limited numbers of sensors are located at the most advantageous locations on the structure. Otherwise, the measured modal properties will be incomplete and an accurate structural identification will become impossible. It is appropriate to mention here that optimal sensor configurations (i.e., optimal number as well as optimal positions of sensors) must be individually determined for each structure to be estimated.

It is quite obvious that the accuracy of the data increases with the increase in the number of sensors being utilized. However, the number of sensors that can be attached to a real structure is limited by economic constraints. Therefore, algorithms that address the issue of limited instrumentation and its effects on resolution and accuracy are important from the standpoint of experimental modal analysis.

1.1. Previous work

The optimal sensor placement techniques, based on their theoretical formulation, can be broadly categorized as statistical and non-statistical based techniques. While the information theory based techniques, information entropy based and variance based methods comes under the category of statistical methods, most of the energy based methods comes under non-statistical methods.

Considerable research has been carried out on the information theory based methods as they are found to be more effective for optimal sensor placement on practical engineering structures.

Kammer (1991), Udwadia (1994), Kirkegaard and Brincker (1994), Penny, *et al.* (1994), Hemez and Farhat (1994), Cobb and Liebst (1996), Reynier and Abou-Kandil (1999) and Shi, *et al*. (2000) have proposed information theory based approaches in order to provide rational solutions to several issues encountered in the problem of selecting the optimal sensor configuration. In these methods, the optimal sensor configuration is taken as the one that maximizes some norm (determinant or trace) of the Fisher information matrix (FIM) or its variants. Cobb and Liebst (1996) and Shi, *et al*. (2000) have reported optimal sensor placement for the purpose of detecting structural damage. Heredia and Esteva (1998) and Heredia, *et al*. (1999) have proposed information based approaches to handle large model uncertainties expected in model updating. The optimal sensor configuration is chosen as the one that minimizes the expected Bayesian loss function involving the trace of the inverse of the FIM for each model. Kammer and Tinker, (2004) proposed a modified information based approach for optimal placement of triaxial sensor sets. A new alternative approach has been recently proposed by Kammer (2005), which is based on sensor set expansion in contrast to all the earlier information based methods which are based on sensor set reduction. This method iteratively expands an initial set of sensors, which are absolutely essential for the specific problem on hand, instead of reducing a large candidate set. This method significantly reduces the computational effort for structures with large number of possible candidate sensor locations.

Information entropy (Jaynes 1978) is considered as criteria to identify optimal sensor locations by Papadimitriou, *et al*. (2000) Metallidis, *et al*. (2003) and Papadimitriou (2004, 2005). The optimal sensor configuration is selected as the one, which minimizes the information entropy norm corresponding to structural testing. Information entropy is a direct measure of the uncertainty in the model parameters. Yuen, *et al*. (2001) have proposed optimal sensor placement technique for model updating and health monitoring of structures, considering the uncertainty in the unmeasured excitation encountered in practical applications for which data are to be taken either from ambient vibration tests or from other uncertain excitations such as earthquake and wind. Information entropy based framed work has been used by

Yuen, *et al*. (2001) for identifying the optimal sensor locations.

From computational point of view, the optimal sensor placement algorithms can be solved either using heuristic procedures like EFI method or by using combinatorial optimisation algorithms like genetic algorithms by formulating an appropriate fitness function. Yao, *et al*. (1993), Guo, *et al*. (2004), Worden and Burrows (2001) and Papadimitriou (2004) have used genetic algorithms to obtain optimal sensor configurations. It has been reported that GA requires up to 30 times the computational time, when compared to heuristic information based approaches. The quality of solutions obtained using GA is marginally superior with sensor configurations differing either at one or two sensor locations when compared to the heuristic information based methods. A Pareto optimal formulation using GA has been proposed by Papadimitriou (2005) by considering information entropy as optimality criteria. The multi-objective optimisation problem formulated using GA attempts to identify optimal sensor locations that simultaneously minimize appropriately defined entropy indices for multiple model classes.

1.2. Present work

It can be summarized from this brief review that the statistical based methods (both information theoretic and information entropy) for optimal sensor placement on engineering structures are being extensively investigated. Among the computational algorithms, the heuristic procedures like EFI (i.e. sequential elimination of DOF) are being predominantly used in various disciplines of engineering due to their simplicity and computational efficiency. On the other hand, the GA based combinatorial optimisation techniques are reported to be highly compute intensive, with marginal gain in performance when compared to the heuristic procedures. Moreover, the GA based algorithms have been tested only for problems with smaller number of possible candidate sensor locations in most of the earlier works, with the exception of Yao, *et al*. (1993). In view of this, the scalability and gain of these algorithms (when compared to information theory based heuristic approaches) for problems with larger candidate senor configurations is not assured. It should be mentioned here that Yao, *et al*. (1993) have reduced the candidate sensor locations to 200 of the space station model with larger number of possible candidate sensor locations, before using GA for optimizing the sensor positions. Similarly, efficacy of the information entropy based approaches is also demonstrated using only problems with smaller set of possible candidate sensor configurations. Therefore, most of the OSP techniques applied to practical engineering problems with large possible candidate sensor locations belong to the category of information theory based heuristic approaches. Keeping these things in view, it is proposed to evaluate the information theory based heuristic approaches along with some selective energy based non-statistical methods in this paper.

Several alternative information theory based schemes have been proposed in the literature for OSP and applied to several aerospace, civil and mechanical structures. However, the characteristic evaluation of these OSP algorithms and their comparative performances, while applied to different civil structures has not been reported so far. Keeping this in view, some of the selective OSP algorithms reported in the literature and applied to various specific problems in aerospace, civil and mechanical engineering are considered for evaluation in this paper.

We have considered five different classes of OSP methods for evaluation in this paper. The first class of methods use the effective independence method (EFI) (Kammer 1991, Kammer and Brillhart 1996) for in-orbit modal identification of large space structures, based on maximisation of determinant of Fisher Information Matrix (FIM). The other technique in this class is the effective independence with driving point residue (EFI-DPR) method (Imamovic 1998) which was originally developed for large mechanical structures like engine casings. EFI-DPR method is developed by synthesizing EFI and Energy approaches.

The second class of methods are the energy based methods (Heo, *et al*. 1997, Hemez and Farhat 1994) which essentially makes use of the EFI scheme of OSP, but attempts to maximize the kinetic energy or strain energy content of the signals acquired. These methods are used in the literature for large-scale civil and aero structures.

The third class of OSP methods is the variance method (Fedorov and Hackl 1994), which is based on the most informative subset (MIS). The fourth and fifth class of methods considered in this paper are eigenvalue vector product (EVP) method (Doebling 1996, Larson, *et al*. 1994) and Non-optimal driving point (NODP) method (Imamovic 1998), which are primarily energy based methods. These methods tend to select the sensor locations that maximize the vibration energy content of the signal acquired.

2. Methods for optimal sensor placement (OSP)

In this section various OSP methods considered for evaluation in this paper are briefly discussed.

2.1. Effective independence based OSP methods

2.1.1. Effective independence (EFI) algorithm

The effective independence algorithm (Kammer 1991, Kammer and Brillhart 1996) is based on the spatial independence concept that guarantees the linear independency of the identified modes. For the purpose of test and analysis mode shape correlation, the analyst must obtain measurements at locations on the structure that render the extracted test-mode partitions linearly independent. The test modes must be spatially differentiable. If the test-mode shapes are not spatially independent, test and analysis mode shape correlation using orthogonality and cross-orthogonality computations (Chen and Garba 1985) cannot be performed. This is because the test modes and the corresponding modal partitions obtained using finite element method (FEM) will be indistinguishable. Spatial independence implies that at any instant of time, the sensor output equation (Udwadia and Garba 1985, Udwadia 1994) is given by

$$y = \Phi\, q + w \tag{1}$$

where y is a vector of measured structural responses from the sensors, Φ is the matrix of FEM target modes partitioned to the sensor locations, and q is the coefficient of response vector. In other words, the vector q represents the individual contribution of each item in Φ to y. w is a vector representing the measurement of noise and all other effects not modelled by the FEM. In actual computation, w is assumed to have a Gaussian distribution with zero mean and covariance matrix $\Psi = \sigma^2.\ \mathrm{I}$, where I is the identity. In other words, w assumes noises with equal variances. It is important to note that if the noise modified modes shapes are linearly independent, the original target modes will also be independent. In other words, the optimal sensor locations will not alter with noise.

If the candidate sensor set contains 's' locations, but available resources limit the sensor configuration to '$m \ll s$' sensors, then the issue is to place the 'm' sensors within the 's' candidate locations while maintaining as much independent information as possible and thus obtaining the best estimates of the modal states. The least square method is the most widely used approach for the estimation problems such as the present problem. Applying the least square method to Eq. (1), to compute the target states $\hat{q}$, yields

$$\hat{q} = [\Phi^T\Phi]^{-1}\Phi^T y \tag{2}$$

The best estimate implies that the covariance matrix of the estimate errors will be a minimum. The covariance matrix of the estimate error results in

$$P = E[(q-\hat{q})(q-\hat{q})^T] = [[\Phi^T\Phi]^{-1}\Phi^T \cdot \psi \cdot \Phi[\Phi^T\Phi]^{-1}] = Q_0^{-1} \tag{3}$$

where E [ ] denotes the expected value of the quantity in the brackets, and ψ is the covariance matrix of the noise vector, w, in equation (1). The matrix Q_0, in equation (3) is called the Fisher Information Matrix (FIM), and it must be maximized in order to minimize the error of estimation. It is assumed that the noise at different sensors is uncorrelated, but possesses the same variance σ^2. With this, the fisher information matrix can be written as

$$Q = \frac{1}{\sigma^2}\Phi^T\Phi = \frac{1}{\sigma^2}Q_0 \tag{4}$$

In order to minimize P, a suitable norm of Q must be maximized. Determinant of FIM is considered as the matrix norm to arrive at optimal sensor placement. By considering the contribution of each degree of freedom, Q_0 can be expressed as

$$Q_0 = \sum_{i=1}^{s}\Phi^{i^T}\Phi^i = \sum_{i=1}^{s}Q^i \tag{5}$$

where Φ^i is the i^{th} row of the modal partition Φ corresponding to the i^{th} degree of freedom or sensor location. Eq. (5) illustrates that as each degree of freedom is added to or subtracted from the candidate set, information is added or subtracted from the FIM. The number of degrees of freedom in the candidate sensor set can be reduced by eliminating locations that do not contribute significantly to the independent information contained within the target-mode partitions. Redundant information can be deleted. With this back ground, the EFI algorithm for optimal sensor placement can be formulated as an iterative algorithm, which evaluates the candidate location sensor contributions employing effective independence distribution (EID) vector given as

$$E_D = [\Phi\,\Psi] \otimes [\Phi\,\Psi]\lambda^{-1}\{1\}_k \tag{6}$$

where symbol '$\otimes$' indicate element wise multiplication. $\{1\}_k$ is a vector of size ($k \times 1$) with each element as 1. ψ and λ are the eigenvector matrix and eigenvalue of FIM respectively. In order to ensure maximisation of the determinant of FIM, the sensor position having the smallest E_D coefficient is eliminated at each iteration. This process is repeated until the number of candidate sensors 'm' equals to the desired fixed number M. The EFI algorithm is given in Fig. 1.

