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Determining minimum analysis conditions of scale ratio change to evaluate modal damping ratio in long-span bridge

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Abstract. Damping ratio and frequency have influence on dynamic serviceability or instability such as vortex-induced vibration and displacement amplification due to earthquake and critical flutter velocity, and it is thus important to make determination of damping ratio and frequency accurate. As bridges are getting longer, small scale model test considering similitude law must be conducted to evaluate damping ratio and frequency. Analysis conditions modified by similitude law are applied to experimental test considering different scale ratios. Generally, Nyquist frequency condition based on natural frequency modified by similitude law has been used to determine sampling rate for different scale ratios, and total time length has been determined by users arbitrarily or by considering similitude law with respect to time for different scale ratios. However, Nyquist frequency condition is not suitable for multimode system with noisy signals. In addition, there is no specified criteria for determination of total time length. Those analysis conditions severely affect accuracy of damping ratio. The focus of this study is made on the determination of minimum analysis conditions for different scale ratios. Influence of signal to noise ratio is studied according to the level of noise level. Free initial value problem is proposed to resolve the condition that is difficult to know original initial value for free vibration. Ambient and free vibration tests were used to analyze the dynamic properties of a system using data collected from tests with a two degree-of-freedom section model and performed on full bridge 3D models of cable stayed bridges. The free decay is estimated with the stochastic subspace identification method that uses displacement data to measure damping ratios under noisy conditions, and the iterative least squares method that adopts low pass filtering and fourth order central differencing. Reasonable results were yielded in numerical and experimental tests.

Keywords: damping ratio; sampling rate; total time length; signal to noise ratio; free initial value problem; scale ratio

1. Introduction

Dynamic response is evaluated for stability assessment in its current state. Evaluation of damping ratios helps to make the prediction of dynamic instability accurate. Whereas, assessment of damping ratio is difficult to be performed in multimode system with noisy signals. As bridges are getting longer, small scale model test must be conducted to evaluate damping ratio and frequency considering minimum analysis conditions that have influence on the accuracy of system identification technique (Chun 2017). All parameters used in aeroelastic analysis can be modified by using similitude law (Buckingham 1914, Buckingham 1915, Simiu and Scanlan 1996). Generally, experimental tests are conducted with the variable considering this similitude law. However, minimum analysis conditions (sampling rate and total time length) are difficult to follow similitude law and thus should be determined for different scale ratios. In addition, first vertical and torsional mode mainly affect serviceability or instability of long-span cable bridges. For this reason, these two modes are used to compute modal damping and frequency for all numerical models and experimental tests.

The variable considering similitude law have been applied to small scale model test. When scale ratios changed, all parameters used in analysis of dynamic system can be modified by using similitude law. In this case, Nyquist frequency condition has been used to determine sampling frequency related to analysis conditions. For example, Nyquist frequency should be determined considering sampling rate is more than two times the natural frequency (Grenander 1959, Condon and Ransom 2016). In addition, Nyquist frequency condition based on natural frequency modified by similitude law has been used to evaluate modal damping ratios for small scale bridge model. However, Nyquist frequency condition is insufficient to be applied to multimode systems with noisy signal and systems modified by similitude law. In case of total time length, the determination of total time length has been conducted by users arbitrarily or by considering

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similitude law for different scale ratios. Therefore, analysis conditions, which are both sampling rate and total time length, should be determined.

The accuracy of SI techniques is important to evaluate modal damping ratios and frequencies. Especially, signal to noise ratio affects the accuracy of global solutions (modal damping ratio and frequency). Previous researches (Chowdhury and Sarkar 2003, Bartoli *et al.* 2009, Chun *et al.* 2017) indicated that errors of global solution were less than around 5% when it comes to 20% NL (25% NL for Bartoli *et al.*). Because those studies are related with flutter phenomenon, noise level (NL) described in Eq. (9) is assumed to be higher than typical damping evaluation. Therefore, numerical verifications of this study are performed with NL 10 that is maximum signal to noise ratio (Magalhaes *et al.* 2010). In addition, modal complexity causes the estimation of modal parameter to be difficult. Modal complexity means the level of damping non-proportionality described in Table 1 and presents the difficulty in orthogonality of modes for damping coefficient and stiffness matrix. For this reason, the influence of modal complexity was investigated for the stochastic subspace identification (SSI) method and the iterative least squares (ILS) method described in section 2.

Free vibration is the most useful way to obtain damping ratio. Especially, initial value problem (IVP) described by previous research has been developed for extraction of flutter derivatives with noisy signal under free vibration test. IVP also can be used to predict damping ratio and natural frequency. As a matter of fact, IVP is difficult to obtain dynamic properties without the initial condition because the initial condition is mainly used to obtain a transient solution (that consists of a modal damping ratio and natural frequency) of dynamic systems in free vibration tests (Craig and Kurdila 2006). In general, the initial condition, which consists of displacement and velocity, is difficult to obtain accurately because of environmental conditions, geometrical limitations and device reasons. Therefore, this study presents a method using the free initial value problem (FIVP) that uses the value of the current state without the original initial condition, so the method only requires current initial conditions, i.e., displacement and velocity at the current time in IVP.

In this study, the ILS method is used to compute damping ratios in IVP. In addition, the covariance-driven stochastic subspace identification (SSI-COV) that is a kind of operational modal analysis is used to compare with the ILS method and to compute the damping ratio under ambient vibrations. To identify modal parameters in SSI, the Hankel matrix, which consists of a cross-correlation matrix, is extracted (Overscjee and Moor 1996, Peeters and Roeck 2001). In addition, the iterative least squares (ILS) method initiated by Chowdhury and Sarkar (2003) was used to compute modal damping and natural frequency for the free initial value problem in IVP. The ILS method was developed to overcome difficulty in extracting flutter derivatives for the Modified Ibrahim Time Domain (MITD) method that was described by Sarkar (1992). The displacement-based problem of a dynamic system generally adopts discretization techniques that use either central or compact differencing. Central differencing performs direct discretization. For example, velocity is obtained only by displacement. However, compact differencing uses the same contiguous variables. For example, velocity is obtained by both displacements and velocities that include adjacent velocities. Compact differencing is more accurate than central differencing but its performance is inversely proportional to data size (this means difficulty in calculation) when obtaining velocity and acceleration (Chu and Fan 1998). This study uses fourth order central differencing that is highly accurate ($O(\Delta t)^4$) and less affected by data size. Moreover, low or high pass filtering helps eliminate errors that distort discrete data of the exact solution. This study adopted zero phase filtering (Matlab function - filtfilt) that minimizes the phase shift of filtered signals in the ILS method (James *et al.* 2003).

In this study, the focus was made on the determination of minimum analysis conditions for different scale ratios and the analysis for the influence of damping ratio and frequency according to the level of NL. In addition, FIVP described in section 3 is proposed to resolve the condition that is difficult to know original initial value due to noisy signal under free vibration. To implement this objectivity, numerical models are classified into five types according to scale ratios (0, 70, 120, 200, 300). Signal to noise ratio is also classified into three types according to NL (3, 6, 10). Numerical analysis and section model tests were performed by developing two degree-of-freedom (DOF) models and adopting the SSI and ILS methods. In addition, another experiment was conducted with multiple modes on full bridge 3D model tests that applied a similitude law (real main span of 700 m and scale ratio of 120). Numerical analysis showed the independence of initial values with respect to time by using FIVP. Experimental tests were performed with free vibration test using the free initial value condition and ambient vibration test.

2. System identification technique for modal damping and frequency

2.1 The free decay of free and ambient vibration

Displacement data was used to predict the free decay of free and ambient vibrations. Velocity and acceleration were discretized by displacement filtered with zero phase filtering. Fourth order central differencing was adopted for discretization with respect to time. The initial condition (displacement and velocity) was essential to compute for the motion of structures in structural dynamics. However, it is difficult to get an exact measurement of the initial condition because environmental factors and device limitations, etc., would interfere accurate measurement. This study presents a free initial condition method that allows free measurement of original initial conditions.

To precisely simulate dynamic responses, the Nyquist frequency condition must be considered. In general, the ability to capture the movement of a device should be over two times the reference frequency, which is the natural frequency of each mode (James *et al*. 2003). In the present work, the sampling rate of recorded data was more than five times the highest frequency among all modes. All the applications presented in section 7 are based on the use of a laser sensor device (linear response from 0–200 *Hz*). Acceleration data was verified with a one-axial load cell for each DOF that enabled the comparison of simulated and measured data.

