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A damage localization method based on the singular value decomposition (SVD) for plates

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Abstract. Boundary effect and the noise robustness are the two crucial aspects which affect the effectiveness of the damage localization based on the mode shape measurements. To overcome the boundary effect problem and enhance the noise robustness in damage detection, a simple damage localization method is proposed based on the Singular Value Decomposition (SVD) for the mode shape of composite plates. In the proposed method, the boundary effect problem is addressed by the decomposition and reconstruction of mode shape, and the noise robustness in enhanced by the noise filtering during the decomposition and reconstruction process. Numerical validations are performed on plate-like structures for various damage and boundary scenarios. Validations show that the proposed method is accurate and effective in the damage detection for the two-dimensional structures.

Keywords: damage localization; singular value decomposition; plate

1. Introduction

Damage localization via mode shapes is one of the most important branches in the vibration-based damage detection methods (Deraemaeker *et al.* 2010). The mode shape, the eigenvector of the inspected structures, is more suitable for damage localization compared with the frequency (the eigen-value) owing to its distributive property. It is known that the presence of damage will result in the occurrence of singularities in the corresponding mode shapes (Cao *et al*. 2011, Zhou *et al.* 2016). Thereafter the damage localization problem can be converted to the singularity detection problem.

Based on mode shape measurements, various technologies have been proposed focusing on the damage localization in the past decade. Some famous technologies, like mode curvature (containing the its extensions and improvements) (Qiao *et al.* 2008, Radzienski *et al*. 2011, Xu *et al.* 2015, Cao *et al.* 2014, Cao *et al.* 2014), continuous wavelet transform (Yang *et al.* 2016a, Liu *et al*. 2016), discrete wavelet transform (Xiang and Liang 2012, Katunin 2015b, 2010, Ji *et al.* 2019), fractural dimension method (Yang *et al.* 2012, Qiao and Cao 2008), hybrid method (Khatir *et al.* 2016, Zhou and Abdel Wahab 2016, Zhou *et al*. 2016, Xiang *et al.* 2014), modal updating method (Gillich *et al.* 2014, Gillich *et al.* 2014, Abdel Wahab 2001), the multi-step method (Xiang *et al.* 2014, Soman *et al.* 2014, Jiawei *et al*. 2012, Yun *et al.* 2009), wave propagation methods (Miao *et al.* 2012, Yu and Giurgiutiu 2005), and entropy method (Yang *et al*. 2016, Yang *et al*. 2016) are widely used in different areas. These methods, however, require the artificial boundary extension since the finite length of mode shape data. These artificial extensions may introduce additional singularity on boundaries and affects the damage detection further. For some Fourier spectrum-based methods, like (Yang *et al*. 2016b), the extension requirement can be stricter and more complex. The singular value decomposition (SVD) provides an efficient path for damage feature extraction in absence of boundary effect as it is based on the direct decomposition and reconstruction of mode shape vector/matrix. In this work, we propose an SVD-based method for the title problem.

SVD is an important factorization of a real or complex matrix in linear algebra. It is the generalization of the eigen-decomposition of a positive semi-definite normal matrix (for example, a symmetric matrix with positive eigen-values) to any matrix via an extension of the polar decomposition. Specially, it is also widely used for damage detection or structural healthy monitoring purpose from different views. The SVD are usually employed to filtering the interference induced by environmental factors. In the reference (Liu *et al.* 2015, Ruotolo and Surace 1999), SVD is applied in the guided wave signal processing to get a robust ultrasonic damage detection performance under complex environmental condition. The SVD can be also used as a damage index. In the reference (Vanlanduit *et al.* 2005), Vanlanduit *et al*. employed a robust SVD to detect damage in structures from measurements taken under different conditions. Galvanetto *et al*. (Galvanetto and Violaris 2007) proposed a proper orthogonal decomposition to process the time history recorded by different sensors, and then got a proper orthogonal decomposition for the damage localization of mode shapes. These methods are

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based on the SVD of time signal or space-time signal. It should be mentioned that the spatial signal, namely mode shape can be decomposed by SVD when some necessary processing technologies are conducted. Recently, Jiang *et al*. (2017) proposed a Hankel matrix based SVD method to detect damage in beam and plate. In their methodology, the mode shape is firstly decomposed by SVD, and then a Hankel matrix is created by the periodic fold of the eigen-vector to create a matrix for damage localization. However, in the present work, we found the periodic fold is not required for damage detection. In addition, the periodic fold will induce the boundary effect although it is helpful for noise suppression. Different with Jiang and Xiang's work (Jiang *et al*. 2017), we proposed a periodic fold-free and artificial padding-free SVD method for damage localization in plates.

The rest parts of this paper are organized as follows. Section 2 introduces theoretical developments of this work, containing the employed models, the analysis of noisy mode shapes for damaged composite plates, and the basic scheme of the present algorithm. Section 3 mainly focuses on the numerical validations and comparisons of the proposed methodology in various noise conditions and different plate shapes. Experimental validations are presented in Section 4. Conclusions and discussions are presented in Section 5.

2. Theoretical developments

The mode shape of plate $\mathbf{W}$ can be seen as a matrix whose rows and columns are related to the physical positions. By aid of singular value decomposition (SVD), we can get the following formulation

$$\mathbf{W} = \mathbf{USV} \tag{1}$$

where $\mathbf{U}$ is the vector related to columns and the $\mathbf{V}$ is the vector related to the rows of $\mathbf{W}$

$$\mathbf{U} = \left[\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{m-1}, \mathbf{u}_m\right] \tag{2}$$

$$\mathbf{V} = \left[\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_{n-1}, \mathbf{v}_n\right] \tag{3}$$

The entries are all column vectors. $\mathbf{S}$ is called the eigen-value of the matrix. For a square matrix ($m = n$), $\mathbf{S}$ is diagonal

$$\mathbf{S} = diag\left\{s_1, s_2, ..., s_{n-1}, s_n\right\} \tag{4}$$

In another aspect, Eq. (1) can be seen as the summation of different components

$$\mathbf{W} = \sum_i \mathbf{W}_i = \sum_i s_i \mathbf{u}_i \otimes \mathbf{v}_i^T \tag{5}$$

The value of s_i determines the contribution of the reconstruction sub-mode shape $\mathbf{u}_i \otimes \mathbf{v}_i$ for $\mathbf{W}$.

