Smart Structures and Systems, *Vol. 21, No. 4 (2018) 449-467*
DOI: https://doi.org/10.12989/sss.2018.21.4.449

Simultaneous identification of damage in bridge under moving mass by Adjoint variable method

Akbar Mirzaee*, Reza Abbasnia[a] and Mohsenali Shayanfar[b]

Department of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran, Islamic Republic of Iran

(Received June 9, 2017, Revised March 10, 2018, Accepted March 13, 2018)

Abstract. In this paper, a theoretical and numerical study on bridge simultaneous damage detection procedure for identifying both the system parameters and input excitation mass, are presented. This method is called 'Adjoint Variable Method' which is an iterative gradient-based model updating method based on the dynamic response sensitivity. The main advantage of proposed method is inclusion of an analytical method to augment the accuracy and speed of the solution. Moving mass is a model which takes into account the inertia effects of the vehicle. This interaction model is a time varying system and proposed method is capable of detecting damage in this variable system. Robustness of proposed method is illustrated by correctly detection of the location and extension of predetermined single, multiple and random damages in all ranges of speed and mass ratio of moving vehicle. A comparison study of common sensitivity and proposed method confirms its efficiency and performance improvement in sensitivity-based damage detection methods. Various sources of errors including the effects of measurement noise and initial assumption error in stability of method are also discussed.

Keywords: damage detection; finite element model updating; sensitivity; Ill posed problem; simultaneous identification; Adjoint variable method

1. Introduction

Structural Health Monitoring (SHM) is necessary for various aerospace, mechanical, and civil engineering applications to evaluate the status of the structure for performing its authorized tasks. Bridges have limited lifetime and start to degrade as soon as they are under service. Processes such as corrosion, fatigue, erosion, abrasion and overloads degrade them until they aren't fit for their intended use.

Damage detection is the most important application of SHM. In general, all existing methods can be divided into two groups: local and global approaches. Local monitoring methods locate and identify small defects in accessible and small inspection zones via ultrasonic testing (Staszewski 2003, Ostachowicz *et al.* 2009) or statistical classification techniques (Silva *et al*. 2008, Nair *et al*. 2006). These methods do not require structural modeling and are outside the scope of this paper.

Vibration characteristics of the bridge are related to the occurring of damage in the structural elements. This theory is based on the fact that changes in the stiffness and mass properties of the bridges can result in the changes of dynamic characteristics of bridges.

The developments in the field of damage detection by using the vibration data of civil engineering structures have been recently reviewed by several authors.

Doebling *et al.* (1998) provided a comprehensive review on the damage detection methods by examining changes in the dynamic properties of a structure. Zou *et al.* (2000) summarized the methods on vibration-based damage detection and health monitoring for composite structures especially in delamination modeling techniques and delamination detection.

Damage detection usually requires a mathematical model on the structure in conjunction with experimental parameters of the structure. The identification approaches are mainly based on the change in the natural frequencies (Cawley and Adams 1979, Friswell *et al.* 1994, Narkis 1994), mode shapes (Pandey *et al.*1991, Ratcliffe 1997, Rizos *et al*.1990) or measured modal flexibility (Pandey and Biswas 1994, Doebling *et al.* 1996, Lim 1991, Wu and Law 2004).

The performance of damage detection procedures based on vibration data depends on that the in-built analytical procedures to be accurate enough to discern even small changes in responses due to damages in the structure. That is mean the analytical models employed here need to be more accurate than when are used in a routine response analysis. It should be noted that the bridge–vehicle system is a time-varying system. Consequently, frequency domain parameters of bridge structure such as natural frequencies, mode shapes, modal damping and frequency response function can't use to detect damages in such systems directly. Of course, if the vehicle-bridge interaction will be

*Corresponding author, Ph.D.
E-mail: mirzaee_akbar@yahoo.com
[a] Associate Professor
E-mail: abbasnia@iust.ac.ir
[b] Associate Professor
E-mail: shayanfar@iust.ac.ir

Copyright © 2018 Techno-Press, Ltd.
http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

ignored and treats the moving vehicle as a moving load, the system will be time invariant in nature and frequency domain parameters could be used for damage detection purposes. In this case, the errors due to ignoring the vehicle -bridge interaction would also introduce unknown errors into damage detection procedures. In fact, it is not clear on how the frequency domain parameters of bridge structure could be extracted from vibration which is induced by vehicular traffic if one includes vehicle–bridge interaction effects into the analysis. The focus of the present study is developing a time-domain approach to detect damages in bridge structure by analyzing the combined system of bridge and vehicle.

For moving masses, the analysis is most often performed in the time domain via a direct comparison of the simulated responses with measured responses.

Majumder and Manohar (2003, 2004) propose a method for damage identification of linear and non-linear beams excited by a moving oscillator; the beam and the oscillator are treated together as a single coupled and time-varying system. Sieniawska *et al.* (2009) use a static substitute of the equation of motion for identifying the structural linear parameters from the dynamic response of the structure under a moving load with a determined constant magnitude.

Identification of moving loads is important not only for assessment of pavements or bridges but also in traffic studies, in design code calibration, for traffic control, etc. Several techniques have been developed which solve one of these identification problems separately: either they identify the damage while assuming load characteristics to be known or they identify the moving load while the structure is assumed to be undamaged.

Identification of moving loads has been studied extensively in the past two decades (Yu and Chan 2007). A direct measurement of the dynamic axle loads of vehicles is expensive, difficult and subject to bias. Therefore the techniques of indirect identification from measured responses have been studied, as they can be performed easier and at lower costs. Chan, Law *et al*. have proposed four general methods for indirect identification which are the time-domain method (TDM) (Law *et al*. 1997), the frequency-time domain method (FTDM) (Law et al. 1999), Interpretive Method I (IMI) (Chan and O'Connor 1990) and Interpretive Method II (IMII) (Chan *et al.* 1999). All of them require the parameters of the bridge model to be known in advance. Each method has its advantages and limitations which are compared in Chan *et al.* (2001). The numerical ill-conditioning of the problem seems to be the main factor that decreases the accuracy of the identification results. To improve the accuracy, techniques based on the Singular Value Decomposition (SVD) have been investigated and adopted for the inverse computation (Yu and Chan 2003). Some regularization methods of SVD have been used e.g., Law *et al.* (2001), Zhu and Law (2006) use the Tikhonov regularization, while Law and Fang (2001), González *et al.* (2008) couple it with the dynamic programming approach. In general, all these methods require a model of the structure in order to build the equation of the motion, even if some of them allow to be used for identification of parameters besides the moving load such as the prestressing force (Law *et al*. 2008) or parameters of the vehicle model (Jiang *et al*. 2004).

In real applications unknown damages and unknown moving loads can be coexisted and influence the response of the system together; it seems that their simultaneous identification is a relatively unexplored field. In the case of unknown excitations and unknown structural damages, the related identification problems are inherently coupled so both factors together influence the structural response and cannot be identified separately. Hoshiya and Maruyama (1987) apply a weighted global iteration procedure and the extended Kalman filter for simultaneous identification of a moving load and modal parameters of a simply supported beam. Lu and Law (2007) identify damage and parameters of non-moving impulsive or sinusoidal force excitations in a two-step identification process using a limited number of measurements. Zhang *et al.* (2009) present a method for simultaneous identification of structural physical parameters and an unknown support excitation. Zhu and Law (2007) propose a method for simultaneous identification of moving loads and damages using a two-step approach that separately adjusts the loads and the damage factors in each iteration of the optimization process; the number of sensors is one less than the number of the beam elements.

Zhang *et al.* (2010) addressed simultaneous identification of damages and nonmoving excitation forces in truss structures; a moving force could be identified only by a simultaneous identification of all load-time histories in all involved degrees of freedom (DOFs).

In this paper, a novel simultaneous sensitivity-based damage detection method referred to as "Adjoint variable method", is developed. The sensitivities of dynamic response with respect to the structural physical parameters and the input excitation mass are calculated simultaneously in this study. Perturbations in the structural parameters are identified together with the input excitation masses using an iterative algorithm.

The outline of the work is as follows: Inverse problems along with model updating are briefly introduced in section2. The basic theory of sensitivity analysis is addressed in section3 and the proposed algorithm will be presented in section4. Numerical simulations are presented in section 5 with studies on the effect of different factors which may affect the accuracy of the proposed analysis in practice. Conclusion will be drawn in the last section.

2. Finite element model updating and inverse problem

Since many algorithms of damage detection are based on the difference between modified model before occurrence of damage and after that, problems such as parameter identification and damage detection are closely related to model updating. Discrepancy between two models is used for detecting and quantifying of damage in the structure.

A key step in model-based damage identification is the updating of the finite element model of the structure in such a way that the measured responses can be reproduced by the FE-model. A general flowchart of this operation is given in Fig. 1. The identification procedure presented in this paper is a sensitivity based model updating routine. Sensitivity coefficients are the derivatives of the system responses with respect to the physical parameters or input excitation mass, and are needed in the cost function of the flowchart of Fig. 1.

2.1 Finite element modeling of bridge vibration under moving mass

The moving load model is the simplest model that can be supposed that has been frequently adopted by researchers in studying the vehicle-bridge interaction. With this model, the fundamental dynamic specifications of the bridge caused by the moving action of the vehicle can be captured with an adequate degree of accuracy.

However, the effect of interaction between the bridge and the moving vehicle was just ignored. For this reason, the moving load model is good only for the case where the mass of the vehicle is small relative to that of the bridge and only when the vehicle response is not of interest.

For cases where the inertia of the vehicle cannot be ignored, a moving mass model should be adopted instead. For a general finite element model of a linear elastic time-invariant structure, the equation of motion is given by

$$[\mathrm{M}]\{z_{,tt}\} + [\mathrm{C}]\{z_{,t}\} + [\mathrm{K}]\{z\} = [\mathrm{B}]\{\mathrm{F}\} \quad (1)$$

Where $[\mathrm{M}] = [\mathrm{M_b}] + [\mathrm{M_v}]$ is the total mass matrix of the system in which $[\mathrm{M_b}]$ and $[\mathrm{M_v}]$ are mass matrix of bridge and vehicle respectively and $[\mathrm{M_v}] = [\mathrm{BB}]\{\mathrm{M}\}$ where {M} is a vector of applied masses with matrix [BB] mapping these masses to the associated DOF of the structure. [K] and [C] are stiffness and damping matrices. $z_{,tt}$ and $z_{,t}$ and z are acceleration, velocity and displacement vectors respectively for the whole structure and the force vector can be defined as {F}={M}g which is mapping by matrix [B] to the associated DOF of the structure. Rayleigh damping in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used.

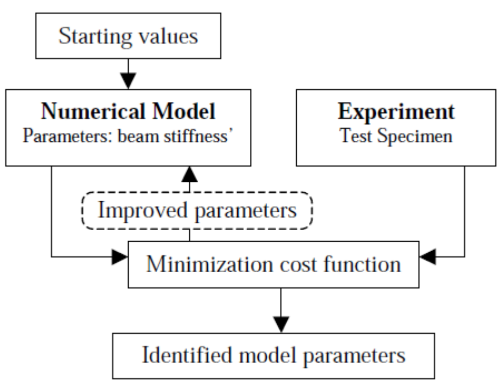


Fig. 1 General flowchart of a FEM-updating

$$[\mathrm{C}] = a_0[\mathrm{M_b}] + a_1[\mathrm{K}] \quad (2)$$

Where a_0 and a_1 are constant to be determined from two modal damping ratios. If a more accurate estimation of the actual damping is required, a more general form of Rayleigh damping, the Caughey damping model can be adopted.

As equation 1 shows, moving masses in a bridge-vehicle system not only excite the supporting structure via their gravities but also modify its inertial properties so the differential equation of motion is time-varying. In order to solve this equation, the numerical integration of the equation of motion is repeated as well as updating the mass matrix in each time step.

