

# Structural damage detection using a multi-stage improved differential evolution algorithm (Numerical and experimental)

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**Abstract.** An efficient method utilizing the multi-stage improved differential evolution algorithm (MSIDEA) as an optimization solver is presented here to detect the multiple-damage of structural systems. Natural frequency changes of a structure are considered as a criterion for damage occurrence. The structural damage detection problem is first transmuted into a standard optimization problem dealing with continuous variables, and then the MSIDEA is utilized to solve the optimization problem for finding the site and severity of structural damage. In order to assess the performance of the proposed method for damage identification, an experimental study and two numerical examples with considering measurement noise are considered. All the results demonstrate the effectiveness of the proposed method for accurately determining the site and severity of multiple-damage. Also, the performance of the MSIDEA for damage detection compared to the standard differential evolution algorithm (DEA) is confirmed by test examples.

**Keywords:** damage detection; natural frequency; optimization method; multi-stage improved differential evolution algorithm; modal testing

## 1. Introduction

Many structural systems may experience some local damage during their lifetime. Moreover, neglecting the local damage may cause to reduce the age of structural systems or even an overall failure of the structures. Thus local damage should be detected in an early stage before developing. As a result, structural health monitoring and damage identification is a vital topic in industry and engineering that has drawn wide attention from various engineering fields such as civil, mechanical, and aerospace engineering. The fundamental law is that damage will change the stiffness, mass, and damping properties of a structure. Such a change would lead to changes in the static and dynamic responses of the structure. This rule enables us to identify the damage by comparing the response data of the structure before and after damage. During the last decades, many methods have been introduced to detect eventual damage in the structural systems. One type of methods utilizes optimization algorithms for solving the damage detection problem.

Many successful applications of damage detection using optimization algorithms have been reported in the literature. Mares and Surace (1996) used the genetic algorithm (GA) to maximize an objective function in order to identify macroscopic structural damage in elastic structures from measured natural frequencies and mode shapes. A procedure for detecting the damage in beam-type structures based on a micro genetic algorithm using incomplete and noisy modal test data was proposed by Au *et al.* (2003). A Vibration-

based damage detection method in experimental beams using genetic algorithm from measured natural frequencies, mode shape and modal strain energy has been presented by Kim *et al.* (2007). Experimental results revealed that the damage detection is the most accurate when frequency changes combined with modal strain-energy changes are used as the modal features for the proposed method. An application of GA for determining the damage site and extent of flexible bridges maximizing a correlation coefficient, named the multiple damage location assurance criterion (MDLAC) has been proposed by Koh and Dyke (2007). A crack detection method in beam-like structures based on binary and continuous genetic algorithms and a model of the damaged structure has been proposed by Vakil-Baghmisheh *et al.* (2008). Structural damage detection in continuum structures using successive zooming genetic algorithm (SZGA) from natural frequencies has been presented by Kwon *et al.* (2008). It was concluded that the proposed method can find out the exact structural damage of the monitored structure and reduce the time and amount of computation. A two-stage damage detection approach based on subset selection and genetic algorithms has been proposed by Yun *et al.* (2009). In the first stage, the subset selection method was applied for the identification of the multiple damage locations. In the second stage, the damage severities of the identified damaged elements were determined applying SSGA to solve the optimization problem. A two-stage method of determining the location and severity of multiple-beam-type structure damage by using the information fusion technique and micro-search genetic algorithm (MSGGA) has been presented by Guo and Li (2009). A hybrid particle swarm optimization-simplex algorithm (HPSOS) for damage identification in truss-type structures using frequency domain data has been proposed by Begambre and Laier

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(2009). The damage detection in plate structures based on pattern search and genetic algorithms from the modal data has been presented by Ghodrati *et al.* (2011). Structural damage detection using an efficient correlation-based index (ECBI) and a modified genetic algorithm (MGA) has been introduced by Nobahari and Seyedpoor (2011). An application of the bee algorithm (BA) to the problem of crack detection in beams was introduced by Moradi *et al.* (2011). In order to find the location and extent of structural damage, a multi-stage particle swarm optimization (MPSO) assuming a discrete nature for damage variables has been introduced by Seyedpoor (2011). Vakil-Baghmisheh *et al.* (2012) used the hybrid particle swarm–Nelder–Mead (PS–NM) algorithm to minimize an objective function in order to identify crack in cantilever beams from measured natural frequencies. A self-adaptive multi-chromosome genetic algorithm (SAMGA) for localizing and quantifying the damage of truss structures was presented by Villalba and Laier (2012). Nouri Shirazi *et al.* (2014) used the modified particle swarm optimization (MPSO) to minimize an objective function (ECBI) in order to identify structural damage from changes of natural frequencies. A mixed particle swarm-ray optimization together with harmony search (HRPSO) for localizing and quantifying the structural damage was proposed by Kaveh *et al.* (2014). A method for structural damage identification based on chaotic artificial bee colony (CABC) algorithm has been presented by Xu *et al.* (2015). In their study, residuals of natural frequencies and modal assurance criteria (MAC) were used to establish the objective function, then ABC and CABC were utilized to solve the optimization problem. The simulation results indicated that the CABC algorithm can identify the local damage better in comparison with ABC and other evolutionary algorithms, even with noise corruption. A procedure for detecting the crack in cantilever beams based on a modified particle swarm optimization (MPSO) using measured natural frequencies was proposed by Jena and Parhi (2015). The ant colony optimization (ACO) for structural damage identification has been used by Braun *et al.* (2015). In their study, the inverse problem of identification of structural stiffness coefficients of a damped spring-mass system was presented. An application of modified cuckoo optimization algorithm (MCOA) for determining the crack site and extent of cantilever Euler–Bernoulli beam has been presented by Moezi *et al.* (2015). In their study, the determination of a crack location and depth in experimental cantilever beams was formulated as an optimization problem and the location and depth of crack were found by minimizing an objective function which is based on the weighted squares difference of the measured and calculated natural frequencies. It was concluded that the MCOA yields better results than cuckoo, and GA–Nelder–Mead algorithms. The differential evolution algorithm (DEA) for structural damage identification using natural frequencies has been used by Seyedpoor *et al.* (2015). Li and Lu (2015) used the multi-swarm fruit fly optimization algorithm (MFOA) to minimize an objective function in order to identify structural damage from the first several natural frequencies and mode shapes. A two-stage damage

detection method for truss structures using a modal residual vector based indicator and differential evolution algorithm has been presented by Seyedpoor and Montazer (2016). In the first stage, a modal residual vector based indicator (MRVBI) was introduced to locate the potentially damaged elements and reduce the damage variables of a truss structure. Then, in the second stage, a differential evolution (DE) based optimization method was implemented to find the actual site and extent of damage in the structure. Simulation results showed the high performance of the method for accurately identifying the damage location and severity of trusses with considering the measurement noise. Eroglu and Tufekci (2016) used the GA to minimize an error function in order to identify the crack location and depth in experimental beams from measured natural frequencies. A new finite element formulation was presented for straight beams with an edge crack, including the effects of shear deformation, and rotatory inertia. An improved hybrid Pincus–Nelder–Mead optimization algorithm (IP–NMA) for structural damage identification using natural frequencies has been proposed by Nhamage *et al.* (2016). It was concluded that the IP–NMA yields better results than Pincus–Nelder–Mead optimization algorithm (P–NMA), and meta-heuristic harmony search (HS) algorithm, emphasizing its capacity in damage diagnosis and assessment. A hybrid self-adaptive firefly–nelder–mead algorithm (SA–FNM) for structural damage detection has been presented by Pan *et al.* (2016). A particle swarm optimization (PSO) for crack identification in beams using measured natural frequencies has been proposed by Zhang *et al.* (2016). The structure with crack was first modeled by multi-variable wavelet finite element method (MWFEM) so that the vibration parameters of the first three natural frequencies in arbitrary crack conditions can be obtained, which was named as the forward problem. Second, the structure with crack was tested to obtain the vibration parameters of first three natural frequencies by modal testing and advanced vibration signal processing method. Then, the analyzed and measured first three natural frequencies were combined together to obtain the location and size of the crack by using PSO. An application of GA to the problem of crack detection in experimental beams was presented by Ravanfar *et al.* (2016). In their study, a vibration-based damage detection algorithm using a damage indicator called Relative Wavelet Packet Entropy (RWPE) was applied to determine the location and severity of damage. A damage detection method for truss structures using simplified dolphin echolocation algorithm based on modal data has been proposed by Kaveh *et al.* (2016). Wang and Jing (2017) used an improved bacterial optimization algorithm (IBOA) in order to identify fault in complex structures. In their study, a novel method for the damage identification of complex structures based on an optimized virtual beam-like structure approach was proposed. A complex structure can be regarded as a combination of numerous virtual beam-like structures considering the vibration transmission path from vibration sources to each sensor.