For the sake of convenience, we can further relate the physical coordinate (x, y) with the matrix $\mathbf{W}$ and the ones derived from $\mathbf{W}$ as the rows and columns of them can be easily mapped to (x, y). The inspected damaged mode shape of plate can be seen as the combination of three parts

$$\mathbf{W}(x,y) = \mathbf{W}_T(x,y) + \mathbf{W}_D(x,y) + \mathbf{N}(x,y) \tag{6}$$

where $\mathbf{W}_T(x, y)$ refers to the tendency of mode shape, which is the main part of $\mathbf{W}(x, y)$ (corresponding to the largest s_i). However, the damage information is rarely contained in this part. The second component is $\mathbf{W}_D(x, y)$, corresponding to the damage related sub-mode shapes. Compared with $\mathbf{W}_T(x, y)$, the energy carried by $\mathbf{W}_D(x, y)$ is lower, thus the corresponding eigen-value is smaller than the one of $\mathbf{W}_T(x, y)$. The component $\mathbf{N}(x, y)$ refers to the influence of noise. Usually, the contribution noise is relatively small. Thereafter, the noise effect can be filtered by selecting some crucial sub-mode shape to reconstruct $\mathbf{W}(x, y)$. In order to extract the damage information, filtering the sub-mode shapes with the largest and the smallest s_i can be an alternative path. After the filtering, the mode shape can be reconstructed by the summation of $\mathbf{W}_{Di}(x, y)$ easily. This is the basic logic of the present work. Pay attention to above process, we can see that the computation of these equation does not involve any additional extended mode shape information on boundary, thus the boundary effect can be overcome without padding. This is main advantage of the present method compared with spatial wavelet and other spatial decomposition methods

3. Numerical validations

3.1 Validations for square plate

Damage is considered by the stiffness reduction of the intact plates. The outlines of the damage investigated in this work are presented in Fig. 1. The basic model is a 0.5 m×0.5 m×0.004 m plate, on which two different defects are implemented:

Line damage: $l'_x = 0.21$ m, $L = 0.15$ m.

Area damage: $l''_x = l''_y = 0.29$ m, $L_x = 0.045$ m and $L_y = 0.09$ m.

In the damaged area, the local thickness is reduced to 95% of the original thickness. The specimens in simulations are assumed as the transversally isotropic 24-layered (0°/15°.../0°) glass-fiber reinforced plastic composite plates.

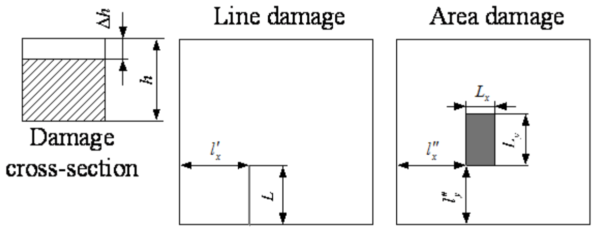
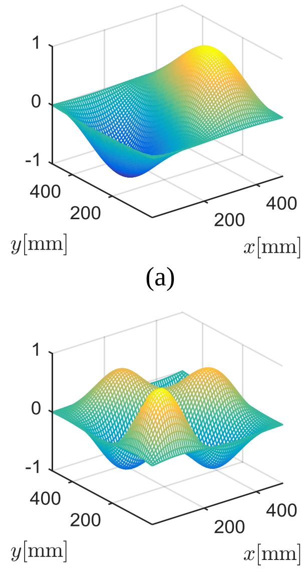


Fig. 1 The illustrations of damage



(a)

(b)

Fig. 2 The inspected mode shapes: (a) mode (2,1) and (b) mode (2,3)

The material parameters are: the Young's modulus are E_1 = 38.293GPa and E_2 = 10.141GPa, the shear modulus G_{12} = 3.533 GPa, the mass density is 1784 kg/m^3, and the Poisson's ratio $\mu_{12} = \mu_{21} = 0.366$. By means of laminated plate theory , stiffness and mass matrix can be obtained.

Validations of the developed method are conducted based on the damage models in a progressive manner. To present a comprehensive comparison, the spatial continuous wavelet transform (CWT) is selected to provide reference solutions. The following aspects are also considered into comparisons:

(a) The influence of noise. The noise is added by aid of the MATLAB function "awgn" (adding white Gaussian noise), in which the signal to noise ratios (*SNR*) is employed as one of the inputs.

(b) The influence of modal order. Two typical modes for plates, namely mode (2, 1) and mode (2, 3) are investigated. There the number indicts the wavenumber of mode shape.

Fig. 2 shows the inspected damaged mode shapes (line damage). The singularity induced by damage is relatively weak so that it cannot be revealed by the direct observation of mode shapes. Thus, further inspection based on some singularity extraction technology is required.

Although the formulation given in Eq. (7) provides an alternative way to filter the influence of mode shape and noise, the criterion to determine filtered mode is still a tough task. Thus, the filtering process presented in Eq. (7) is conducted by two steps:

Step 1: Filter the tendency of mode shape. The tendency of mode shape can be filtered out by the following equitation

$$\mathbf{W}_T(x,y) = \sum_{i\in T} s_i \mathbf{u}_i \otimes \mathbf{v}_i^T, \text{ where } \frac{\sum_{i\in T} s_i}{\sum_i s_i} \geq 0.95 \quad (7)$$

the above equation defines the border of $\mathbf{W}_T$. Based on Eq. (7), a coarsely filtered mode shape can be obtained

$$\mathbf{W}_R(x,y) = \mathbf{W}(x,y) - \mathbf{W}_T(x,y) = \mathbf{W}_D(x,y) + \mathbf{N}(x,y) \quad (8)$$

$\mathbf{W}_R(x, y)$ is the remained mode shape, which is mainly comprised of damage feature and noise.