2.2 Objective functions

The approach minimizes the difference between response quantities (acceleration response) of the measured data and model predictions. This problem may be expressed as the minimization of J, where

$$\begin{gathered} J(\theta) = \|z_m - z(\alpha)\|^2 = \epsilon^T\epsilon \\ \epsilon = z_m - z(\alpha) \end{gathered} \quad (3)$$

Here z_m and $z(\alpha)$ are the measured and computed response vectors, α is a vector of all unknown parameters, and ϵ is the response residual vector.

2.3 Nonlinear model updating for damage detection

When the parameters of a model are unknown, they must be estimated using measured data. Since the relationship between the acceleration responses $z_{,tt}$ and the fractional stiffness parameter α is nonlinear, a nonlinear model updating technique like the Gauss-Newton method is required. This kind of method has the advantage that the second derivatives which can be challenging to compute are not required. The Gauss-Newton method in the damage detection procedure can be described in terms of the acceleration response at the i^{th} DOF of the structure as follows

$$z_{,tt-dl}(\alpha_d) = z_{,tt-ul}(\alpha^0) + S(\alpha^0) \times \Delta\alpha^1 + S(\alpha^0 + \Delta\alpha^1) \times \Delta\alpha^2 + \cdots \quad (4)$$

The superscript 0, 1, 2 denote the iteration numbers. Index u denotes the initial state while index d denotes the final damage state. $z_{,tt-dl}$ and $z_{,tt-ul}$ are vectors of the acceleration response at the i^{th} DOF of the damaged and intact states, respectively. The damage identification equation for $(k+1)^{th}$ iteration is

$$\Delta z_{,tt}{}^k = S^k \times \Delta\alpha^{k+1} \quad (5)$$

Where S^k and $\Delta z_{,tt}{}^k$ are obtained from the k^{th} iteration. The iteration in Eq. (5) starts with an initial value α^0 leading to $\Delta z_{,tt}^0 = z_{,tt-dl} - z_{,tt-ul}(\alpha^0)$ and $S^0 = S(\alpha^0)$. The parameter vector is updated as $\alpha^k = \alpha^0 + \sum_{i=1}^{k} \Delta\alpha^i$. Sensitivity matrix $S^k = S(\alpha^k)$ and the residual vector $\Delta z_{,tt}{}^k = z_{,tt-dl} - z_{,tt-ul}(\alpha^0) - \sum_{i=0}^{k-1} S^i \Delta\alpha^{i+1}$, $(k = 1,2,\ldots)$ of the next iteration are then computed from results in the previous iterations.

The acceleration response vector $z_{,tt-ul}$ from the physical intact structure is computed from the associated analytical model via dynamic analysis. $z_{,tt-dl}$ is the acceleration response of the damaged structure model. In general, the measured acceleration responses (including measurement errors) from the damaged structure are obtained as $z_{,tt-dl}$.

The iteration is terminated when a pre-selected convergence condition is met. The final identified damaged vector becomes (Ratcliffe 1997)

$$\Delta\alpha = \Delta\alpha^1 + \Delta\alpha^2 + \cdots + \Delta\alpha^n \tag{6}$$

2.4 Regularization

Like many other inverse problems, Eq. (5) is ill-conditioned and regularization techniques are needed to provide bounds to the solution. The aim of regularization in the inverse analysis is to promote certain regions of parameter space where the model realization must be existed. The two most widely used regularization methods are Tikhonov regularization (Friswell and Penny 1994) and truncated singular value decomposition (Friswell and Mottershead 1995, Ricles and Kosmatka 1992). In the Tikhonov regularization, the new cost function is defined as

$$J(\Delta\alpha^{k+1}, \lambda) = \left\|S^k . \Delta\alpha^{k+1} - \Delta z^k_{,tt}\right\|_2^2 + \lambda^2 \left\|\Delta\alpha^{k+1}\right\|_2^2 \tag{7}$$

The regularization parameter $\lambda \geq 0$ controls the extent of two errors contribution in the cost function of Eq. (7) and the fractional change stiffness $\Delta\alpha^{k+1}$ is obtained by minimizing the cost function in Eq. (7)

The regularized solution can be written in the following form by minimizing the function in Eq. (7)

$$:\Delta\alpha^{k+1} = (\left(S^k\right)^T S^k + \lambda^2 I)^{-1} (S^k)^T \Delta z^k_{,tt} \tag{8}$$

To express the contribution of the singular values, the corresponding vectors in the solution clearly and the role which regularization parameter plays as a filter factor, the sensitivity matrix is decomposed on the other handthe singular value decomposition (SVD) applies to the sensitivity matrix S^k to obtain

$$S^k = U\Sigma V^T \tag{9}$$

Where $U \in R^{nt\times nt}$ and $V \in R^{m\times m}$ are orthogonal matrices satisfying $U^T U = I_{nt}$ and$V^T V = I_m$, and matrix $\sum$ has the size of $nt \times m$ with the singular values $\sigma_i (i = 1, 2, \dots, m)$ on the diagonal arranged in a decreasing order such that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$ and zeros elsewhere.

The regularized solution in Eq. (8) can be written as

$$\Delta\alpha^{k+1} = \sum\nolimits_{i=1}^{m} f_i \frac{U_i^T \Delta z^k_{,tt}}{\sigma_i} V_i \tag{10}$$

Where $f_i = \sigma_i^2/(\sigma_i^2 + \lambda^2)$ $(i = 1, 2, \dots, m)$ are defined as filter factors. So, the solution norm $\left\|\Delta\alpha^{k+1}\right\|_2^2$ and the residual norm $\left\|S^k . \Delta\alpha^{k+1} - \Delta z^k_{,tt}\right\|_2^2$ can be expressed as follows

$$\eta^2 = \left\|\Delta\alpha^{k+1}\right\|_2^2 = \sum_{i=1}^{m} (\frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \frac{U_i^T \Delta z^k_{,tt}}{\sigma_i})^2 \tag{11}$$

$$\rho^2 = \left\|S^k . \Delta\alpha^{k+1} - \Delta z^k_{,tt}\right\|_2^2 = \sum_{i=1}^{m} (\frac{\lambda^2}{\sigma_i^2 + \lambda^2} U_i^T \Delta z^k_{,tt})^2 \tag{12}$$

These two equations should be balanced by choosing an appropriate regularization parameter.

2.5 Element damage index

In damage detection inverse problem, it is assumed that the stiffness matrix of whole elements decrease uniformly by damages, and the flexural rigidity, EI_i of i^{th} beam element becomes $\beta_i EI_i$ when there is damage. The fractional change in stiffness of an element can be expressed as follows (Zhu and Hao 2007).

$$\Delta K_{bi} = \left(K_{bi} - \widetilde{K}_{bi}\right) = (1 - \beta_i) K_{bi} \tag{13}$$

Where K_{bi} and $\widetilde{K}_{bi}$ are the i^{th} element stiffness matrices in undamaged and damaged state of the beam, respectively. ΔK_{bi} is the stiffness reduction of the i^{th} element. A positive value of $\beta_i \in [0,1]$ will indicate reduction in the element stiffness. When $\beta_i = 1$ the i^{th} element is undamaged and when $\beta_i = 0$ the i^{th} element is completely damaged.

The stiffness matrix of the damaged structure is obtained by assembling the entire element stiffness matrix $\widetilde{K}_{bi}$ of the structure.

$$K_b = \sum_{i=1}^{N} A_i^T \widetilde{K}_{bi} A_i = \sum_{i=1}^{N} \beta_i A_i^T K_{bi} A_i \tag{14}$$

Where A_i is the extended matrix of element nodal displacement which facilitates forming the global stiffness matrix by assembling the local constituent element stiffness matrix.

2.6 Input mass identification

The sensitivity-based analysis method without considering the second and higher order effects is adopted in this study. Based on Gauss-Newton method will have the follow equation

$$\{\delta z_{,tt}\} = [S_m]\{\delta M\} \tag{15}$$

The physical parameters of the intact structure are used as an approximation to calculate the sensitivity matrix $[S_m]$ as we are not certain about the true state of the damage structure. $[S_m]$ is the sensitivity matrix which is the change of acceleration response with respect to the change of mass in time domain such that$\{\delta M\}$ is the vector of perturbation in mass. The Eq. (15) can be solved by Tikhonov method as follows

$$\delta M = ((S_M)^T S_M + \lambda^2 I)^{-1} (S_M)^T \delta z_{,tt} \tag{16}$$

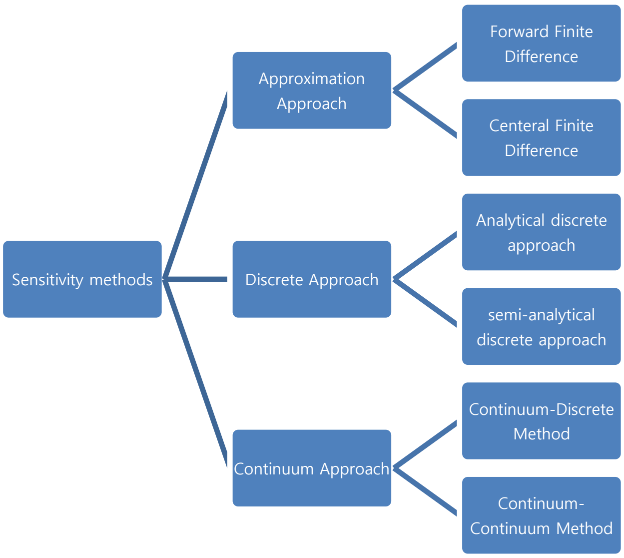


Fig. 2 Different Approaches for sensitivity analysis

2.7 Damage identification

The forces which is obtained by previous sections can use for local damage detection. So by using Gauss-Newton method will have follow equation

$$\{\delta z_{,tt}\} = [S_S]\{\delta\alpha\} \tag{17}$$

$[S_S]$ is the sensitivity matrix which is the change of acceleration response with respect to the physical parameter in time domain, $\{\delta\alpha\}$ is the vector of physical parameter perturbation. The physical parameter can also be obtained by using the Tikhonov regularization method as follows

$$\delta\alpha = ((S_S)^T S_S + \lambda^2 I)^{-1}(S_S)^T \delta z_{,tt} \tag{18}$$

3. Sensitivity analysis of transient dynamic response

The purpose of sensitivity analysis is to quantify the effects of parameter variations on calculated results. Terms such as influence, importance, ranking by importance, and dominance are all related to sensitivity analysis.

The most important difficulty in sensitivity base damage detection methods is calculation of sensitivity matrix. Calculation of this massive matrix is repeated in each iteration and according to its dimensions, is so time-consuming so has a significant effect on the efficiency of damage detection method.

3.1 Methods of structural sensitivity analysis

When the parameter variations are small, the traditional way can be used to assess their effects on calculated responses by using the perturbation theory, either directly or indirectly, via variational principles. The basic aim of perturbation theory is to predict the effects of small parameter variations without actually calculating the perturbed configuration but rather by using solely unperturbed quantities.

Various methods employed in sensitivity analysis are listed in Fig. 2. In general, three approaches are used to obtain the sensitivity matrix: the approximation, discrete, and continuum approaches.

In the approximation approach, sensitivity matrix is obtained by either the forward finite difference or the central finite difference method.

In the discrete method, sensitivity matrix is obtained by design parameter derivatives of the discrete governing equation. For this process, it is necessary to take the derivative of the stiffness matrix. If this derivative is obtained analytically using the explicit expression of the stiffness matrix with respect to the variable, it is an analytical method, since the analytical expressions of stiffness matrix are used. However, if the derivative is obtained using a finite difference method, the method is called a semi analytical method. The design parameter represents a structural parameter that can affect the results of the analysis.

The design parameter sensitivity information can be computed either with the direct differentiation method or with the adjoint variable method.

In the continuum approach, the design parameter derivative of the variational equation is taken before it is discretized. If the structural problem and sensitivity equations are solved as a continuum problem, then it is called the continuum-continuum method. The continuum sensitivity equation is solved by discretization in the same way that structural problems are solved. Since differentiation is taken at the continuum domain and is then followed by discretization, this method is called the continuum-discrete method.