In this study, a multi-stage improved differential evolution algorithm (MSIDEA) is introduced to identify

multiple-structural damage. For this, the problem of structural damage detection is first transformed into the standard form of an optimization problem dealing with continuous damage variables. The MSIDEA is utilized as an optimization solver for finding the site and severity of damaged elements. An experimental test and two numerical examples with considering measurement noise are considered to show the performance of the proposed method. The results show that the MSIDEA can provide a robust tool for determining the site and extent of multiple-damage accurately and quickly.

## 2. Optimization based damage detection method

Structural damage detection using non-destructive methods has received a significant attention during the last years. The fundamental law is that damage will change the stiffness properties of a structure. Such a change would lead to change in the dynamic response of the structure. This rule enables us to identify the damage by comparing the dynamic response of the structure before and after damage. The damage detection problem can be interpreted to find a set of damage variables minimizing or maximizing a correlation index between response data of a structure before and after damage (Mares and Surace 1996, Au *et al.* 2003, Koh and Dyke 2007, Vakil-Baghmisheh *et al.* 2008, Guo and Li 2009, Begambre and Laier 2009, Xu *et al.* 2015, Li and Lu 2015, Seyedpoor and Montazer 2016). Therefore, the problem can be transformed into an optimization problem (Gholizadeh 2015, Gholizadeh and Shahrezaei 2015, Gholizadeh and Poorhoseini 2016) as

$$\begin{aligned} \text{Find: } & X^T = \{x_1, x_2, \dots, x_n\} \\ \text{Minimize: } & \text{Obj}(X) \\ \text{Subject to: } & X^l \leq X \leq X^u \end{aligned} \quad (1)$$

where  $X^T = \{x_1, x_2, \dots, x_n\}$  is a damage variable vector containing the location and size of  $n$  unknown damages of structure elements;  $X^l$  and  $X^u$  are the lower and upper bounds of the damage vector and  $\text{Obj}(X)$  is an objective function that need to be minimized.

In many researches, various correlation indices were chosen as the objective function. In this study, an efficient correlation-based index (ECBI) introduced by Nobahari and Seyedpoor (2011) is used as the objective function for the optimization given by

$$\begin{aligned} ECBI(X) = -\frac{1}{2} \left[ \frac{|\Delta F^T \cdot \delta F(X)|^2}{(\Delta F^T \cdot \Delta F)(\delta F^T(X) \cdot \delta F(X))} \right. \\ \left. + \frac{1}{n_f} \sum_{i=1}^{n_f} \frac{\min(f_i(X), f_{di})}{\max(f_i(X), f_{di})} \right] \end{aligned} \quad (2)$$

In the objective function,  $\Delta F$  is the change of frequency vector of damaged structure with respect to the frequency vector of healthy structure. The  $\Delta F$  can be defined as

$$\Delta F = \left\{ \Delta f_i = \frac{f_{hi} - f_{di}}{f_{hi}} \right\}, \quad i = 1, 2, \dots, n_f \quad (3)$$

where  $f_{hi}$  and  $f_{di}$  are the  $i$ th component of healthy frequency vector ( $F_h$ ) and damaged frequency vector ( $F_d$ ) of the structure, respectively. The number of total frequencies considered for damage detection is denoted by  $n_f$ .

Also,  $\delta F(X)$  is the change of frequency vector of an analytical model with respect to the frequency vector of healthy structure. The  $\delta F(X)$  can be defined as

$$\delta F(X) = \left\{ \Delta f_i(X) = \frac{f_{hi} - f_i(X)}{f_{hi}} \right\}, \quad i = 1, 2, \dots, n_f \quad (4)$$

where  $f_i(X)$  is the  $i$ th component of an analytical frequency vector ( $F(X)$ ) of the structure.

The ECBI varies from a minimum value  $-1$  to a maximum value  $0$ . It will be minimal when the vector of analytical frequencies becomes identical to the frequency vector of the damaged structure, that is,  $F(X) = F_d$ .

## 3. The proposed optimization algorithm

The selection of an efficient algorithm for solving the damage optimization problem is a critical issue, because the damage identification problem has many local solutions. In this study, a multi-stage improved differential evolution algorithm (MSIDEA) is proposed to properly solve the damage detection problem. In the remaining part of this section, the original differential evolution algorithm (DEA) is briefly described at first and then the proposed MSIDEA is discussed.

### 3.1 Differential Evolution Algorithm (DEA)

In 1997, a new algorithm, DEA, was proposed by Storn and Price (1997) to solve optimization problems. The ability to finding the global solution and solving the nonlinear problems with a non-differentiable objective function is the main advantages of the algorithm. The framework of DEA is similar to a standard GA, however, the classical crossover and mutation operators in GA have been replaced by alternative operators and consequently came up to a suitable differential operator. DEA can be implemented very easily and requires two parameters tuning. The main steps of original DEA can be summarized as follows (Storn and Price 1997, Das *et al.* 2008):

#### Step 1) Initialization

In this step, the initial parameters, constants and initial population are set. Like other algorithms, DEA begins to search from an initial population. The positions of the  $np$  number of particles (initial population) are initialized randomly.

$$\begin{aligned} X_i &= (x_i^1, \dots, x_i^d, \dots, x_i^n) \\ X^l &\leq X_i \leq X^u \\ i &= 1, 2, \dots, np \end{aligned} \quad (5)$$

where  $x_i^d$  represents the position of the  $i$ th particle in the  $d$ th dimension, while  $n$  is the dimension of the problem;  $X^l$  and  $X^u$  are the lower and upper vectors of a damage variable vector, respectively and  $np$  is the number of initial population that must be at least 4.

**Step 2) Mutation**

For a given vector  $X_i (i=1, 2, \dots, np)$ , a mutant vector is defined by a particular combination of three different current solutions as

$$M_i = X_{r1} + F \cdot (X_{r2} - X_{r3}) \quad (6)$$

$$r1 \neq r2 \neq r3 \neq i$$

where the three different indices  $r1, r2$  and  $r3 \in \{1, 2, \dots, np\}$  are randomly chosen to be different from index  $i$ . Also,  $F \in [0, 1]$  is a real and constant factor which controls the amplification of the differential variation  $(X_{r2} - X_{r3})$ .

**Step 3) Crossover**

In order to increase the diversity of the perturbed parameter vector, crossover is introduced by producing the trial vectors  $U_i (i=1, 2, \dots, np)$  as

$$U_{ji} = \begin{cases} M_{ji} & \text{if } (\text{rand}_{ji} \leq CC \text{ or } j = \text{irnd}_i) \\ X_{ji} & \text{if } (\text{rand}_{ji} > CC \text{ and } j \neq \text{irnd}_i) \end{cases} \quad (7)$$

$$j = 1, 2, \dots, n$$

where  $\text{rand}_{ji}$  is a uniformly random number  $\in [0, 1]$ ,  $CC$  is the crossover constant  $\in [0, 1]$  and  $\text{irnd}_i$  is a random integer  $\in \{1, 2, \dots, n\}$ .