Step 2: Filter the noise. In order to filter noise, the SVD is conducted again for $\mathbf{W}_R(x, y)$

$$\mathbf{W}_R(x,y) = \sum_i \mathbf{W}_{Ri}(x,y) = \sum_i s_i \mathbf{u}_i \otimes \mathbf{v}_i^T \quad (9)$$

For $\mathbf{W}_R(x, y)$, $\mathbf{W}_D(x, y)$ is not as determinant as $\mathbf{W}_T(x, y)$ for $\mathbf{W}(x, y)$, especially under strong noise interference. For different noise level, the energy distributions between damage feature and noise are distinct, and thus the noise filter criterion could not be a fixed number like 0.95 in Eq. (7). To address this problem, a correlation-based filter criterion is proposed. For the basic SVD component $\mathbf{W}_{Ri}(x, y)$ (i = 1, 2, ... , n), we can calculate the correlation coefficient (without spatial lag) matrix between the neighbor components

$$R_i = \iint \mathbf{W}_{Ri}(x,y)\mathbf{W}_{R(i+1)}(x,y)dxdy = \sum_x \sum_y \mathbf{W}_{Ri}(x,y)\mathbf{W}_{R(i+1)}(x,y) \quad (10)$$

where R_i describes the similarity of $\mathbf{W}_{Ri}(x, y)$ and $\mathbf{W}_{R(i+1)}(x, y)$. Theoretically, the mean value of R_i should be smaller for correlation between noise determinant components. Accordingly, R_i should achieve its maximum at damage location (very small at intact position) when the correlation is calculated by two damage feature determinant components. Thereafter, the components $\mathbf{W}_{Ri}(x, y)$ corresponding to the first five largest R_i are selected to reconstruct the damage index by summation.

3.1.1 Line damage condition

To assess the performance of the proposed method on different mode shapes, the mode (2, 3) is employed as the object. Comparison between the developed method and the modal curvature is given in Fig. 5. Conclusions are similar with the ones obtained for mode (2, 1). The present SVD-based method reveals a more distinct damage outline and its noise robustness is also higher than the modal curvature method. Damage maps given by the modal curvature is easily to be identified as a node defect, however, the developed method identified a more accurate exterior of damage, which agrees well with the physical conditions. Compare the slices obtained by the two methods, we can see the present method performs well when $SNR \geq$ 50dB, in contrast, the modal curvature slices lose efficacy of damage detection when SNR < 80dB.

3.1.2 Area damage condition

To demonstrate the performance on different damage, validations are conducted on the area damage condition which is shown in Fig. 1. Damage maps and the corresponding horizontal slices at y = 373 mm given by the two methods are shown in Figs. 6 and 7 for comparison. Induced by the left and right side of the damage area, there should be two step offsets near 290 mm and 335 mm. From the results comparison of mode (2, 1) and mode (2, 3)

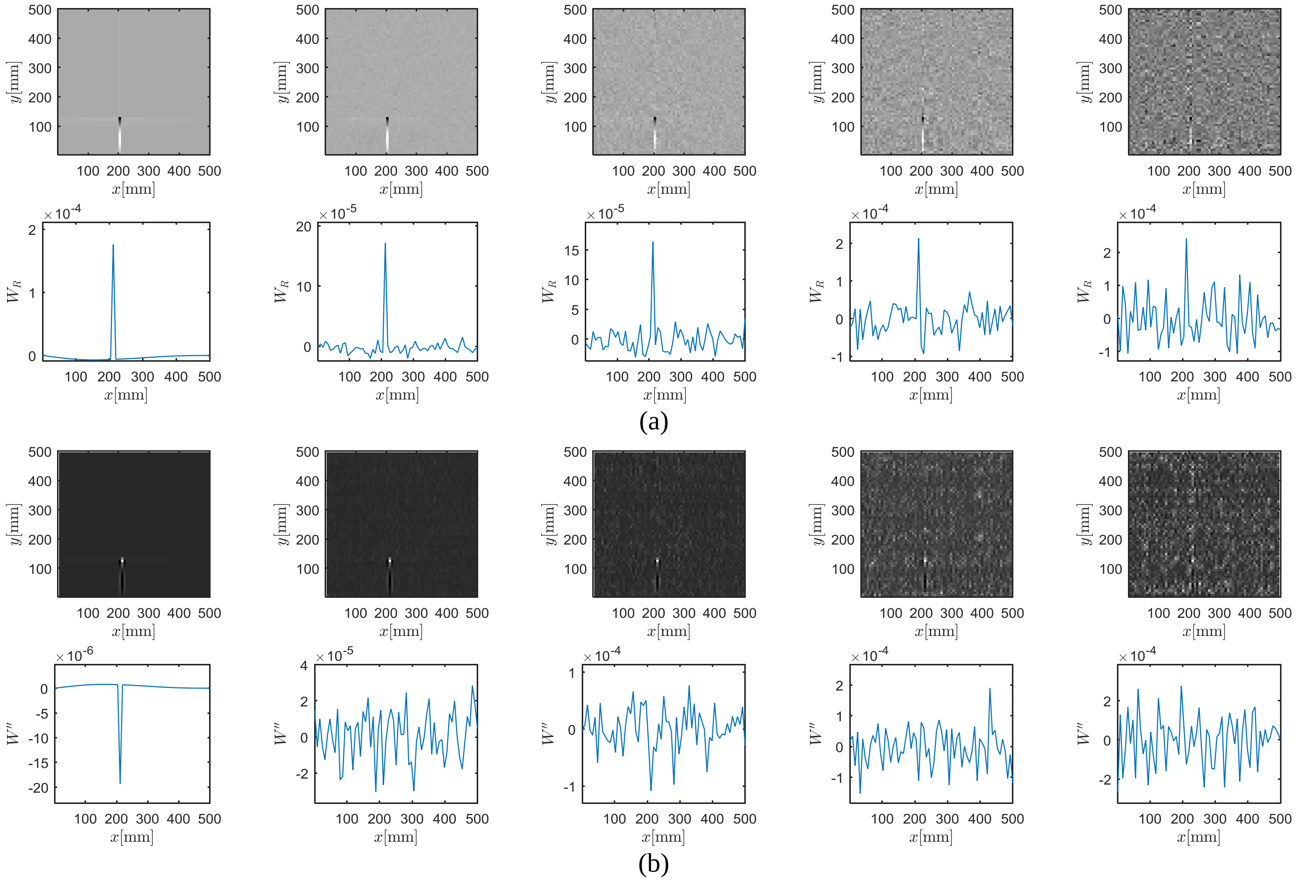
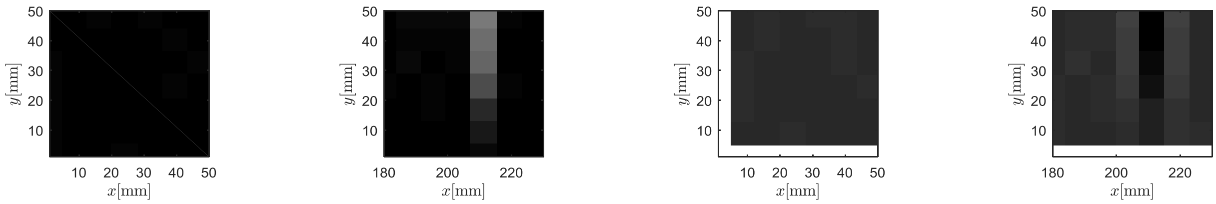


