Smart Structures and Systems, *Vol. 22, No. 6 (2018) 743-751*
DOI: https://doi.org/10.12989/sss.2018.22.6.743

Buckling analysis of nanocomposite plates coated by magnetostrictive layer

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(Received October 13, 2018, Revised November 22, 2018, Accepted November 24, 2018)

Abstract. In this project, buckling response of polymeric plates reinforced with carbon nanotubes (CNTs) and coated by magnetostrictive layer was studied. The equivalent nanocomposite properties are determined using Mori-Tanak model considering agglomeration effects. The structure is simulated with first order shear deformation theory (FSDT). Employing strains-displacements, stress-strain, the energy equations of the structure are obtained. Using Hamilton's principal, the governing equations are derived considering the coupling of mechanical displacements and magnetic field. Using Navier method, the buckling load of the sandwich structure is obtained. The influences of volume percent and agglomeration of CNTs, geometrical parameters and magnetic field on the buckling load are investigated. Results show that with increasing volume percent of CNTs, the buckling load increases. In addition, applying magnetic field, increases the frequency of the sandwich structure.

Keywords: buckling analysis; magnetostrictive; CNTs; agglomeration; magnetic field

1. Introduction

The application of sandwich structures in many industries is rising due to their excellent properties such as high strength, low weight and resistance to fatigue. One of the special types of these structures is truncated conical shell with application in aerospace, marine and automobile industries. Due to their practical interest, sandwich structures have been the subject of numerous works.

In the field of sandwich structures, Shariyat (2009) studied the dynamic buckling of piezo laminated plates under thermo-electro-mechanical loads. Pandit *et al.* (2010) proposed a finite element model for bending and vibration analysis of laminated sandwich plates. The analysis of bending, buckling and free vibration response of laminated plate was presented by Ferreira *et al*. (2011). Malekzadeh and Shojaee (2013) investigated the buckling behavior of quadrilateral laminated plates reinforced by carbon nanotubes (CNTs). Malekzadeh and Zarei (2014) performed free vibration analysis of quadrilateral laminated CNTs reinforced plates based on FSDT. Marjanović and Vuksanović (2014) carried out free vibration and buckling analysis of laminated composite and sandwich plates. Li *et al.* (2015) researched the dynamic buckling behavior of laminated composite plates under an axial step load. Li *et al.* (2016) performed the analysis of the buckling and vibro-acoustic response of the laminated composite plates in thermal environment. The post-buckling analysis of bi-axially compressed laminated nanocomposite plates was presented by Zhang *et al.* (2016). Large amplitude vibration problem of laminated composite spherical shell panel under combined temperature and moisture environment was analyzed by Mahapatra *et al.* (2016a). The nonlinear free vibration behaviour of laminated composite spherical shell panel under the elevated hygrothermal environment was investigated by Mahapatra and Panda (2016b). Mahapatra *et al.* (2016c) studied the geometrically nonlinear transverse bending behavior of the shear deformable laminated composite spherical shell panel under hygro-thermo-mechanical loading. Nonlinear free vibration behavior of laminated composite curved panel under hygrothermal environment was investigated by Mahapatra *et al.* (2016d). Nonlinear flexural behaviour of laminated composite doubly curved shell panel was investigated by Mahapatra *et al.* (2016e) under hygro-thermo-mechanical loading by considering the degraded composite material properties through a micromechanical model. Moradi-Dastjerdi and Malek-Mohammadi (2016) studied bi-axial behavior of nanocomposite sandwich plates reinforced by CNTs Fan and Wang (2016) carried out nonlinear bending and post-buckling analysis of hybrid laminated plates containing CNTs reinforced composite layers in thermal environments. Yu *et al.* (2016) studied free vibration and buckling response of laminated composite plates with cutouts. Free vibration analysis of anti-symmetric laminated composite and soft core sandwich plates was studied by Sayyad and Ghugal (2017). Zhao *et al*. (2017) proposed a finite element formulation on basis of piecewise shear deformation theory to assess vibrational behavior of laminated composite and sandwich plates in thermal environments. Pramod *et al.* (2017) appraised static and free vibration response of cross-ply laminated plates with simply supported boundary conditions. Lei *et al.* (2017) presented a geometrically nonlinear analysis of CNTs reinforced laminated composite plate using meshless method. Zhang and Selim (2017) focused on free vibration analysis of CNTs reinforced thick laminated composite plate according to Reddy's higher order shear deformation theory. Hajmohammad *et al*. (2017) investigated dynamic buckling of laminated

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http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

viscoelastic sandwich plate with CNT-reinforced layers and viscoelastic piezoelectric layers at the top and bottom face sheets. Wave propagation in a piezoelectric sandwich plate with viscoelastic nanocomposite core subjected to a magnetic field and viscoelastic piezoelectric layers subjected to an electric field was studied by Kolahchi et al. (2017a). In another work by Kolahchi *et al.* (2017b) Optimization of embedded piezoelectric sandwich nanocomposite plates for dynamic buckling analysis was presented based on Grey Wolf algorithm. The flexural behaviour of the laminated composite plate embedded with two different smart materials (piezoelectric and magnetostrictive) and subsequent deflection suppression were investigated by Dutta *et al*. (2017). Shokravi and Jalili (2017) presented nonlocal temperature-dependent dynamic buckling analysis of embedded sandwich micro plates reinforced by functionally graded carbon nanotubes (FG-CNTs). Suman *et al.* (2017) studied static bending and strength behaviour of the laminated composite plate embedded with magnetostrictive (MS) material numerically using commercial finite element tool. Vibration and buckling analysis of laminated sandwich truncated conical shells with compressible or incompressible core were presented by Nasihatgozar and Khalili (2018). Shokravi (2018) studied dynamic buckling of the smart subjected to blast load subjected to electric field.