Major disadvantage of the finite difference method is the accuracy of its sensitivity results. Depending on perturbation size, sensitivity results are quite different. The continuum-continuum approach is so limited and is not applicable for complex engineering structures because the simple and classical problems can be just solved analytically. The discrete and continuum-discrete methods are equivalent in frame elements. (Choi and Kim 2005). In this paper two different analytical discrete methods, including Direct Differential Method (DDM) and Adjoint Variable Method (AVM) are presented and efficiency of proposed method is investigated with compared with DDM method.

3.2 Direct dfferentiation method

The direct differentiation method (DDM) is a general, accurate and efficient method to compute Finite element response sensitivities to the model parameters. This method directly solves for the design dependency of a state variable, and then computes performance sensitivity using the chain rule of differentiation. This method clearly shows the implicit dependence on the design parameter so a very simple sensitivity expression can be obtained. Consider a structure in which the generalized stiffness and mass matrices have been reduced by considering the boundary conditions. By applying the differentiation to the both sides of Eq. (1) with respect to the ith excitation mass, M_i, the equation will be changed as follows

$$[M]\left\{\frac{\partial z_{,tt}}{\partial M_i}\right\}+[C]\left\{\frac{\partial z_{,t}}{\partial M_i}\right\}+[K]\left\{\frac{\partial z}{\partial M_i}\right\}=g[B]-[BB]\{z_{,tt}\} \quad (19)$$

By applying the differentiation to the both sides of Eq. (1) again with respect to the jth physical parameter, α_j, of the jth element, and assuming Rayleigh damping formula in calculation, the equation will be changed as follows

$$[M]\left\{\frac{\partial z_{,tt}}{\partial \alpha_j}\right\}+[C]\left\{\frac{\partial z_{,t}}{\partial \alpha_j}\right\}+[K]\left\{\frac{\partial z}{\partial \alpha_j}\right\}=-\frac{\partial [K]}{\partial \alpha_j}\{z\}-a_1\frac{\partial [K]}{\partial \alpha_j}\{z_{,t}\} \quad (20)$$

Where a_1 is the coefficient for Rayleigh damping. Note that Eqs. (19) and (20) have the same form as Eq. (1). The response sensitivities can also be obtained by Newmark method. The initial values of the dynamic responses and the sensitivities can be taken equal to zero.

3.3 Adjoint variable method

Sensitivity analysis can be performed very efficiently by usingdeterministic methods based on adjoint functions. The use of adjoint functionsfor analyzing the effects of small perturbations in a linear system was introducedby Wigner (1945).

This method, constructs an adjoint problem that solves for the adjoint variable, which contains all implicitly dependent terms.

For the dynamic response of structure, the following form of a general performance measure will be considered

$$\psi = g(z(T),b)+\int_0^T G(z,b)dt \quad (21)$$

where the final time T is determined by a condition in the form

$$\Omega\big(z(T),z_{,t}(T),b\big)=0 \quad (22)$$

It is presumed that Eq. (22), uniquely determines T, at least locally. This requires that the time derivative of Ω is nonzero at T, as

$$\Omega_{,t}=\frac{\partial\Omega}{\partial z}z_{,t}(T)+\frac{\partial\Omega}{\partial z}z_{,tt}(T)\neq 0 \quad (23)$$

When final time T is prescribed before the response analysis, the relation in Eq.(22), need not be considered.

To obtain the design sensitivity of ψ, define a design variation in the form

$$b_\tau = b+\tau\delta b \quad (24)$$

Design **b** is perturbed in the direction of **δb** with the parameter **τ**. Substituting $\mathbf{b_\tau}$ into Eq. (21), the derivative of Eq. (21), can be evaluated with respect to **τ** at **τ = 0**. Leibnitz's rule of differentiation of an integral may be used to obtain the following expression

$$\begin{aligned}\psi' = \frac{\partial g}{\partial b}\delta b+\frac{\partial g}{\partial z}\left[z'(T)+z_{,t}(T)T'\right]+G(z(T),b)T' \\ +\int_0^T\left[\frac{\partial G}{\partial z}z'+\frac{\partial G}{\partial b}\delta b\right]dt\end{aligned} \quad (25)$$

Where

$$z' = z'(b,\delta b)\equiv\frac{d}{d\tau}z(t,b+\tau\delta b)|_{\tau=0}=\frac{d}{db}[z(t,b)]\delta b$$

$$T' = T'(b,\delta b)\equiv\frac{d}{d\tau}T(b+\tau\delta b)\Big|_{\tau=0}=\frac{dT}{db}\delta b$$

Note that since the expression in Eq. (21), that determines **T** depends on the design, **T** will also depend on the design. Thus, terms arise in Eq. (25), that involve the derivative of **T** with respect to the design. In order to eliminate these terms, differentiate Eq. (22), with respect to τ and evaluate it at τ = 0 in order to obtain:

$$\frac{\partial\Omega}{\partial z}\left[z'(T)+z_{,t}(T)T'\right]+\frac{\partial\Omega}{\partial z_{,t}}\left[z'_{,t}(T)+z_{,tt}(T)T'\right]+\frac{\partial\Omega}{\partial b}\delta b=0 \quad (26)$$

This equation may also be written as

$$\begin{aligned}\Omega_{,t}T' = \left[\frac{\partial\Omega}{\partial z}z_{,t}(T)+\frac{\partial\Omega}{\partial z_{,t}}z_{,tt}(T)\right]T' \\ = -\left(\frac{\partial\Omega}{\partial z}z'(T)+\frac{\partial\Omega}{\partial z_{,t}}z'_{,t}(T)+\frac{\partial\Omega}{\partial b}\delta b\right)\end{aligned} \quad (27)$$

Since it is presumed by Eq. (21), that $\Omega_{,t}\neq 0$, then

$$T' = -\frac{1}{\Omega_{,t}}\left(\frac{\partial\Omega}{\partial z}z'(T)+\frac{\partial\Omega}{\partial z_{,t}}z'_{,t}(T)+\frac{\partial\Omega}{\partial b}\delta b\right) \quad (28)$$

Substituting the result of Eq. (28), into Eq. (25), the following is obtained

$$\begin{aligned}\psi' = [\frac{\partial g}{\partial z}-\left(\frac{\partial g}{\partial z}z_{,t}(T)+G(z(T),b)\frac{1}{\Omega_{,t}}\frac{\partial\Omega}{\partial z}\right]z'(T) \\ -\left[\frac{\partial g}{\partial z}z_{,t}(T)+G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial\Omega}{\partial z_{,t}}z'_{,t}(T) \\ +\int_0^T\left[\frac{\partial G}{\partial z}z'+\frac{\partial G}{\partial b}\delta b\right]dt+\frac{\partial g}{\partial b}\delta b \\ -\left[\frac{\partial g}{\partial z}z_{,t}(T)+G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial\Omega}{\partial b}\delta b\end{aligned} \quad (29)$$

Note that ψ' depends on z' and $z'_{,t}$ at T, as well as on z' within the integration.

In order to write ψ' in Eq. (26), explicitly in terms of a design variation, the adjoint variable technique can be used. In the case of a dynamic system, all terms in Eq. (1), can be multiplied by $\lambda^T(t)$ and integrated over the interval [0,T], to obtain the following equation which is expressed by λ

$$\int_0^T \lambda^T\left[M(b,t)z_{,tt} + C(b)z_{,t} + K(b)z - F(t,b)\right]dt = 0 \qquad (30)$$

Since this equation must hold for arbitrary λ, which is now taken to be independent of the design, substitute b_τ into Eq. (30), and differentiate it with respect to τ in order to obtain the following relationship

$$\int_0^T \left[\lambda^T M(b,t)z'_{,tt} + \lambda^T C(b)z'_{,t} + \lambda^T K(b)z' - \frac{\partial R}{\partial b}\delta b\right] dt = 0 \quad (31)$$

Where

$$R = \tilde{\lambda}^T F(t,b) - \tilde{\lambda}^T M(b,t)\tilde{z}_{,tt} - \tilde{\lambda}^T C(b)\tilde{z}_{,t} - \tilde{\lambda}^T K(b)\tilde{z} \qquad (32)$$

With the superposed tilde (~) denoting variables that are held constant during the differentiation with respect to the design in Eq. (31).

Since Eq. (31), contains the time derivatives of z', integrate the first two integrands separately in order to move the time derivatives to λ, as

$$\begin{aligned}\lambda^T M(b,T)z'_{,t}(T) &- \lambda_{,t}^T(T)M(b,T)z'(T) - \lambda^T(T)M_{,t}(b,T)z'(T) \\ &+ \lambda^T(T)C(b)z'(T) \\ &+ \int_0^T \Big\{[\lambda_{,tt}^T M(b,t) - \lambda_{,t}^T(C(b) - 2M_{,t}(b,t)) \\ &+ \lambda^T(K(b) + M_{,tt}(b,t))]z' - \frac{\partial R}{\partial b}\delta b\Big\} dt \\ &= 0\end{aligned} \qquad (33)$$

The adjoint variable method expresses the unknown terms in Eq. (29), in terms of the adjoint variable (λ). Since Eq. (33), must hold for arbitrary functions λ(t), λ may be chosen so that the coefficients of terms involving $z'(T)$, $z_{,t}'(T)$ and z' in Eqs. (29), and (33), are equal. If such a function λ(t) can be found, then the unwanted terms in Eq. (29), involving $z'(T)$, $z_{,t}'(T)$ and z' can be replaced by terms that explicitly depend on δb in Eq. (33), To be more specific, choose a λ(t) that satisfies the following

$$M(T)\lambda(T) = -\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega^T}{\partial z_{,t}} \qquad (34)$$

$$\begin{aligned}M(b,T)\lambda_{,t}(T) = (C^T(b) - M_{,t}^T(T))\lambda(T) - \frac{\partial g^T}{\partial z} \\ + \left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega^T}{\partial z}\end{aligned} \qquad (35)$$

$$M(t)\lambda_{,tt} - \bar{C}^T(b,t)\lambda_{,t} + \bar{K}(b,t)\lambda = \frac{\partial G^T}{\partial z}, \qquad 0 \le t \le T \qquad (36)$$

In which

$$\bar{C}(b,t) = C(b) - 2M_{,t}(t)$$
$$\bar{K}(b,t) = K(b) + M_{,tt}(t)$$

Note that once the dynamic equation of Eqs. (1) and (22), are used to determine T, then z(T), $z_{,t}(T)$, $\frac{\partial \Omega}{\partial z}$, $\frac{\partial \Omega}{\partial z_{,t}}$ and $\Omega_{,t}$ may be evaluated. Eq. (23), can then be solved for λ(T) since the mass matrix M(T) is nonsingular. Having determined λ(T), all terms on the right of Eq. (35), can be evaluated, and the equation can be solved for $\lambda_{,t}(T)$. Thus, a set of terminal conditions on λ has been determined. Since M(b) is nonsingular, Eq. (36), may then be integrated from T to 0, yielding the unique solution λ(t). Taken as a whole, Eq. (34), through Eq. (36), may be thought of as a terminal value problem.

Since the terms involving a variation in the state variable in Eqs. (29), and (33), are identical, substitute Eq. (33), into Eq. (29), to obtain

$$\begin{aligned}\psi' = \frac{\partial g}{\partial b}\delta b + \int_0^T \left[\frac{\partial G}{\partial b} + \frac{\partial R}{\partial b}\right] dt\delta b - [\frac{\partial g}{\partial z} z_{,t}(T) \\ + G(z(T),b)]\frac{1}{\Omega_{,t}}\frac{\partial \Omega}{\partial b}\delta b \equiv \frac{\partial \psi}{\partial b}\delta b\end{aligned} \qquad (37)$$

Every term in this equation can now be calculated. The terms $\frac{\partial g}{\partial b}$, $\frac{\partial G}{\partial b}$ and $\frac{\partial \Omega}{\partial b}$represent explicit partial derivatives with respect to the design. The term $\frac{\partial R}{\partial b}$, however, must be evaluated from Eq. (32), thus requiring λ(t). Note also that since design variation δb does not depend on time, it is taken outside the integral in Eq. (37).