**Step 4) Selection**

For final selection, the trial vector  $U_i$  and target vector  $X_i$  are compared. If the vector  $U_i$  yields a smaller objective function value than  $X_i$ , then  $X_i$  is set to  $U_i$ ; otherwise, the old value  $X_i$  is retained.

**Step 5) Check convergence (Repeat steps 2 to 5)**

In this step, solution convergence is controlled. If the solution is converged, then the optimization is stopped otherwise return to step 2. The best fitness value at the final iteration is considered as the global fitness while the position of the corresponding particle at specified dimensions is taken as the global solution of the problem. The flowchart of DEA can be simply shown in Fig. 1.

**3.2 The multi-stage improved differential evolution algorithm (MSIDEA)**

In this study to improve and speed up the optimization process of DEA, two schemes are used. Firstly, instead of the basic mutation scheme for DEA, the mutation is performed as (Das *et al.* 2008)

$$M_i = X_{\text{best}} + F \cdot (X_{r1} + X_{r2} - X_{r3} - X_{r4}) \quad (8)$$

where  $X_{\text{best}}$  is the best vector of the current population and the four different indices  $r1, r2, r3$  and  $r4 \in \{1, 2, \dots, np\}$  are randomly chosen to be different from index  $i$ .

Secondly, a random variation scheme is used to change the mutation constant,  $F$  of DEA as (Chien *et al.* 2009)

$$F = s \cdot \sqrt{r^2 \cdot dd} - b \quad (9)$$

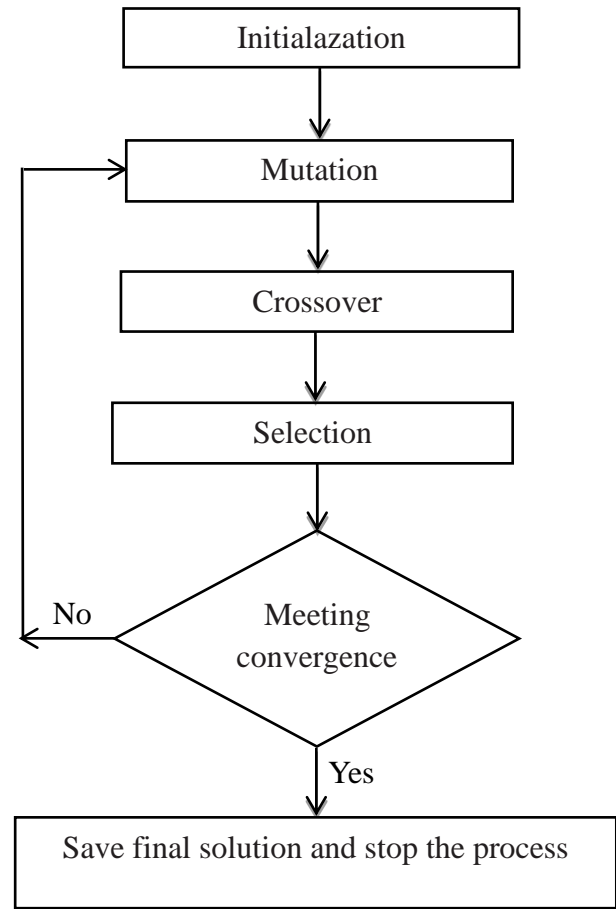


Fig. 1 The flowchart of DEA

where  $r$  is a uniformly random number  $\in [0, 1]$ . Value of the coefficients  $s, b$  and  $dd$  in the expression for  $F$ , was set to 1.5, 0.2 and 0.5, respectively by the authors based on empirical observations.

Also, a multi-stage technique is introduced here to accurately detect the multiple structural damages by improved differential evolution algorithm (IDEA). Based on this algorithm, the location of damaged elements of a structure identified in each optimization stage is imposed on the next optimization stage while the effects of healthy elements on the succeeding stage are neglected. By this approach, all healthy elements are successively eliminated during some stages and the algorithm converges to the correct location and extent of damaged elements. During the optimization stages, the dimensions of optimization problem are decreased gradually and this makes the time and total computational cost of the optimization reduce. The step by step summary of the multi-stage improved differential evolution algorithm (MSIDEA) is as follows:

**Step 1)** Set the initial number of damage variables,  $n$  to the total number of structural elements. Randomly generate the initial position vectors of particles distributed throughout the design space bounded by the specific limits:  $X^l \leq X_i \leq X^u, i = 1, 2, \dots, np$ .

**Step 2)** Employ the IDEA to find the optimal solution,  $X_{IDEA}^T = \{x_1, x_2, \dots, x_n\}$ .

**Step 3)** Find the locations of healthy elements, that is, for all components of damage vector  $X_{IDEA}$  find  $i: x_i = 0$ , and also determine the total number of healthy elements,  $h$ .

**Step 4)** Eliminate the healthy elements from the set of damage variables and reduce the dimension of optimization problem from  $n$  to  $n-h$ .

**Step 5)** Employ a new IDEA stage to find the optimal solution of current stage, i.e.  $X_{IDEA}^T = \{x_1, x_2, \dots, x_{n-h}\}$ .

**Step 6)** Check the convergence by comparing the optimal solutions of two successive optimization stages. If two vectors are identical go to step 7, otherwise go to step 3.

**Step 7)** Save the best (final) optimal solution and stop the optimization process.

According to steps 1 to 7, the flowchart of the MSIDEA can be simply shown in Fig. 2.

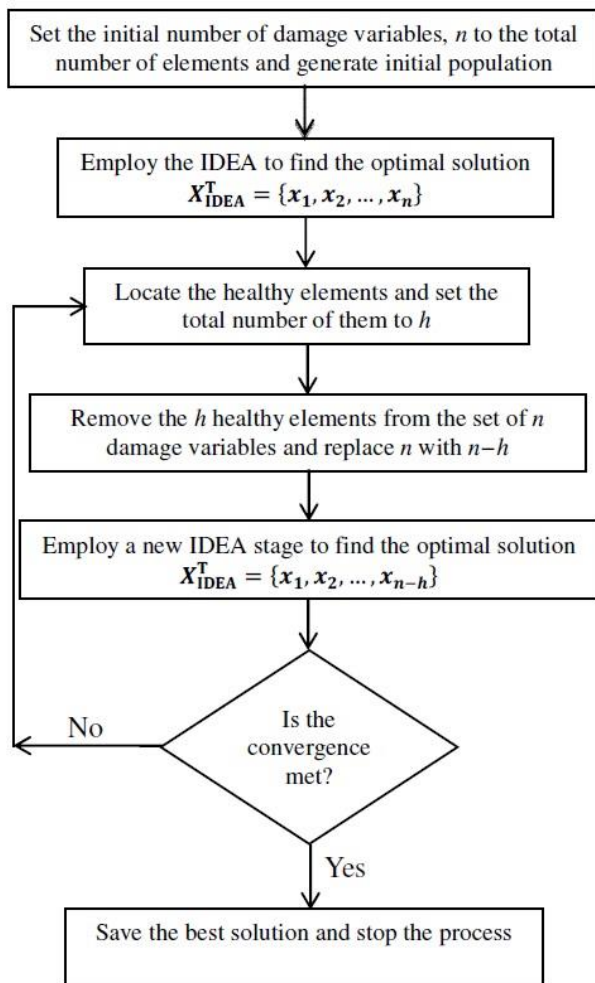


Fig. 2 The flowchart of the MSIDEA

## 4. Experimental test and numerical examples

In order to show the capabilities of the proposed method for identifying the damage, an experimental test and two numerical examples with considering measurement noise selected from the literature are considered. The experimental example is a 10-element cantilevered beam, the first numerical example is a 47-bar planar truss and the second numerical example is a 63-element space frame. In 47-bar planar truss and 63-element space frame, the effect of measurement noise on the efficiency of the method is considered.