Fig. 3 Comparison of the presented method and the modal curvature for linear damage plate, mode (2, 1) *SNR* = ∞, 80, 70, 60, 50dB from left to right: damage indexes and the corresponding horizontal slices at *y* = 75 mm given by (a) the presented method and (b) modal curvature method



(a) (b)

Fig. 4 Comparison of zoom-out view: (a) the presented method and (b) modal curvature method

shown in Figs. 6 and 7, it is observed that the presented method performs better either on damage maps or the corresponding slices. Damage maps given by modal curvature method blurs very fast with the degeneration of *SNR*, and the damage feature is hardly found in the corresponding slices when *SNR* < 80dB. In contrast, the developed SVD-based method shows the stronger noise robustness.

3.1.3 Intact condition

Validation is also conducted on an intact plate. The aim of this verification is to show that the developed method will not give false alarm for intact plate. Usually, a foregone measurement for the intact structure will serve as the baseline for classical damage detection method, and then the difference or distance between the inspected mode shape and the baseline can be related to the presence and location of damage. However, the baseline is not required in the present method and the modal curvature method. Without the baseline, the singularity caused by boundary effect and measurement uncertainty may be easily deemed as the evidence of damage. Thus, it is important to verify the performance of the proposed method for intact structure. Fig. 8 gives the corresponding damage maps for mode (2, 1) and mode (2, 3) with different noise level. For noise-free case, the present damage map displays a smooth waveform similar to mode shape. However, for noise-contaminated cases, there is only noise-like waveform presented as no damage occurs.