2.1.2. EFI-DPR method

EFI method discussed earlier can lead to the selection of sensor locations with low energy content as optimal locations. This may ultimately result in possible loss of information. Keeping this in view, a modification to EFI is suggested (Worden and Burrows 2001) to eliminate this problem. In this modified EFI method termed as EFI-DPR method, the candidate sensor contribution of the EFI is multiplied by the corresponding driving point residue (DPR) coefficient (Imamovic 1998) given by

1. Set desired number of sensors as M
2. Use finite element method to construct the eigenvector matrix (Φ) of the given structure with N active degrees of freedom.
3. K = N-M

   FOR i = 1, K

   i. Formulate FIM matrix Q

   ii. Solve eigen value problem $(Q-\lambda I)\Psi = 0$

   iii. Compute E_D vector using equation (6)

   iv. Find r such that $E_{D_r} = \min_i (E_{D_i})$

   v. Remove node r and update Φ with reduced number of sensors

   ENDFOR

Fig. 1 Effective independence (EFI) algorithm

$$\mathrm{DPR}_i = \sum_{k=1}^{N} \frac{\Phi_{ik}^2}{\omega_k} \tag{7}$$

where ω_k is the k^{th} target mode frequency and N is the desired number of modes. The modified E_D vector can be written as

$$\mathrm{DPR}\_E_{D_i} = [\Phi\Psi] \otimes [\Phi\Psi]\lambda^{-1}\{1\}_i{}^*\mathrm{DPR}_i \tag{8}$$

It can be observed that DPR acts as a weighting factor for the E_D.

2.1.3. EFI expansion

It has been shown by Kammer (2004) that the Fisher Information matrix can be decomposed into the contributions from each candidate sensor location in the form

$$Q = \sum_{i=1}^{n_c} \phi_i^T \phi_i = \sum_{i=1}^{n_c} Q_i \tag{9}$$

where n_c is the number of candidate sensors, ϕ_i is the i^{th} row of the target partition matrix associated with the i^{th} candidate sensor location. For structures with large initial candidate sensor set, the iterative procedure involved in effective independence method leads to significant computational cost. Further, if a few specific sensor locations are absolutely essential during dynamic testing, there is no provision in the effective independence method to accommodate these locations within the chosen optimal sensor configuration. In order to overcome the two above mentioned problems, sensor set expansion method is proposed (Kammer 2005), where the sensor set is expanded iteratively by choosing in each iteration, the best ranked DOF with respect to EFI value and added to the initially chosen sensor set. This iterative process is repeated till the desired number of candidate sensor locations is obtained. The effective independence values of the candidate sensor set will be re-ranked and sorted in each iteration. The E_D^i can be written as (Kammer 2005).

$$E_D^i = \phi_i Q_0^{-1} \phi_i^T \tag{10}$$

where $Q_0 = \sum_{i=1}^{n} \phi_i^T \phi_i$ is the FIM corresponding to the initially selected 'n' sensors at the beginning and will be progressively updated with the chosen DOF EFI contribution during sensor set expansion.

2.2. Energy based OSP methods

2.2.1. NRG method

In the NRG method (Hemez and Farhat 1994), the optimal sensor locations are identified based on nodal energy of the structure. The strain energy distribution (E_r) in a structure is a direct measure of its load carrying capability. It provides two critical informations. First the modes that contribute the most to storing the energy are indicated by large energies:

$$E_r = \Phi_r^T K \Phi_r \tag{11}$$

where K is the stiffness matrix, $r = 1, 2, \ldots.\ m_1$ and m_1 is the number of target mode shapes. When the modes Φ_r are derived from finite element analysis, the energies E_r are simply equal to the eigen values ω_r^2. On the other hand, they estimate the measured strain energies when identified data Φ^m is used in Eq. (11) instead of analytical modes. The energy distribution per structural member can be obtained as

$$E_r^i = [L^{(i)}\Phi_r]^T K^i [L^{(i)}\Phi_r] \tag{12}$$

where $i = 1, 2, \ldots.\ N_e$ and $r = 1\ldots..m_1$. N_e is the total number of elements. $L^{(i)}$ is a localization operator, which extracts from the global vector Φ_r, the degrees of freedom (DOF) that are connected to the i^{th} finite element. Physically, large energies E_r^i point to elements that are critical for the integrity of the structure because they store large ratios of the strain energy of the r^{th} mode. Once the energies related to each element is computed for all the target modes, the nodal energy of the structure can be computed by distributing the sum of total elemental energy to each of the DOF in the element. The nodal energy vector can be sorted and the top 'p' DOFs are considered for the placement of 'p' sensors.

2.2.2. Energy matrix rank optimisation (EMRO) method

The EMRO method (Hemez and Farhat 1994) for optimal placement of sensors is similar to the EFI method. The main difference is that the objective of EMRO is to find a reduced configuration of sensor placements which maximizes the measure of the strain energy of the structure rather than the determinant of the FIM Q. The Fisher information matrix, Q is weighted with the FE stiffness matrix, K, generating the strain energy matrix E_r defined in Eq. (11).

The stiffness matrix is decomposed into lower (L) and upper (U) triangular cholesky factor matrices

$$K = LU \tag{13}$$

$$\varphi = U\Phi \tag{14}$$

The strain energy matrix can be represented as the product of the matrix φ and its transpose,similar to the FIM Q, as described below by the expression:

$$E = \varphi^T \varphi \tag{15}$$

Therefore after evaluating φ, the optimal sensor placement process using EMRO is identical tothat described for the EFI algorithm described in Fig. 1. Since EMRO is based on strain energy criteria, we call it hereafter as Strain Energy Matrix Rank Optimisation (SEMRO).

Similarly, EMRO based on Kinetic energy (Heo, *et al.* 1997) called hereafter as Kinetic Energy

Matrix Rank Optimisation (KEMRO) can be formulated by using the kinetic energy expressioninstead of strain energy given in Eq. (11).

$$KE_r = \Phi_r^T M \Phi_r \tag{16}$$

In KEMRO, K is replaced with M, the mass matrix, in Eq. (13).

2.3. Eigenvalue vector product method

The eigenvalue vector product (EVP) method is an energy based technique (Doebling 1996, Larson, *et al*. 1994). It involves evaluation of the vector EVP calculated using the following expression:

$$\mathrm{EVP}_i = \prod_{j=1}^{m_1} \Phi_{ij} \tag{17}$$

where 'm_1' is the number of target modes. This technique selects the sensors with the largest EVP values in order to prevent the choice of sensors placed on nodal lines of a vibration mode and to maximize their vibration energy. Once the EVP vector is computed, the EVP values are sorted and the best 'p' DOFs are chosen for placement of 'p' sensors.

2.4. Non-optimal driving point based method

The non-optimal driving point (NODP) based method (Imamovic 1998) is energy based technique and is generally used to find the optimal excitation point. It is based on the concept that the amount of vibration energy of any mode shape depends on the relative positions of the excitation source and the mode shape nodal lines. However, the amount of the vibration energy measured by the sensors is a function of the relative positions of the sensors and mode shape nodal lines. Therefore, the NODP based method can be used to identify the optimal sensor placements as well. In this method, the NODP vector is computed and similar to EVP, thevalues of NODP are sorted and the best 'p' DOFs are chosen for placement of 'p' sensors. The NODP vector can be computed as

$$\mathrm{NODP}_i = \min(\Phi_{i1}, \Phi_{i2}, \ldots\ldots \Phi_{ir}) \tag{18}$$

where r is desired number of target modes.

2.5. Method based on variance

The variance method (VM) is an evolution of the most informative subset (MIS) technique (Fedorov and Hackl 1994) according to physical and mathematical considerations on the coefficients of the covariance matrix [C] (Duda 1997). The most informative subset (MIS) technique is a statistics based method that relies on a covariance analysis approach and was basically developed for problems related to evaluation and approximation of a phenomenon by making use of finite number of observations. In the present context, the phenomenon investigated by the MIS technique is the evaluation of the mode shapes in non-instrumented locations of the structure. Therefore, supposing that each target mode shape is evaluated (observed) in 'M' ($y_M = \Phi_1, \Phi_2, \Phi_3, \ldots\ldots, \Phi_M$) of the possible '$n$' locations, the mode shapes need to be estimated in the remaining 'n–M' locations ($y_p = \Phi_{M+1}, \Phi_{M+2}, \Phi_{M+3}, \ldots\ldots, \Phi_{M+p}$) ($n$=$M$+$p$). This problem can be solved using the best linear unbiased estimator of y_p (Fedorov and Hackl 1994) given below:

$$\hat{y}_p = C_{pp} C_{MM}^{-1} y_M \tag{19}$$

If the matrix Y is assumed equal to Φ_{FE}^T, the matrices C_{pp} and C_{MM} are the diagonal matrices of its covariance matrix (Fedorov and Hackl 1994).

$$\mathrm{Cov(Y)} = \begin{bmatrix} C_{MM} & C_{Mp} \\ C_{pM} & C_{pp} \end{bmatrix} = \begin{bmatrix} \mathrm{Cov}(y_M) & C_{Mp} \\ C_{pM} & \mathrm{Cov}(y_p) \end{bmatrix} \tag{20}$$