Since Eq. (37), must hold for all δb, the design derivative vector of ψ is

$$\begin{aligned}\frac{d\psi}{db} = \frac{\partial g}{\partial b}(z(T),b) + \int_0^T [\frac{\partial G}{\partial b}(z,b) \\ + \frac{\partial R}{\partial b}(\lambda(t), z(t), z_{,t}(t), z_{,tt}(t), b)]dt \\ - \frac{1}{\Omega_{,t}}\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{\partial \Omega}{\partial b}\end{aligned} \qquad (38)$$

4. Proposed method

C.E Inglis proposed an approximation solution for considering the effects of vehicles moving over large-span bridges. He introduced an assumption according to which the gravitational effects of the load may be separated from the inertial ones. In the calculation, the force is considered as moving load along the beam while the mass of the vehicle acts at a definite, constant point x_0. Using this method, one can reduce system time dependent matrices to

$$M(t) \approx M\left(\frac{T}{2}\right) \bar{C}(b,t) \approx C(b) - 2M_{,t}\left(\frac{T}{2}\right) \bar{K}(b,t) \approx K(b) + M_{,tt}\left(\frac{T}{2}\right)$$

So, Eq. (36) can be rewritten as

$$M(\frac{T}{2})\lambda_{,tt} - \bar{C}^T\left(b,\frac{T}{2}\right)\lambda_{,t} + \bar{K}\left(b,\frac{T}{2}\right)\lambda = 0, \qquad 0 \le t \le T \qquad (39)$$

That is a linear equation.

While structural vibration responses are used for damage detection, by assuming G=0, Eq. (39), is a free vibration of beam with terminal conditions. Solving Eq.

(39), for a single degree of freedom system is as follow $m\lambda_{,tt} - c\lambda_{,t} + k\lambda = 0$ with terminal conditions: $\lambda(T), \dot{\lambda}(T)$

$$\lambda_T(t) = e^{\xi\omega(t-T)}\left(A_1\sin(\omega_D t) + B_1\cos(\omega_D t)\right) \tag{40}$$

$$\begin{cases} A_1 = \left(\dfrac{\lambda_{,t}(T)}{\omega_D} - \dfrac{\xi}{\sqrt{1-\xi^2}}\lambda(T)\right)\cos(\omega_D T) + \lambda(T)\sin(\omega_D T) \\ B_1 = \dfrac{\lambda(T)}{\cos(\omega_D t)} - A_1\tan(\omega_D T) \end{cases} \tag{41}$$

In which

$$\xi = {}^{c}/_{2m\omega} = {}^{c}/_{c_{cr}} < 1 \quad \text{and} \quad \omega_D = \omega\sqrt{1-\xi^2}$$

When time **T** is known, the coefficients of the characteristic equation of **T'** and thereupon **Ω** will be zero, so the terminal conditions are as follow

$$\lambda(T) = 0 \tag{42}$$

$$\lambda_{,t}(T) = M^{-1}(b) \times \left(-\frac{\partial g^T}{\partial z}\right) \tag{43}$$

Substitute Eqs. (42) and (43), into Eq. (41), to obtain

$$\begin{cases} A_1 = \dfrac{\lambda_{,t}(T)}{\omega_D}cos(\omega_D T) \\ B_1 = -\dfrac{\lambda_{,t}(T)}{\omega_D}sin(\omega_D T) \end{cases} \tag{44}$$

Note that $\frac{\partial g}{\partial z}$ like A_1 and B_1 is depend on time T, so terminal values for different amounts of **T** are not similar and adjoint equation should be calculated for all amounts of **T** separately. So

$$\begin{aligned} \lambda_T(t) = e^{\xi\omega(t-T)}\Bigg(&\frac{\lambda_{,t}(T)}{\omega_D}cos(\omega_D T)\,sin(\omega_D t) \\ &- \frac{\lambda_{,t}(T)}{\omega_D}sin(\omega_D T)\,cos(\omega_D t)\Bigg) \\ &= P_T f(t) + Q_T g(t) \end{aligned} \tag{45}$$

In which: $P_T = e^{-\xi\omega T}\frac{\lambda_{,t}(T)}{\omega_D}cos(\omega_D T) f(t) = e^{\xi\omega t}sin(\omega_D t)\, Q_T = -e^{-\xi\omega T}\frac{\lambda_{,t}(T)}{\omega_D}sin(\omega_D T) g(t) = e^{\xi\omega t}cos(\omega_D t)$

4.1 Sensitivity matrix for physical parameter

By using the Eq. (38), assuming **T** is known and **G=0** because of using structural vibration data, the Eq. (46), can be obtained as follows

$$\frac{d\psi}{db} = \int_0^T \frac{\partial R}{\partial b}dt \tag{46}$$

In this equation:
$R = \tilde{\lambda}^T F(t) - \tilde{\lambda}^T M\tilde{z}_{,tt} - \tilde{\lambda}^T C(b)\tilde{z}_{,t} - \tilde{\lambda}^T K(b)\tilde{z}$ *And* $C = a_0 K(b) + a_1 M$ is Rayleigh damping matrix, so

$$\frac{\partial R}{\partial b} = -\widetilde{\lambda^T} a_0 \frac{\partial K}{\partial b}\widetilde{z_{,t}} - \widetilde{\lambda^T}\frac{\partial K}{\partial b}\tilde{z} \tag{47}$$

And finally, the component of sensitivity matrix in time **T** is

$$\frac{d\psi}{db}(T) = \int_0^T \left(-\widetilde{\lambda^T} a_0 \frac{\partial K}{\partial b}\widetilde{z_{,t}} - \widetilde{\lambda^T}\frac{\partial K}{\partial b}\tilde{z}\right)dt \tag{48}$$

Solving the above equations directly is not possible in a multi degree of freedom problem. So, for this purpose, the variables should be changed as follows

$$\{\lambda\} = [\phi]\{Y\} \tag{49}$$

In this equation, matrix [φ] denotes the vibration modes (modal matrix).

The terminal conditions of above equations are as follows

$$\{Y(T)\} = M^{-1}[\phi]^T[m]\{\lambda(T)\} \tag{50}$$

$$\{Y_{,t}(T)\} = M^{-1}[\phi]^T[m]\{\lambda_{,t}(T)\} \tag{51}$$

By substituting Eq. (50) in Eq. (36) and multiplying $[\phi]^T$ in both sides, the new equation in modal space is as follow

$$[M]\{Y_{,tt}\} - [C]\{Y_{,t}\} + [K]\{Y\} = \{0\} \tag{52}$$

Each of $[M], [C]\,and\,[K]$ matrix is diagonal, so

$$M_i\{Y_{,tt_i}\} - C_i\{Y_{,t_i}\} + K_i\{Y_i\} = \{0\} \tag{53}$$

$$\begin{aligned} \frac{d\psi}{db}(T) = -\int_0^T &\langle Y\rangle \times [\phi]^T \times a_0\left[\frac{\partial k}{\partial b}\right] \times \{z_{,t}\} + \langle Y\rangle \times [\phi]^T \\ &\times \left[\frac{\partial k}{\partial b}\right] \times \{z\}dt \end{aligned} \tag{54}$$

Consider: $[\phi]^T \times a_0\left[\frac{\partial k}{\partial b}\right] \times \{z_{,t}\} = \{zz_{,t}\}$ and $[\phi]^T \times \left[\frac{\partial k}{\partial b}\right] \times \{z\} = \{zz\}$

Eq. (54) can be expressed as Eq. (55)

$$\frac{d\psi}{db}(T) = -\int_0^T \langle Y\rangle \times \{zz_{,t}\} + \langle Y\rangle \times \{zz\}dt \tag{55}$$

Variable Y in modal space can be written based on Eq. (45) as follows

$$\{Y\} = \{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\} \tag{56}$$

By substituting Eq. (56) in Eq. (55) a new expression is derived to calculate the sensitivity.

$$\begin{aligned} \frac{d\psi}{db}(T) = -\int_0^T &(\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \times \{zz_{,t}\} \\ &+ (\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \\ &\times \{zz\}dt \end{aligned} \tag{57}$$

So Eq. (57) can be rewritten as follow

$$\begin{aligned} \frac{d\psi}{db}(T) = -\int_0^T &\langle P(T)\rangle \times (\{f(t)\}.\{zz_{,t}\} + \{f(t)\}.\{zz\}) \\ &+ \langle Q(T)\rangle \\ &\times (\{g(t)\}.\{zz_{,t}\} + \{g(t)\}.\{zz\})dt \end{aligned} \tag{58}$$

Consider following parameters

$$A = \int_0^T \{f(t)\}.\{zz_{,t}\}\,dt \quad B = \int_0^T \{g(t)\}.\{zz_{,t}\}\,dt \quad C = \int_0^T \{f(t)\}.\{zz\}\,dt \quad D = \int_0^T \{g(t)\}.\{zz\}\,dt$$

So, Eq. (58) is presented asfollows

$$\frac{d\psi}{db}(T) = -\langle P(T)\rangle \times (\{A\} + \{C\}) - \langle Q(T)\rangle \times (\{B\} + \{C\}) \tag{59}$$

Solving Eq. (59) directly is too time consuming, because in each time step all terms in Eq. (59) should be recalculted. Therefore, an incremental solution is developed as follow

$$\{A_{T+\Delta T}\} = \int_0^{T+\Delta T} \{f(t)\}.\{zz_{,t}\}\,dt = \int_0^{T} \{f(t)\}.\{zz_{,t}\}\,dt + \int_T^{T+\Delta T} \{f(t)\}.\{zz_{,t}\}\,dt \tag{60}$$

$$\{A_{T+\Delta T}\} = \{A_T\} + \{\delta A\}\ ,\{\delta A\} = \int_T^{T+\Delta T} \{f(t)\}.\{zz_{,t}\}\,dt \cong \left\{f\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz_{,t}\left(T + \frac{\Delta T}{2}\right)\right\} \tag{61}$$

Similar to Eq. (61) for other parameters we have

$$\{\delta B\} = \int_T^{T+\Delta T} \{g(t)\}.\{zz_{,t}\}\,dt \cong \left\{g\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz_{,t}\left(T + \frac{\Delta T}{2}\right)\right\} \tag{62}$$

$$\{\delta C\} = \int_T^{T+\Delta T} \{f(t)\}.\{zz\}\,dt \cong \left\{f\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz\left(T + \frac{\Delta T}{2}\right)\right\} \tag{63}$$

$$\{\delta D\} = \int_T^{T+\Delta T} \{g(t)\}.\{zz\}\,dt \cong \left\{g\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz\left(T + \frac{\Delta T}{2}\right)\right\} \tag{64}$$

And finally, the sensitivity expression in time $T + \Delta T$ is as follow

$$\frac{d\psi}{db}(T + \Delta T) = -\langle P(T + \Delta T)\rangle \times (\{A_{T+\Delta T}\} + \{C_{T+\Delta T}\}) - \langle Q(T + \Delta T)\rangle \times (\{B_{T+\Delta T}\} + \{D_{T+\Delta T}\}) \tag{65}$$

4.2 Sensitivity matrix for excitation mass

By deriving the Eq. (32) with respect to the parameters of the i^{th} excitation mass, the equation changes in to the follows

$$\frac{\partial R}{\partial b} = \frac{\partial R}{\partial M_i} = g\widetilde{\lambda^T} B \tag{66}$$

And component of sensitivity matrix in time **T** is

$$\frac{d\psi}{dM_i}(T) = g\int_0^T \widetilde{\lambda^T} B\,dt \tag{67}$$

By using modal space and the Eq. (56)

$$\frac{d\psi}{dM_i}(T) = g\int_0^T \langle Y\rangle \times [\phi]^T \times [B]\,dt \tag{68}$$

Consider: $[\phi]^T \times [B] = \{BB\}$

By subsituing the Eq. (56) into Eq. (68), a new expression is derived to calculate the sensitivity.