To the best of authors' knowledge, this paper is the first to investigate the buckling analysis of the nanocomposite plates coated by magnetostrictive layer. The mathematical model is developed on the basis of the FSDT and using Hamilton's principle. The Navier method is applied to obtain the buckling load of the system. The effects of various parameters like geometric constants, volume fraction and agglomeration of CNTs and magnetic field on the buckling load of the structure are examined.

2. Formulation

Fig. 1 shows a nanocomposite plate reinforced by agglomerated CNTs coated by magnetostrictive layer. The length and width of the struature are *a* and *b*, respectively and thickness of the nanocomposite and magnetostrictive layers are indicated by h_n and h_m , respectively.

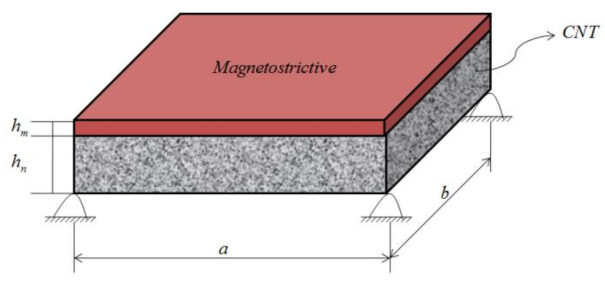

Fig. 1 A nanocomposite plate reinforced by agglomerated CNTs coated by magnetostrictive layer

2.1 FSDT

There are many new theories for modeling of different structures. Some of the new theories have been used by Tounsi and co-authors (Bessaim 2013, Bouderba 2013, Belabed 2014, Ait Amar Meziane 2014, Zidi 2014, Hamidi, 2015, Bourada 2015, Bousahla *et al*. 2016a ,b, Beldjelili 2016, Boukhari 2016, Draiche 2016, Bellifa 2015, Attia 2015, Mahi 2015, Ait Yahia 2015, Bennoun 2016, El-Haina, 2017, Menasria 2017, Chikh 2017).

Based on FSDT, the displacement fields can be written as (Reddy 2002)

$$\begin{aligned} U(\mathrm{x,y,z,t}) &= u_0(x,y,t) + z\,\psi(x,y,t), \\ V(\mathrm{x,y,z,t}) &= v_0(x,y,t) + z\,\phi(x,y,t), \\ W(\mathrm{x,y,z,t}) &= w_0(x,y,t), \end{aligned} \tag{1}$$

where u_0, v_0 and w_0 are mid-plane displacements and ψ as well as ϕ indicate the rotations. The strain relations of the structure are

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \psi}{\partial x} \\ \varepsilon_{yy} &= \frac{\partial V}{\partial y} = \frac{\partial v}{\partial y} + z\frac{\partial \phi}{\partial y} \\ \varepsilon_{zz} &= \frac{\partial W}{\partial z} = 0 \\ \gamma_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial y} + z\frac{\partial \psi}{\partial y} + \frac{\partial v}{\partial x} + z\frac{\partial \phi}{\partial x}\right) \\ \gamma_{xz} &= \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left(\psi + \frac{\partial w}{\partial x}\right) \\ \gamma_{yz} &= \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\left(\phi + \frac{\partial w}{\partial y}\right) \end{aligned} \tag{2}$$

2.2 Basic relations

The stress relations for the nanocomposite and magnetostrictive layers can be expressed as

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\gamma_{xz} \\ 2\gamma_{yz} \\ 2\gamma_{xy} \end{pmatrix} \tag{3}$$

$$\begin{aligned} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix} &= \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{pmatrix} - \begin{pmatrix} 0 & 0 & \bar{e}_{31} \\ 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{e}_{36} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ h_z \end{pmatrix}, \\ \begin{pmatrix} \tau_{yz} \\ \tau_{xz} \end{pmatrix} &= \begin{pmatrix} C_{44} & 0 \\ 0 & C_{55} \end{pmatrix} \begin{pmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{pmatrix}, \end{aligned} \tag{4}$$

where Q_{ij} and C_{ij} are stiffness constants of the nanocomposite and magnetostrictive layers, respectively;