The covariance of the estimation error can be evaluated as:

$$D_{pp} = \mathrm{Cov}(\hat{y}_p - y_p) = C_{pp} - C_{pM} C_{MM}^{-1} \mathrm{C}_{Mp} \tag{21}$$

D_{pp} can be used to measure the efficiency of the estimator measuring the dynamic response in the specified '*M*' locations. Taking determinant as efficient measure of the estimation, the best '*M*' sensor positions will be those that minimise the determinant of D_{pp}.

$$\mathrm{Cov(Y)} = \begin{bmatrix} C_{MM} & C_{Mp} \\ C_{pM} & C_{pp} \end{bmatrix} = |C_{MM}||C_{pp} - C_{pM} C_{MM}^{-1} C_{Mp}| = |C_{MM}||D_{pp}| \tag{22}$$

The minimization of the D_{pp} determinant is equivalent to the maximisation of a measure of the C_{MM} determinant or the maximisation of the determinant of the $[\Phi_{FE}^T]$ covariance matrix for a set of M sensor positions. However, the maximization of the determinant of Cov $[\Phi_{FE}^T]$ guarantees the rank of the target mode shape matrix Φ restricted to the *M selected* locations to be equal to N, but it does not assure that the signal strength is maximized. Maximisation of the signal strength as well as maximisation of determinant of C_{MM} can be simultaneously achieved, if the off-diagonal terms of C_{MM} are close to zero and the diagonal terms are tending towards one. This is equivalent to the maximisation of the function V_r defined as

$$V_r = \sum_{i=1}^{N} \frac{c_{ii}}{\sum_{i \neq j} c_{ij}} \tag{23}$$

where c_{ii} is the i^{th} diagonal term and $\sum_{i \neq j} c_{ij}$ is the sum of non-diagonal terms of i^{th} row of matrix C_{MM}. This method involves sorting the vector V_r and the top ranked 'p' DOFs are considered for the placement of 'p' sensors. N is the number of possible candidate sensor locations.

3. Evaluation parameters for OSP algorithms

In order to evaluate the effectiveness of various OSP techniques discussed in this paper, we employ the following measures.

The first criterion assesses the capability of each OSP technique to capture the dynamic response of a structure by measuring the mean square error (MSE) between the FE model mode shapes obtained using finite element method and the mode shape obtained by a cubic spline interpolation (CSI) of the displacement measured at the optimal sensor locations dictated by each of the algorithm. The total mean square error (TMSE) for all target modes is computed using the following expression.

$$\text{TMSE} = \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} (\Phi_{ij}^{FE} - \Phi_{ij}^{CSI})^2}{n} \tag{24}$$

where n is the number of possible candidate set of sensor locations, m is the number of mode shapes considered. The second criterion is based on the concept that the strength of the signals acquired associated with modal characteristics should be as high as possible in order to reduce the noise. In order to comply with this aspect, the Fisher information matrix (FIM) determinant behaviour is recorded during the sensor selection and presented interms of percentages of its initial value against the progressive number of the omitted candidate sensors. Maximising the determinant of the Fisher information matrix will maximize the spatial independence of the target modal partitions and maximise the corresponding signal strength (Kammer 2004).

The third criterion considered is the measure of maximisation of kinetic energy of the structure with reduced sensor configuration. In order to evaluate this aspect, the determinant behaviour of the FIM weighted with mass matrix is recorded during sensor selection and presented interms of percentages of its initial value against the progressive number of omitted candidate sensors.

The fourth criterion considered is the accuracy of information obtained by each of the sensor configuration. The accuracy of the information can be monitored during iterative process by tracking the trace values of Fisher Information matrix. A larger trace value implies that, the sensor configuration possessing a smaller estimate error covariance matrix yielding better state estimates (Kammer 1991). Hence the trace values of FIM should be maximized. The behaviour of each of the algorithm considered with respect to the percentage reduction in trace of FIM when compared to its initial value, during progressive elimination of candidate sensor locations is evaluated.

The condition number of the FIM is the fifth evaluation criteria considered in this paper. Smaller condition number signifies less sensitivity in the estimates to analytical modeling error.

While a cursory comparison of the modal frequencies may provide some initial information, it is rather insufficient by itself. A more objective way of comparing the numerical results to the experimental results involves the comparison of the mode shapes obtained by each method. A tool called modal assurance criteria (MAC) is an effective way of comparing two sets of structural dynamic data and devising a correlation measure. MAC is defined as

$$\text{MAC} = \frac{\left|(\Phi^a)^T(\Phi^e)\right|^2}{\left|(\Phi^e)^T(\Phi^e)\right|\left|(\Phi^a)^T(\Phi^a)\right|} \tag{25}$$

where 'a' refers to analytical data and 'e' refers to experimental data and Φ is the mode shape. MAC will yield a value greater than 0.90 and close to 1 if the experimental and analytical mode shapes are well correlated. For uncorrelated modes, the value will be less than 0.005.

4. Numerical studies

Numerical experiments have been conducted to evaluate various OSP techniques discussed in this

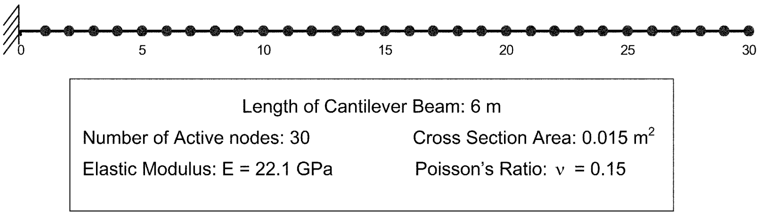


Fig. 2 Cantilever beam and its finite element idealization

paper. Since the prime objective is to characterize various algorithms with respect to their effectiveness in capturing vibration behaviour of civil structures, we have considered five distinct types of structures starting with a beam, microwave tower, tall building frame, rectangular plate (concrete slab bridge), and finally a complex bridge structure.

4.1. Numerical example 1

The first numerical example considered is a cantilever beam. The finite element idealization of the beam and material properties are shown in Fig. 2. All the active 30 nodes in the beam are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors. For the present problem, the desired number of sensors is considered as 5. The first five modes are considered for identification.

Fig. 3 shows the optimal locations of sensors obtained using all the OSP methods presented in this paper. A close look at the results indicate that, each of the method suggest different locations. The effectiveness of each of the sensor configurations in maximizing the information, signal strength, and spatial distribution is evaluated by the following detailed studies.

Some of the OSP methods considered for evaluation like EVP, NRG, VAR, and NODP tend to give the optimal sensor locations, which are rather clustered in one part of the structure. In order to maintain fairly good spatial distribution of sensors across the structure for an improved capture of the vibration mode shapes, we discretise the structure into number of segments and choose the local maximal DOFs in each of the segment as the optimal sensor locations. Fig. 4 shows the behaviour of various OSP techniques considered in this paper for the cantilever beam. The mean square error (MSE) with respect

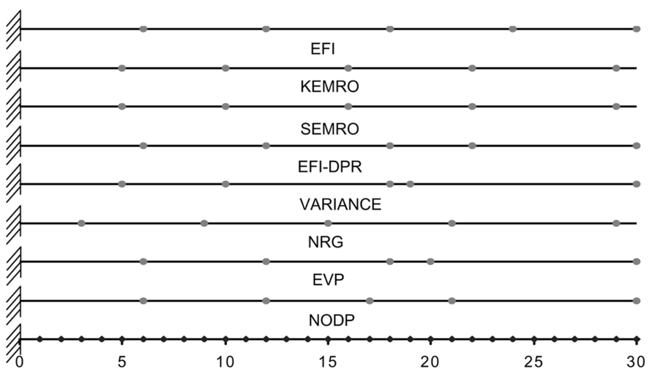
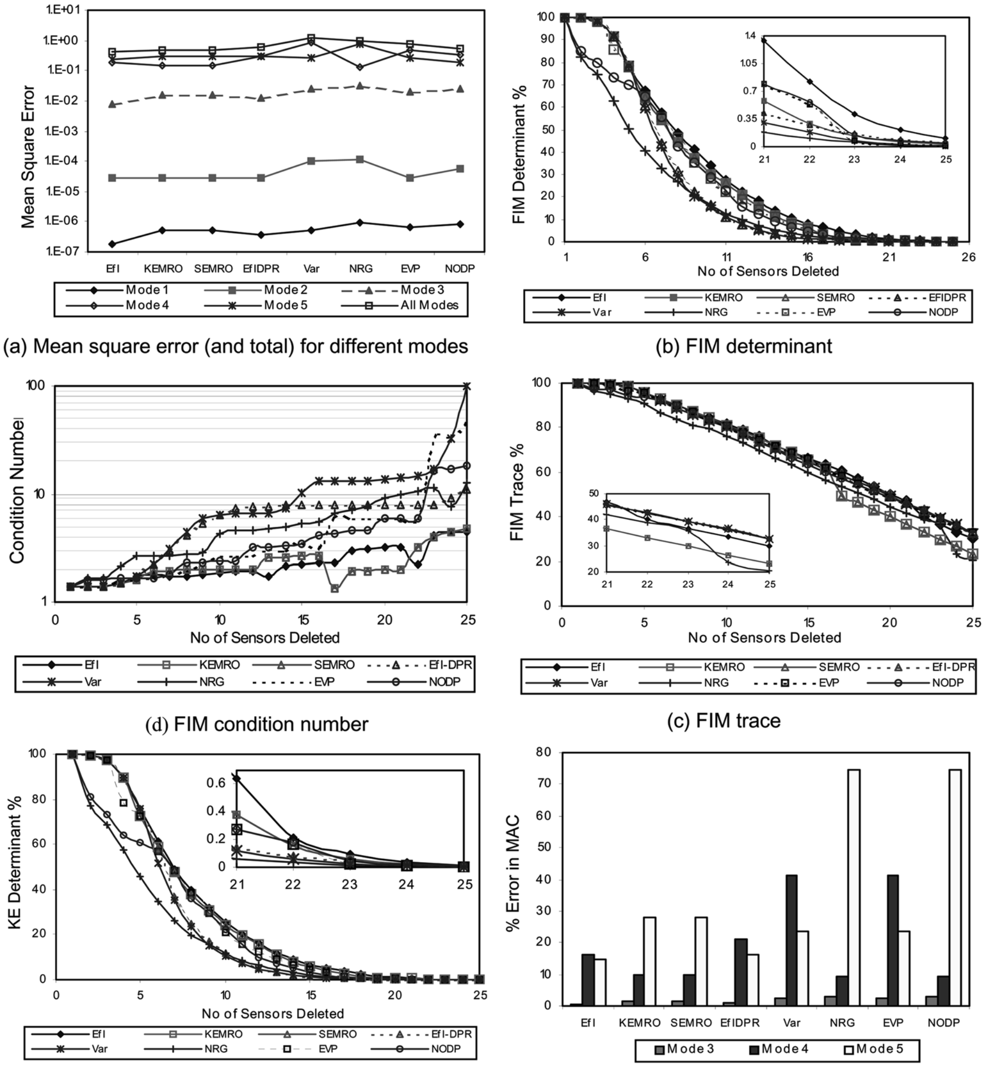