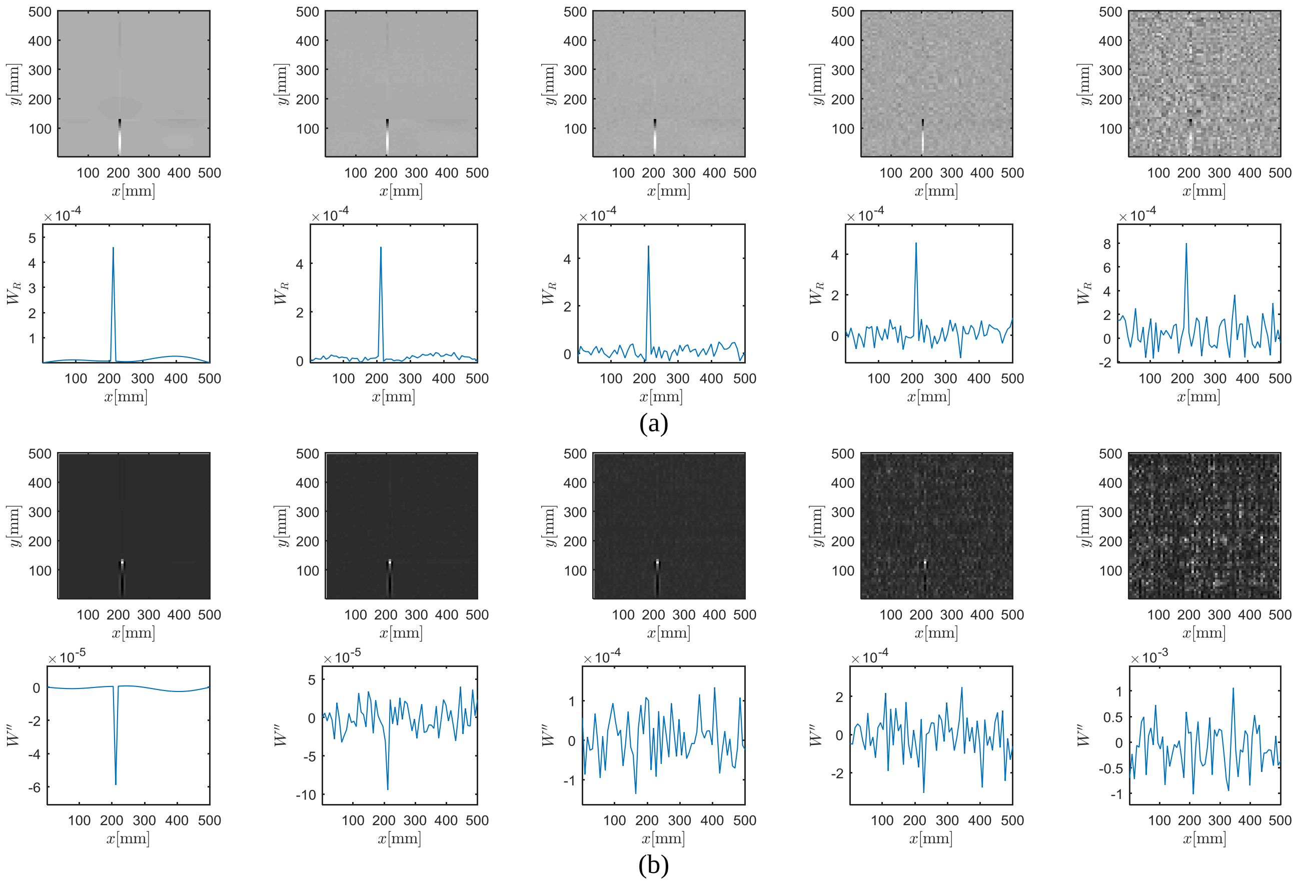
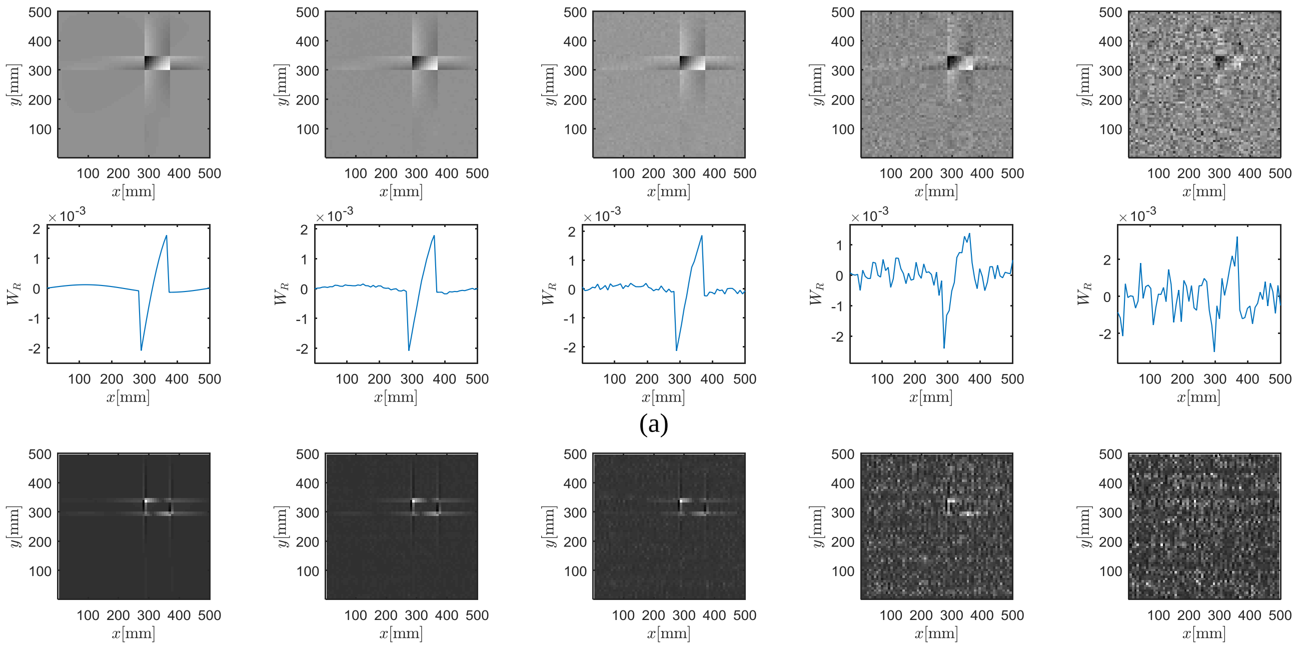


Fig. 5 Comparison of the presented method and the modal curvature for linear damage plate, mode (2, 3) $SNR = \infty$, 80, 70, 60, 50dB from left to right: damage indexes and the corresponding horizontal slices at y = 75 mm given by (a) the presented method and (b) modal curvature method