$\bar{e}_{ij}$ are magnetic constants and $h_z = \frac{\partial \Psi}{\partial x}$ is the magnetic field. Mechanical analysis of nanostructures has been reported by many researchers (Zemri 2015, Larbi Chaht 2015, Belkorissat 2015, Ahouel 2016, Bounouara 2016, Bouafia 2017, Besseghier 2017, Bellifa 2017, Mouffoki 2017, Khetir 2017). The stiffness constants of the nanocomposite layer (Q_{ij}) can be obtained based on Mori-Tanaka model as (Shi and Feng 2004)

$$E = \frac{9KG}{3K+G}, \tag{5}$$

$$\upsilon = \frac{3K-2G}{6K+2G}. \tag{6}$$

where K and G are bulk and shear modulus which can be defined as

$$K = K_{out}\left[1+\frac{\xi\left(\frac{K_{in}}{K_{out}}-1\right)}{1+\alpha(1-\xi)\left(\frac{K_{in}}{K_{out}}-1\right)}\right], \tag{7}$$

$$G = G_{out}\left[1+\frac{\xi\left(\frac{G_{in}}{G_{out}}-1\right)}{1+\beta(1-\xi)\left(\frac{G_{in}}{G_{out}}-1\right)}\right], \tag{8}$$

where ξ and ζ are agglomeration parameters and

$$K_{in} = K_m + \frac{(\delta_r - 3K_m\chi_r)C_r\zeta}{3(\xi - C_r\zeta + C_r\zeta\chi_r)}, \tag{9}$$

$$K_{out} = K_m + \frac{C_r(\delta_r - 3K_m\chi_r)(1-\zeta)}{3[1-\xi-C_r(1-\zeta)+C_r\chi_r(1-\zeta)]}, \tag{10}$$

$$G_{out} = G_m + \frac{C_r(\eta_r - 3G_m\beta_r)(1-\zeta)}{2[1-\xi-C_r(1-\zeta)+C_r\beta_r(1-\zeta)]}, \tag{11}$$

$$G_{in} = G_m + \frac{(\eta_r - 3G_m\beta_r)C_r\zeta}{2(\xi - C_r\zeta + C_r\zeta\beta_r)}, \tag{12}$$

where C_r are volume percent of CNTs and

$$\chi_r = \frac{3(K_m+G_m)+k_r-l_r}{3(k_r+G_m)}, \tag{13}$$

$$\beta_r = \frac{1}{5}\left\{\begin{array}{l}\frac{4G_m+2k_r+l_r}{3(k_r+G_m)}+\frac{4G_m}{(p_r+G_m)} \\ +\frac{2[G_m(3K_m+G_m)+G_m(3K_m+7G_m)]}{G_m(3K_m+G_m)+m_r(3K_m+7G_m)}\end{array}\right\}, \tag{14}$$

$$\delta_r = \frac{1}{3}\left[n_r+2l_r+\frac{(2k_r-l_r)(3K_m+2G_m-l_r)}{k_r+G_m}\right], \tag{15}$$

$$\eta_r = \frac{1}{5}\left[\begin{array}{l}\frac{2}{3}(n_r-l_r)+\frac{4G_m p_r}{(p_r+G_m)} \\ +\frac{8G_m m_r(3K_m+4G_m)}{3K_m(m_r+G_m)+G_m(7m_r+G_m)} \\ +\frac{2(k_r-l_r)(2G_m+l_r)}{3(k_r+G_m)}\end{array}\right]. \tag{16}$$

where k_r، l_r، n_r، p_r، m_r are Hill constants and

$$\alpha = \frac{(1+\upsilon_{out})}{3(1-\upsilon_{out})}, \tag{17}$$

$$\beta = \frac{2(4-5\upsilon_{out})}{15(1-\upsilon_{out})}, \tag{18}$$

$$\upsilon_{out} = \frac{3K_{out}-2G_{out}}{6K_{out}+2G_{out}}. \tag{19}$$

$$K_m = \frac{E_m}{3(1-2\upsilon_m)}, \tag{20}$$

$$G_m = \frac{E_m}{2(1+\upsilon_m)}. \tag{21}$$

2.3 Governing equations

Utilizing energy method, the potential energy can be given as

$$\begin{aligned}U = \frac{1}{2}\int [&N_{xx}\frac{\partial u}{\partial x}+M_{xx}\frac{\partial \psi}{\partial x}+N_{yy}\frac{\partial v}{\partial y} \\ &+M_{yy}\frac{\partial \phi}{\partial y}+N_{xz}(\psi+\frac{\partial w}{\partial x})+ \\ &N_{yz}(\phi+\frac{\partial w}{\partial y})+N_{xy}\frac{\partial u}{\partial y}+ \\ &M_{xy}\frac{\partial \psi}{\partial y}+N_{xy}\frac{\partial v}{\partial x}+M_{xy}\frac{\partial \phi}{\partial x}]dA\end{aligned} \tag{22}$$

where the stress resultants are

$$\begin{Bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\tau_{xy}\end{Bmatrix}dz, \tag{23}$$

$$\begin{Bmatrix} N_{xz} \\ N_{yz} \end{Bmatrix} = k' \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz , \tag{24}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} z\,dz , \tag{25}$$

where k' is shear correction factor. The external work due to the in plane loads are

$$W = -\frac{1}{2}\int [N_x^m (\frac{\partial W}{\partial x})^2 + N_y^m (\frac{\partial W}{\partial y})^2] dA \tag{26}$$

where N_x^m and N_y^m are in plane loads in the *x* and *y* directions, respectively. Applying Hamilton's principle, we have the following governing equations