$$\frac{d\psi}{dM_i}(T) = g\int_0^T (\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \times \{BB\}\,dt \tag{69}$$

So the Eq. (69) can be rewritten as follow

$$\frac{d\psi}{dM_i}(T) = g\int_0^T \langle P(T)\rangle \times (\{f(t)\}.\{BB\}) + \langle Q(T)\rangle \times (\{g(t)\}.\{BB\})\,dt \tag{70}$$

Consider following parameters

$$E = \int_0^T \{f(t)\}.\{BB\}\,dt \quad F = \int_0^T \{g(t)\}.\{BB\}\,dt$$

So, the Eq. (70) is presented as

$$\frac{d\psi}{dM_i}(T) = g(\langle P(T)\rangle \times \{E\} + \langle Q(T)\rangle \times \{F\}) \tag{71}$$

Using incremental solution is as follow

$$\{E_{T+\Delta T}\} = \int_0^{T+\Delta T} \{f(t)\}.\{BB\}\,dt = \int_0^{T} \{f(t)\}.\{BB\}\,dt + \int_T^{T+\Delta T} \{f(t)\}.\{BB\}\,dt \tag{72}$$

$$\{E_{T+\Delta T}\} = \{E_T\} + \{\delta E\}\ ,\{\delta E\} = \int_T^{T+\Delta T} \{f(t)\}.\{BB\}\,dt \cong \left\{f\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{BB\left(T + \frac{\Delta T}{2}\right)\right\} \tag{73}$$

Similar to Eq. (73) for other parameter we have

$$\{\delta F\} = \int_T^{T+\Delta T} \{g(t)\}.\{BB\}\,dt \cong \left\{g\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{BB\left(T + \frac{\Delta T}{2}\right)\right\} \tag{74}$$

And finally the sensitivity expression in time $T + \Delta T$ is

$$\frac{d\psi}{dM_i}(T + \Delta T) = g(\langle P(T + \Delta T)\rangle \times \{E_{T+\Delta T}\} + \langle Q(T + \Delta T)\rangle \times \{F_{T+\Delta T}\}) \tag{75}$$

4.3 Computational algorithm

The computational algorithm that leads to the determination of sensitivity matrix for physical parameters and excitation mass is as below:

- Step1: calculate $\lambda_{,t}(T)$ from Eq. (43)
- Step2: calculate $\omega, \omega_D and \phi$ and consider i=1

- Step3: Calculate the $\frac{\partial K}{\partial \alpha_i}$ and $zz_{,t}, zz$ and consider j=1 for the ith element
- Step4: Calculate $\lambda_{,t}(T)$ from step1 and $Y_{,t}(T)$ from Eq. (51), T_n=Δt and T_o=0 for the j th sensor and corresponding DOF
- Step5: Consider A=B=C=D=E=F=0
- Step6: Calculate $T_m = T_0 + \frac{\Delta t}{2}$ and $P(T_n) - Q(T_n) - f(T_m) - g(T_m)$ from Eq. (45)
- Step7: Calculate $\{\delta\mathrm{A}\}, \{\delta\mathrm{B}\}, \{\delta\mathrm{C}\}, \{\delta\mathrm{D}\}, \{\delta\mathrm{E}\}$ and $\{\delta\mathrm{F}\}$ from Eq. (60-63 and 73-74)
- Step8: Calculate $\frac{\mathrm{d}\psi}{\mathrm{d}\alpha_\mathrm{i}}(\mathrm{T_n})$ from Eq. (65) and $\frac{\mathrm{d}\psi}{\mathrm{d}M_i}$ from Eq. (75)
- Step9: If $T_n < T_{final}$ consider $T_0 = T_n$, $T_n = T_n + \Delta t$ and go to step5 otherwise go to next.
- Step10: If $j < numberofsensors$ consider j=j+1 and go to step 4 otherwise go to next step.
- Step11: If $i < numberofelements$ consider i=i+1 and go to step 3 otherwise finish.

4.4 Procedure of iteration for mass identification and damage detection

The proposed method requires measurement from two states of the structure. The first set of measurement from the undamaged structure serves to update the system parameters with a known set of mass input. While in the measurement on the second state of structure, both the excitation mass and the damaged structure are unknown, and the following iterative algorithm is used in the identification (Lu and Law 2007). The updated finite element model and excitation mass in ith iteration step serves as the reference model in the subsequent comparison.

(A) Iteration for excitation mass parameters

Starting with an initial guess on the unknown mass parameter vector M_0 and the set of physical parameters α_0 from the updated FE model of the structure, the procedure of iteration is given as:

- Step 1: The Eq. (1) is solved at $j = k + 1$ iteration step with the initial force vector and the vector of the undamaged system by using the ß-Newmark method so the displacement vector $\{z\}$, acceleration vector $\{z_{,tt}\}$ and the error vector$\{\delta z_{,tt}\}$ are calculated.
- Step 2: The sensitivity matrix $[S_M]$ of the response with respect to the mass is obtained from Eq. (77) and proposed algorithm for $j = k + 1$ iteration step with the mass vector $\{M_k\}$ obtained from a previous step.
- Step 3: Calculate the$\{M_{k+1}\}$ from Eq. (16).
- Step 4: Repeat Steps 1~3 until the following convergence criteria is satisfied.

$$\frac{\|\mathrm{M_{k+1}} - \mathrm{M_k}\|}{\|\mathrm{M_{k+1}}\|} \times 100\% \le \mathrm{Tol1} \qquad (76)$$

- Step 5: The final vector $\{M_{k+1}\}$ obtained is taken as the modified set of force $\{\mathrm{M}\}$ for the second stage of iteration.

(B) Iteration for the physical parameters of the structure

By using the modified excitation mass parameter vector $\{\mathrm{M}\}$ obtained from (A) iterative algorithm, a set of physical parameters is then obtained as below:

- Step 6: The vector of physical parameters$\{\alpha_s\}$ from the updated finite element model of the structure is taken as a set of initial values. The Eq. (1) is solved at $j = k + 1$ iteration stepby using the ß-Newmark method so the displacement vector$\{z\}$, acceleration vector$\{z_{,tt}\}$ and the error vector $\{\delta z_{,tt}\}$ are calculated.
- Step 7: The sensitivity matrix $[S_s]$ of the response with respect to the different physical parameters of the structure is obtained from Eq. (65) and proposed method at $j = k + 1$ iteration step with the initial physical parameter vector $\{\alpha_k\}$ obtained from a previous step.
- Step 8: Find $\{\alpha_{k+1}\}$ from Eq. (18).
- Step 9: Repeat Steps 6–8 until the following convergence condition are satisfied.

$$\frac{\|\alpha_{\mathrm{k+1}} - \alpha_\mathrm{k}\|}{\|\alpha_\mathrm{k}\|} \times 100\% \le \mathrm{Tol2} \qquad (77)$$

$$\frac{\|\mathrm{Response_{k+1}} - \mathrm{Response_k}\|}{\|\mathrm{Response_k}\|} \times 100\% \le \mathrm{Tol3} \qquad (78)$$

- Step 10: The final obtained vector $\{\alpha_{k+1}\}$ is taken as the modified set of physical parameters $\{\alpha_k\}$ for the next loop of iteration for the force parameters.

The identified excitation mass obtained in (A) should be further improved using the updated physical parameters obtained in (B) and repeating Steps 1–5 and the vector of physical parameters should also be further improved using the modified excitation mass and repeating Steps 6–10.

This iteration procedure continuous until the following convergence condition criteria is satisfied.

$$\frac{\|\mathrm{M_{i+1}} - \mathrm{M_i}\|}{\|\mathrm{M_{i+1}}\|} \times 100\% \le \mathrm{Tol4} \qquad (79)$$

$$\frac{\|\alpha_{\mathrm{i+1}} - \alpha_\mathrm{i}\|}{\|\alpha_\mathrm{i}\|} \times 100\% \le \mathrm{Tol5} \qquad (80)$$

The convergence of this computation strategy has been proved by Li and Chen (1999).

All tolerances are set equal to 1×10^{-6} in this study except otherwise specified.

5. Numerical results

To illustrate the formulations presented in the previous sections, the system shown in Figs. 3 and 9 are considered, and capabilities of proposed method are investigated.

The relative error percentage for physical parameter REPP and excitation mass REPPM based on the identified results are calculated from Eq. (81), where $\|.\|$ is the norm of matrix, $\mathrm{E_{Identified}}$ and $\mathrm{E_{True}}$ are the identified and the true elasticity modulus respectively and $\mathrm{M_{Identified}}$ and $\mathrm{M_{True}}$ are the identified and the true excitation mass, respectively.

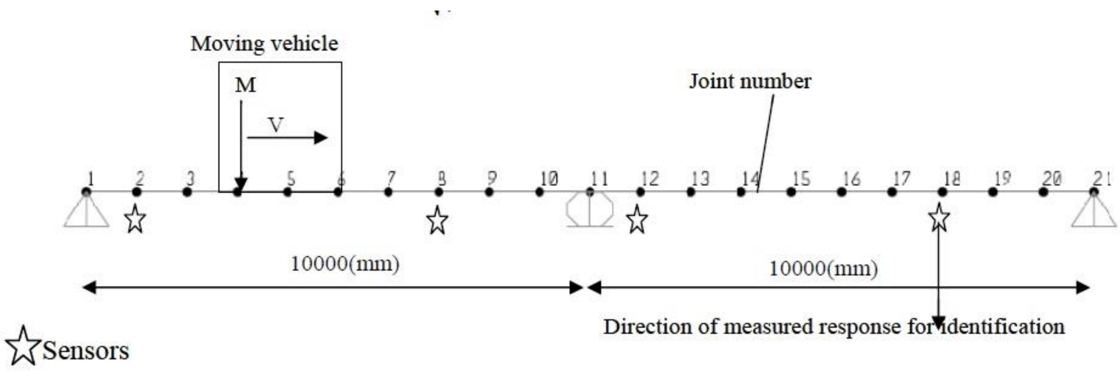


Fig. 3 Multi span bridge model used in damage detection

Table1 Damage scenarios for multi-span bridge

Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M1-1	Single	14	20%	Nil
M1-2	Multi	3,7,11,15,18	2%,6%,5%,2%,8%	Nil
M1-3	Random	All elements	Random damage in all elements with an average of 5%	Nil
M1-4	Estimation of undamaged state	All elements	5% reduction in all elements	Nil

$$\mathrm{RPEP} = \frac{\|\mathrm{E_{Identified}} - \mathrm{E_{True}}\|}{\|\mathrm{E_{True}}\|} \times 100\% \quad \& \quad RPEM = \frac{\|\mathrm{M_{Identified}} - \mathrm{M_{True}}\|}{\|\mathrm{M_{True}}\|} \times 100\% \tag{81}$$

Since the true value of elasticity modulus is unknown, REPP and REPPM can just be used for investigating the efficiency of method.

5.1. Multi span model

A two-span bridge as shown in Fig. 3 is studied to illustrate the proposed method. It consists of 20 Euler–Bernoulli beam elements with 21 nodes each with two DOFs. The mass per unit volume of material is 7.8×10^3 kg/m^3 and the elasticity modulus of material is 2.1×107 N/cm^2. The total length of bridge is 20 m the height and width of the frame section are respectively 200 and 200 mm. The first five un-damped natural frequencies of the intact bridge are 29.3829, 45.8299, 117.3834, 148.1623 and 265.0938 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 4.6413×10^{-6}.

The transverse point load P has a constant velocity, $V = L/T$, where T is the traveling time across the bridge and L is the total length of the bridge.

The integration parameters $\beta = 1/4$ and $\gamma = 1/2$ are used for ß-Newmark method which lead to calculate the approximate constant-average acceleration. Speed ratio is defined as follow

$$\alpha = \frac{V}{V_{cr}} \tag{82}$$

In which V_{cr} is critical speed ($V_{cr} = \frac{\pi}{l}\sqrt{\frac{EI}{\rho}}$), V is moving load speed and ρ is mass per unit length of beam.