### 4.1 Experimental test

To validate the proposed method, a modal test is performed on the 10-element steel cantilevered beam, as shown in Fig. 3. The details of the geometric and material properties of the beam are given in Table 1.

The crack (damage) in the beam similar to a phenomenal cut is generated using an angle grinder. Two cracks are simply shown in Fig. 4.

The cantilevered beam is excited by a hammer (Dytran Impact Hammer 5800B5) at the distance of 900 mm from the fixed end and the dynamic response of the beam is measured using one 2.5 gr miniature accelerometer (Dytran Uniaxial Accelerometer 3035BG) placed at 800 mm from the fixed end. The response measurements are acquired using a signal analyzer (AVANT Lite Dynamic Signal Analyzer Frond End MI-6004).

Table 1 The properties of experimental steel beam

Boundary conditions	Cantilever
Material	Steel (CK 45)
Young's modulus (E)	186.55 GPa
Mass density ( $\rho$ )	7598.04 kg/m <sup>3</sup>
Beam length (L)	1000 mm
Beam width (w)	20 mm
Beam depth (d)	10 mm



Fig. 3 Experimental set up





Fig. 4 Close-up of two cracks in the beam

The frequency response function (FRF) of the beam is acquired using the Data Acquisition and Analysis software. To extract natural frequencies of the cantilever beam from FRF, Modal Genius-330 software are utilized.

The finite element model of the experimental beam having 10 elements and 20 degrees of freedom, are also shown schematically in Fig. 5. The finite element model of the beam is build using Euler-Bernoulli beam elements. The crack in the finite element model is simulated as a relative reduction in the depth of beam section (Sinha *et al.* 2002).

The modal test is conducted on the beam without crack, with a single crack at 350 mm from the fixed end (element 4 in the finite element model) having the crack depth 3 mm (damage extent 0.3) and also with double cracks including previous crack as well as a crack at 650 mm from the fixed end (element 7 in the finite element model) having the crack depth 3 mm (damage extent 0.3). The first four acquired experimental natural frequencies are given in Table 2.

Considering Table 2, two damage scenarios of experimental test, listed in Table 3, are considered and the first four natural frequencies are used for damage detection by the method.

Table 2 The natural frequencies of the beam without crack, with one crack and two cracks

Mode	No crack	$d_{c1}=3$ mm $x_1=350$ mm	$d_{c1}=3$ mm $x_1=350$ mm and $d_{c2}=3$ mm $x_2=650$ mm
	Experimental natural frequencies (Hz)	Experimental natural frequencies (Hz)	Experimental natural frequencies (Hz)
1	8.31	7.92	7.91
2	50.67	49.91	49.53
3	140.38	139.18	137.27
4	278.63	276.29	275.9

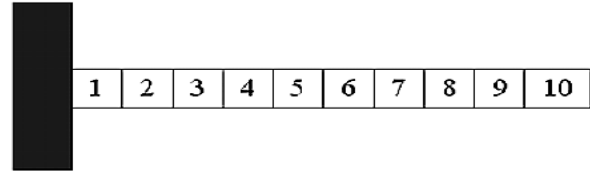


Fig. 5 The finite element model for the experimental beam

Table 3 Two different experimental damage scenarios induced in the 10-element beam

Scenario 1		Scenario 2	
Element number	Damage extent	Element number	Damage extent
4	0.3	4	0.3
-	-	7	0.3

Table 4 The specifications of DEA and MSIDEA

Algorithm	Parameter	Description	Value
DEA	$np$	The number of particles	50
	$maxiter$	The maximum number of iterations	1500
	$F$	The mutation factor	1
	$cc$	The crossover constant	0.5
MSIDEA	$np$	The number of IDEA particles	15
	$maxiter$	The maximum iterations performing by IDEA	150
	$max\_stage$	The maximum number of optimization stages	2
	$cc$	The crossover constant	0.3

The specifications of DEA and the proposed MSIDEA for applying to the damage detection problem are also given in Table 4.

The convergence of the DEA is met when the maximum number of iterations is attained. Also, the convergence of the MSIDEA is met when all optimization stages are attained. In order to consider the stochastic nature of the optimization process using two algorithms, 10 independent sample runs are made for each damage scenario. The damage identification results of damage scenario 1 using two algorithms are given in Tables 5(a) and 5(b), respectively.

The average damage ratios for scenario 1 using two algorithms are also shown in Figs. 6(a) and 6(b), respectively. The damage identification results of damage scenario 2 are given in Tables 6(a) and 6(b), respectively. The average damage ratios for scenario 2 are also shown in Figs. 7(a) and 7(b), respectively.

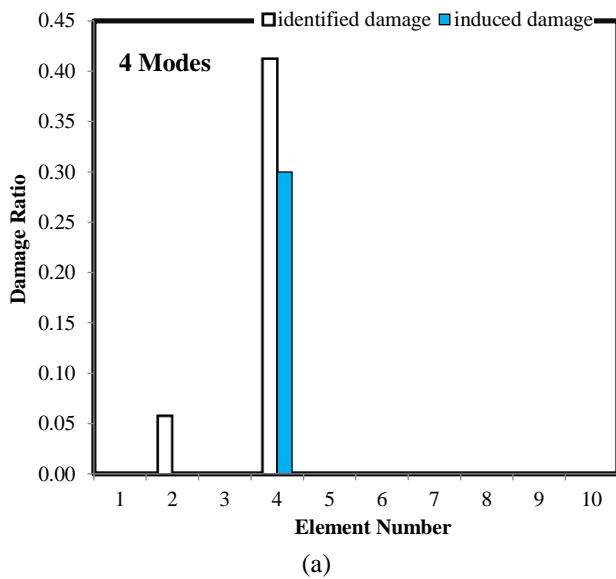
All of the results shown in the tables and figures demonstrate that both DEA and MSIDEA can obtain a good solution, however, the best solution in term of the total number of finite element analyses (FEAs) required are obtained by the MSIDEA. The average number of FEAs requiring for scenarios 1 and 2 by MSIDEA are 4530, while the average number of FEAs needing for scenarios 1 and 2 by DEA is 75050. It is revealed that the MSIDEA has a better performance when compared to the DEA.

Table 5a The damage detection results of 10-element beam for scenario 1 via DEA

Element numbers									
Run number	1	2	3	4	5	...	9	10	Required modal analyses
1	0	0.058	0	0.413	0		0	0	75050
2	0	0.059	0	0.413	0		0	0	75050
3	0	0.058	0	0.413	0		0	0	75050
4	0	0.058	0	0.413	0		0	0	75050
5	0	0.058	0	0.413	0		0	0	75050
6	0	0.058	0	0.413	0		0	0	75050
7	0	0.058	0	0.413	0		0	0	75050
8	0	0.058	0	0.413	0		0	0	75050
9	0	0.058	0	0.413	0		0	0	75050
10	0	0.058	0	0.413	0		0	0	75050
Average	0	0.058	0	0.413	0		0	0	75050
Actual damage	0	0	0	0.3	0		0	0	-
									-1

Table 5b The damage detection results of 10-element beam for scenario 1 via MSIDEA