$$\begin{aligned}
\delta U :& \frac{\partial}{\partial x}(Q_{11}\frac{\partial u}{\partial x}h_n + Q_{12}\frac{\partial v}{\partial y}h_n + C_{11}\frac{\partial u}{\partial x}h_m \\
&+C_{12}h_m\frac{\partial v}{\partial y} + C_{11}\frac{\partial \phi}{\partial x}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) \\
&+C_{12}\frac{\partial \phi}{\partial y}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2)) \\
&+\frac{\partial}{\partial y}(Q_{21}\frac{\partial u}{\partial x}h_n + Q_{22}\frac{\partial v}{\partial y}h_n + C_{21}\frac{\partial u}{\partial x}h_m \\
&+C_{21}\frac{\partial \psi}{\partial x}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) + \\
&C_{21}\frac{\partial \phi}{\partial y}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) + C_{22}\frac{\partial v}{\partial y}h_m) = 0
\end{aligned} \tag{27}$$

$$\begin{aligned}
\delta V :& \frac{\partial}{\partial x}(Q_{66}\frac{\partial u}{\partial y}h_n + Q_{66}\frac{\partial v}{\partial x}h_n + C_{66}\frac{\partial u}{\partial y}h_m \\
&+C_{66}\frac{\partial \psi}{\partial y}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) \\
&+C_{66}\frac{\partial v}{\partial x}h_m + C_{66}\frac{\partial \phi}{\partial x}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) \\
&+\frac{\partial}{\partial y}(Q_{66}\frac{\partial u}{\partial y}h_n + Q_{66}\frac{\partial v}{\partial x}h_n + C_{66}\frac{\partial u}{\partial y}h_m \\
&+C_{66}\frac{\partial \psi}{\partial y}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) + C_{66}\frac{\partial v}{\partial x}h_m \\
&+C_{66}\frac{\partial \phi}{\partial x}((\frac{h_n/2+h_m}{2})^2 - (\frac{h_n}{2})^2) = 0
\end{aligned} \tag{28}$$

$$\begin{aligned}
\delta W :& \frac{\partial}{\partial x}(Q_{44}\psi h_n + Q_{44}\frac{\partial w}{\partial x}h_n + C_{55}\psi h_m \\
&+C_{55}\frac{\partial w}{\partial x}h_m) + \frac{\partial}{\partial y}(Q_{55}\phi h_n + Q_{55}\frac{\partial w}{\partial y}h_n + C_{44}\phi h_m \\
&+C_{44}\frac{\partial w}{\partial y}h_m) + N_x^m\frac{\partial^2 W}{\partial x^2} + N_y^m\frac{\partial^2 W}{\partial y^2} + N_{xy}^m\frac{\partial^2 W}{\partial x\partial y} = 0
\end{aligned} \tag{29}$$

$$\begin{aligned}
\delta \psi :& \frac{\partial}{\partial x}(Q_{11}\frac{\partial \psi}{\partial x}\frac{h_n^3}{12} + Q_{12}\frac{\partial \phi}{\partial y}\frac{h_n^3}{12} \\
&+C_{11}\frac{\partial \psi}{\partial x}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) + \\
&+C_{12}\frac{\partial \phi}{\partial y}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) - (\frac{h_n}{2})^2)) \\
&-(Q_{44}\psi h_n + Q_{44}\frac{\partial w}{\partial x}h_n + C_{55}\psi h_m + C_{55}\frac{\partial w}{\partial x}h_m) \\
&+\frac{\partial}{\partial y}(Q_{66}\frac{\partial \psi}{\partial y}\frac{h_n^3}{12} + Q_{66}\frac{\partial \phi}{\partial x}\frac{h_n^3}{12} \\
&+C_{66}\frac{\partial \psi}{\partial y}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) \\
&+C_{66}\frac{\partial \phi}{\partial x}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) = 0
\end{aligned} \tag{30}$$

$$\begin{aligned}
\delta \phi :& \frac{\partial}{\partial y}(Q_{21}\frac{\partial \psi}{\partial x}\frac{h_n^3}{12} + Q_{22}\frac{\partial \phi}{\partial y}\frac{h_n^3}{12} + + \\
&C_{21}\frac{\partial \psi}{\partial x}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) \\
&+C_{22}\frac{\partial \phi}{\partial y}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) \\
&-(Q_{55}\phi h_n + Q_{55}\frac{\partial w}{\partial y}h_n + C_{44}\phi h_m + C_{44}\frac{\partial w}{\partial y}h_m) \\
&+\frac{\partial}{\partial x}(Q_{66}\frac{\partial \psi}{\partial y}\frac{h_n^3}{12} + Q_{66}\frac{\partial \phi}{\partial x}\frac{h_n^3}{12} + + \\
&C_{66}\frac{\partial \psi}{\partial y}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) \\
&+C_{66}\frac{\partial \phi}{\partial x}((\frac{h_n/2+h_m}{3})^3 - (\frac{h_n}{3})^3) = 0
\end{aligned} \tag{31}$$

3. Solution method

Based on Navier method, we have (Samaei *et al.* 2011)

$$u(x,y,t) = u_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{32}$$

$$v(x,y,t) = v_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \tag{33}$$

$$w(x,y,t) = w_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{34}$$

$$\psi(x,y,t) = \psi_{x0} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{35}$$

$$\phi(x,y,t)=\psi_{y0}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \tag{36}$$

Substituting Eqs. (32)-(36) into Eqs. (27)-(31) yields

$$\left\{\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix}\right\}\begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ \psi_{x0} \\ \psi_{y0} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{37}$$

Setting the determinate of the Eq. (37) yields the buckling load.