Fig. 3 Optimal sensor locations obtained using various OSP techniques for cantilever beam



(a) Mean square error (and total) for different modes (b) FIM determinant

(d) FIM condition number (c) FIM trace

(e) Kinetic energy determinant (f) % Error in MAC

Fig. 4 Characteristic evaluation of different OSP techniques for a cantilever beam

to each mode for all the OSP methods and also the total mean square error (TMSE) of all modes are shown in Fig. 4(a). A close look at Fig. 4(a) indicates that the MSE is generally comparable for all the methods, even though it is marginally higher for the NRG, EVP and variance methods. It can also be observed from mode wise plots of the MSE that the error is increasing with the increase in the mode number. As mentioned earlier, the trace, determinant and the condition number of FIM are monitored during iteration sequence. Fig. 4(b) shows the FIM determinant behaviour that is recorded during the sensor selection and displayed interms of percentage of its initial value against the progressive number of the omitted candidate sensors. The zoomed plot for the last five iterations is also shown separately within the figure. A close look at Fig. 4(b) indicates that FIM determinant of EFI is clearly superior to other methods. The EFI determinant is closely followed by EFI-DPR, NODP, EVP, KEMRO, and

SEMRO in the same order. The FIM determinant of NRG and variance method is the least of all the methods considered here. This clearly indicates that the sensor configurations obtained using EFI method clearly retain more information than the sensor configurations obtained with other methods.

Fig. 4(c) compares the FIM trace behaviour that is recorded during sensor selection for all the OSP methods and displayed interms of percentage of its initial value against the progressive number of omitted sensors. It can be observed from Fig. 4(c) that the EFI method maintains higher trace values when compared to other methods up to 20 iterations. However, the trace values of EFI are marginally dipped between 20 and 25 iterations when compared to NODP, EFI-DPR and variance methods. Further, it can also be observed that the trace values of KEMRO and SEMRO methods is comparable with EFI up to 16 iterations and then there is a substantial dip in the trace values. It can be concluded from the above observations that NODP, EFI-DPR and variance methods yields sensor configurations possessing a smaller estimate of error covariance matrix yielding better state estimates. The EFI closely follows the above methods, even though it exhibits superior performance at the earlier stages of iterations. The SEMRO and KEMRO methods performance is rather poor.

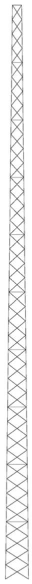
The condition number of FIM should be as low as possible in order to exhibit less sensitivity in the estimates to analytical modeling error. The condition number of FIM matrix during progressive deletion of sensors for various OSP methods is shown as a semi-log plot in Fig. 4(d). A close look at Fig. 4(d) clearly indicates that condition number of EFI method is low when compared to other methods, while the condition number of SEMRO and KEMRO is comparable. The EVP, NODP and variance methods exhibit high condition number of FIM matrix and hence are highly sensitive to modelling error.

In addition to determinant of FIM, it is proposed to compute the determinant of KE ($\Phi^T M \Phi$) for all the methods in order to evaluate the maximisation of energy retention characteristics of each of the method. Fig. 4(e) shows the KE determinant behaviour that is recorded during the sensor selection and displayed interms of percentage of its initial value against the progressive number of the omitted candidate sensors. The zoomed plot for the last five iterations is also shown separately within the figure. A close look at Fig. 4(e) indicates that even though there is considerable variation in the performance of various methods up to 24 iterations, all the methods converge to nearly same points at 25^{th} iteration. In the initial iterations, the performance of EFI and KEMRO interms of maximisation of energy retention is comparable and also clearly superior to other OSP techniques considered in this paper.

The modal assurance criterion (MAC) is expected to yield a value of 1 if the mode shapes are perfectly correlated. The FE model mode shape and the mode shape obtained by CSI of the displacement measured at the optimal sensor locations are used for calculating percentage error in MAC for each mode. The results obtained using various OSP techniques are presented in the form of a bar chart in Fig. 4(f). Since the error is negligibly small for the first two modes, they are not shown. It can be observed from Fig. 4(f) that the mode shapes obtained using EFI, EFI-DPR, KEMRO and SEMRO have good correlation with the analytically obtained mode shapes. The rest of the methods show significant error, especially for the last mode shape.

4.2. Numerical example 2

The second numerical example considered is a 40 m tall microwave tower. The details of the tower are shown in Fig. 5. Forty nodes on the tower are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors. For the present problem, the desired number of sensors is considered as five. The first five modes are considered for identification. Fig. 6 shows the optimal sensor locations suggested by all the eight methods being



Height of tower: 40 m
Elastic modulus: 205 GPa
Poisson's ratio: 0.33
Number of active nodes: 40

Area No	Area (cm^2)
1	45.65
2	34.77
3	25.12
4	15.39
5	13.79
6	8.66
7	6.25
8	4.79
Leg Area	
Bays	Area #
1 to 8	1
9 to 16	2
17 to 24	3
25 to 32	4
33 to 40	5
Braces Area	
Bays	Area #
1 to 8	6
9-24,33,34	7
25 to 32	8
35 to 40	8

Fig. 5 40 m high microwave tower

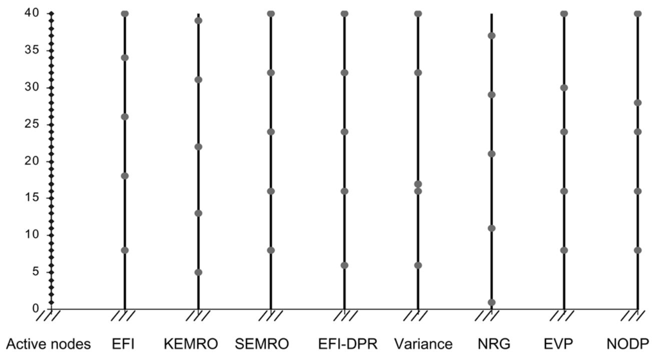
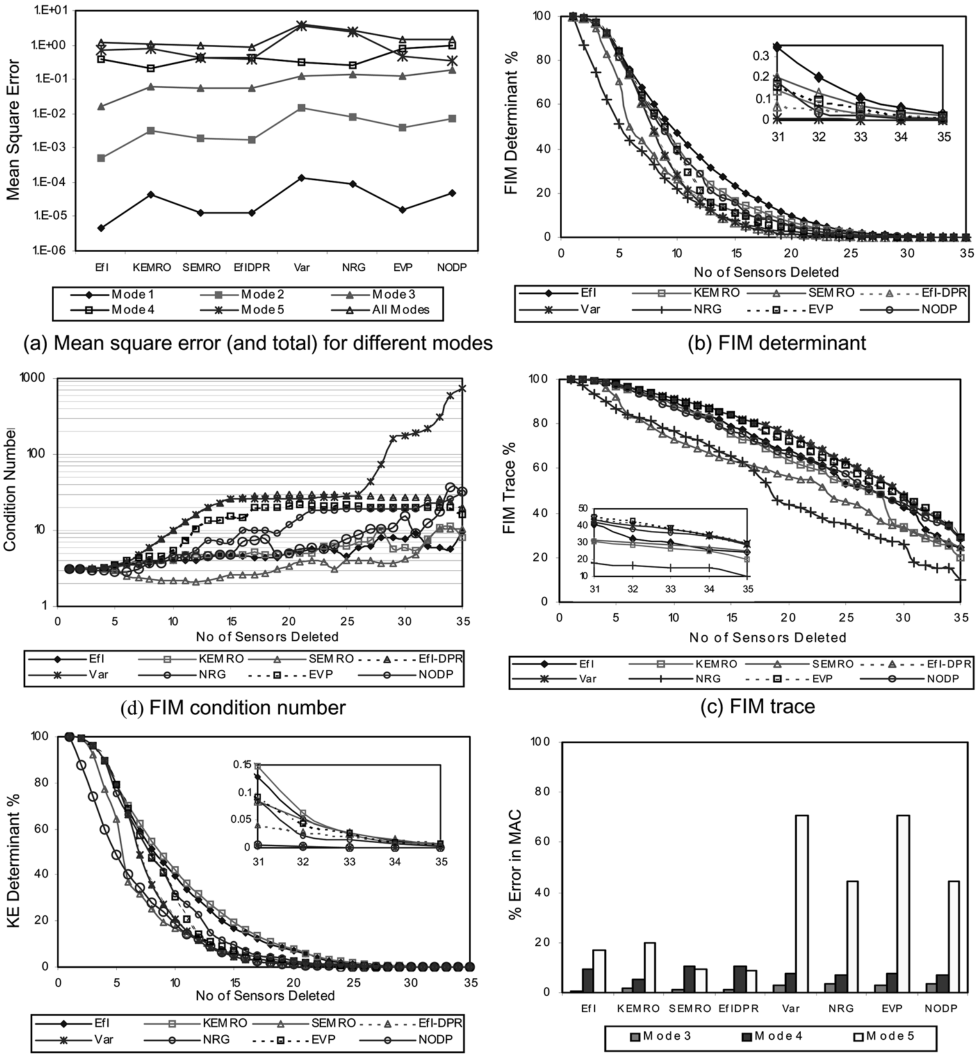


Fig. 6 Optimal sensor locations obtained using various OSP techniques for 40 m Tower

evaluated in this paper. A close look at Fig. 6 indicates that the sensor locations suggested by various OSP techniques are distinct. The behaviour of various OSP techniques is shown in Fig. 7 and the following observations can be made from it.