The measured data was then processed by performing system identification that used output-only data. (Juang and Pappa 1985, Magalhaes *et al*. 2010, Chun *et al*. 2017, Chun 2017) In this study, system identification for modal parameters was based on the ILS and SSI methods. Both methods computed the system matrix for modal parameters derived by a complex eigen-solution of non-proportional damping (Craig and Kurdila 2006). Free vibration tests consisting of imposed initial condition or temporal forced vibration were conducted with the ILS and SSI methods, and ambient vibration tests were performed with the SSI method.

2.2 Stochastic subspace identification (SSI) method

The SSI method can estimate modal parameters by applying singular value decomposition (SVD) of Hankel matrix that consists of a correlation matrix for output data (Peeters and Roeck 2001).

$$\begin{aligned} x(k+1) &= Ax(k)+w(k) \\ y(k) &= Cx(k)+v(k) \\ H_{ij} &= R_k,\ O_{i-l}^{+} = O_{i-l}^{-}A \end{aligned} \tag{1}$$

The Hankel matrix (H_{ij}) consists of a cross-correlation matrix of each measurement (R_k) and an observability matrix (O_i) is obtained by the Hankel matrix decomposed by SVD. O_{i-l}^{+} and O_{i-l}^{-} are obtained by removing the first or last l rows from the observability matrix. System matrix (A) is extracted by the process that considers an adjacent block matrix of the observability matrix. Each variable denotes that C is the output matrix, $x(k)$ is the input vector, $y(k)$ is the output vector, $w(k)$ is the load matrix, and $v(k)$ is the white Gaussian noise matrix.

It is noted that the SSI method works powerfully in multiple-input and multiple-output (MIMO) state-space models. Thus, the SSI method is suitable for tests of field structure and 3D prototype model that can be recorded in many locations. The output matrix can be freely constructed by user selection, measuring displacement, acceleration, or a combination of both (Cho *et al*. 2015).

The SSI method can efficiently perform modal estimates of the dynamic properties of an ambient vibration environment with stationary conditions and normally distributed random loads. Modal parameters must be determined by stabilization diagram that is obtained by singular value decomposition (SVD), satisfying critical damping ratio and modal assurance criterion (MAC) conditions. In this study, stabilization criteria values were 1% for frequencies, 5% for damping ratios and 99.9% for MAC. This method is applied to full bridge 3D model that contains complicated modal parameters.

The SSI method for structural vibrations is generally applied with acceleration data but can also be used with displacement data. The method that does not use the original initial condition is compatible with both the SSI and ILS methods that adopt the free initial value condition. These two methods were applied to numerical and experimental tests, satisfying sufficient sampling rates and total time length (Magalhaes *et al*. 2010).

2.3 Iterative least squares (ILS) method

The ILS method performs the identification of the system matrix using a state–space equation and least square (LS) method that used iterative algorithms. Dynamic equations of free decay are expressed in an effective form that can be organized into four differential equations (Ghilani and Wolf 2006). This is called a state–space equation.

$$\begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} \ or\ \underline{\dot{X}} = [A]\cdot \underline{X} \tag{2}$$

$[A]$ is the mechanical matrix of the state space form and the solution of the first order matrix differential equation is shown below.

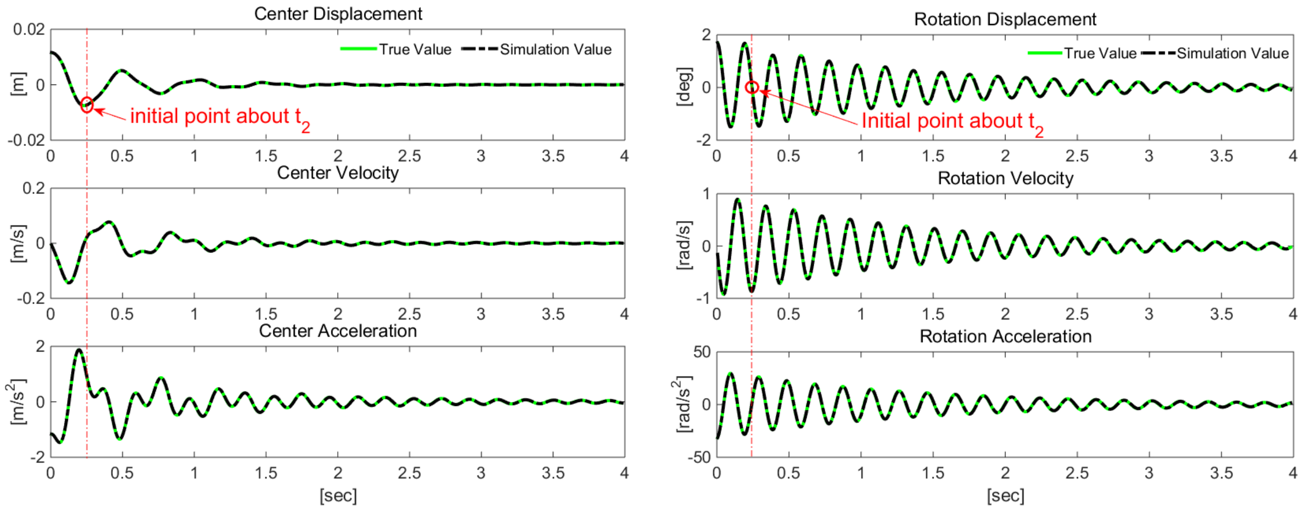
$$\underline{X} = e^{[A]t}\underline{X}_0 \tag{3}$$

Eq. (3) is used to illustrate the derivation of the free initial condition denoted as $\underline{X}_0$. The ILS method performs an iterative procedure to obtain the system matrix as shown below.

$$\begin{aligned} &[A]^i = \left(\underline{\dot{X}}\cdot \underline{X}^{i^T}\right)\left(\underline{X}\cdot \underline{X}^{i^T}\right)^{-1} \\ &\underline{X}^i = e^{[A]^i t}X_0 \qquad\qquad for\ i = 0, 1, 2, \cdots \\ &R_i = \max\left[abs([A]^i - [A]^{i-1})\right] \end{aligned} \tag{4}$$

$\underline{X}$ is the vector of displacement and velocity related to the recorded and then discretized data for each DOF. $\underline{\dot{X}}$ is the differentiation process with regard to the time of $\underline{X}$. $[A]^i$ is the system matrix that consists of the mechanical stiffness and damping coefficient matrix. Index i denotes the number of iterative procedures and $\underline{X}^0$ is the measured data. The initial value of the system matrix ($[A]^0$) is generally obtained by the LS method that is linear model. ILS is similar to the LS method but uses an updated value ($\underline{X}^i$) computed from the iterative process and state-space equation. The system matrix was obtained with iteration level (R_i) under 10^{-6}. The displacement in the vertical and lateral directions was expressed in meters while rotation is expressed in radians when solving whole equations.

The error ratio of the modal parameter of ILS method is under 5% when the noise to signal ratio is 20% as mentioned in the introduction. Therefore, estimation of the modal solution from the ILS method is accurate in adverse



(a) Vertical DOF (b) Rotational DOF

Fig. 1 Comparison between true and simulated values for 2DOF in time domain using the ILS method and the original initial condition (system 1)

conditions. The fit of exponential decay was determined after the application of an optimization process that searched for the optimal time interval with minimum residual error between measured and simulated values.

3. Free initial value problem (FIVP) in free vibration

3.1 Time shift and initial value problem

If the time variable of system 2 is related to that of system 1, then the time variable of system 2 consists of the time variable of system 1 and Δt. The subindex denotes each system that has different initial values but consists of the same model.

$$t_2 = t_1 - \Delta t \tag{5}$$

System 1 uses the solution of the state–space equation expressed below. Let the solution of system 1 at time Δt be equal to initial condition of system 2.

$$\underline{X}(\Delta t) = e^{[A]\Delta t}\underline{X}_1 = \underline{X}_2 \tag{6}$$

We can derive the relationship between systems 1 and 2 using Eq. (6) as shown below

$$\begin{aligned} &\underline{X}(t_1) = e^{[A]t_1}\underline{X}_1 = e^{[A](t_1-\Delta t)}\underline{X}_2 \\ &e^{[A](t_1-\Delta t)}\underline{X}_2 = e^{[A]t_2}\underline{X}_2 \\ &e^{[A]t_1}\underline{X}_1 = e^{[A]t_2}\underline{X}_2 \end{aligned} \tag{7}$$

From Eq. (7) the solution of system 1 can be expressed with the initial condition of system 2. These relationships can make conversion useful between different systems. Specifically, the current initial condition can be used to compute the system matrix even though the original initial condition was not measured. X_1, X_2 are the initial conditions with respect to each system. This relationship is shown in Eq. (7), hereinafter referred to as the free initial condition. In section 3.2, the comparison of between true and simulated values will be verified using the original and free initial conditions.