5.1.1 Damage scenarios

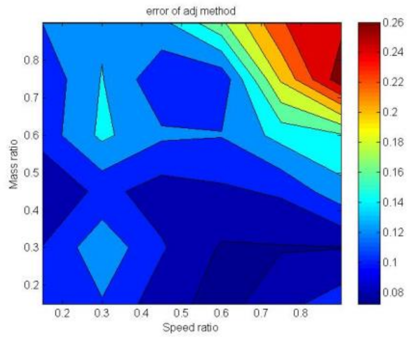
Three damage scenarios of single, multiple and random damages in the bridge without considering the noise effects are studied which are shown in Table1.

Local damage is simulated with a reduction in the elasticity modulus of element material. The sampling rate is 10000 Hz and 670 data of the acceleration response (degree of indeterminacy is 20) collected along the z-direction at nodes 2, 5, 8, 12, 15 and 18 which are used in the identification.

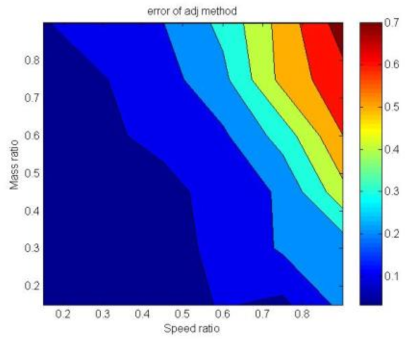
Scenario 1 studies the single damage state. The iterative solution converges in all speed and mass ratio of moving vehicle to bridge ranges, but changes in relative error significantly increase by increasing in relative speed and mass of moving vehicle.

As shown in Fig. 4 in both methods, the minimum error is related to the least relative mass and speed of moving vehicle. For AVM method, the relative percentage of error REP in this case is equal to 0.08 and it increases by increasing the speed ratio. As shown in this figure, for relative mass ratio of 0.15 the REP for speed parameter of 0.95 is equal to 0.1. It’s remarkable that the error ratio significantly increases by increasing the mass ratio.

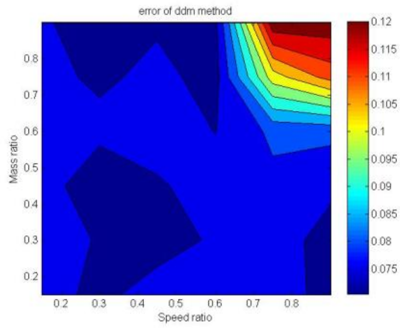
For mass ratio of 0.75 and relative speed equal to 0.15; the relative percentage of error is equal to 0.1% but in relative speed equal to 0.9 it reaches to 0.32% whereas the REP for relative speed of 0.9 and relative mass of 0.9 significantly increases to 1.5%.



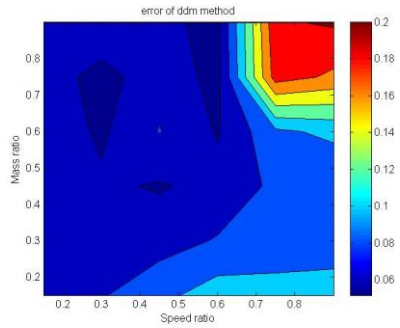
(a) Scenario1-AVM method



(b) Scenario2-AVM method



(c) Scenario1-DDM method



(d) Scenario2-DDM method

Fig. 4 REP contours with respect to speed and mass ratio in model1

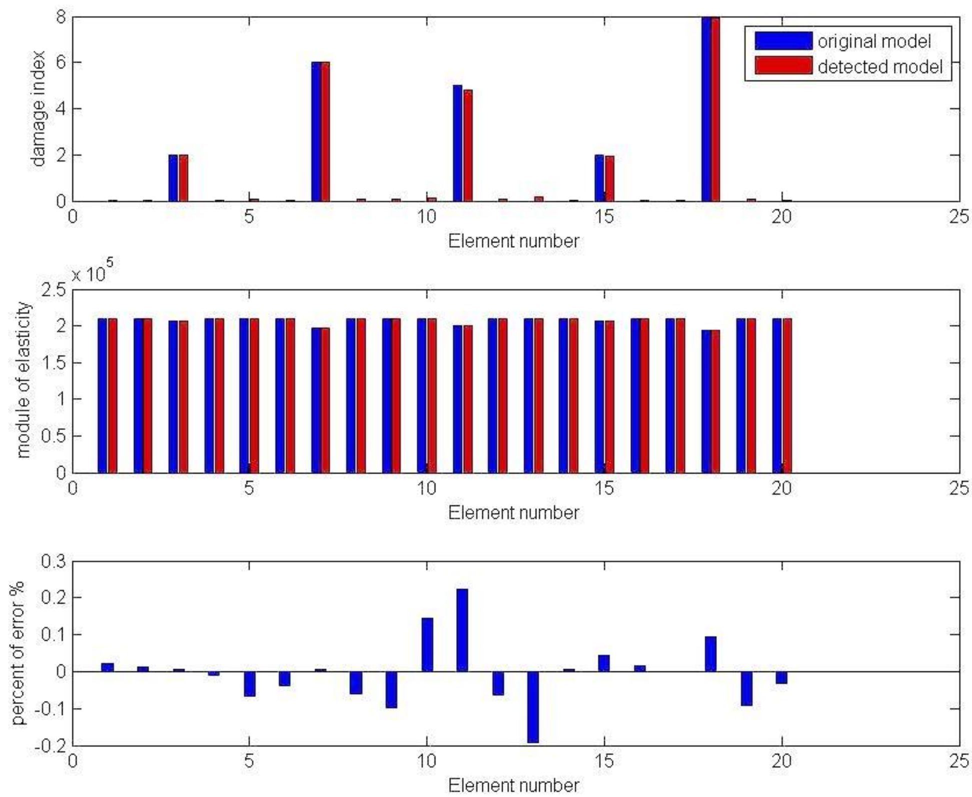


Fig. 5 The extent and damage location in elements 3, 7, 11, 15,18 and the value of error percentage in different elements with AVM method

The REP changes for DDM method are more stable with respect to mass and speed ratio variation. For range of mass ratio less than or equal to 0.75, the error of two methods is almost identical but with increasing mass ratio to 0.9 and relative speed greater than 0.45, error of AVM method is significantly greater than DDM method.

Scenario 2 studies the multiple damages with different amount of measured responses for the identification and Scenario 3 studies the random damages for damage detection. These scenarios also converge in all speed parameter ranges and their results are similar to the first scenario. One more scenario with model error is also included as Scenario 4. This scenario doesn't consist of any simulated damage in the structure, it has identical reduction of the initial elastic modulus of all elements as much 5% in the inverse identification.

The extent and damage locations are detected correctly in all scenarios (Fig. 5) by using both described methods, including DDM and proposed method. The REP parameters are shown in Fig. 4 which is in acceptable range.

Further studies on scenario 4 shows that both methods are sensitive to initial model error and for maximum 20% initial error can be converged and a relatively good finite element model is therefore needed for the damage detection procedure.

5.1.2 Effect of noise

To evaluate the sensitivity of results to the noise, noise-polluted measurements are simulated by adding a noise vector to the correspondingacceleration vector whose root-mean-square (RMS) value is equal to a certain percentage of the RMS value of the noise-free data vector. The components of all noise vectors which are obtained by Gaussian distribution are in the form of uncorrelated with a zero mean and unit standard deviation. Then on the basis of the noise-free acceleration $z_{,tt_{nf}}$; the noise-polluted acceleration $z_{,tt_{np}}$ of the bridge at location x can be simulated by

$$z_{,tt_{np}} = z_{,tt_{nf}} + RMS\left(z_{,tt_{nf}}\right) \times N_{level} \times N_{unit} \qquad (84)$$

Where RMS ($z_{,tt_{nf}}$) is the r.m.s value of the noise-free acceleration vector $z_{,tt_{nf}} \times N_{level}$ is the noise level, and N_{unit} is a randomly generated noise vector with zero mean and unit standard deviation. (Jiang *et al*. 2004)

In order to study effect of noise in stability of sensitivity methods, scenario2 (Speed ratio of moving load is considered to be fix and equal with 0.3 and mass ratio is equal to 0.5) is considered and different levels of noise pollution are investigated, and RPEP changes with increasing number of loops for iterative procedure has been studied.

Results are illustrated in Fig. 6 for DDM and AVM methods. These contours show that both AVM and DDM methods are sensitive to noise and if noise level becomes greater than 1.4% these methods lose their effectiveness and are not able to detect damage. So, in cases with noise level greater than 1.5%, a denoising tool alongside sensitivity methods should be used.

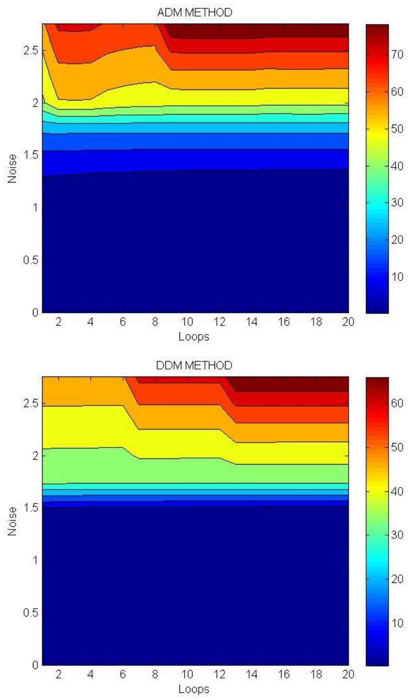


Fig. 6 RPEP contours with respect to noise level and loops

5.1.3 Efficiency of proposed method

In order to compare and quantification the performance of different methods and evaluate the proposed method, relative efficiency parameter (REP) is defined as

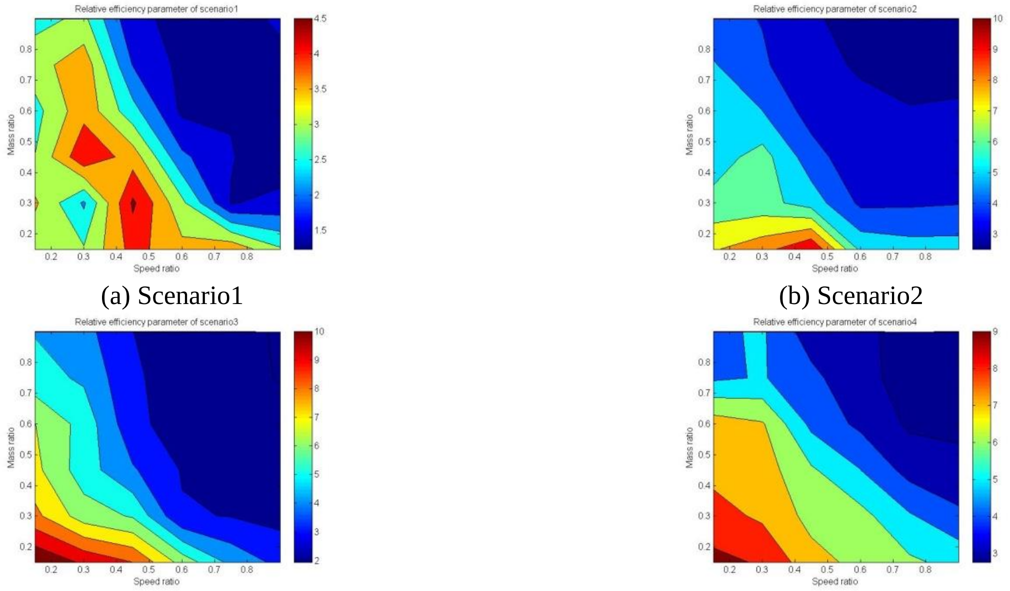
$$REP = ST_{DDM}/ST_{ADM} \qquad (85)$$

In which, ST is the solution time of DD method. In fact this parameter represents the computation cost of method.