Element numbers									
Run number	1	2	3	4	5	...	9	10	Required modal analyses
1	0	0.059	0	0.413	0		0	0	4530
2	0	0.059	0	0.412	0		0	0	4530
3	0	0.058	0	0.413	0		0	0	4530
4	0	0.058	0	0.413	0		0	0	4530
5	0	0.058	0	0.413	0		0	0	4530
6	0	0.058	0	0.413	0		0	0	4530
7	0	0.059	0	0.413	0		0	0	4530
8	0	0.058	0	0.413	0		0	0	4530
9	0	0.058	0	0.413	0		0	0	4530
10	0	0.058	0	0.413	0		0	0	4530
Average	0	0.058	0	0.413	0		0	0	4530
Actual damage	0	0	0	0.3	0		0	0	-
									-1



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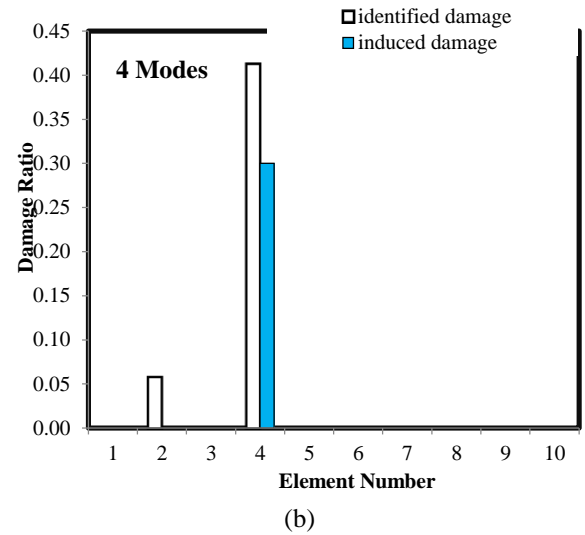


Fig. 6 (a) Final damage ratios of the 10-element beam for scenario 1 via DEA and (b) Final damage ratios of the 10-element beam for scenario 1 via MSIDEA

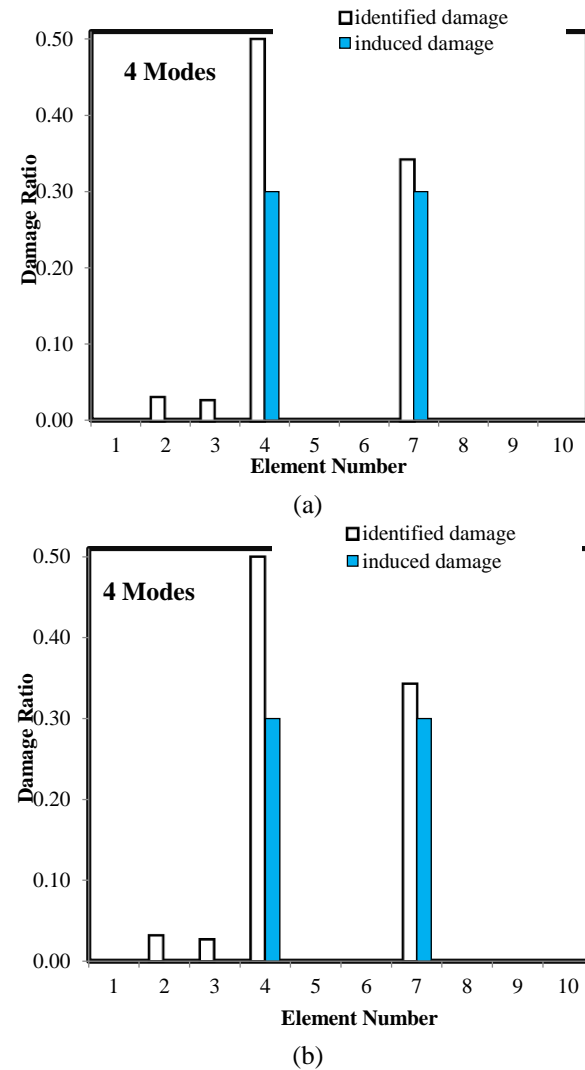


Fig. 7 (a) Final damage ratios of the 10-element beam for scenario 2 via DEA and (b) Final damage ratios of the 10-element beam for scenario 2 via MSIDEA

Table 6a The damage detection results of 10-element beam for scenario 2 via DEA

Element numbers									Required modal analyses	ECBI
Run number	1	2	3	4	...	7	...	10		
1	0	0.032	0.028	0.5		0.343		0	75050	-
2	0	0.032	0.028	0.5		0.343		0	75050	-
3	0	0.032	0.028	0.5		0.343		0	75050	-
4	0	0.032	0.028	0.5		0.343		0	75050	-
5	0	0.032	0.028	0.5		0.343		0	75050	-
6	0	0.032	0.028	0.5		0.343		0	75050	-
7	0	0.032	0.028	0.5		0.343		0	75050	-
8	0	0.032	0.028	0.5		0.343		0	75050	-
9	0	0.032	0.028	0.5		0.343		0	75050	-
10	0	0.032	0.028	0.5		0.343		0	75050	-
Average	0	0.032	0.028	0.5		0.343		0	75050	-
Actual damage	0	0	0	0.3		0.3		0	-	-1



Table 7 Four different damage scenarios induced in 47-bar planar truss

Scenario 1		Scenario 2		Scenario 3		Scenario 4	
Element number	Damage extent	Element number	Damage extent	Element number	Damage extent	Element number	Damage extent
10	0.3	30	0.3	10	0.3	40	0.3
-	-	-	-	30	0.3	41	0.2

Table 8 The damage detection results of 47-bar planar truss for scenario 1 via DEA

Element numbers						Required modal analyses	ECBI
Run number	1	...	10	...	47		
1	0		0.18		0	41360	-0.9794
2	0		0.226		0	51440	-0.9823
3	0		0.243		0	46000	-0.986
4	0		0.307		0	54880	-0.9864
5	0		0.187		0	41520	-0.9727
6	0		0.293		0	42480	-0.9719
7	0.02		0.267		0	53440	-0.9957
8	0		0.231		0	44400	-0.9806
9	0		0.339		0	42320	-0.9716
10	0		0.269		0	55280	-0.9867
Average	0.002		0.244		0	47312	-0.9813
Actual damage	0		0.3		0	-	-1

Table 9 The damage detection results of 47-bar planar truss for scenario 1 via MSIDEA

Element numbers						Required modal analyses	ECBI
Run number	1	...	10	...	47		
1	0		0.441		0	22575	-0.9922
2	0		0.585		0	22575	-0.9853
3	0		0.281		0	22575	-0.9931
4	0		0.72		0	22575	-0.9851
5	0		0.27		0	22575	-0.9869
6	0		0.609		0.02	22575	-0.976
7	0		0.443		0	22575	-0.9858
8	0		0.277		0	22575	-0.9942
9	0		0.347		0	22575	-0.9732
10	0		0.326		0	22575	-0.9963
Average	0		0.43		0.002	22575	-0.9868
Actual damage	0		0.3		0	-	-1

The convergence of the DEA is met when the objective function reaches -0.9973 or the objective function does not considerably change after 100 successive iterations. The convergence of the MSIDEA is met when the objective function reaches -0.9973 or the all optimization stages are attained. In order to consider the stochastic nature of the optimization process using DEA and MSIDEA, 10 independent sample runs are made for each damage scenario. The solutions of DEA and MSIDEA for damage scenarios 1 to 4 are given in Tables 8–15 and Figs. 9–16, respectively.