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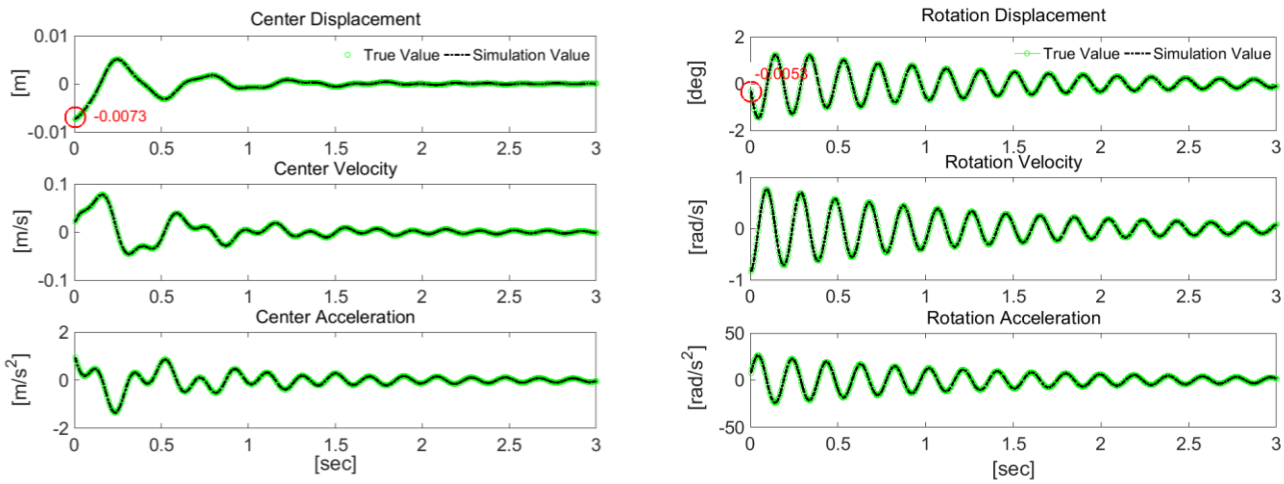
3.2 Numerical simulation

The accuracy of the free initial condition must be verified. Thus, Eq. (8) which follows MKS units is used to simulate numerical response shown in Figs. 1 and 2. This properties extracted by operational data were used to reflect real environmental condition (Bogunovic Jakobsen and Hjorth-Hansen 1995). Vertical and torsional mode mainly affect serviceability or instability of long-span cable bridges. For this reason, these two modes are used in this study. Fig. 1 indicates the model that presents damping ratios 0.1559% and 0.0226%, and natural frequencies 2.0157 *Hz* and 5.1325 *Hz* for modes 1 and 2. In addition, mass is 2.6526 and 0.0189 for each mode.

$$K = \begin{bmatrix} 420.100 & -59.181 \\ 1.755 & 19.659 \end{bmatrix} \qquad C = \begin{bmatrix} 8.931 & -0.080 \\ 0.435 & 0.039 \end{bmatrix} \tag{8}$$



(a) Vertical DOF (b) Rotational DOF

Fig. 2 Comparison between true and simulated values for 2DOF in time domain using the ILS method and the free initial condition (system 2)

Table 1 The system properties for models that have different damping ratio

Model	Modal complexity	Non-proportionality of damping	Mode 1		Mode 2	
			f (*Hz*)	*ξ* (%)	*f* (*Hz*)	*ξ* (%)
SMHD	None	0.01	0.2404	2.6834	0.6131	1.5849
SMLD	None	0.01	0.2404	0.3001	0.6131	0.3001

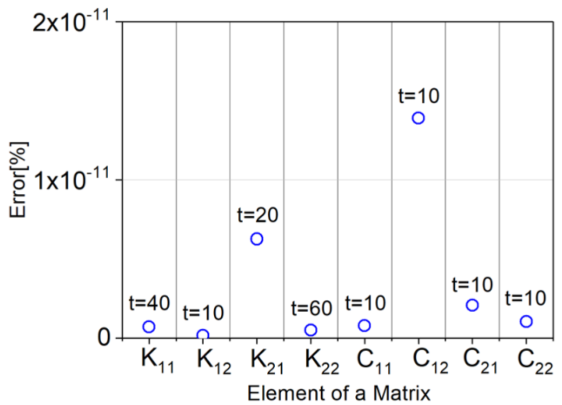


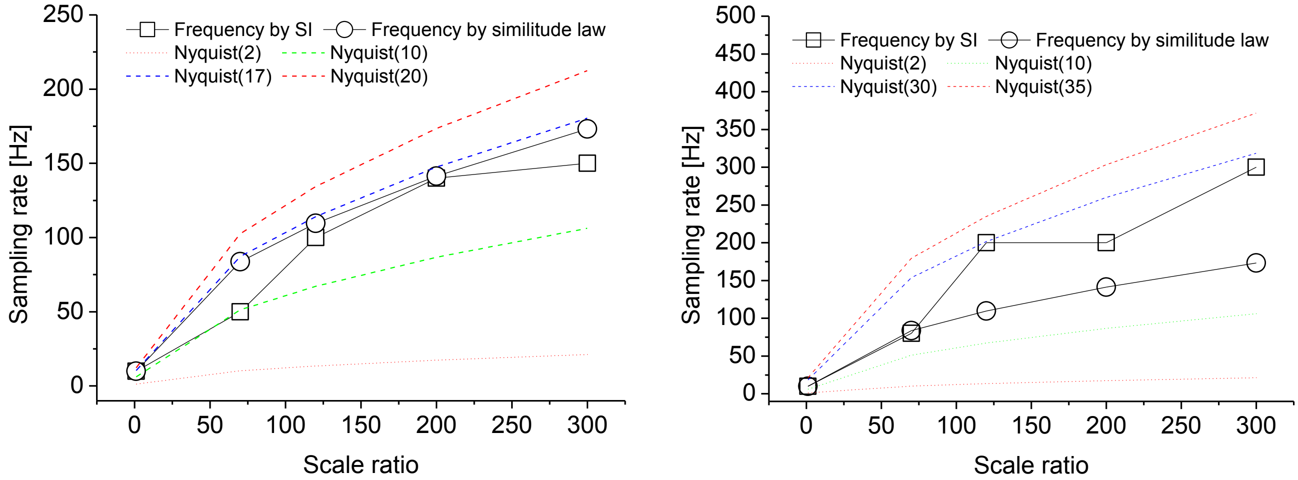
Fig. 3 The accuracy of the free initial value problem compared with the original initial value problem (Each number on the circle-symbol indicates the time [s] of occurrence)

The original initial displacement was 0.0116 m and 1.7189° (0.03 rad) and the original velocity was zero for each DOF (system 1). The simulated data using the original initial condition is shown in Fig. 1. The current initial value (system 2) was -0.0073 m and -0.0053° for each DOF. The simulated data using the current initial value is shown in Fig. 2. Both true and simulated values precisely matched the free initial condition as well as the original initial condition.

The initial time was divided into 10 s intervals from 10-60 s in terms of the time of the original system. The numbers for the time designated in Fig. 3 present the time that occurs maximum error among whole initial time cases and then this time can be equal to the initial time of new systems. Therefore, the free initial condition is suitable for any free vibration when the original initial condition is unknown regardless of the initial time. Additionally, the errors shown in Fig. 3 result from the discretization process because the maximum difference between the free initial value and original initial value problem is nearly zero (less than 10^{-11}).