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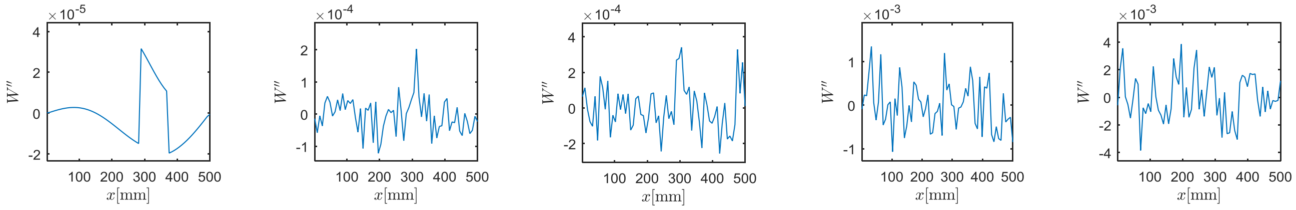
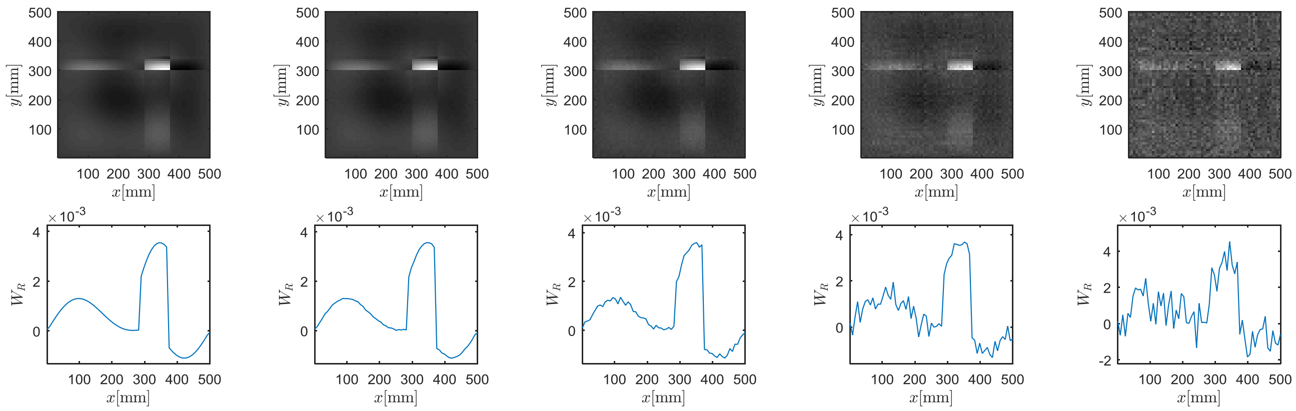
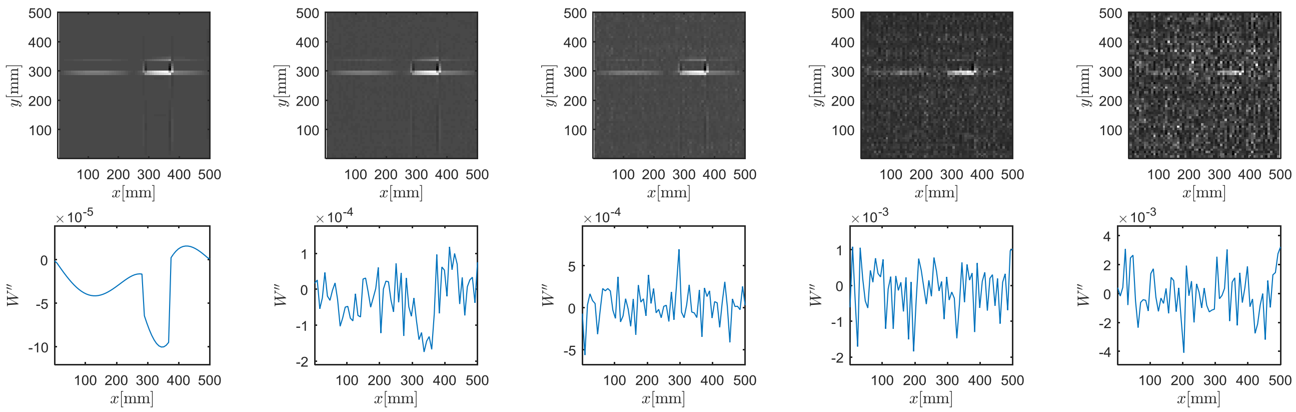


(b)

Fig. 6 Comparison of the presented method and the modal curvature for area damage plate, mode (2, 1) *SNR* = ∞, 80, 70, 60, 50dB from left to right: damage indexes and the corresponding horizontal slices at *y* = 373 mm given by (a) the presented method and (b) modal curvature method

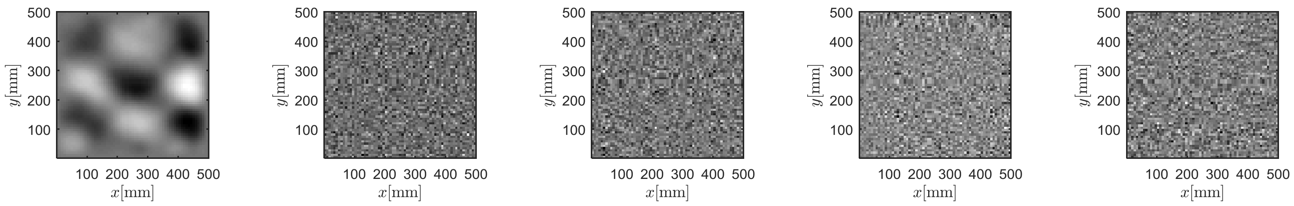


(a)



(b)

Fig. 7 Comparison of the presented method and the modal curvature for area damage plate, mode (2, 3) *SNR* = ∞, 80, 70, 60, 50dB from left to right: damage indexes and the corresponding horizontal slices at *y* = 373 mm given by (a) the presented method and (b) modal curvature method



(a)

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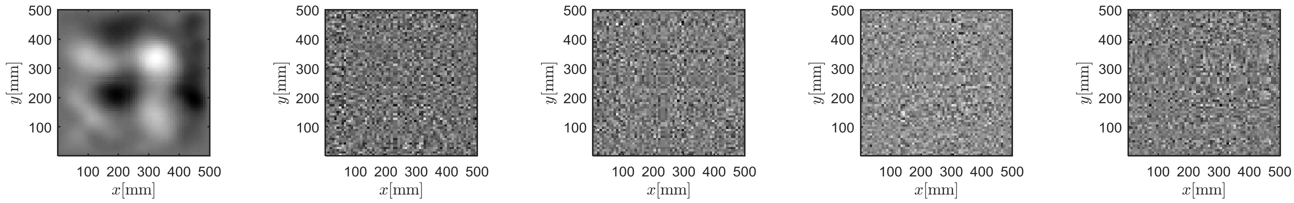


(b)

Fig. 8. Damage maps obtained by the presented method for intact plate: (a) mode (2, 1) and (b) mode (2, 3). $SNR = \infty$, 80, 70, 60, 50dB from left to right