Table 10 The damage detection results of 47-bar planar truss for scenario 2 via DEA

Element numbers						Required modal analyses	ECBI
Run number	1	...	30	...	47		
1	0		0.263		0	37440	-0.9955
2	0		0.268		0	42800	-0.9962
3	0.04		0.356		0	17600	-0.9975
4	0		0.294		0	35680	-0.9967
5	0		0.27		0	47280	-0.9956
6	0.01		0.396		0	18320	-0.9973
7	0		0.267		0	31440	-0.9973
8	0.078		0.35		0	16160	-0.9973
9	0		0.303		0	19520	-0.9973
10	0		0.371		0	19360	-0.9974
Average	0.014		0.314		0	28560	-0.9968
Actual damage	0		0.3		0	-	-1

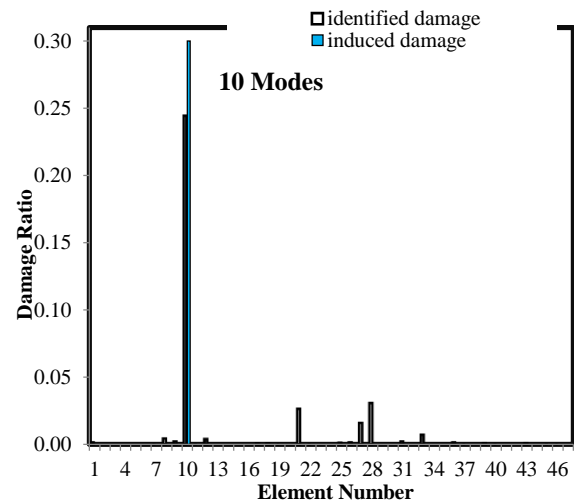


Fig. 9 Final damage ratios of the 47-bar planar truss for scenario 1 via DEA

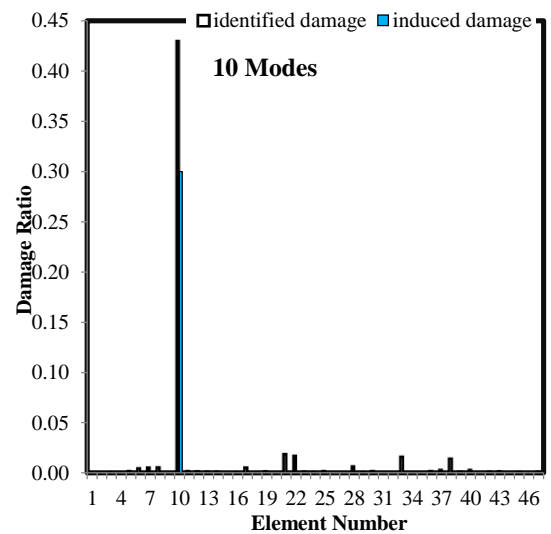


Fig. 10 Final damage ratios of the 47-bar planar truss for scenario 1 via MSIDEA

Table11 The damage detection results of 47-bar planar truss for scenario 2 via MSIDEA

Element numbers						
Run numbers	1	...	30	...	47	
	Required modal analyses					ECBI
1	0		0.262		0	4515 -0.9983
2	0		0.238		0	4515 -0.999
3	0		0.274		0	22575 -0.9965
4	0		0.263		0	4515 -0.9996
5	0		0.263		0	22575 -0.9955
6	0		0.271		0	4515 -0.9994
7	0		0.265		0	22575 -0.9939
8	0.03		0.263		0	13545 -0.9975
9	0		0.303		0	22575 -0.9958
10	0		0.28		0	22575 -0.9934
Average	0.003		0.268		0	14448 -0.9969
Actual damage	0		0.300		0	- -1

Table 12The damage detection results of 47-bar planar truss for scenario 3 via DEA

Element numbers								Required modal analyses	ECBI
Run number	1	...	10	...	30	...	47		
1	0		0.309		0.308		0	40960	-0.9969
2	0		0.293		0.29		0	27200	-0.9975
3	0		0.315		0.291		0	30640	-0.9973
4	0		0.281		0.299		0	40000	-0.9973
5	0		0.259		0.295		0	29360	-0.9976
6	0		0.308		0.264		0	32480	-0.9974
7	0		0.299		0.253		0	50320	-0.9972
8	0		0.29		0.304		0	28960	-0.9974
9	0.02		0.249		0.342		0	26800	-0.9974
10	0		0.325		0.321		0	27600	-0.9973
Average	0.002		0.293		0.297		0	33432	-0.9973
Actual damage	0		0.3		0.3		0	-	-1

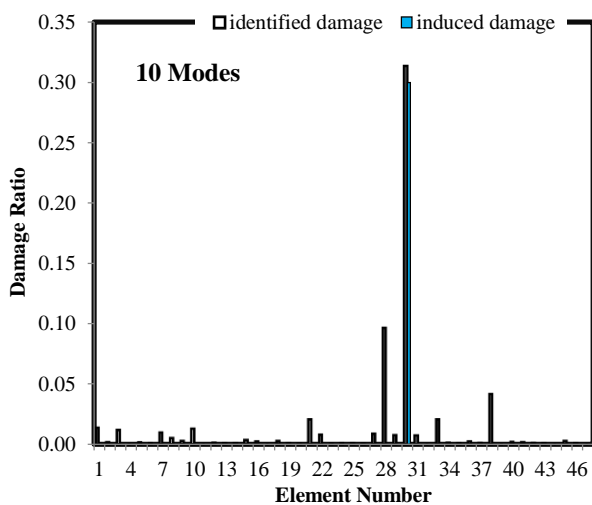


Fig. 11 Final damage ratios of the 47-bar planar truss for scenario 2 via DEA

Table13 The damage detection results of 47-bar planar truss for scenario 3 via MSIDEA

Element numbers								Required modal analyses	<i>ECBI</i>
Run number	1	...	10	...	30	...	47		
1	0		0.303		0.301		0	4515	-0.9979
2	0		0.288		0.296		0	9030	-0.9978
3	0		0.312		0.29		0	9030	-0.9978
4	0		0.293		0.28		0	9030	-0.9988
5	0		0.28		0.286		0	13545	-0.9977
6	0		0.293		0.298		0	9030	-0.9986
7	0		0.284		0.28		0	4515	-0.9988
8	0		0.307		0.285		0	9030	-0.9988
9	0		0.305		0.271		0	9030	-0.9974
10	0		0.287		0.315		0	9030	-0.9983
Average	0		0.295		0.29		0	8579	-0.9982
Actual damage	0		0.3		0.3		0	-	-1

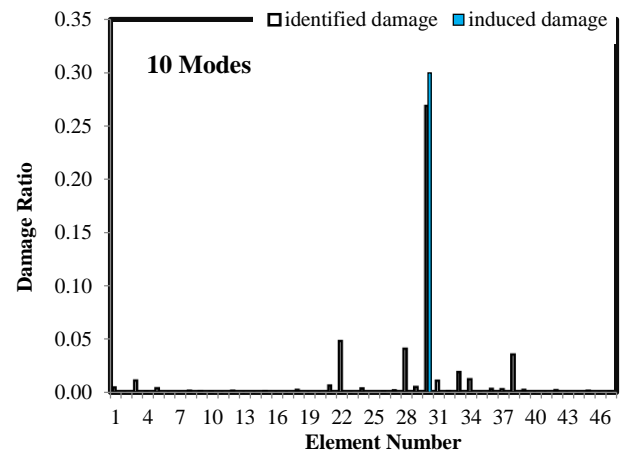


Fig. 12 Final damage ratios of the 47-bar planar truss for scenario 2 via MSIDEA

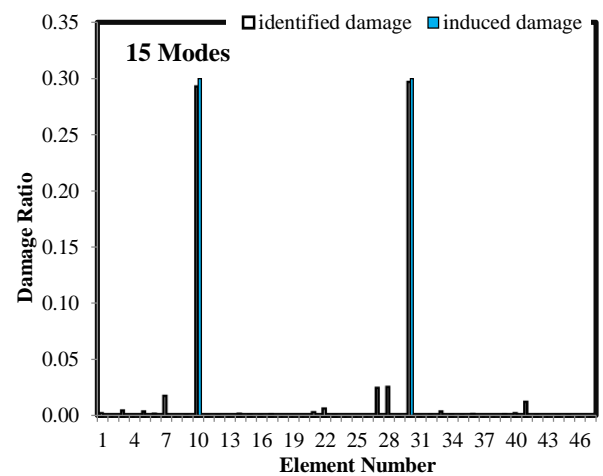


Fig. 13 Final damage ratios of the 47-bar planar truss for scenario 3 via DEA

Table 14 The damage detection results of 47-bar planar truss for scenario 4 via DEA