4. Numerical results

In this section, a polymeric plate is assumed with Young's modulus of $E_m = 0.8\,GPa$ which is reinforced by CNTs with Young's modulus of $E_r = 1\,TPa$ and coated by magnetostrictive layer with Young's modulus of $E_{magnet} = 20GPa$. The length to width of the sandwich structure is $a/b = 2$.

At the first, the results are validated with neglecting the CNTs and magnetostrictive layer. As shown in Table 1, the buckling load a plate with different solution methods is presented. It can be found that the results of this work are the same as those reported by Guo *et al.* (2015).

The second validation is about buckling of plates reinforced by CNTs without magnetostrictive plate. The dimesionless buckling load of the simply supported nanocomposite plate is illustrated in Table 2. It is observed that the results are math with those reported by Lei *et al*. (2013).

Table 1 Validation of this work with Guo *et al*. (2015)

Solution	Buckling load
Exact (Guo *et al*. 2015)	4.000
Finite element method (FEM) (Guo et al. 2015)	4.011
Boundary element method (BEM) (Guo *et al*. 2015)	4.041
Dual reciprocity method (DRM) (Guo et al. (2015))	3.999
Spline finite strip method (SFSM) (Guo *et al*. 2015)	4.000
Spline finite strip method (SFSM) (Guo *et al*. 2015)	4.000
Radial point interpolation method (RPIM) (Guo *et al*. 2015)	4.017
Differential quadrature element method (DQEM) (Guo *et al*. 2015)	3.997
Discrete singular convolution (DSC) (Guo *et al*. 2015)	4.011
Present	4.006

Table 2 Validation of this work with Lei *et al.* (2013)

Mode	Lei *et al.* (2013)	Present
1	14.1073	14.1068
2	23.3149	23.3143
3	25.6506	25.6501
4	27.0498	27.0491

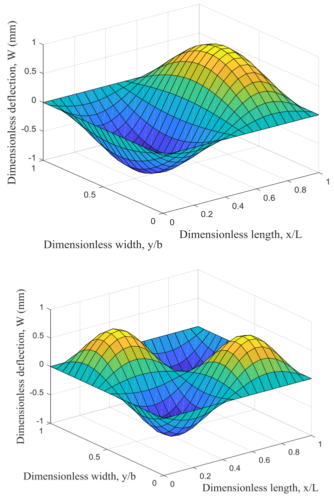

Fig. 2 (a) axial and circumferential first modes (b) axial first and circumferential second modes (c) circumferential first and axial second modes (d) axial and circumferential second modes

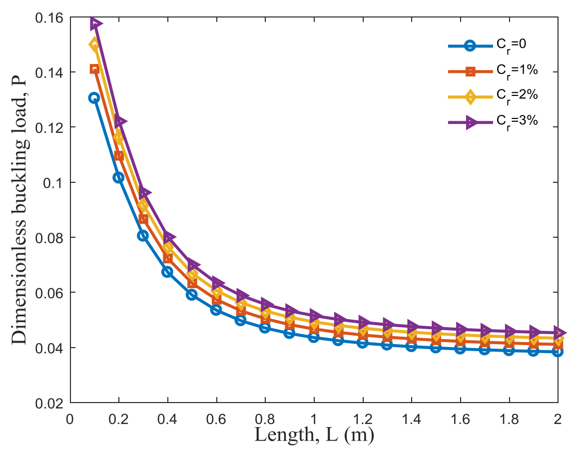

Fig. 3 The effect of CNT volume percent on the dimensionless buckling load versus length of the sandwich structure

Figs. 3 and 4 illustrate the effects of CNT volume percent and agglomeration on the dimensionless buckling load versus length of the structure. It can be found that with enhancing the sandwich structure length, the dimensionless buckling is decreased due to reduction in the stiffness of the structure. With increasing the CNT volume percent, the dimensionless buckling load is increased. In addition, considering agglomeration of CNTs leads to reduce the dimensionless buckling load. It is due to this fact that with increasing the CNTs volume percent, the stiffness is increased and with assuming agglomeration, the stability and rigidity of the sandwich structure decreases.

The effect of magnetic field on the dimensionless buckling load versus length of the sandwich structure is shown in Fig. 5. It can be observed that with increasing the magnetic field, the dimensionless buckling load is improved. It is since with increasing the magnetic field, the stiffness of the structure increases.

Figs. 6 and 7 demonstrate the effect of nanocomposite and magnetostrictive layers thickness on the dimensionless buckling load versus length of the sandwich structure, respectively. It is found that with enhancing the nanocomposite and magnetostrictive layers thickness, the dimensionless buckling load is increases. It is because with enhancing the nanocomposite and magnetostrictive layers thickness, the stiffness of the structure increases.