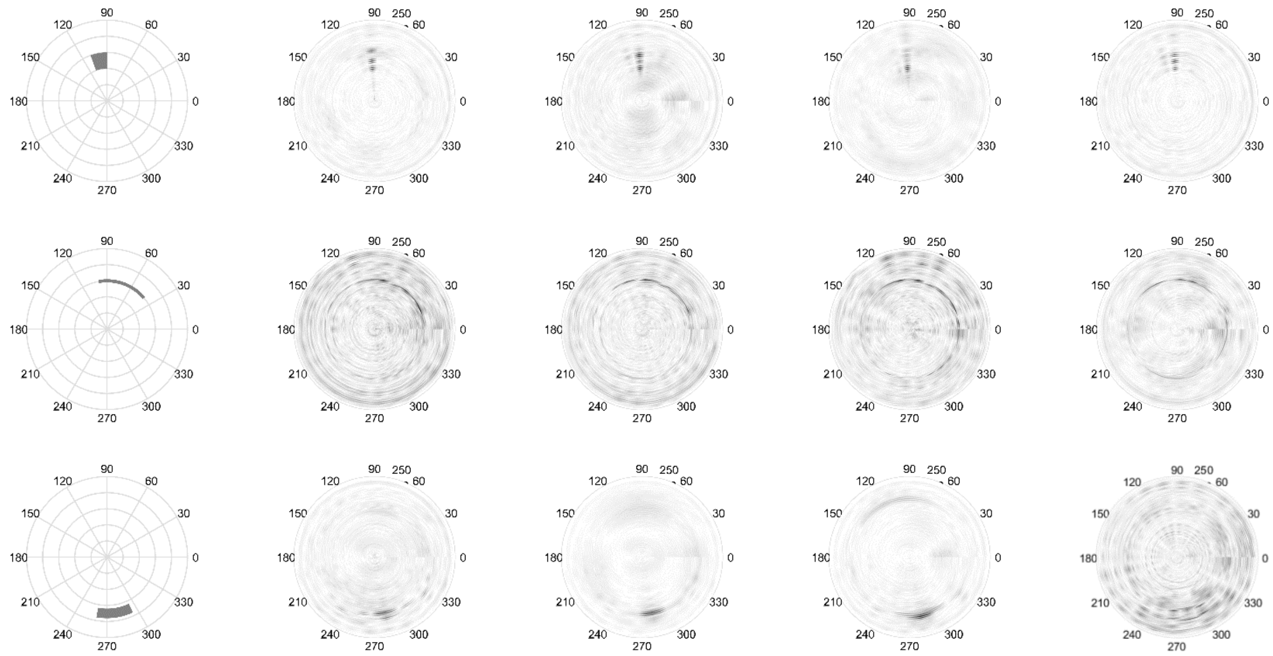


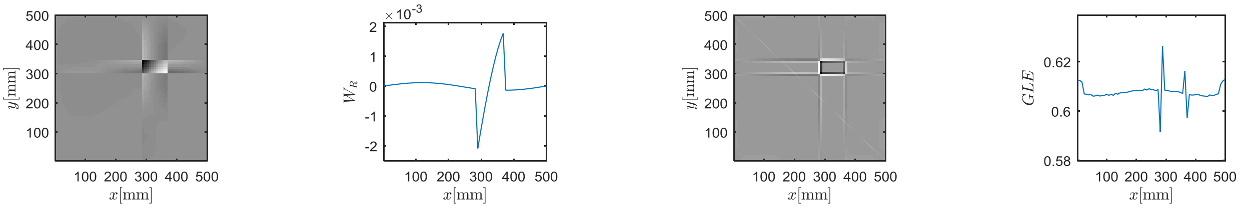
Fig. 9 Schemes of damage locations in circular plates: the 1st row refers to the surface damage 1, the 2nd row refers to the surface damage 2, and the 3rd row refers to the delamination. The results for the 1st-4th mode shapes are presented from left to right

3.2 Validations for circle plate

The problems of SVD method presented above are described for the rectangular or square plates. However, in many cases of engineering structures is necessary to detect damage for circle plates. Therefore, the method is described in polar coordinates to validate the detecting capability of the present method for complex structures. The basic formulation of the process is same with above theoretical development. Converting the Cartesian coordinate to polar coordinates, the same procedure can be conducted. The basic model is the defined in the benchmark (Katunin 2015b, Katunin and Przystałka 2014) for laminated circle plates. The dimensional parameters are radius r = 250 mm, the thickness t = 2.4 mm. The inspected plate is mode by 12-layerd glass-fiber reinforced polymer laminate, whose material parameters are Young’s modules E_1 = 38.293GPa, E_2 = 10.141GPa, the shear modules G_{12} = 3.533GPa. The Poisson’s ratio is 0.366 and the mass density is 1784 kg/m^3. The benchmark provided three typical damage scenes as show in Fig. 9, the surface damage 1, surface damage 2 and delamination case. The damage identification results are also given together. A comparison with the original damage, the developed method performs well on localization, either for various damage types or mode shapes. In addition, the boundary effect is not seen the presented results as they are calculated by the decomposition and reconstruction of SVD directly, special singularity extraction methodologies like wavelet or modal curvature are not employed and the boundary padding are not required.

3.3 Further comparison

In above sections, we have compared the presented method with the classical modal curvature method. The modal curvature method is a well-established method in practice, however, not a typical advanced singularity extraction method with boundary effect problem. Hence a further comparison with the two-dimensional generalized local entropy method (GLE) (Yang *et al.* 2014, Yang *et al.*



(a) (b)

Fig. 10 Comparisons between the presented method (a) and the 2D-GLE method (b) for mode (2, 1)

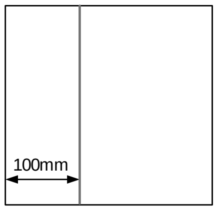


Fig. 11 The damaged plates used in experimental investigations

2016) is conducted here to show the superiority of SVD method on boundary issue. The inspected object is selected as the area damage plate, mode (2, 1), for which the SNR = ∞ is used to make a more objective comparison. Results are presented in Fig. 10, from which one can see the evident boundary effect (near 0 mm and 500 mm) in 2D-GLE result. It should be emphasized that the presented GLE results is obtained by a carefully designed boundary padding. The presented method performs better on boundary.