Element numbers							Required modal analyses	ECBI
Run number	1	...	40	41	...	47		
1	0		0.49	0		0	41200	-0.9689
2	0.01		0.297	0.97		0	59520	-0.9971
3	0		0.359	0.26		0	20880	-0.9973
4	0		0.317	0.196		0	25840	-0.998
5	0		0.3	0.195		0	22320	-0.9977
6	0		0.309	0.21		0	22400	-0.9977
7	0		0.308	0.21		0	30480	-0.9973
8	0		0.284	0.178		0	43600	-0.9971
9	0		0.352	0.25		0	23680	-0.9975
10	0		0.343	0.232		0	24240	-0.9975
Average	0.001		0.336	0.193		0	31416	-0.9946
Actual damage	0		0.3	0.2		0	-	-1

Table 15 The damage detection results of 47-bar planar truss for scenario 4 via MSIDEA

Element numbers							Required modal analyses	ECBI
Run number	1	...	40	41	...	47		
1	0		0.275	0.237		0	22575	-
2	0.01		0.279	0.203		0	9030	-0.998
3	0		0.289	0.216		0	9030	-
4	0		0.29	0.192		0	9030	-0.998
5	0		0.303	0.196		0	22575	-
6	0		0.307	0.198		0	9030	-
7	0		0.3	0.198		0	9030	-
8	0		0.287	0.185		0	4515	-
9	0		0.249	0		0	22575	-0.99
10	0		0.298	0.196		0	22575	-
Average	0.001		0.288	0.182		0	13997	-
Actual damage	0		0.3	0.2		0	-	-1

All of the results shown in the tables and figures demonstrate that the best solutions in terms of actual damage identification and the total number of FEAs required are obtained by means of the MSIDEA. The average number of FEAs requiring for scenarios 1, 2, 3 and 4 of MSIDEA are 22575, 14448, 8579 and 13997, respectively, while the average number of FEAs needing for DEA are 47312, 28560, 33432 and 31416, respectively. It is revealed that the MSIDEA has a better performance when compared to the DEA and a great number of FEAs are saved.

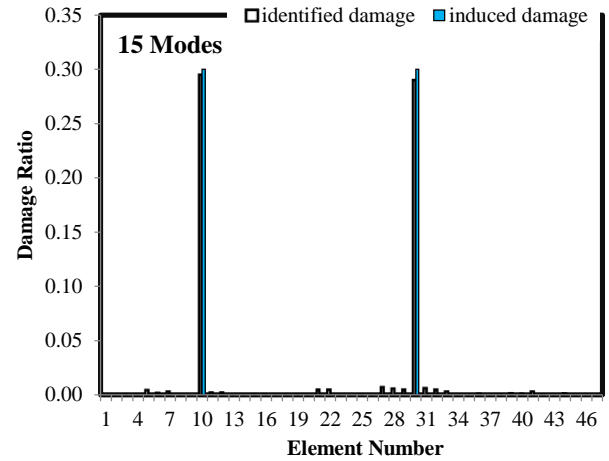


Fig. 14 Final damage ratios of the 47-bar planar truss for scenario 3 via MSIDEA

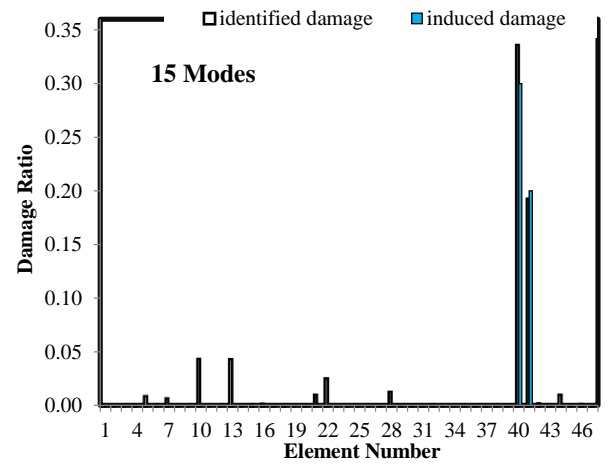


Fig. 15 Final damage ratios of the 47-bar planar truss for scenario 4 via DEA

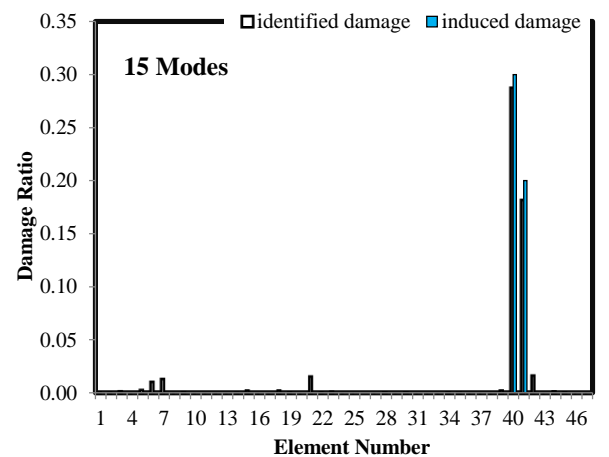


Fig. 16 Final damage ratios of the 47-bar planar truss for scenario 4 via MSIDEA

#### 4.3 Sixty three-element space frame

A six-story space frame (Seyedpoor 2011, Seyedpoor *et al.* 2015) as depicted in Fig. 17 is considered as the second numerical example. The structure has 63 elements and 36 nodes. The frame is modeled using the finite element method, leading to 180 degrees of freedom. The sections used for the beams and columns are W-shape. The area, inertia moments,  $I_z$ ,  $I_y$  and  $I_o$  of each element are  $0.02 \text{ m}^2$ ,  $0.02 \text{ m}^4$ ,  $0.01 \text{ m}^4$  and  $0.01 \text{ m}^4$ , respectively. The modulus of elasticity is 200 GPa and the material density is  $7850 \text{ kg/m}^3$ .

The beams and columns have length  $L_1=7.32\text{m}$  and  $L_2=3.66 \text{ m}$ , respectively. Damage in the structure is also simulated as a relative reduction in the elasticity modulus of individual elements. Three different damage scenarios are considered as listed in Table 16. For identifying the damage scenario 1, the first 11 natural frequencies and for identifying the damage scenarios 2 and 3, the first 15 natural frequencies of the structure are considered. The measurement noise is considered here by polluting the natural frequencies using a standard error of  $\pm 0.15 \%$ .

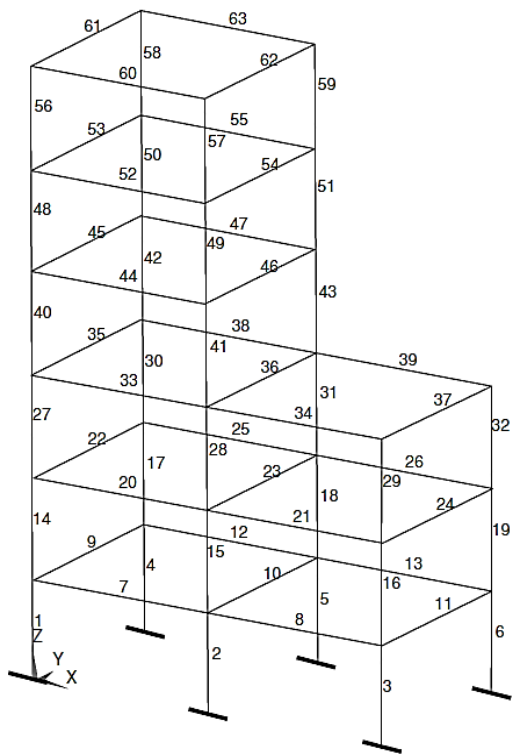


Fig. 17 The finite element model for the 63-element space frame

Table 16 Three different damage scenarios induced in 63-element space frame

Scenario 1		Scenario 2		Scenario 3	
Element number	Damage extent	Element number	Damage extent	Element number	Damage extent
10	0.3	3	0.4	9	0.1
-	-	6	0.4	10	0.2
-	-	-	-	11	0.3

In this example, the DEA could not converge to an appropriate solution, accordingly only the results of MSIDEA have been reported here. For identifying all the damage scenarios using MSIDEA, particle numbers, the maximum number of iterations and the maximum number of optimization stages are set to 30, 150 and 4, respectively. Also, for identifying the damage scenarios,  $cc$  is set to 0.3.