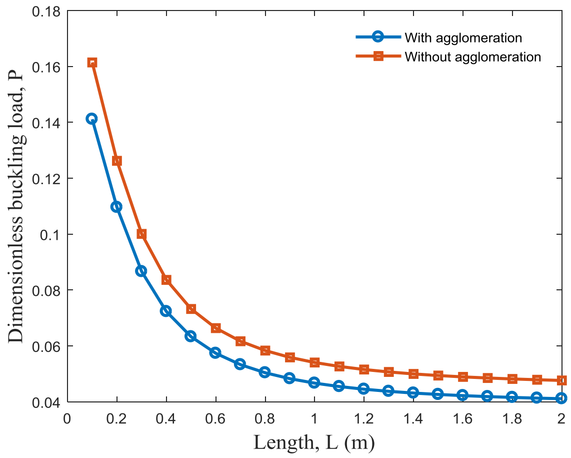

Fig. 4 The effect of CNT agglomeration on the dimensionless buckling load versus length of the sandwich structure

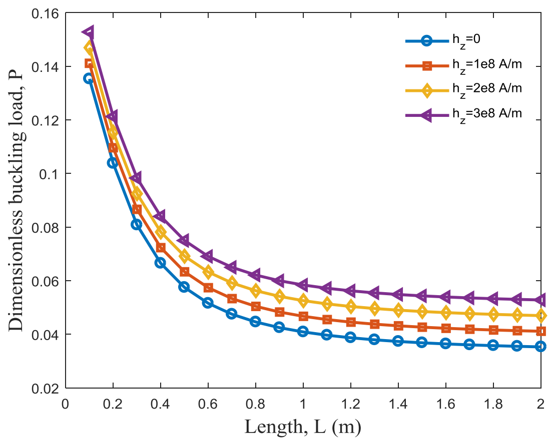

Fig. 5 The effect of magnetic field on the dimensionless buckling load versus length of the sandwich structure

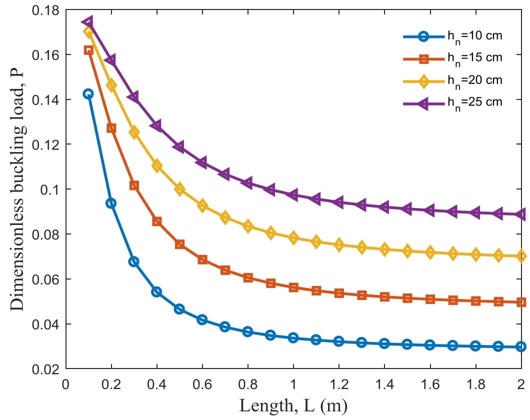

Fig. 6 The effect of nanocomposite layer thickness on the dimensionless buckling load versus length of the sandwich structure

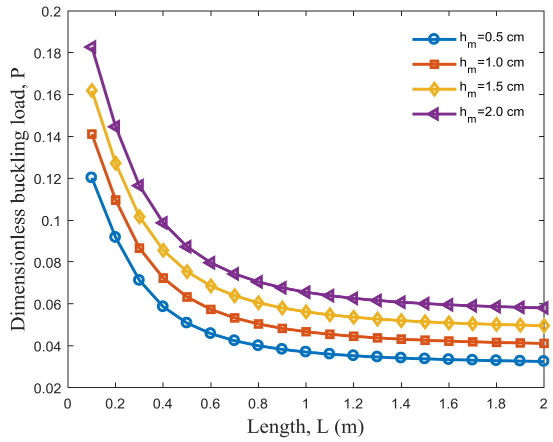

Fig. 7 The effect of magnetostrictive layer thickness on the dimensionless buckling load versus length of the sandwich structure

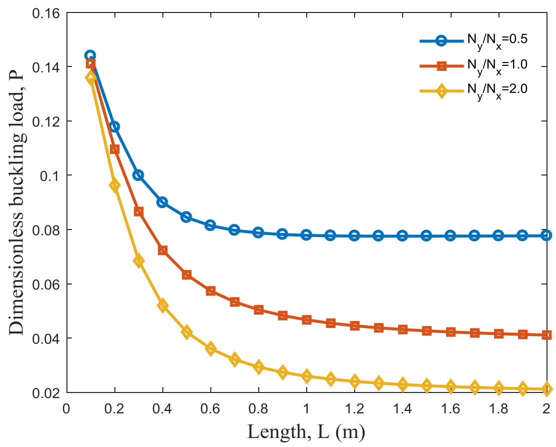

Fig. 8 The effect of lateral to axial load ratio on the dimensionless buckling load versus length of the sandwich structure

The effect of lateral to axial load ratio on the dimensionless buckling load versus length of the sandwich structure is presented in Fig. 8. It is shown that with increasing the lateral to axial load ratio, the dimensionless buckling load is decreased.