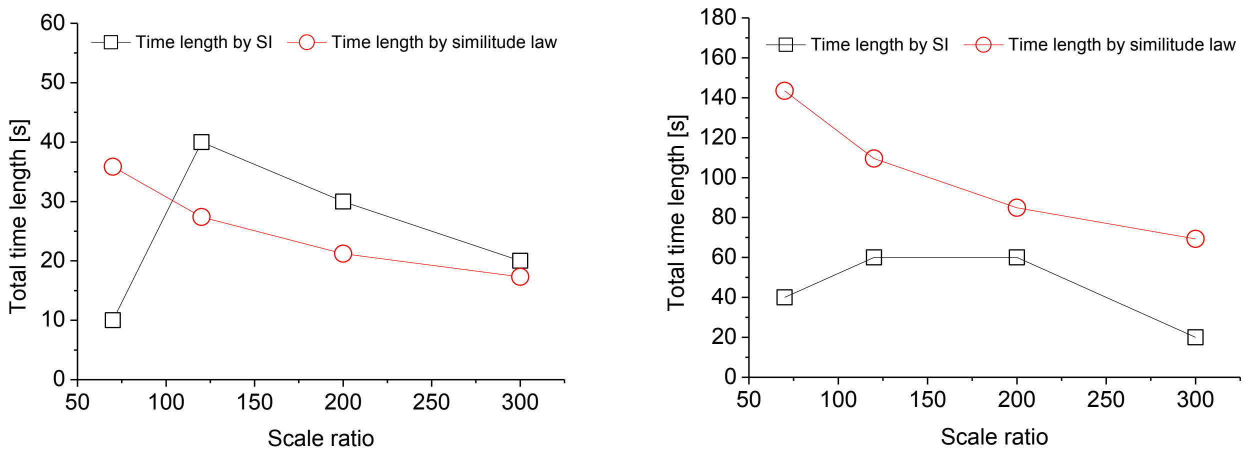
4. The effect of dynamic property with signal to noise ratio

Signal to noise ratio has influence on extracting solution in SI technique and greatly affects accuracy of damping ratios. Table 1 presents the system properties for models that have different damping ratio. SMHD (seperated model with high damping case) model presents high damping ratio used in typical structures. In addition, SMLD (seperated model with low damping case) model presents low damping ratio used as design value in long-span bridges (KSCE 2006). Those two models have none modal complexity



(a) SMHD (b) SMLD

Fig. 4 Minimum analysis conditions for sampling rate with NL 10



(a) SMHD (b) SMLD

Fig. 5 Minimum analysis conditions for total time length with NL 10

that means separated frequency between two modes and have non-proportionality of damping that satisfies orthogonality condition for damping coefficient matrix. Analysis of those two models is conducted by the ILS method. In addition, this analysis is performed with several total time length (1, 2, 5, 10, 20, 30, 40, 50 min) and sampling rate (3, 5, 10, 12, 24 *Hz*). Two model is analyzed with three noise level (NL) that consist of 3, 6 and 10.

Table 2 indicates that low damping condition is difficult to identify damping ratios than high damping condition. Although damping ratios are easily evaluated in low NL, damping ratios were accurately extracted in NL 10. All those minimum conditions were computed when errors were less than 6%. Each case was simulated 100 times.

The noise level (NL), noise to signal ratio, indicates the number of errors contained in a signal expressed as a percentage. NL was defined by Sarkar (1992) as follows

$$NL(\%) = \frac{\sqrt{E(\eta_i^2)}}{\sqrt{E(Y_i^2) - E(\eta_i^2)}} \times 100 \tag{9}$$

Table 2 Minimum analysis conditions according to NL

NL	SMHD		SMLD	
	Sampling rate	Total time length	Sampling rate	Total time length
3	3	1	10	5
6	3	5	10	10
10	10	5	10	20

*NL: Noise level, a unit of sampling rate is [*Hz*], a u nit of total time length is [min]

where, $\underline{Y}$ is noisy signal vector, $\underline{\eta}$ is total noise vector, Y_i, η_i are the elements of vectors $\underline{Y}$, $\underline{\eta}$ respectively, and $E(\cdot)$ denotes the expected value. All the numerical models in this study are based on theoretical values with 10% NL generated with random white Gaussian noise containing all information on the occurred frequency without bias. The NL is conservative in cases of well-conducted ambient vibration tests.

Table 3 Minimum analysis conditions with NL10 according to scale ratio

Scale ratio	SMHD		SMLD	
	Sampling rate [*Hz*]	Total time length [s]	Sampling rate [*Hz*]	Total time length [s]
1	10	300	10	1200
70	50	10	80	40
120	100	40	200	60
200	140	30	200	60
300	150	20	300	20

5. Minimum analysis conditions for different scale ratios

As bridge is getting longer, experimental model should be designed with small scale model. Minimum analysis conditions for different scale ratios should be determined for making global solution stable. In this section, all analysis are performed under NL 10 that guarantees conservative analysis condition and use numerical model described in Table 1 that presents the system properties for models that have different damping ratio. Analysis of those two models was conducted by ILS method as shown in Figs. 4 and 5. This analysis was performed with several scale ratios: For scale ratio 1, analysis was performed with several total time length (1, 2, 5, 10, 20, 30, 40, 50 min) and sampling rate (2, 3, 5, 10, 12, 24 *Hz*); For scale ratio 70, analysis was performed with several total time length (5, 10, 20, 30, 40, 80, 144, 300 s) and sampling rate (20, 50, 80, 100, 150, 200 *Hz*); For scale ratio 120, analysis was performed with several total time length (5, 7, 10, 20, 40, 60, 180, 300 s) and sampling rate (20, 30, 50, 100, 200, 300 *Hz*); For scale ratio 200, analysis was performed with several total time length (4, 5, 7, 14, 30, 60, 120, 240 s) and sampling rate (30, 60, 140, 200, 300, 350 *Hz*); For scale ratio 300, analysis was performed with several total time length (3, 4, 5, 10, 20, 40, 60, 180 s) and sampling rate (34, 68, 150, 200, 300, 400 *Hz*). All those minimum conditions were also computed when error rates were less than 6%. Low damping ratio condition needs conservative analysis condition than high damping ratio. From those results, minimum analysis conditions can be derived by comparison between the result obtained by SI technique and Nyquist frequency condition. Each case was simulated 100 times.

Table 3 synthesizes minimum condition of total time length and sampling rate for different damping ratios. The system that has low damping ratio should be identified with conservative condition. In other words, low damping ratio condition needs higher total time length and sampling rate for extracting damping ratio. Square mark indicates total time length extracted by the ILS method and circle mark presents total time length adopting similitude law applied to reference total time length (for scale ratio 1) in Figs. 4 and 5. Sampling rate and total time length by similitude law indicate that total time length and sampling rate for scale ratio 1 are changed by applying similitude law for each scale ratio. In case of sampling rate, proposed sampling rate can be defined by considering Nyquist frequency concept. Nyquist frequency condition means that signals can be simulated when sampling rate is more than twice the reference frequency of the system. In this study, Nyquist (k) was used to define a specific sampling rate. Nyquist (k) means that sampling rate is k times the reference frequency of a system. As shown in Figs. 4(a) and 4(b), Each of Nyquist (17) and Nyquist (30) is a critical line in high and low damping case, respectively. To guarantee reliability, conservative criteria was determined using Nyquist (20) and Nyquist (35) for each case. When the damping ratio has unknown values, Nyquist (35) should be applied to the sampling rate.

In case of total time length (Fig. 5), tendency of SMHD and SMLD is different. The fact that total time length by similitude law is lower than by SI technique shows that similitude law should not be applied to total time length. Therefore, it is noted that minimum total time length must be determined by SI technique.

6. Numerical verification under free vibration

6.1 Description of numerical model

Verification of the free initial value problem was conducted with a simple model that had no interference in each mode as shown in Eq. (8). An SM1 model with a high damping ratio is constructed to verify proposed minimum analysis conditions described in section 5. The free initial value problem is also applied to additional models having modal complexities and closed frequencies for each mode described in Tables 4 and 5. Modal complexity refers to the level of damping non-proportionality described by Magalhaes *et al.* (2010). Non-proportionality is defined as the ratio between the sum of the absolute values of the off-diagonal elements and the sum of the absolute values of the diagonal elements of the modal coordinate damping matrix. The proportionality of the system is important in order to orthogonalize and diagonalize the system matrix in terms of different mode shapes (Nagarajaiah and Yang 2015). Therefore, if non-proportionality is high, it is difficult to accurately estimate the dynamic properties of modal damping ratio and frequency. This section presents four numerical models, shown in Table 5, that adopt the ILS method and free initial condition. Table 5 that is theoretical solution presents the natural frequency and modal damping ratios for each mode.