In Fig. 7 changes of REP parameter with respect to velocity and mass ratio is illustrated. As shown in this figure, as much as velocity and mass ratio decreases, The REP parameter increases. Summary of this figure is shown in Table2. According to this table, the efficiency parameter is between 1.2327 to 10.9170 and its average is 4.2073.

In Fig. 8 changes of REP average with respect to velocity and mass ratio is shown. As illustrated in this figure, increasing these two ratios, the REP parameter decreases almost linear. For example, in mass ratio equal to 0.15 average of REP is about 7.5 but increasing mass ratio to 0.9 causes this amount reduces to 3.2.

In addition, accuracy of AVM method reduces significantly for mass ratio greater than 0.9 and in velocity ratio greater than 0.45. So, in this range, using AVM method is not recommended. It is noteworthy that in real bridges, Including highways and railway bridges, maximum ratio of moving vehicle to bridge is much lower than this ratio. So the adjoint variable method is extremely successful for real time structures and computational cost for this method is about 23.8% of other sensitivity-based finite element model updating method.



(a) Scenario1 (b) Scenario2

(c) Scenario3 (d) Scenario4

Fig. 7 REP contours with respect to speed and mass ratio in model1

Table 2 REP ranges in different scenarios for model1

Damage scenario	Max REP	Min REP	average
M1-1	4.5928	1.2327	2.5669
M1-2	10.0151	2.5146	4.5142
M1-3	10.9170	1.9474	4.4732
M1-4	9.2783	2.7705	5.2750
Total	10.9170	1.2327	4.2073

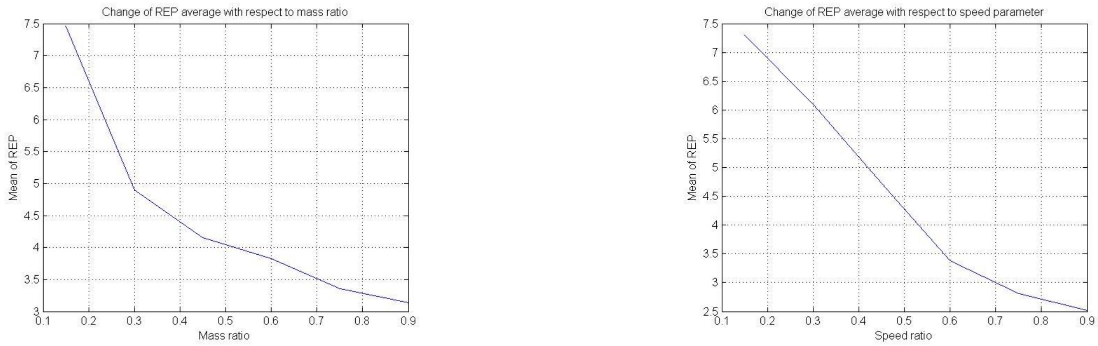


Fig. 8 Average of REP changes with respect to speed and mass ratio in model1-scenario3

5.2. Plane grid model

A plane grid model of bridge is studied as another numerical example to illustrate the effectiveness of the proposed method. The finite element model of the structure is shown in Fig. 9 the structure is modeled by 46 frame elements and 32 nodes with three Dof's at each node for the translation and rotational deformations. The mass density of material is 7.8×10^{-9} kg/mm^3 and the elastic modulus of material is 2.1×10^{-5} N/ mm^2. The first five un-damped natural frequencies of the intact bridge are 45.59, 92.77, 181.74, 259.73 and 399.07 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 2.364×10^{-5}.

5.2.1 Damage scenarios

Three damage scenarios of single, multiple and random damages in the bridge without measurement noise are studied and they are shown in Table3.

The sampling rate is 14000 Hz and 1150 data of the acceleration response (degree of indeterminacy is 25) collected along the z-direction at nodes 4, 11, 21 and 27 are used.

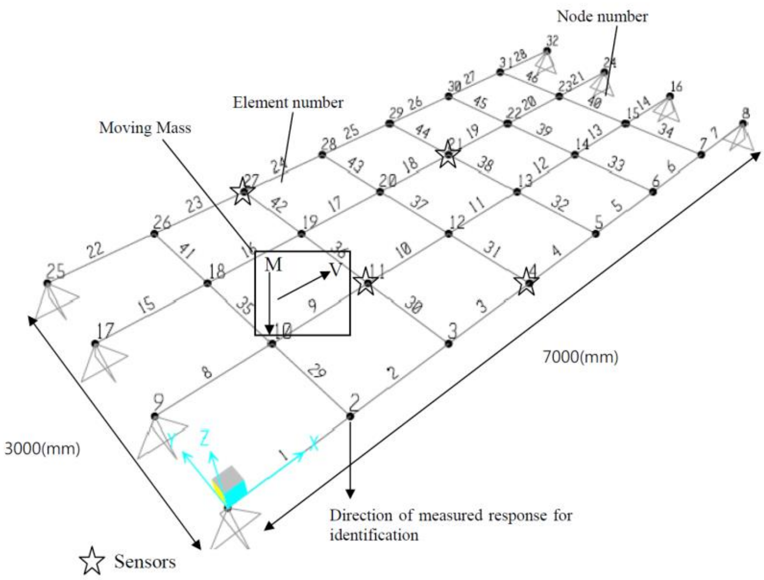
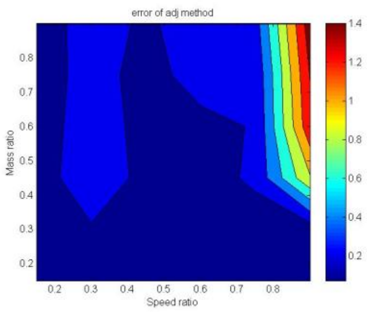


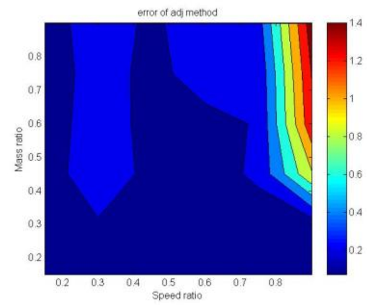
Fig. 9 Plane grid bridge model used in detection procedure

Table 3 Damage scenarios for grid model

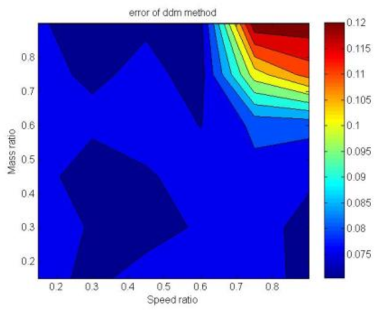
Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M2-1	Single	41	7%	Nil
M2-2	Multi	5,7,12,15,24,37	4%,11%,6%,2%,10%,16%	Nil
M2-3	Random	All elements	Random damage in all elements with an average of 5%	Nil
M2-4	Estimation of undamaged state	All elements	6% reduction in all elements	Nil



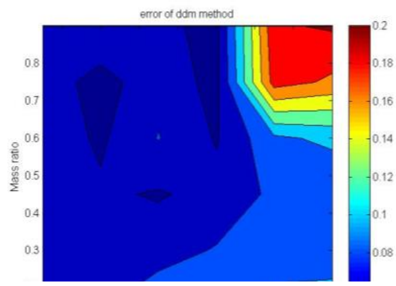
(a) Scenario1-AVM method



(b) Scenario2-AVM method



(c) Scenario1-DDM method



(d) Scenario2-DDM method

Fig. 10 RPE contours with respect to speed and mass ratio in model2

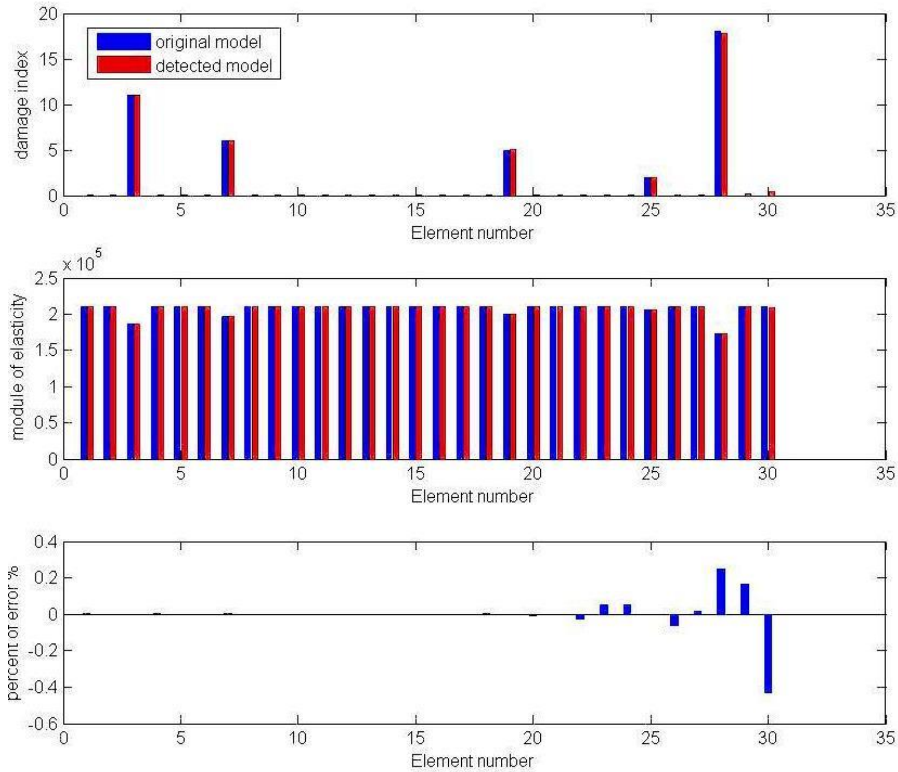


Fig. 11 Detection of damage location and amount in elements 5, 7, 12, 15, 24 and 37 and distribution of error in different elements with AVM scheme

Similar to the previous model, scenario 1 studies the single damage scenario and the iterative solution converges in all speed parameters and mass ratio and changes of relative error significantly increase by increasing in relative speed and mass of moving vehicle.

Fig. 10 shows that in both methods, the minimum error is related to the least relative mass and moving speed. And for higher amounts of mass ratio, it increases significantly for AVM method. For DDM method, The RPE parameter is more stable with respect to mass and speed ratio variation. For range of mass ratio equal to 0.75, the error of two methods is almost identical but with increasing mass ratio to 0.9 and relative speed greater than 0.45, error of AVM method is significantly greater than DDM method.

Scenario 2 is on multiple damages with different amount of measured responses for the identification and Scenario3 is on random damages for the identification. These scenarios also converge in all speed parameter ranges and similar results with the first scenario were obtained. One more scenario with model error is also included as Scenario 4. This scenario consists of no simulated damage in the structure, but with the initial elastic modulus of material of all the elements under-estimated by 6% in the inverse identification.

Using both described methods, including DDM and proposed method, the damage locations and amount are identified correctly in all the Scenarios (Fig. 12).

5.2.2 Effect of noise

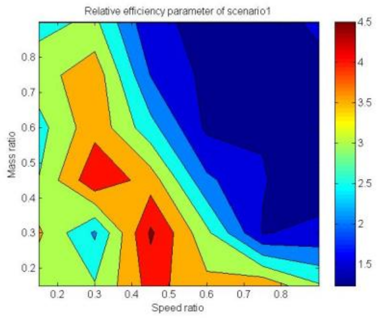
In order to study effect of noise in stability of sensitivity methods, scenario3 (Speed ratio of moving load is considered to be fix and equal with 0.3 and mass ratio is equal to 0.5) is considered and different levels of noise pollution are investigated, and RPE changes with increasing number of loops for iterative procedure has been studied. Fig. 13 Shows that both AVM and DDM methods are sensitive to noise and if noise level becomes greater than 2.8% and 2.5% for AVM method and DDM method respectively, these methods lose their effectiveness and are not able to detect damage. So, in cases with noise level greater than mentioned values, a de-noising tool such as wavelet transform alongside sensitivity methods should be used. The wavelet transform is mainly attractive because of its ability to compress and encode information to reduce noise or to detect any local singular behavior of a signal.