5. Conclusions

Buckling analysis of the nanocomposite plate coated by magnetostrictive layer was presented in this work. The nanocomposite plate was reinforced by CNTs considering agglomeration based on Mori-Tanaka model. Based on FSDT, the governing equations were derived considering coupling of mechanical displacements and magnetic field. Utilizing Navier method, the buckling load was calculated and the effects of CNTs volume percent and agglomeration, geometrical parameters and magnetic field were shown. The results show that with increasing the CNT volume percent, the dimensionless buckling load was increased. In addition, considering agglomeration of CNTs leads to reduce the dimensionless buckling load. It can be observed that with increasing the magnetic field, the dimensionless buckling load was improved. It was found that with enhancing the nanocomposite and magnetostrictive layers thickness, the dimensionless buckling load was increases.

References

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct*., **20**(5), 963-981.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct*., **18**(1), 187-212.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Bég, O.A. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.: Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A. and Hassan, S. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J Braz. Soc. Mech. Sci. Eng*., **38**(1), 265-275.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech*., **62**(6), 695-702.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mat. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A. and Tounsi, A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater.*, **15**(6), 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Hassan, S. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst.*, **19**(6), 601-614.

Bouafia, K.H., Kaci, A., Houari M.S.A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst*., **19**(2), 115-126.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016a), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech*., **60**(2), 313-335.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct*., **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016b), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech*., **58**(3), 397-422.

Boukhari, A., Atmane, H.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech*., **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct*., **20**(2), 227 - 249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct*., **18**(2), 409-423.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst*., **19**(3), 289-297.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng*., **11**, 671-690.

Dutta, G., Panda. S.K., Mahapatra, T.R. and Singh, V.K. (2017), "Electro-magneto-elastic response of laminated composite plate: A finite element approach", *Int. J. Appl. Comput. Math*., **3**, 2573-2592.

El-Haina, F., Bakora, A., Bousahla, A.A. and Hassan, S. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech*., **63**(5), 585-595.

Fan, Y. and Wang, H. (2016), "Nonlinear bending and postbuckling analysis of matrix cracked hybrid laminated plates containing carbon nanotube reinforced composite layers in thermal environments", *Compos. Part B: Eng.*, **86**, 1-16.

Ferreira, A.J.M., Roque, C.M.C., Carrera, E., Cinefra, M. and Polit, O. (2011), "Two higher order Zig-Zag theories for the accurate analysis of bending, vibration and buckling response of laminated plates by radial basis functions collocation and a unified formulation", *J. Compos. Mat.*, **45**, 2523-2536.

Guo, L., Zhang, H., Lu, R. and Yu, G. (2015), "Magnetoelectric analysis of a bilayer piezoelectric/magnetostrictive composite system with interfacial effect", *Compos. Struct*., **134**, 285-293.

Hajmohammad, M.H., Zarei, M.S., Nouri, A. and Kolahchi, R. (2017), "Dynamic buckling of sensor/functionally graded-carbon nanotubes reinforced laminated plates/actuator based on sinusoidal-viscopiezoelasticity theories", *J. Sandw. Struct.*

Mater., In press.

Khetir, H., Bouiadjra, M.B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech*., **64**(4), 391-402.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Nouri, A. (2017a), "Wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator based on refined Zigzag theory", *Int. J. Mech. Sci.*, **130**, 534-545.

Kolahchi, R., Keshtegar, B. and Fakhar, M.H. (2017b), "Optimization of dynamic buckling for sandwich nanocomposite plates with sensor and actuator layer based on sinusoidal visco-piezoelasticity theories using Grey Wolf algorithm", *J. Sandw. Struct. Mater*., In press.

Larbi Chaht, F., Kaci, A., Houari M.S.A. and Hassan, S. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct*., **18**(2), 425-442.

Lei, Z.X., Liew, K.M. and Yu, J.L. (2013), "Buckling analysis of functionally graded carbon nanotube-reinforced composite plates using the element-freekp-Ritz method", *Compos. Struct*., **98**, 160-168

Li, N., Han, Z.J. and Lu, G.Y. (2015), "Research on the dynamic buckling of composite laminated plates with one edge fixed and three edges simply-supported", *Appl. Mech. Mat*., **751**, 189-194.

Li, X., Yu, K., Han, J., Song, H. and Zhao, R. (2016), "Buckling and vibro-acoustic response of the clamped composite laminated plate in thermal environment", *Int. J. Mech. Sci.*, **119**, 370-382.

Lei, Z.X., Zhang, L.W. and Liew ,K.M. (2017), "Meshless modeling of geometrically nonlinear behavior of CNT-reinforced functionally graded composite laminated plates", *Appl. Math. Comput.*, **295**, 24-46.

Mahapatra, T.R. and Panda, S.K. (2016a), "Nonlinear free vibration analysis of laminated composite spherical shell panel under elevated hygrothermal environment: A micromechanical approach", *Aerosp. Sci. Technol*., **49**, 276-288.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016b), "Nonlinear flexural analysis of laminated composite panel under hygro-thermo-mechanical loading—A micromechanical approach", *Int. J. Comput. Meth*., **13**, 1650015.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016c), "Nonlinear hygro-thermo-elastic vibration analysis of doubly curved composite shell panel using finite element micromechanical model", *Mech. Adv. Mater. Struct*., **23**, 1343-1359.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016d), "Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel", *Int. J. Mech. Mater. Des*., **12**, 153-171.