Table 4 The system properties for all numerical models

Model	M [kg]		K [N/m]		C [$N \cdot s/m$]	
SM1	2.6526	0	420.100	-59.181	0.040	-0.080
	0	0.0189	1.755	19.659	0.435	0.039
CM1	1250	26.3	100000	0	327.5	-72.5
	26.3	1250	0	100000	-72.5	327.5
CM2	1250	26.3	100000	1050	327.5	-72.5
	26.3	1250	-1050	100000	-72.5	327.5
CM3	1250	26.3	100000	2000	327.5	-72.5
	26.3	1250	-2000	100000	-72.5	327.5

*SM: Separated model for frequency; CM: Closed model for frequency

Table 5 Numerical models and dynamic properties for non-proportionality of damping and adjacent frequency

Model	Modal complexity	Non-proportionality of damping	Mode1		Mode2	
			f (Hz)	ζ (%)	f (Hz)	ζ (%)
SM1	Strong	15.73	2.0179	2.3570	5.1268	2.2658
CM1	None	0.00	1.4088	1.1286	1.4387	1.8080
CM2	Some	0.50	1.4105	1.0800	1.4370	1.8564
CM3	Strong	0.95	1.4152	0.8548	1.4325	2.0812

The SM model indicates well separated frequency while CM model indicates closed frequency. An SM1 model with a damping ratio will be verified as shown in Table 6. Because the SM1 model has well separated frequency, it is useful for checking and comparisons with several conditions despite of its strong non-proportionality of damping.

6.2 Simulation of free decay

An SM1 model, based on experimental settings that had been conducted in wind tunnel test of Korea University, was constructed with a scale ratio 70. Fig. 6 presents the modal natural frequency and damping ratio for case 3e, which has strong modal complexity, which is simulated 100 times and shows accurate results for all modes (mean values 2.3729% and 2.2630%, and standard deviations 0.0526 and 0.0578, respectively, for two modes in the case of damping ratios). It is found that estimation of damping is performed well in strong non-proportionality condition and frequency estimation is less dependent to noise signal. In Fig. 7, the minimum total time length and sampling rate could be determined as 40 s and 80 Hz within 6% error, respectively. From Mode2_20 in Fig. 7(a), even though the total time length is enough to identify system parameters, it is noted that the damping ratio can be distorted under the conditions of insufficient sampling rate. From Table 7, it is found that proposed minimum analysis conditions gives conservative bound and helps results to be evaluated reasonably.

Table 6 The cases of numerical model for SM1

Sampling frequency [Hz]	Total time length (s)							
	5	10	20	30	40	80	144	300
20(⊟)	1a	1b	1c	1d	1e	1f	1g	1h
40(◇)	2a	2b	2c	2d	2e	2f	2g	2h
80(△)	3a	3b	3c	3d	3e	3f	3g	3h
100(✳)	4a	4b	4c	4d	4e	4f	4g	4h
200(⊖)	5a	5b	5c	5d	5e	5f	5g	5h

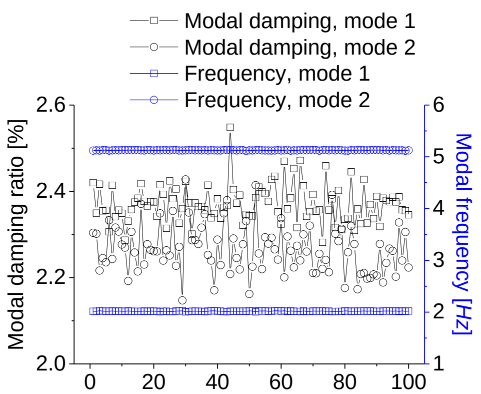
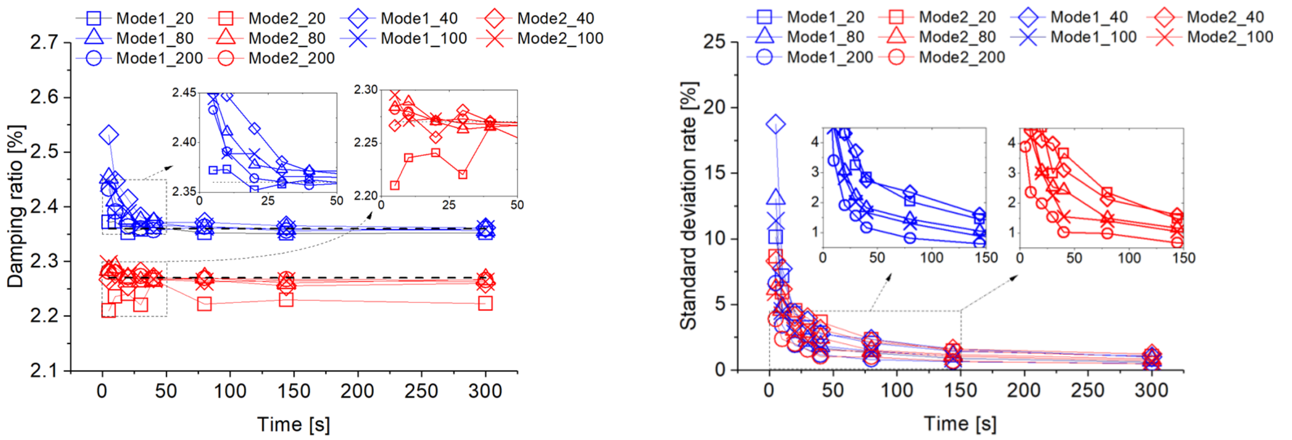


Fig. 6 Numerical simulation of case 3e with modal damping ratio and natural frequency for two modes



(a) Mean values (b) Standard deviation rate values

Fig. 7 Mean and standard deviation rate of modal damping ratio for each case of SM1 (mode 1 represented by blue lines, mode 2 by red lines; dashed lines present theoretical values)

Table 7 Comparison between proposed analysis condition and simulated condition of case 3e for high damping

Scale ratio	SM1	Proposed minimum analysis conditions	
		Nyquist (17)	Nyquist (20)
	Sampling rate [*Hz*]	Sampling rate [*Hz*]	Sampling rate [*Hz*]
70	80	87	102

* Nyquist (k) means that sampling rate is k times the highest reference frequency of a system

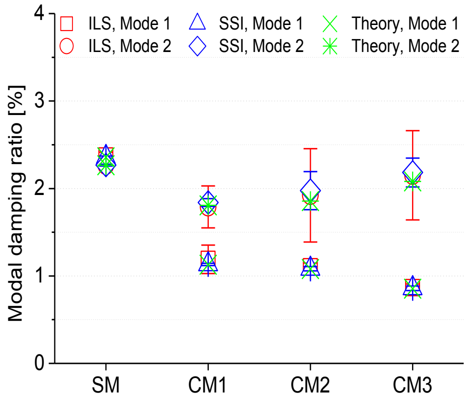


Fig. 8 Mean and standard deviation of modal damping ratio of case 3e with two modes for all numerical models (the ILS method in red, the SSI method in blue, and theoretical values in green)

The condition satisfied with 40 s and 80 *Hz* was used to verify all numerical models shown in Table 4 and experimental tests. Fig. 8 presents the theoretical results and the results extracted by the ILS and SSI method for all numerical models described in Table 4 and shows the well-matched mean values. In addition, standard deviation described in Fig. 8 is high in closed frequency model and higher modes. The SSI method showed good agreement with the precision of damping ratios at high modes for all models.

7. Experimental test

An experimental test was conducted with a 2DOF section model and a full aeroelastic bridge 3D model that adopted the similitude law proposed by Chun (2017) to simulate a performance identical to that of a real bridge. The section model test described in Fig. 9 was conducted with ambient and free vibration tests, applying the ILS and SSI methods with FIVP. Free vibration tests with imposed initial condition and temporal forced vibration were conducted with the ILS and SSI methods, and ambient vibration tests were performed with section models and a full aeroelastic bridge 3D model under turbulent flow coming toward the test model.

Identification of all experimental tests is performed considering proposed minimum analysis conditions. Because experimental test models had low damping, Nyquist (35) condition was applied to sampling rate. Minimum total time length of the section model test was determined as 40 s considering several simulations with the ILS method.