The MSE is smaller for EFI for the first three modes when compared to all other methods. However, when TMSE for all five modes is considered, the errors are comparable for all methods, except NRG and variance which show marginally higher values. The FIM determinant of EFI is higher and is closely followed by SEMRO and NODP methods. The determinant values of EFI-DPR, variance and KEMRO methods are fairly lower. Similarly, the trace of FIM matrix obtained for EFI, SEMRO, EVP is superior to other methods, while KEMRO, NRG are relatively inferior. The condition number of FIM matrices of EFI, KEMRO and SEMRO are found to be lower, while variance, NRG and NODP exhibit high condition number. The determinant of KE is found to be high for KEMRO, EFI and SEMRO, while it is low for NRG, EFI-DPR and NODP.



(a) Mean square error (and total) for different modes

(b) FIM determinant

(d) FIM condition number

(c) FIM trace

(e) Kinetic energy determinant

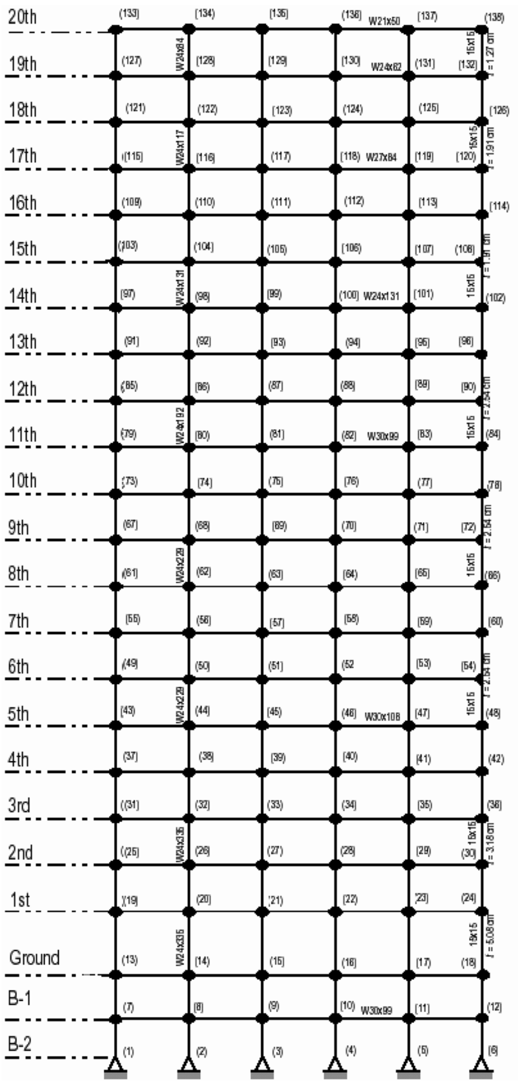
(f) % Error in MAC

Fig. 7 Characteristic evaluation of different OSP techniques for 40 m Tower

From Fig. 7(f), where the mode wise percentage error of MAC for various OSP techniques is shown, it can be observed that EFI, KEMRO, SEMRO and EFI-DPR methods exhibit lower MAC errors, while other methods show significant error especially for the fifth mode shape.

4.3. Numerical example 3

The third numerical example considered is a 22 storey five bay framed structure. The details of the framed structure are shown in Fig. 8. Twenty two nodes on the frame are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors. For the present problem, the desired number of sensors is considered as 5. Five modes are considered for identification here. The optimal sensor locations given by various OSP techniques are



Natural Frequencies (First six values)	0.2473 0.7049 1.2232 1.7305 2.2721 2.4778
Basement heights	3.65m
Ground height	5.49m
1^{st}-19^{th} levels height	3.96m
Bay widths (all)	6.10m

Fig. 8 22 storey 5 bay framed structure

shown in Fig. 9. Similar to earlier numerical examples, here also one can observe that distinct optimal sensor locations are suggested by each of the method considered in the paper. The behaviour of various OSP techniques for the framed structure considered as numerical example is shown in Fig. 10.

From the results furnished in Fig. 10, it can be observed that EFI, KEMRO, SEMRO shows higher FIM determinant values. EFI, EFI-DPR, Variance methods show higher trace values. While condition number of FIM for EFI, KEMRO and SEMRO is lower, Variance, EFI-DPR and NRG exhibit high condition number. The determinant of KE is found to be higher for KEMRO, EFI, and SEMRO and lower for NRG, Variance, and NODP methods. The percentage of MAC error of various OSP techniques given in Fig. 10(f) indicate that EFI, KEMRO, SEMRO and EFI-DPR consistently exhibit low percentage errors, while other methods exhibit very high percentage of error especially for the fifth mode.

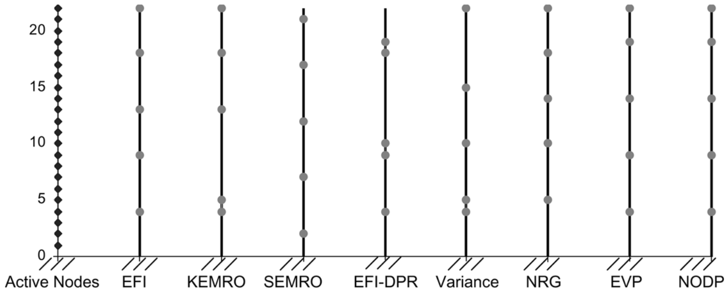


Fig. 9 Optimal sensor locations obtained using various OSP techniques for 22 storey 5-bay framed structure

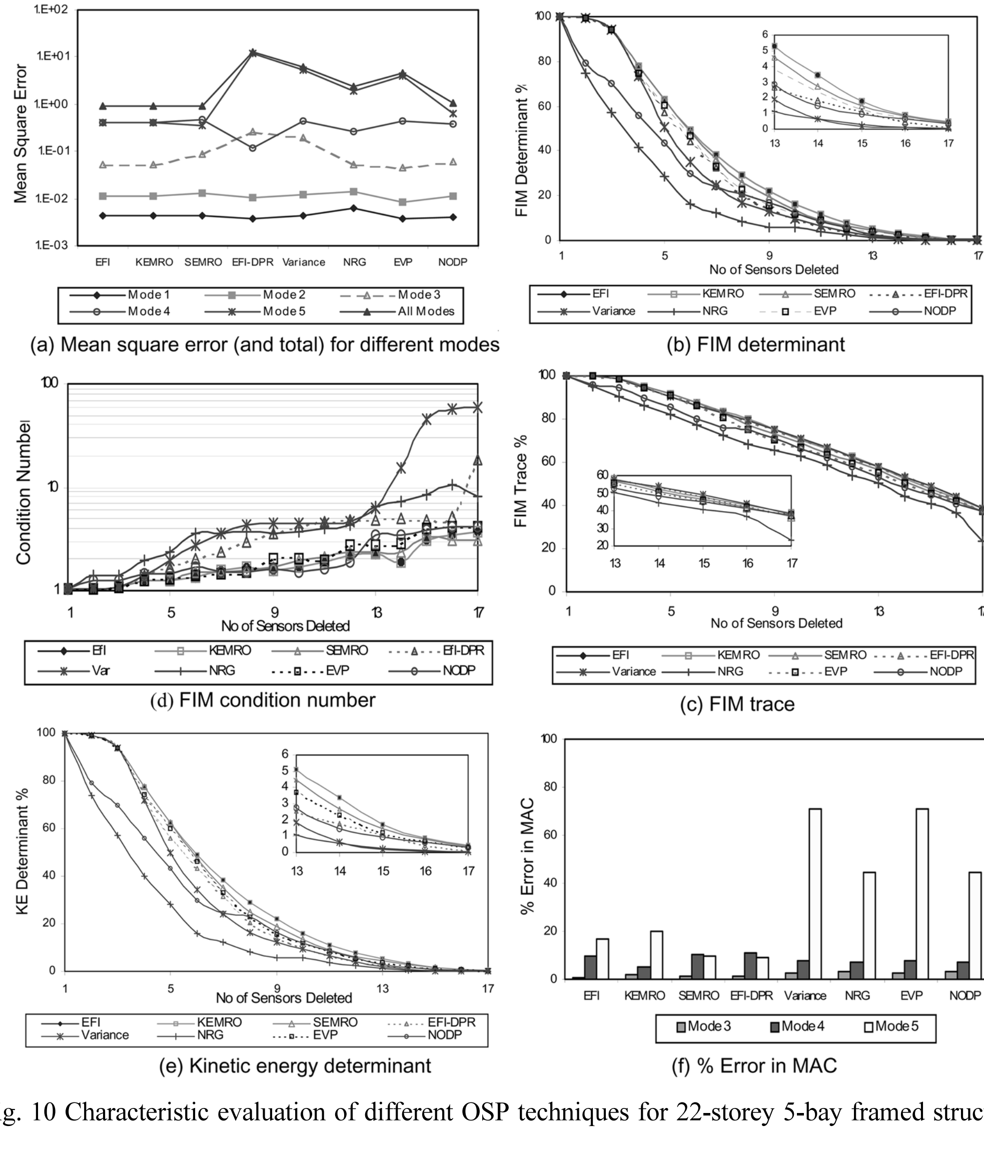


Fig. 10 Characteristic evaluation of different OSP techniques for 22-storey 5-bay framed structure

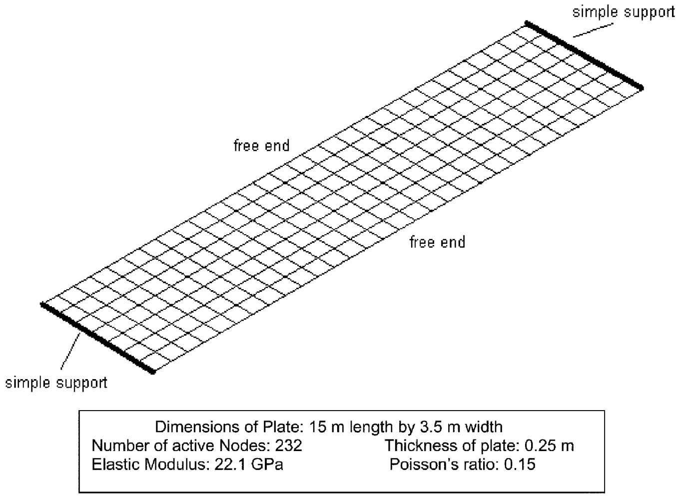


Fig. 11 FEM discretisation of a rectangular plate (concrete slab bridge) supported on short edges