Mahi, A., Bedia, E.A.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model*., **39**, 2489-2508.

Malekzadeh, P. and Shojaee, M. (2013), "Buckling analysis of quadrilateral laminated plates with carbon nanotubes reinforced composite layers", *Thin-Wall. Struct.*, **71**, 108-118.

Malekzadeh, P. and Zarei, A.R. (2014), "Free vibration of quadrilateral laminated plates with carbon nanotube reinforced composite layers", *Thin-Wall. Struct.*, **82**, 221-232.

Marjanović, M. and Vuksanović, D. (2014), "Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations", *Compos. Struct.*, **108**, 9-20.

Menasria, A., Bouhadra, A., Tounsi, A. and Hassan, S. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct*., **25**(2), 157-175.

Meziane, M.A.A., Abdelaziz, H.H. and Tounsi, A.T. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater*., **16**(3), 293-318.

Moradi-Dastjerdi, R. and Malek-Mohammadi, H. (2016), "Biaxial buckling analysis of functionally graded nanocomposite sandwich plates reinforced by aggregated carbon nanotube using improved high-order theory", *J. Sandw. Struct. Mater*., **19**, 736-769.

Mouffoki, A., Adda Bedia, E.A., Houari M.S.A. and Hassan, S. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst*., **20**(3), 369 - 383.

Nasihatgozar, M. and Khalili, S.M.R. (2018), "Vibration and buckling analysis of laminated sandwich conical shells using higher order shear deformation theory and differential quadrature method", *J. Sandw. Struct. Mater*., In press.

Pandit, M.K., Sheikh, A.H. and Singh, B.N. (2010), "Analysis of laminated sandwich plates based on an improved higher order Zigzag theory", *J. Sandw. Struct. Mater*., **12**, 234-241.

Pramod, A.L.N., Natarajan, S., Ferreira, A.J.M., Carrera, E. and Cinefra, M. (2017), "Static and free vibration analysis of cross-ply laminated plates using the Reissner-mixed variational theorem and the cell based smoothed finite element method", *Euro. J. Mech. - A/Solids*, **62**, 14-21.

Reddy, J.N. (2002), *Mechanics of Laminated Composite Plates and Shells*, Theory and Analysis, Second Edition, CRC Press.

Samaei, A.T., Abbasion, S. and Mirsayar, M.M. (2011), "Buckling analysis of a single-layer graphene sheet embedded in an elastic medium based on nonlocal Mindlin plate theory", *Mech. Res. Commun*., **38**, 481-493.

Sayyad, A.S. and Ghugal, Y.M. (2017), "On the free vibration of angle-ply laminated composite and soft core sandwich plates", *J. Sandw. Struct. Mater*., **19**, 679-711.

Shariyat, M. (2009), "Dynamic buckling of imperfect laminated plates with piezoelectric sensors and actuators subjected to thermo-electro-mechanical loadings, considering the temperature-dependency of the material properties", *Compos. Struct.*, **88**, 228–239.

Shi, D.L. and Feng, X.Q. (2004), "The effect of nanotube waviness and agglomeration on the elastic property of carbon nanotube-reinforced composties", *J. Eng. Mat. Tech. ASME*, **126**, 250-270.

Shokravi, M. and Jalili, N. (2017), "Dynamic buckling response of temperature-dependent functionally graded-carbon nanotubes-reinforced Sandwich microplates considering structural damping", *Smart Struct. Syst*., **20**(5), 583-593.

Shokravi, M. (2018), "Dynamic buckling of smart Sandwich beam subjected to electric field based on hyperbolic piezoelasticity theory", *Smart Struct. Syst*., **22**(3), 327-334.

Suman, S.D., Hirwani, C.K., Chaturvedi, A. and Panda, S.K. (2017), "Effect of magnetostrictive material layer on the stress and deformation behaviour of laminated structure", *IOP Conference Series: Materials Science and Engineering*, **178** (1), 012026.

Yu, T., Yin, Sh., Bui, T.Q., Xia, S.H., Tanaka, S. and Hirose, S. (2016), "NURBS-based isogeometric analysis of buckling and free vibration problems for laminated composites plates with complicated cutouts using a new simple FSDT theory and level set method", *Thin-Wall. Struct.*, **101**, 141-156.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech*., **54**(4), 693-710.

Zhao, R., Yu, K., Hulbert, G.M., Wu, Y. and Li, X. (2017), "Piecewise shear deformation theory and finite element

formulation for vibration analysis of laminated composite and sandwich plates in thermal environments", *Compos. Struct.*, **160**, 1060-1083.
Zhang, L.W., Liew, K.M. and Reddy, J.N. (2016), "Postbuckling analysis of bi-axially compressed laminated nanocomposite plates using the first-order shear deformation theory", *Compos. Struct.*, **152**, 418-431.
Zhang, L.W. and Selim, B.A. (2017), "Vibration analysis of CNT-reinforced thick laminated composite plates based on Reddy's higher-order shear deformation theory", *Compos. Struct.*, **160**, 689-705.
Zidi, M., Tounsi, A., and Bég, O.A. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Tech*., **34**, 24-34.

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