7.1 Section model test

The section model test was conducted with ambient and free vibration tests, and the test was carried out five times under different initial conditions. The properties of section models were classified into two groups, consisting of a well-separated model and closed model (natural frequencies 1.95 *Hz* and 3.82 *Hz* for each mode in the well-separated model and 1.958 *Hz* and 1.988 *Hz* for each mode in the

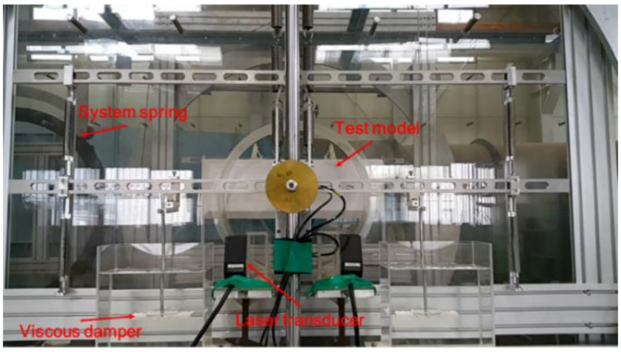
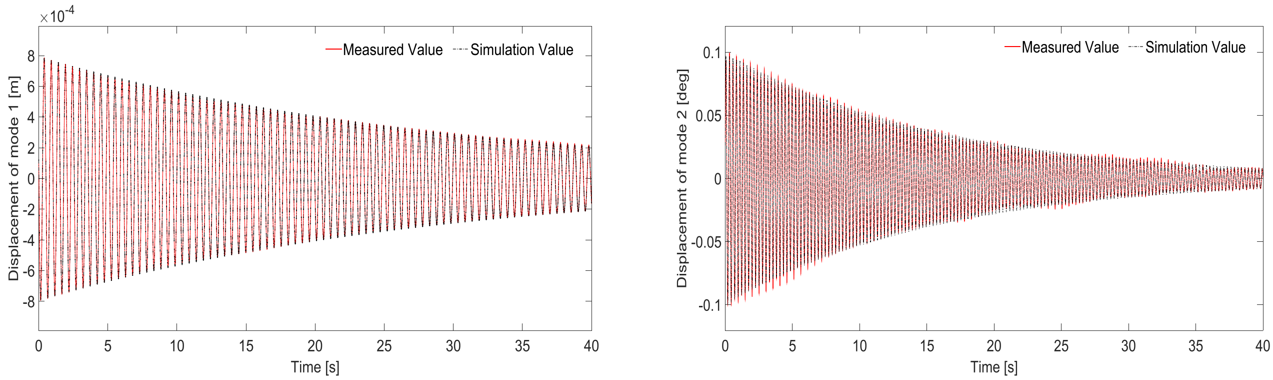


Fig. 9 Test set-up for free and ambient vibrations for vertical and torsional mode in Korea university



(a) Vertical DOF (b) Rotational DOF

Fig. 10 Time history displacement data using the ILS method under 40 s and 120 *Hz* in SMT 2 for all modes (simulation values were compared with measured values)

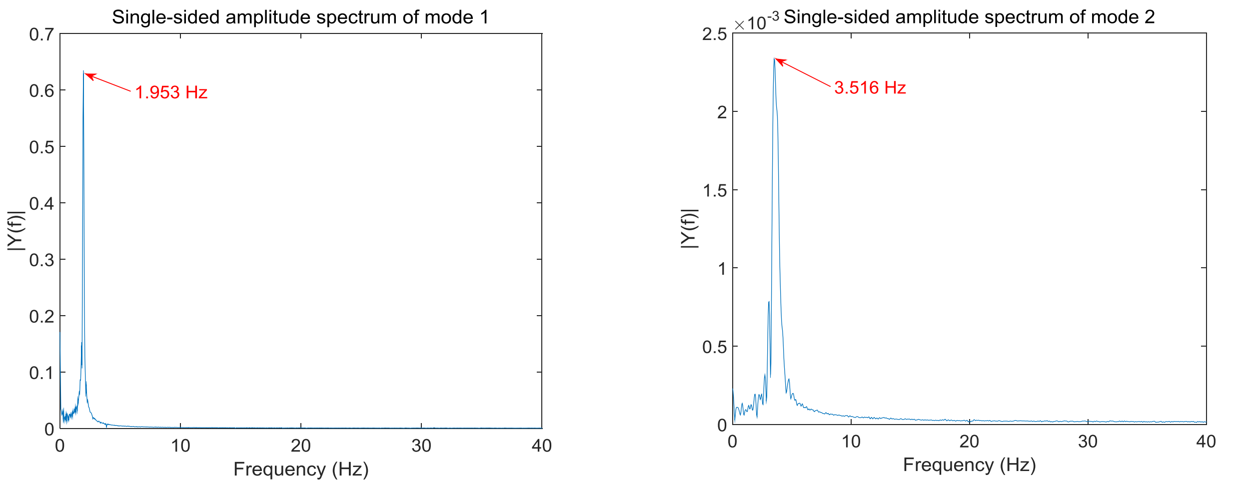
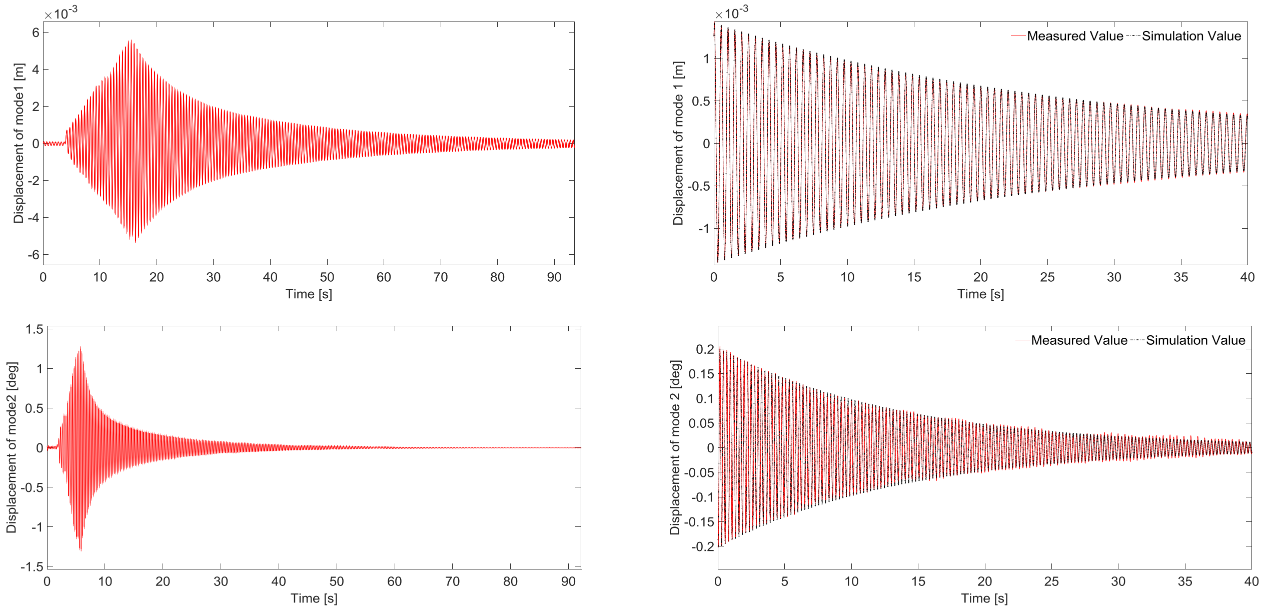


Fig. 11 FFT diagram of subjected forces for each DOF (mode 1 in left, mode 2 in right)

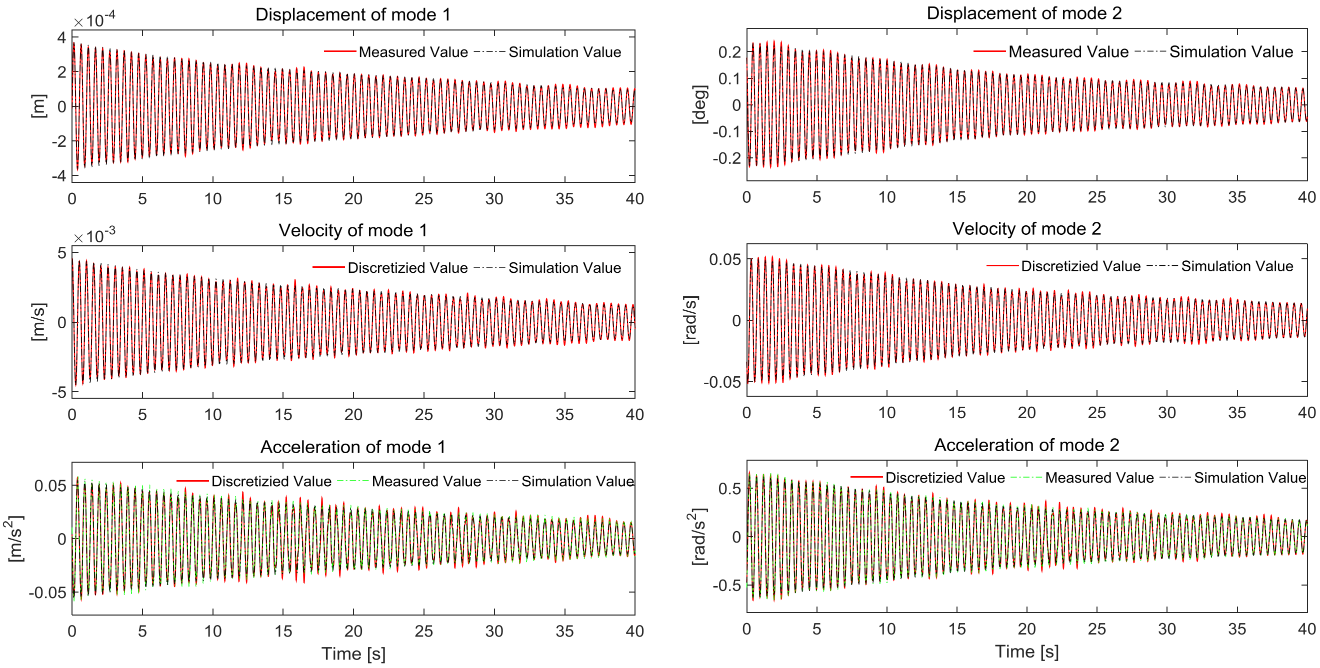
closed model). The target modal damping ratio was approximately 0.3% and 0.265% for each mode in the SMT model (SMT 1-3) and 0.26% and 0.27% for each mode in the CMT model.