4.3. Numerical example 4

A rectangular plate (concrete slab bridge) of size 3.5 m × 15 m supported on the short edges is considered as the fourth numerical example. The FE discretisation of the plate and the material properties are shown in Fig. 11. Two hundred and thirty two DOFs are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors i.e., 16 considered in the present work. Fig. 12 presents the optimal sensor locations obtained using various sensor placement techniques being evaluated in this paper. The optimal locations showed in Fig. 12 shows that each OSP technique chooses a distinct optimal sensor configuration. It can also be observed that the optimal sensor locations suggested by most of the OSP techniques are biased towards the free edges of the plate. The reason for the bias of the optimal sensor locations to the free edges of the plate can be explained by formulating the independence distribution vector E_D, which consists of the diagonal terms of the matrix E (Kammer 1991) given by

$$E = \Phi_s[\Phi_s^T\Phi_s]^{-1}\Phi_s^T \tag{26}$$

where Φ_s is the matrix of FEM target modes partitioned to possible candidate sensor locations. The diagonal term of E represent the contributions of the corresponding sensor locations to the rank of Φ_s or the linear independence of columns. For the plate problem, it is observed that all the diagonal entries of the matrix E corresponding to the DOF of free edges are significantly higher than the remaining interior DOF. This clearly indicates that the DOF of free edges are vital for linear independence of the identified mode shapes, where as sensor locations corresponding to the interior DOF of the plate, which have much lesser diagonal value, can be eliminated with out affecting the independence of modes and thus the identification of target modes. While the optimal sensor locations suggested by NODP, EVP exhibit symmetry, the optimal locations suggested by EFI and KEMRO are nearly symmetric.

Fig. 13 shows the behaviour of all the OSP techniques considered in this paper for solving the plate (slab bridge) problem. It can be observed form Fig. 13(a) that EFI, EFI-DPR exhibits superior

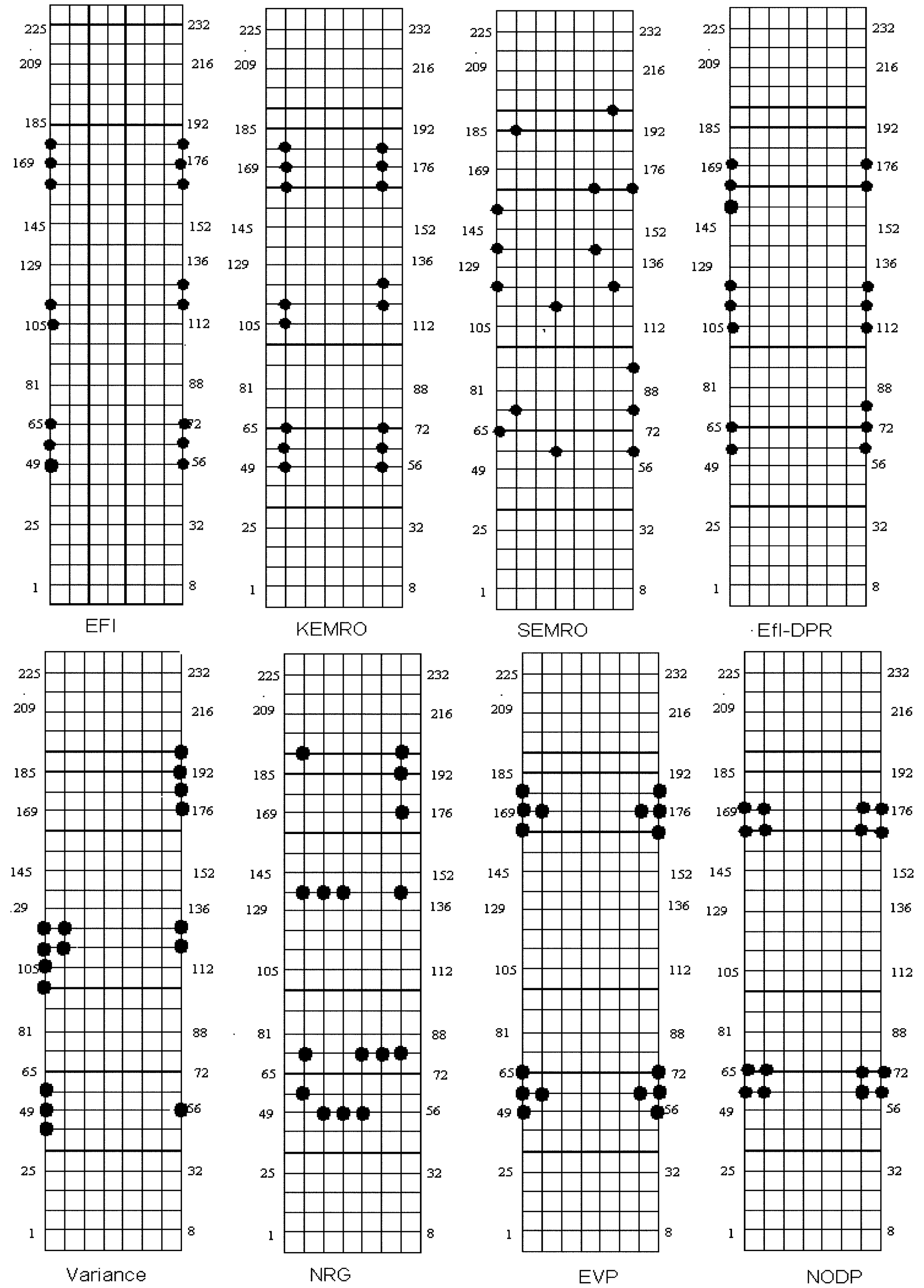


Fig. 12 Optimal sensor locations obtained using various OSP techniques for rectangular plate (concrete slab bridge)

performance interms of minimizing TMSE. The behaviour related to determinant, trace and condition number of FIM and the determinant of KE are shown in Figs. 13(b) to 13(e). A close look at these figures indicates that EFI is consistent and exhibits superior performance when compared to other OSP methods. However, for the plate problems, KEMRO, SEMRO, and EFI-DPR show comparable performance. The percentage error of MAC shown in Fig. 13(f) indicates that MAC errors are quite

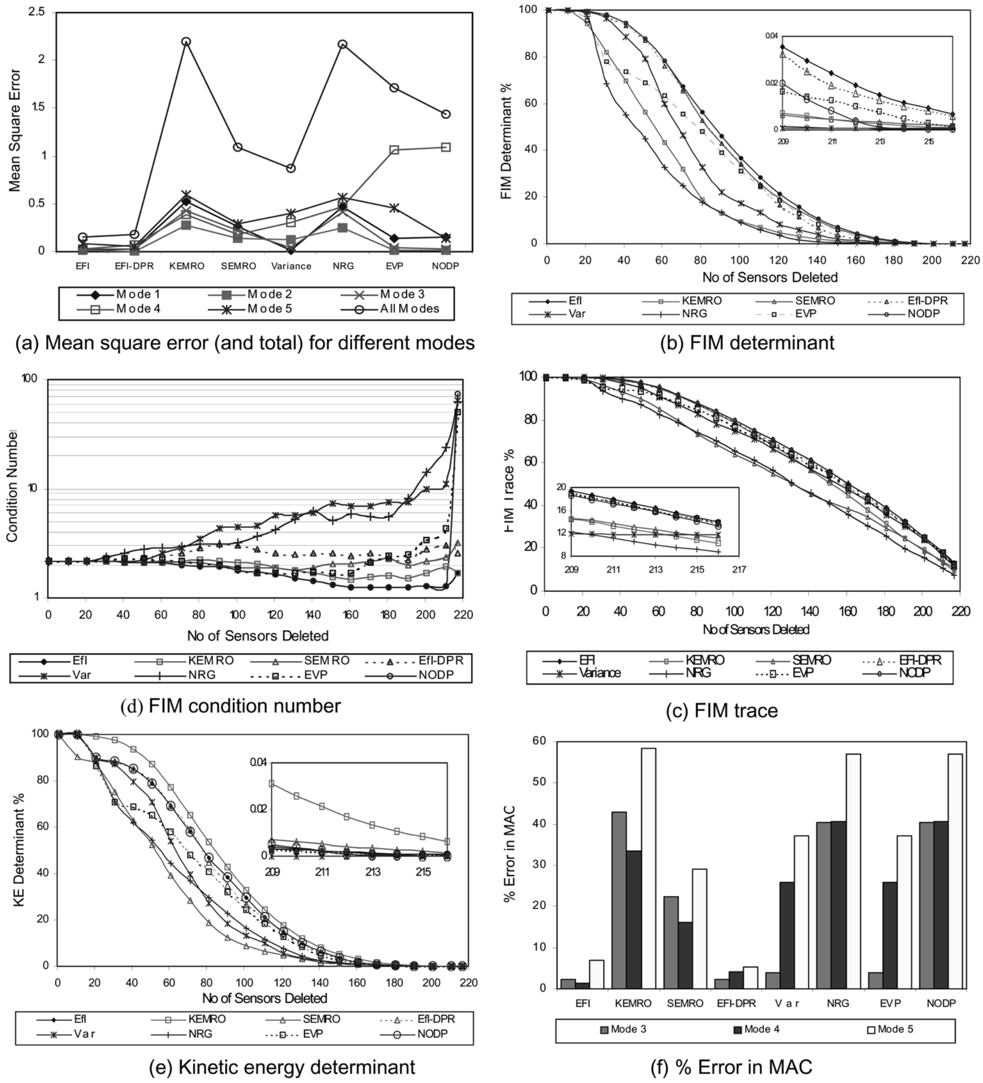


Fig. 13 Characteristic evaluation of different OSP techniques for a rectangular plate (concrete slab bridge)

insignificant in case of EFI and EFI-DPR when compared to other methods, especially for the fourth and fifth modes.