4. Experimental validations

Experimental validations are conducted via the benchmark provided by Prof. Katunin in (Katunin 2015a, b). The corresponding specimens are made by composite plates made from 12-layered laminate with the dimension 300 mm×300 mm×2.5 mm. As shown in Fig. 11, the defect is a through-the-width crack with the width of 1 mm and the depth of 0.5 mm. The plate with an edge-length of 300 mm is clamped in the frame by 24 bolts, in such a way that the internal square of the frame had the edge length of 250 mm.The measurement of mode shapes are obtained by the SLDV Polytec PSV-400 with a vibrometer controller OFV-5000. For the reference signal, a PCB piezotronics 208C03 force sensor is used. The plate is then excited by a pseudo-random noise signal with the frequency band from 0-2000 Hz with the resolution of 0.625 Hz, and the sampling frequency is set as 5120 Hz. More details can be found in (Katunin 2015a, b). These data sets are composed of five individual mode shapes, named "mode1-mode5". Although the mode shapes involved are not the physically 1st-5th mode shapes of these plates, they are simply denoted as the 1st-5th mode shapes for the sake of clarity.

Results obtained by the presented method and the modal curvature method are compared in Fig. 12. To demonstrate the performance of the developed method, the slices extracted from y = 50 mm are also shown in Fig. 12. The boundary effect of the model curvature method is suppressed by deleting the corresponding boundary parts, however, the present results are displayed with no additional processing. It is seen that the presented method performs better than the modal curvature method. For the results of the 1st mode shape, an obvious damage feature and peak can be found in damage map and the slice. The results of the modal curvature method, however, show the damage location faintly and a false singularity can be found in the slice near x = 230 mm. Similar conclusions can be gotten from the comparison of 2nd, 3rd and 5th mode shape results. The 4th mode shape contains limited information for the defect, which can be seen in (Katunin 2015a, b), thus the presented method shows smooth surface and slice for it. The modal curvature method presents a false peak at x = 80mm. These comparisons validate the effectiveness of the developed methodology.

5. Conclusions

The present work provides an SVD-based method damage detection of plate-like structures. The method can address the boundary effect and the noise robustness problems, which affects the performances of vibration-based damage detection. The boundary effect problem is addressed by the decomposition and reconstruction of mode shape, and the noise robustness in enhanced by the noise filtering during the decomposition and reconstruction process. Numerical validations are performed on plate-like structures for various damage and boundary scenarios. Validations show that the proposed method is accurate and effective in the damage detection for the two-dimensional structures.

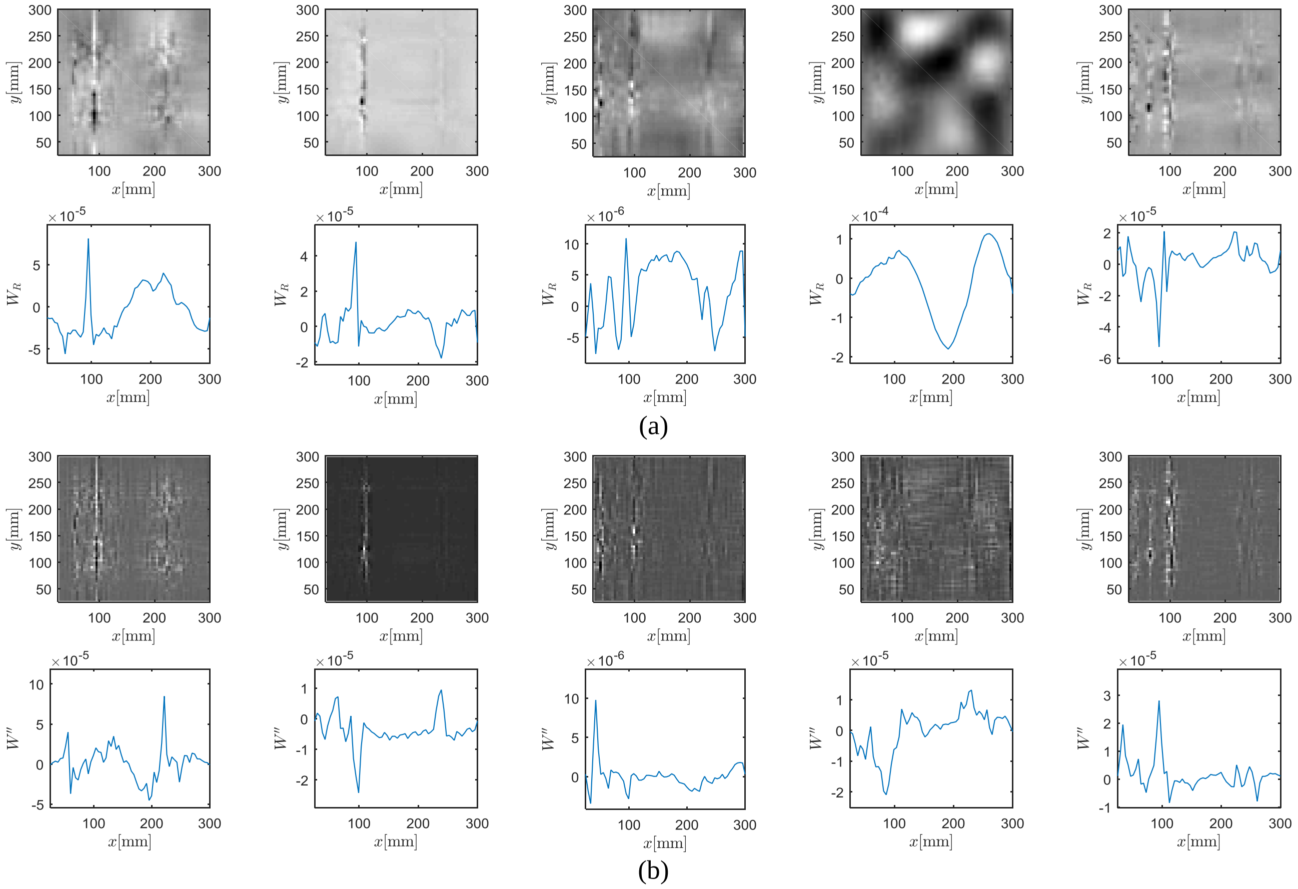


Fig. 12 Comparison of the presented method and the modal curvature: (a) the presented results and (b) the results obtained by the modal curvature method. From left to right locate the 1st-5th modes. The slices are extracted from y = 50 mm

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