A laser transducer device was used to measure the displacement and an axial force transducer was installed to measure acceleration for comparison with simulated acceleration for each DOF as shown in Fig. 9. Because mass of section model can be calculated, acceleration is obtained by axial force transducer. The free vibration tests were carried out under temporal forced vibration conditions and imposed initial conditions. Ambient vibration tests in section model test were conducted under turbulent flow conditions with turbulence intensity under 10%.



(a) Vertical DOF (b) Rotational DOF

Fig. 12 Time history displacement data using the ILS method under 40 s and 120 *Hz* in SMT 3 for all modes (simulation values were compared with measured values)



(a) Vertical DOF (b) Rotational DOF

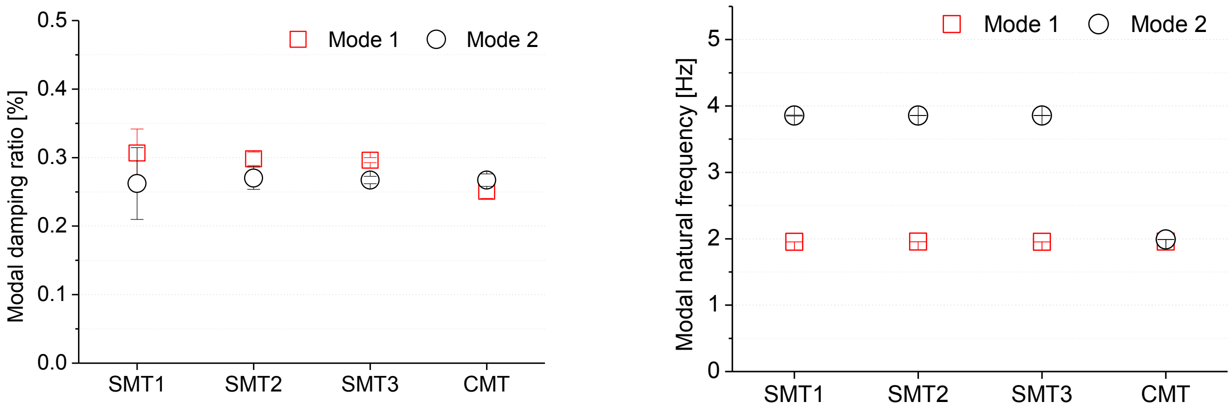
Fig. 13 Time history displacement data using ILS method under 40 s and 120 *Hz* in CMT for all modes (displacement at the top, velocity in the middle, acceleration at the bottom; simulation values were compared with measured values)

In the ILS method, the process to determine the optimal modal damping is used to choice the free initial condition that has the minimum sum of residuals compared with recorded data, for various initial times. For example, if test data is acquired totally for 60 s, modal damping is determined by the solution of optimal signal among several cases that have total time length 40 s and the total number of those cases which is equal to 120 (sampling rate) times the difference between 40 s (total time length) and 60 s (totally recorded time). In this manner, optimal signal was selected through the process that finds the signal having lowest residual among several cases (120×20). The SSI method was used to evaluate modal parameters for turbulent flows. Fig. 10 presents comparisons of both measured and

Table 8 Mean values of section model tests

Model	Type of free decay	Mode1		Mode2	
		f (*Hz*)	ξ (%)	*f* (*Hz*)	ξ (%)
SMT1	Ambient vibration	1.9551	0.2982	3.8566	0.2701
SMT2	Free vibration	1.9539	0.3067	3.8548	0.2622
SMT3	Free vibration (temporal forced vibration)	1.9536	0.2962	3.8543	0.2672
CMT	Free vibration	1.9581	0.2507	1.9880	0.2672

*SMT: Separated model test for frequency; CMT: Closed model test for frequency



(a) Modal damping ratios (b) Modal natural frequencies

Fig. 14 Mean and standard deviations of damping ratio and frequency under 40 s and 120 *Hz* for all models in section model tests

simulated values, estimating the damping ratio 0.3067% and 0.2622% and natural frequency 1.9539 *Hz* and 3.8548 *Hz*, respectively, taken from the average of the five-time measurements. It was found that the free initial condition can be used without any original initial value. The simulated signals for all DOF accurately matched the measured values as shown in Fig. 10.

The temporal forced vibration test (free vibration test) was conducted by applying sinusoidal resonant loads with estimated frequencies of 1.953 *Hz* and 3.516 *Hz* for modes 1 and 2, respectively, as shown in Fig. 11. In addition, Fig. 12 presents time history data of temporal forced vibration for 2DOF. The simulated values presented on the right in Fig. 12 delivered good results for time domain data which can be implemented when modal frequency and modal damping were accurately evaluated for all modes.
Displacement data was used for the ILS method under 50% of maximum displacement after loading.

Fig. 13 presents comparisons of both measured and simulated values, with estimated damping ratios 0.2507% and 0.2672%, and natural frequencies 1.9581 *Hz* and 1.9880 *Hz*, respectively, taken from the average of five-time measurements. The closed frequency case is less accurate than the well-separated model in same conditions according to the results of numerical analysis described in Fig. 6. For this reason, the simulated acceleration was compared with measured acceleration recorded with an axial force transducer. The simulated acceleration was more stable for outlier noise compared to discretized acceleration, and had more similarities with measured acceleration. In general, truncation errors and environmental noise accumulated during the discretization process. Nonetheless, displacement-based problems can provide reliable velocities and acceleration by adopting the free initial condition and the ILS method and low pass filtering methods.

The estimated modal damping and natural frequency are summarized for the four experimental models in Table 8. SMT1-3 had the same experimental settings as those used for different vibration tests and provided similar mean values for the two modes. The ambient vibration test showed higher variations than free vibration for modal damping estimation as shown in Fig. 14. In particular, the temporal forced vibration results had less variation since these were similar to the deterministic problem (noise was relatively smaller than the amplitude of displacement). The SSI method of ambient vibration test (SMT1) was reliable for noise loading similar to turbulent flow conditions. The CMT with closed frequency also provided good quality results. Moreover, natural frequency was accurately predicted for all experimental models regardless of the identification method.