4.5. Numerical example 5

Finally, a bridge structure with multiple longitudinal girders is taken as numerical example to evaluate various OSP techniques. The bridge is of size 8.43 m × 10.4 m with four longitudinal girders. In order to accurately model the bridge, we preferred to use three dimensional finite elements to model the slab and also the longitudinal girders. The isometric view of the bridge and the material properties are shown in Fig. 14. The total number of brick elements in the FE model is 1643. Nine hundred and sixty nodes are considered as possible candidate set of sensor locations and the sensors are placed to

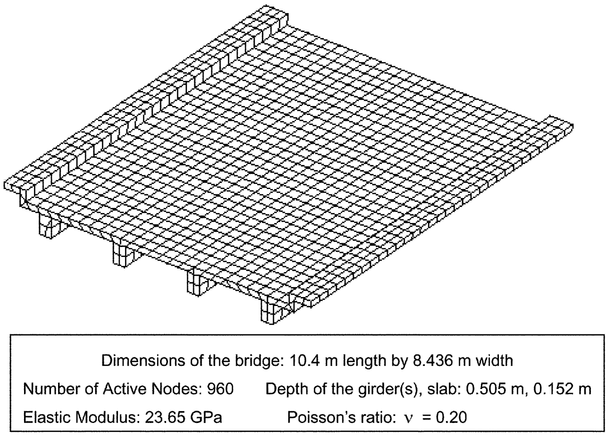


Fig. 14 Isometric view of the concrete girder bridge three-dimensional FE model

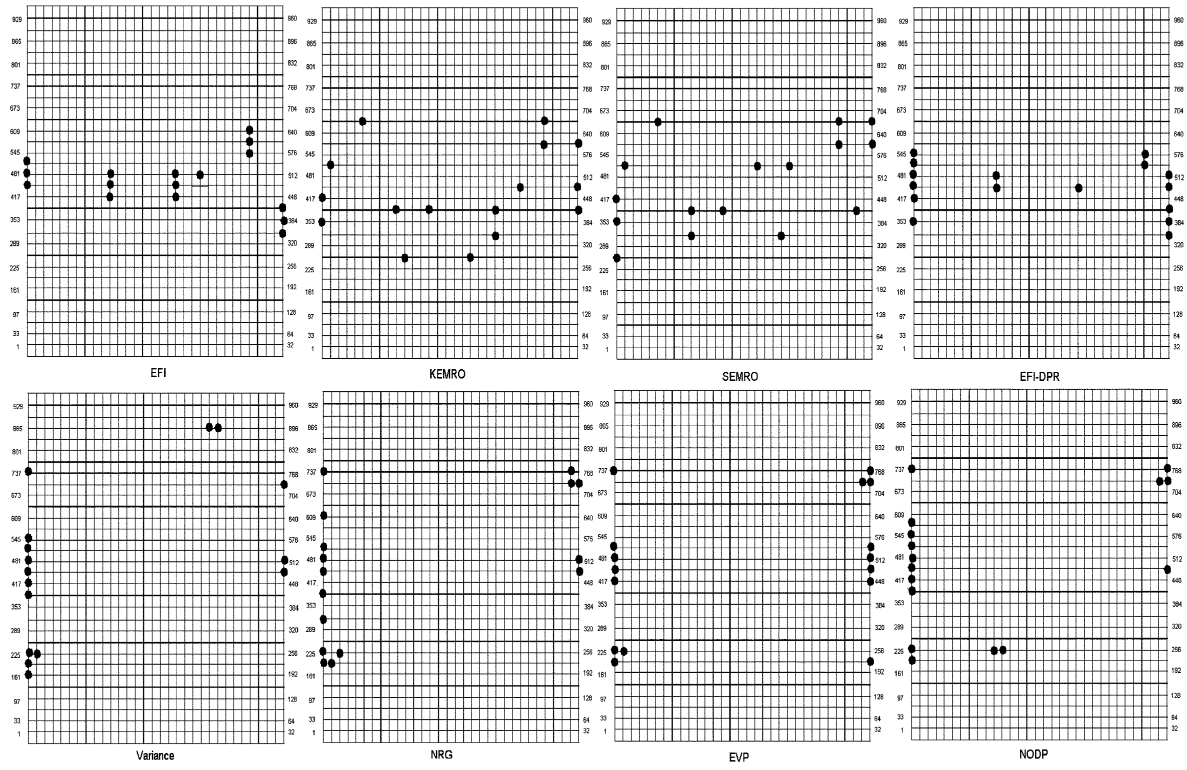


Fig. 15 Optimal sensor locations obtained using various OSP techniques for the concrete girder bridge

measure the vertical displacement (i.e. identification of bending modes) at all the possible candidate sensor locations. The number of sensors is gradually reduced to desired number of sensors i.e., 16 considered in the present work. Fig. 15 presents the optimal sensor locations obtained using various sensor placement techniques being evaluated in this paper.

Fig. 16 shows the characteristic behaviour of all the OSP methods considered. It can be observed from Fig. 16(a) that EFI, EFI-DPR, KEMRO, and SEMRO exhibit superior performance in terms of

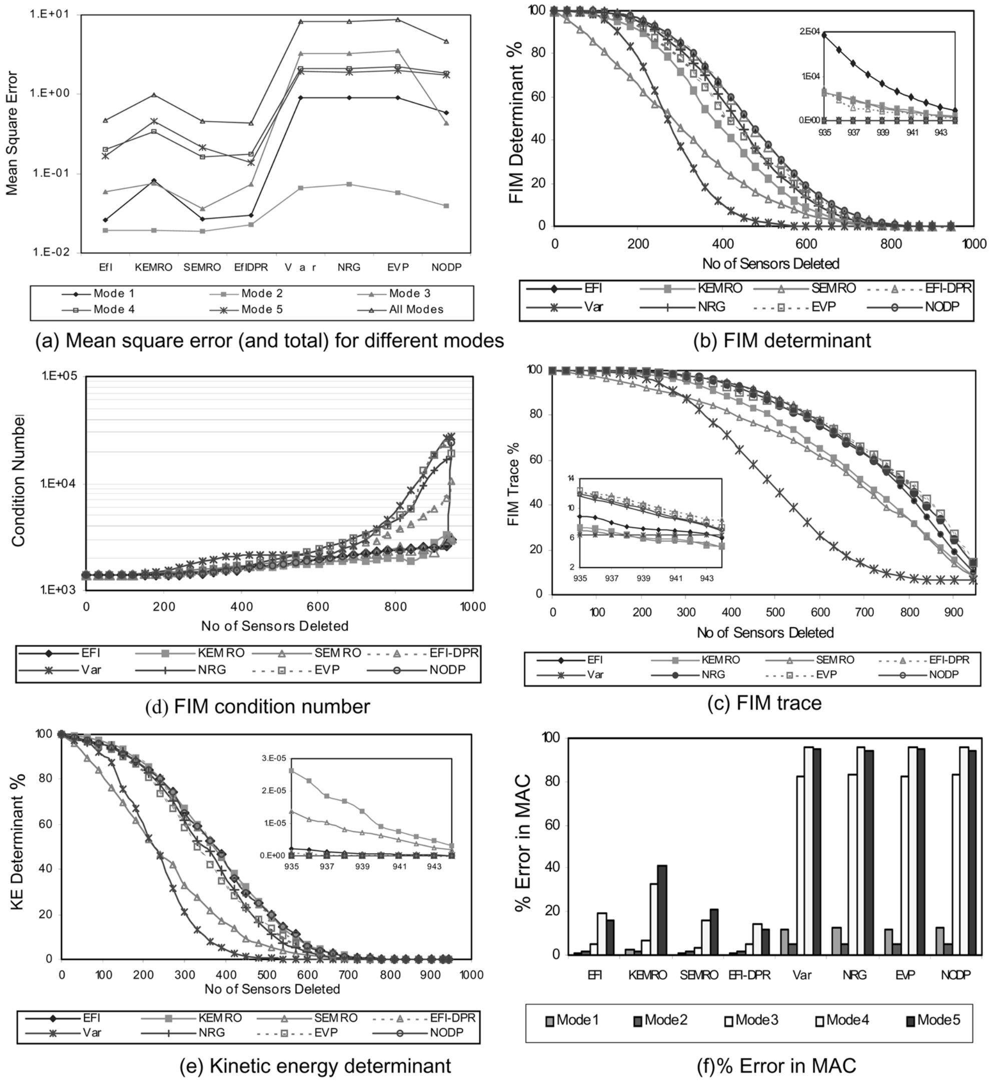


Fig. 16 Characteristic evaluation of different OSP techniques for the concrete girder bridge

minimizing TMSE. The behaviour related to determinant, trace and condition number of FIM and the determinant of KE are shown in Figs. 16(b) to 16(e). A close look at these figures indicates that EFI is consistent and exhibits superior performance when compared to other OSP methods. However, for the bridge problem, KEMRO, SEMRO, and EFI-DPR show comparable performances. The percentage error of MAC is shown in Fig. 16(f). A close look at Fig. 16(f) indicates that MAC errors are quite insignificant in case of EFI and EFI-DPR when compared to other methods, especially for the third, fourth, and fifth modes.

4.6. Optimal number of sensors

We have presented so far the optimal sensor configurations using various OSP techniques for the

Table 1 Optimal number of sensors required for varied levels of accuracy

TMSE	Optimal number of sensors							
	EFI	KEMRO	SEMRO	EFI-DPR	Variance	NRG	EVP	NODP
	Cantilever Beam							
0.0001	9	10	14	25	25	12	15	18
0.001	7	6	8	23	9	6	15	14
0.01	5	6	7	6	6	6	6	6
	Microwave Tower							
0.001	10	17	6	13	26	23	14	12
0.01	5	6	6	13	12	7	11	10
0.1	5	4	5	10	8	4	11	9
	22 Storey Frame							
0.01	8	8	7	6	19	9	7	10
0.1	5	5	5	5	5	4	4	5
0.25	3	3	4	3	3	3	4	4
	Rectangular Plate (concrete slab bridge)							
0.08	78	76	54	89	110	123	37	52
0.09	25	68	46	48	58	121	45	33
0.1	6	60	33	6	26	114	18	22
	Concrete Girder Bridge							
0.05	160	43	42	206	475	153	235	210
0.08	63	31	18	126	445	108	123	127
0.1	19	31	16	68	442	108	112	91
0.15	5	31	13	9	413	108	108	78

given number of sensors. However, It is always desirable to identify optimal number of sensors for the given error tolerance rather than decide upon the number of sensors arbitrarily. Keeping this in view, an effort has been made here to obtain the minimum number of sensors required for each of the OSP algorithm with varied TMSE. Table 1 shows the optimal number of sensors required for all the five numerical examples considered in this paper. A close look at the results given in the table indicates that KEMRO and SEMRO suggest less number of sensors when higher accuracies are desirable, especially for bridge problems. However, EFI suggests least number of sensors than any other method investigated in this paper when a practicable level of accuracy is prescribed.