5.2.3 Efficiency of proposed method

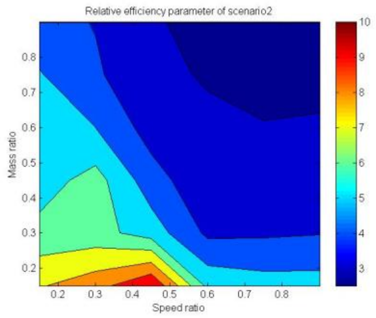
In Fig. 14 changes of REP parameter with respect to the velocity and mass ratio is illustrated. As shown in this figure, as much as velocity and mass ratio decrease, The REP parameter increases.

Summary of this figure is shown in Table4. According to this table, the efficiency parameter is between 1.2327 to 10.9170 and its average is 4.2073.

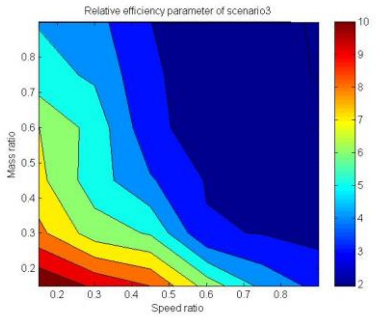
Fig. 12 RPE contours with respect to noise level and loops



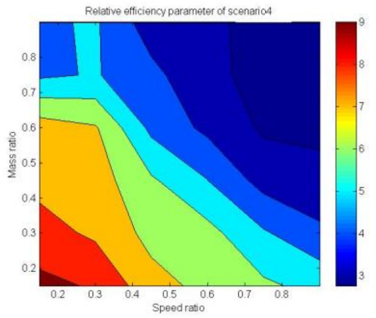
(a)Scenario1



(b)Scenario2



(c)Scenario3



(d)Scenario4

Fig. 13 EP contours with respect to speed and mass ratio in model2

Table 4 REP ranges in different scenarios for model2

Damage scenario	Max REP	Min REP	Average
M2-1	4.5928	1.2327	2.5669
M2-2	10.0151	2.5146	4.5142
M2-3	10.9170	1.9474	4.4732
M2-4	9.2783	2.7705	5.2750
Total	10.9170	1.2327	4.2073

In Fig. 15 changes of REP average with respect to velocity and mass ratio is shown. As illustrated in this figure, increasing these two ratios, the REP parameter decreases. Furthermore, accuracy of AVM method reduces significantly for mass ratio greater than 0.9 and in velocity ratio greater than 0.45. So, for second model similar to firs, in this range, using AVM method is not recommended. Outside of this limit and for real time structures, the adjoint variable method is extremely successful and computational cost for this method is about 23.8% of DDM method.

6. Conclusions

In this paper an iterative sensitivity-based method has been developed to identify both the input excitation mass and the physical parameters of a bridge just from the output of the system. This method can be an efficient tool in the case when the structure for example a highway bridge must be on operation continuously.

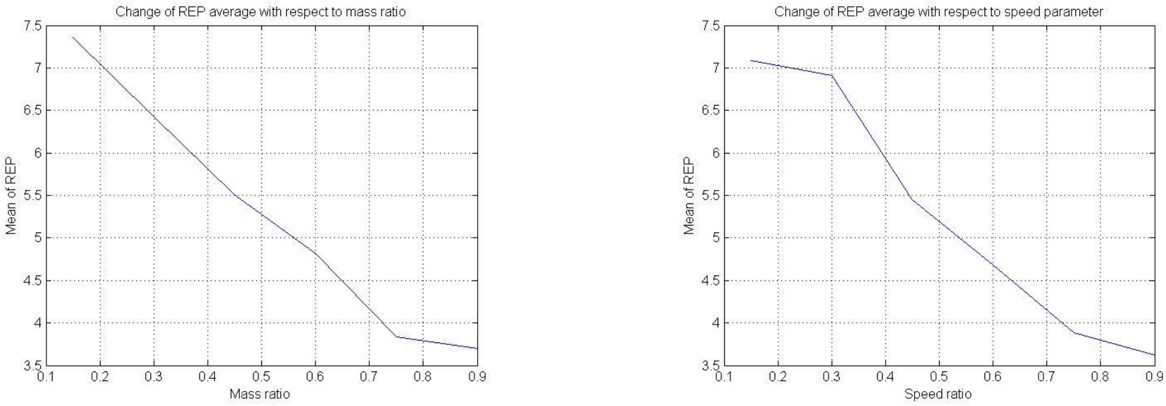


Fig. 14 Average of REP changes with respect to speed and mass ratio in model2-scenario4

In proposed method an incremental solution for adjoint variable equation developed which calculates each elements of sensitivity matrix separately. The main advantage is inclusion of an analytical method to augment the accuracy and speed of the solution.

Numerical simulations demonstrate the efficiency and accuracy of the method to identify location and extent of single, multiple and random damages and unknown input excitation mass simultaneously in different bridge models. Comparison studies confirmed that computational cost for this method is much lower than other traditional sensitivity methods. For modern, practical engineering applications, the cost of damage detection analysis is expensive. So, this method is feasible for large-scale problems.

The drawback of this method is its accuracy and efficiency reduction in mass and speed ratios near to one. It's notable that in real structures this range of speed and mass ratio is not accessible.

References

Alampalli, S. and Fu, G. (1994), "Remote monitoring systems for bridge condition", *Transportation Research and Development Bureau, New York State Department of Transportation,* Client Report 94.

Cawley. P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. Strain Anal.*, **14**, 49-57.

Chan, T., Law, S., Yung, T. and Yuan, X. (1999), "An interpretive method for moving force identification", *J. Sound Vib.*, **219**, **503-524.**

Chan, T., Yu, L., Law. S. and Yung, T. (2001), "Moving force identification studies. II: comparative studies", *J Sound Vib.*, **247**(1), 77-95.

Choi, K.K. and Kim, N.H., (2005), "Structural sensitivity analysis and optimization 1, linear systems", *Springer, USA.*

Doebling, S.W., Peterson, L.D. and Alvin, K.F. (1996), "Estimation of reciprocal residual flexibility from experimental modal data", *Am. Inst. Aeronaut. Astronaut. J.*, **34**, 1678-1685.

Deobling, S.W., et al. (1998), "A review of damage identification methods that examine changes in dynamic properties", *Shock Vib. Digest*, **30**, 91-105.

Friswell, M.I., Penny, J.E.T. and Wilson, D.A.L. (1994), "Using vibration data and statistical measures to locate damage in structures, Modal Analysis", *Int. J. Anal. Exper. Modal Anal.*, **9**, 239-254.

Friswell, M.I. and Mottershead, J.E. (1995), "Finite element model updating in structural dynamics", *Kluwer Academic Publishers, Dordrecht.*

González, A., Rowley, C. and Obrien, E. (2008), "A general solution to the identification of moving vehicle forces on a bridge", *Int. J. Numer. Meth. Eng.*, **75**(3), 335-354.

Hoshiya, M. and Maruyama, O. (1987), "Identification of running load and beam system", *J. Eng. Mech. – ASCE*, **113**(6), 813-824.

Jiang, R.J., Au, F.T, and Cheung, Y.K. (2004), "Identification of vehicles moving on continuous bridges with rough surface", *J. Sound Vib.*, **274**, 1045-1063.

Law, S., Chan, T. and Zeng, Q. (1999), "Moving force identification-a frequency and time domain analysis", *J. Dy. Syst. Meas. Control*, **121**(3), 394-401.

Law, S., Wu, S. and Shi, Z. (2008), "Moving load and prestress identification using wavelet-based method", *J. Appl. Mech.*, **75**(2), **021,014.**

Li, J. and Chen, J.(1999), "A statistical average algorithm for the dynamic compound inverse problem", *Comput. Mech.*, **30**(2), 88-95.

Lim, T.W. (1991), "Structural damage detection using modal test data", *Am. Inst. Aeronaut. Astronaut. J.*, **29**, 2271-2274.

Lu, Z.R. and Law, S.S. (2007), "Identification of system parameters and input force from output only", *Mech. Syst. Signal Pr.*, **21**, 2099-2111.

Majumder, L. and Manohar, C. (2004), "Nonlinear reduced models for beam damage detection using data on moving oscillatorbeam interactions", *Comput. Struct.*, **82**(2-3), **301-314.**

MOSAVI.A (2010), "Vibration-based damage detection and health monitoring of bridges", *PHD thesis, Faculty of North Carolina State University, Raleigh, North Carolina, USA.*

Narkis, Y. (1994), "Identification of crack location in vibrating simply supported beam", *J. Sound Vib.*, **172**, 549-558.

Pandey, A.K., Biswas, M. and Samman, MM. (1991), "Damage detection from change in curvature mode shapes", *.J. Sound Vib.*, **145**, 321-332.

Ratcliffe.C.P. (1997), "Damage detection using a modified Laplacian operator on mode shape data", *J. Sound Vib.*, **204**, 505-517.

Ricles, J.M. and Kosmatka, J.B. (1992), "Damage detection in elastic structures using vibration residual forces and weighted sensitivity", *Am. Inst. Aeronaut. Astronaut. J.*, **30**, 2310-2316.

Rizos,.P.F., Aspragathos, N. and Dimarogonas, A.D. (1990), "Identification of crack location and magnitude in a cantilever beam", *J. Sound Vib.*, **138**, 381-388.

Sieniawska, R., Sniady, P. and Zukowski, S. (2009), "Identification of the structure parameters applying a moving load", *J. Sound Vib*., **319**(1-2), **355-365.**

Silva, S., Dias, J.M. and Lopes .J.V. (2008), "Structural health monitoring in smart structures through time series analysis", *Struct. Health Monit.*, **7**(3), 231-244.

Solís, M., Algaba, M. and Galvín, P. (2013), "Continuous wavelet analysis of mode shapes differences for damage detection", *J. Mech. Syst. Signal Pr*., **40**, 645-666.

Srinivasan, G., Massimo, R. and Sathyaneryona, H. (2011), "Computational techniques for structural health monitoring", *Springer*, *Newyork.*

Staszewski, .W.J. (2003), "Structural health monitoring using guided ultrasonic waves", (Eds., Holnicki-Szulc, J,, Soares, C.A.M.), .*Advances in smart technologies in structural engineering*, *Springer*, *Berlin*, 117-162.

Wu, D. and Law, S.S. (2004), "Model error correction from truncated modal flexibility sensitivity and generic parameters. I: Simulation", *Mech. Syst. Signal Pr.*, **18**(6), 1381-1399.

Yu, L. and Chan, T. (2003), "Moving force identification based on the frequency-time domain method", *J. Sound Vib*., **261**(2), 329-349.

Yu, L. and Chan, T. (2007), "Recent research on identification of moving loads on bridges", *J. Sound Vib*., **305**, 3-21.

Zhang, K., Law, S. and Duan, Z. (2009), "Condition assessment of structures under unknown support excitation", *Earthq. Eng. Eng. Vib*., **8**(1), 103-114.

Zhang, Q., Jankowski, L. and Duan, Z. (2010), "Simultaneous identification of moving masses and structural damage", *Struct. Multidiscip. O.*, **42**, 907-922.

Zhu, X. and Law, S. (2006), "Moving load identification on multispan continuous bridges with elastic bearings", *Mech. Syst. Signal Pr.*, **20**(7), 1759-1782.

Zhu, X. and Law, S. (2007), "Damage detection in simply supported concrete bridge structure under moving vehicular loads", *J. Vib. Acoust*., **129**(1), 58-65.

Zhu, X.Q. and Hao, H. (2007), "Damage detection of bridge beam structures under moving loads. Research program report", *School of Civil and Resource Engineering*, *the University of Western Australia.*

Zou, T., Tong, L. and Steve, G.(2000), "Vibration based model-dependent damage (delamination) identification and health monitoring for composite structures—a review", .*J. Sound Vib*., **230**(2), 357-378.

HJ