Table 9 Mean value of full bridge 3D model tests for multiple modes

Direction of motion	Order of mode	Prototype design		Measurement (ILS)	
		$f\,(Hz)$	ξ (%)	$f\,(Hz)$	ξ (%)
Lateral (L_1)	1	1.9410	0.3	1.9677	0.3339
Vertical (V_1)	1	2.8814	0.3	2.8821	0.3253
Vertical (V_2)	2	5.0374	0.3	4.7124	0.4323
Torsional (θ_1)	1	5.2926	0.3	5.4754	0.3020

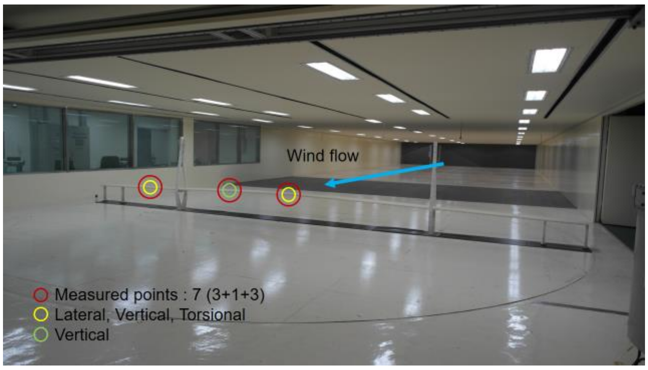


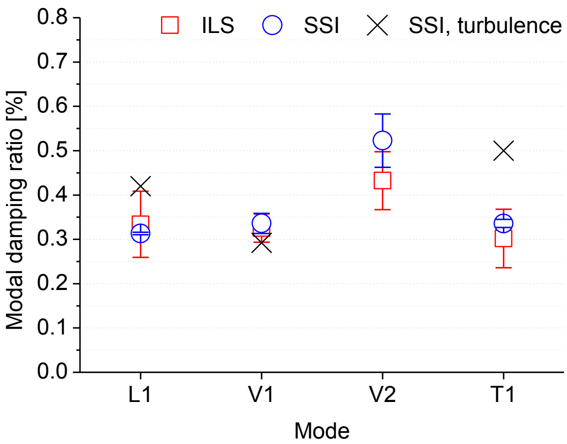
Fig. 15 General view of full bridge 3D model

7.2 Full bridge 3D model test

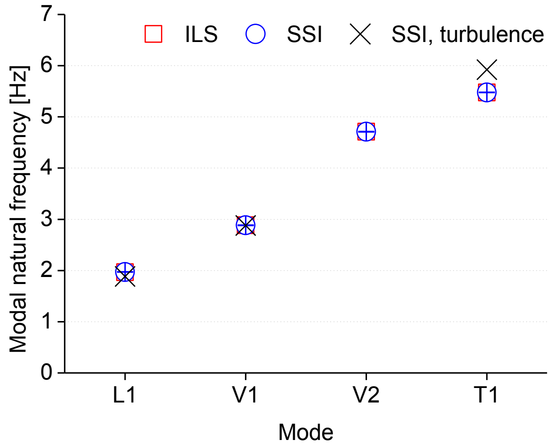
A full bridge 3D model of a composite cable-stayed bridge with a main span of 700 m described in Fig. 15 was designed with Korea University and Pyunghwa engineering consultants associated with the super-long span bridge of the R&D center of Korea Expressway Corporation in Korea. As the main span of the full bridge 3D model becomes longer, a similitude law must be applied for testing in scaled experimental tests. This test model had scale ratio of 120 for a damping ratio of 0.3% regardless of the scale ratio (super long-span bridge is conservatively designed as 0.3%). In the full bridge 3D model test, the free vibration tests imposed by the initial condition were analyzed using the ILS and SSI methods, and the ambient vibration tests were conducted using the SSI method at 1.38 m/s (15.1 m/s in field environment), which generated turbulent flow that represented a turbulence intensity of approximately 15% at the measured location.

Table 9 synthesizes the design values of a prototype for representative modes of the lateral, vertical, and torsional directions. When comparing the difference between designed and measured values, the maximum error was 3.45% for the natural frequency of the vertical mode and 11.3% for the damping ratios of the lateral mode, except for the second mode. Thus, the full bridge 3D model was well constructed since the damping ratio of each representative mode was approximately 0.3% and the natural frequency of each mode provided proper results. The full bridge 3D model tests were measured with 60 s and 200 *Hz*, described in Figs. 16 and 17. When considering proposed analysis condition, sampling frequency should be 200 *Hz* since Nyquist (35) is applied to the highest mode. In addition, optimal total time length was determined as 60 s under Nyquist (35). The estimation of modal damping and natural frequency was summarized for the full bridge 3D model in Table 9 and Fig. 16. The full bridge 3D model that consists of multiple modes faces challenges in estimating the dynamic properties of systems and computes more variations in results compared to the 2DOF section model test with regards to modal damping ratios. The maximum standard deviation of modal damping ratio was around 0.075 for the lateral mode. Therefore, the method presented in this paper properly identified modal damping ratios and natural frequencies. The free vibration test was separately conducted for each mode. The SSI method was used to estimate modal parameters with multiple output under turbulent flow, simultaneously measuring the displacement at target points (the number of measured points was seven). The result of the SSI method under turbulent flow was evaluated by singular value decomposition and stabilization diagram as shown in Fig. 17. The second mode was not captured by the SSI method under turbulent flow. The first mode of each representative direction was similar to damping ratio and natural frequency in free vibration test, whereas damping ratio had rather difference under SSI methods with turbulent flow.

Fig. 18 presents the motion of the superposed mode in the vertical direction composed of V_1 and V_2 as described in table 9. It is found that the superposed time series data is well constructed and FIVP is properly applied to multiple modes.



(a) Modal damping ratios



(b) Modal natural frequencies

Fig. 16 Mean and standard deviation of modal damping ratio and frequency under 60 s and 200 *Hz* for all modes, using the ILS and SSI method in full bridge 3D model tests

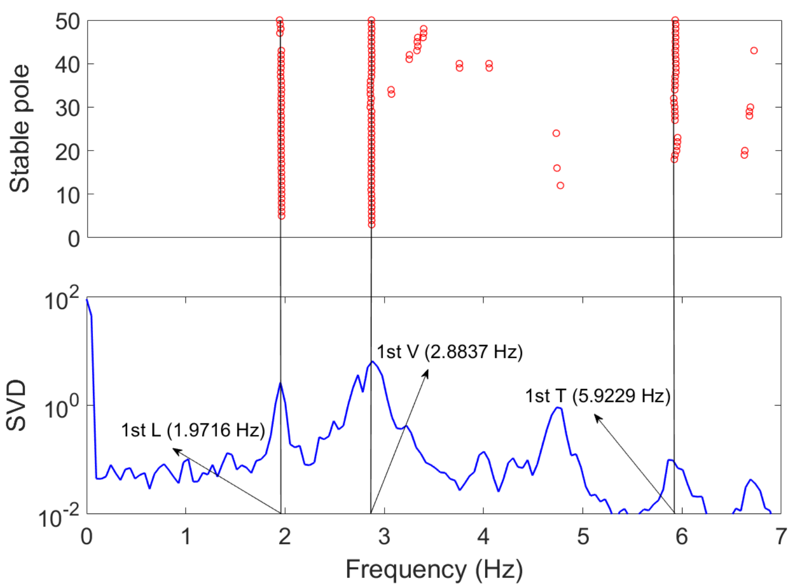


Fig. 17 SVD and stabilization diagram obtained with the SSI method in full bridge 3D model tests

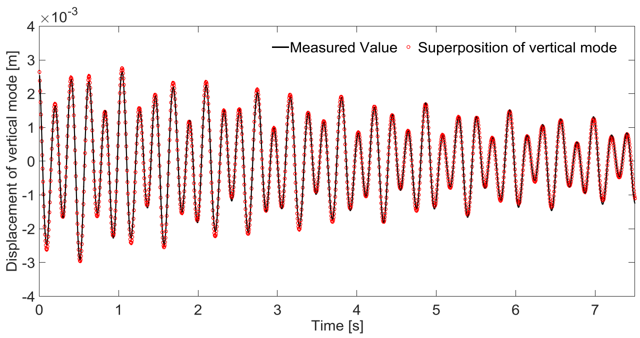


Fig. 18 Time history data of superposed vertical mode using the ILS method and FIVP

8. Conclusions

This paper proposed minimum analysis conditions that can be applied to small scale bridge model. It was derived from the comparison between the sampling frequency considering similitude law and the sampling frequency extracted by the ILS method. Minimum sampling rate was defined considering Nyquist (20) and Nyquist (35) for high damping and low damping model, respectively. Generally, Nyquist (35) is recommended for stable application. In addition, total time length should be determined after stabilization process adopting Nyquist (35).

The second original contribution of present work is proposal and application of FIVP to free vibration test. FIVP was extensively used to evaluate modal damping ratio. FIVP could be helpful in the environments that had difficulty in measuring original initial value under free vibration. FIVP was applied to both the ILS and SSI method and computed reasonable damping ratio and natural frequency.

Finally, extensive validation of proposed minimum analysis conditions was conducted in section model tests and full bridge 3D model tests. Experimental tests consisting of free and ambient vibration tests yielded reasonable results using the ILS and SSI methods. In a full bridge 3D model, the accuracy of damping ratio was decreased in the high mode, while natural frequency was accurate in all modes. The displacement-based ILS method computed reasonable acceleration data under noisy conditions. In this manner, the ILS method is suggested to get acceleration data than discretization technique in displacement-based condition.

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