4.7. Sensitivity of OSP algorithms with varied number of modes

In most cases, it is customary to determine the optimal sensor locations assuming a conservative number of target modes to be extracted during dynamic testing. However, it is always possible that only less number of modes (than target mode) is extracted during dynamic testing in the field. It can easily be verified that the optimal sensor locations are sensitive to the mode shape and certainly the number of modes influence the optimal locations. Keeping this in view, it is proposed to evaluate the sensitivities

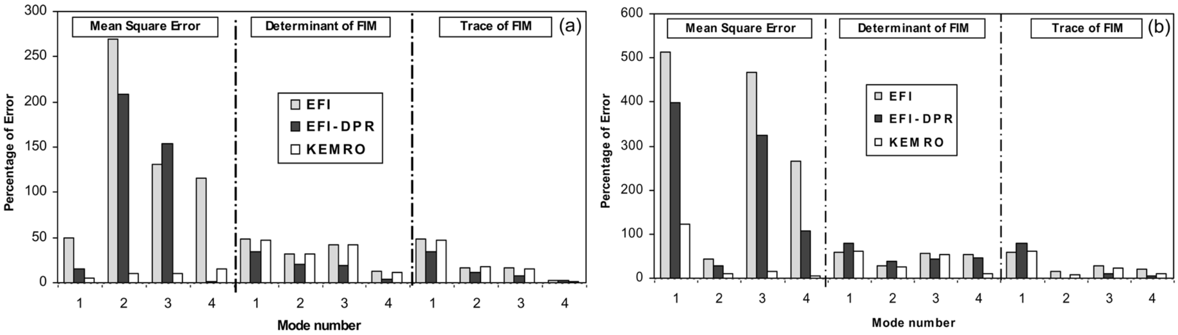


Fig. 17 Sensitivity of selected OSP methods with respect to varied number of target modes. (a) Plate and (b) Bridge

of various OSP techniques with respect to number of modes to be extracted. For this study, only EFI, EFI-DPR and KEMRO are considered. The optimal sensor locations are determined by using these three methods for the last two numerical examples i.e. plate and bridge, considering first five modes. The sensitivities of the three OSP techniques with varied number of modes are shown in the form of percentage error in Fig. 17 for both the numerical examples. A close look at Fig. 17 indicates that EFI-DPR and KEMRO are less sensitive when compared to EFI method. While the error in determinant and trace values of FIM with varied number of modes is comparatively less in EFI-DPR, the KEMRO is

Table 2 Optimal sensor locations obtained using EFI truncation and EFI expansion

Method	Optimal Sensor Locations for Girder	det (FIM)	No. of locations differing
EFI	49, 56, 57, 64, 65, 72, 105, 113, 120, 128, 161, 168, 169, 176, 177, 184	1.6735E-40	-
EFI-EXP-R	49, 57, 64, 65, 72, 105, 112, 113, 120, 121, 128, 161, 168, 176, 177, 184	1.5142E-40	2
EFI-EXP-L	49, 56, 64, 65, 72, 80, 112, 113, 120, 121, 161, 168, 169,176, 177, 184	1.4729E-40	3
EFI-EXP-B	49, 56, 57, 64, 65, 72, 105, 113, 120, 128, 161, 168, 169,176, 177, 184	1.6735E-40	0
EFI-EXP-F	49, 56, 57, 64, 65, 72, 105 ,113, 120, 128, 161, 168, 169, 176, 177, 184	1.6735E-40	0
	Optimal sensor locations for girder bridge		
EFI	352, 384, 416, 427, 435, 449, 459, 467, 481, 491, 499, 502, 513, 572, 604, 636	8.919E-19	-
EFI-EXP-R	352, 384, 416, 427, 435, 448, 459, 467, 481, 491, 499, 502, 513, 572, 604, 636	7.853E-19	1
EFI-EXP-L	352, 384, 416, 427, 435, 449, 458, 459, 467, 481, 491, 499, 513, 572, 604, 636	8.647E-19	1
EFI-EXP-B	352, 384, 427, 435, 449, 459, 460, 467, 469, 480, 481, 491, 572, 604, 636, 668	7.513E-19	4
EFI-EXP-F	384, 416, 417, 427, 435, 448, 449, 459, 467, 470, 480, 481, 492, 513, 604, 636	7.788E-19	5

The locations of initial sensor set are underlined in the table.
EFI: Effective independence method - Truncation.
EFI-EXP-R: Sensor expansion method with random initial set among the sixteen best ranked dof with respect to EFI values
EFI-EXP-L: Sensor expansion method with initial set containing locations with least EFI values among the sixteen best ranked dof with respect to EFI values.
EFI-EXP-B: Sensor expansion method with initial set containing locations with highest EFI values among the sixteen best ranked dof with respect to EFI values.
EFI-EXP-F: Sensor expansion method with initial set containing locations with highest EFI values from the entire possible sensor set locations (all DOFs).

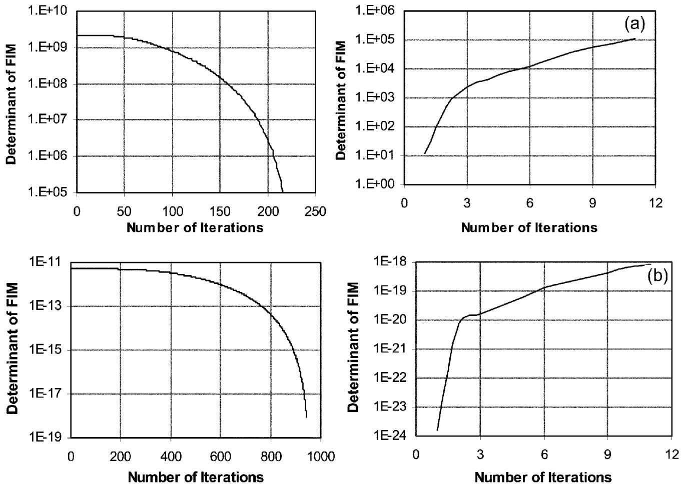


Fig. 18 FEM determinant using sensor set truncation and expansion methods. (a) Plate and (b) Bridge

found to be least sensitive when mean square error is considered. The results are found to be consistent for the two numerical examples considered.

4.8. Sensor set expansion

Numerical studies have been carried out to study the efficacy of the sensor set expansion method for civil structures. For this purpose, the last two numerical examples, i.e., plate and bridge, are considered. In order to have a fair comparison with the truncation method, the initial set of sensors is chosen from the sixteen optimal locations obtained using the EFI truncation method. The number of initial sensors chosen is five. We employed three different approaches to choose the initial sensor set from the best sensor locations. They are random selection, selection of sensors with lowest EFI values and finally sensors with highest EFI values. Apart from these three approaches, we have also considered a fourth approach, in which we choose the initial sensor set from the best ranked DOF with respect to EFI values from all the possible sensor set locations i.e., all DOFs. The results obtained are compared with EFI truncation method and are shown in Table 2 for both the numerical examples considered. The initial sensors chosen for each of the four approaches are clearly marked in the table. A close look at the results indicates that the truncation method is marginally superior to sensor expansion method and sometimes even comparable. It can also be observed that the optimal sensor locations obtained using expansion method differs marginally (i.e. only in few locations) when compared with the truncation method. The reduction in the FIM determinant for sensor set expansion method ranges from 0 to 15% for different initial sensor set selection schemes. The FIM determinant performance of EFI truncation as well as expansion methods is shown in Fig. 18.

5. Conclusions

In this paper, several OSP techniques are evaluated. The desired sensor configurations should be chosen in such a way that they should be able to give maximum information for the reconstruction of modal and dynamic characteristics of the structure being investigated. Our current interest in evaluating various OSP methods is to characterize each method with respect to its applicability to civil engineering structures.

In this paper, we have considered eight OSP techniques and employed on five civil engineering structures. In order to assess the relative performance of each of the OSP methods, we propose six evaluation criteria. The following are the conclusions based on studies carried out in this paper.

i. Effective independence method is found to be consistently superior for all the problems solved in this paper. The sensor configurations obtained using EFI is capable of accurately capturing the dynamic response of the structure with very less mean square error, exhibits high signal strength which minimises the sensitivity with respect to noise, high accuracy, and less sensitivity to analytical modeling errors.

ii. KEMRO and SEMRO methods follow EFI interms of consistent performance when compared to other methods while accurately capturing dynamic response, signal strength, accuracy, etc.

iii. EFI-DPR method is competitive for the plate and bridge problems considered in this paper, while it performs rather poorly for the beam, tower, and frame structures.

iv. For the given MSE error tolerance, the EFI method generally yields lesser number of sensors when compared to other methods when a practical limit is set on level of accuracy. However, when higher levels of accuracy are desired, KEMRO and SEMRO give sensor configurations which are superior to EFI.

v. The numerical studies carried out to investigate the sensitivities of some selective well behaved OSP methods with respect to the varied target modes indicate that EFI is highly sensitive to variation in target modes. While EFI-DPR is found to be less sensitive when trace and determinant values of FIM is considered, KEMRO is found to be less sensitive when MSE is considered.

vi. The sensor set expansion method appears to be slightly inferior but sometimes even comparable with conventional EFI truncation method. However, sensor expansion method results in considerable reduction in computational cost while applying for problems with larger candidate sensor set.

vii. The general limitation of EFI and also KEMRO and SEMRO methods is that the desired number of sensors should not be less than the number of modes considered for identification. In the context of civil engineering structures, this may not be a major problem because in most of the situations, the interest lies in identifying only few lower modes.

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