The convergence of the MSIDEA is met when all optimization stages are attained. In order to consider the stochastic nature of the optimization process, 10 independent sample runs are made for each damage scenario. The damage identification results for damage scenarios 1, 2 and 3 using MSIDEA are given in Tables 17, 18 and 19, respectively. The average damage ratios for scenarios 1, 2 and 3 are also shown in Figs. 18, 19 and 20, respectively.

Table 17 The damage detection results of 63-element frame for scenario 1 via MSIDEA

Element numbers										Required modal analyses	ECBI
Run number	1	...	5	...	10	...	31	...	63		
1	0		0		0.06		0		0	18120	-0.953
2	0		0		0.278		0		0	18120	-0.972
3	0		0		0.197		0		0	18120	-0.978
4	0		0		0.259		0		0	18120	-0.95
5	0		0		0.313		0		0	18120	-0.984
6	0		0		0.018		0		0	18120	-0.963
7	0		0		0.201		0		0	18120	-0.968
8	0		0		0.263		0		0	18120	-0.972
9	0		0		0.131		0		0	18120	-0.971
10	0		0		0.253		0		0	18120	-0.953
Average	0		0		0.197		0		0	18120	-0.966
Actual damage	0		0		0.3		0		0	-	-1

Table 18 The damage detection results of 63-element frame for scenario 2 via MSIDEA

Element numbers										Required modal analyses	ECBI
Run number	1	...	3	...	6	...	31	...	63		
1	0		0.398		0.381		0		0	18120	-0.9994
2	0		0.399		0.397		0		0	18120	-0.9995
3	0		0.386		0.396		0		0	18120	-0.9995
4	0		0.39		0.406		0		0	18120	-0.9997
5	0		0.437		0.306		0		0	18120	-0.9995
6	0		0.355		0.41		0		0	18120	-0.9994
7	0		0.342		0.387		0		0	18120	-0.9995
8	0		0.399		0.404		0		0.03	18120	-0.9992
9	0		0.4		0.394		0		0	18120	-0.9996
10	0		0.399		0.392		0		0	18120	-0.999
Average	0		<b>0.39</b>		<b>0.387</b>		0		0.003	18120	-0.9994
Actual damage	0		0.4		0.4		0		0	-	-1

Table 19 The damage detection results of 63-element frame for scenario 3 via MSIDEA

Element numbers										Required modal analyses	ECBI
Run number	1	...	9	10	11	...	31	...	63		
1	0		0.272	0.411	0.467		0		0	18120	-0.988
2	0		0.079	0.22	0.277		0		0	18120	-0.995
3	0		0.253	0.412	0.53		0		0	18120	-0.993
4	0		0.083	0.174	0.262		0		0	18120	-0.995
5	0		0.115	0.199	0.275		0		0	18120	-0.993
6	0		0.129	0.185	0.258		0		0	18120	-0.99
7	0		0.202	0.441	0.513		0		0	18120	-0.99
8	0		0.113	0.157	0.293		0		0	18120	-0.993
9	0		0.019	0.145	0.188		0		0	18120	-0.99
10	0		0.053	0.233	0.268		0		0	18120	-0.991
Average	0		0.132	0.258	0.333		0		0	18120	-0.992
Actual damage	0		0.1	0.2	0.3		0		0	-	-1

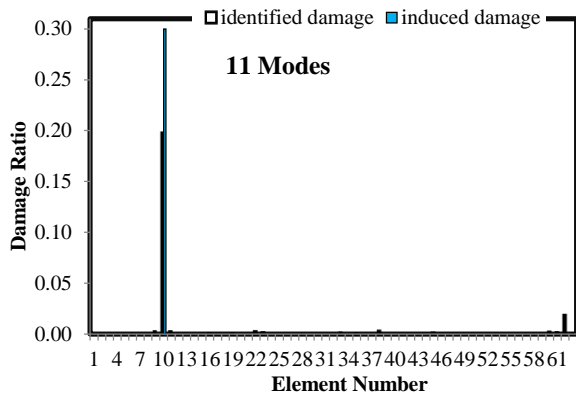


Fig. 18 Final damage ratios of the 63-element frame for scenario 1 via MSIDEA

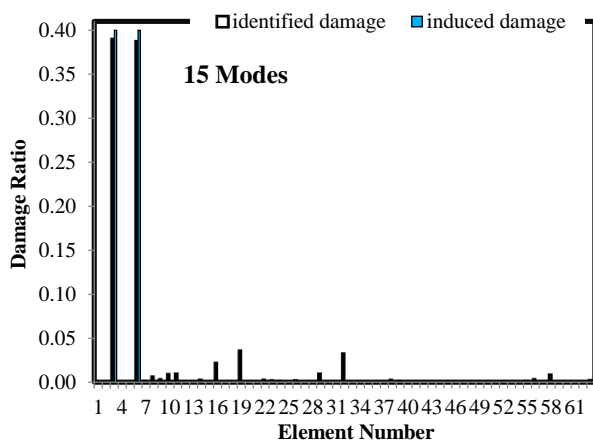


Fig. 19 Final damage ratios of the 63-element frame for scenario 2 via MSIDEA

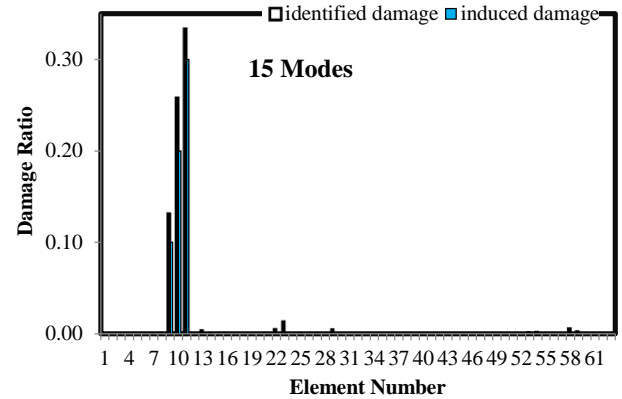


Fig. 20 Final damage ratios of the 63-element frame for scenario 3 via MSIDEA

As can be seen in the tables and figures, the MSIDEA proposed here can accurately detect the damage sites and extent for most of the simulations. The average number of FEAs requiring for scenarios 1, 2 and 3 of MSIDEA are 18120.

## 5. Conclusions

An efficient optimization procedure has been introduced to solve the problem of structural damage detection that is a highly nonlinear problem with a great number of local solutions. The structural damage detection problem is firstly formulated as a standard optimization problem aiming to minimize an *ECBI* for finding real damage variables. The MSIDEA is proposed to properly solve the optimization problem. In order to assess the competence of the proposed approach for structural damage detection, an experimental test and two numerical examples with considering measurement noise are tested. The results demonstrate that the combination of *ECBI* and MSIDEA can provide a robust tool for damage detection. The results of the proposed approach (MSIDEA+*ECBI*) have shown a high performance for the method when compared with actual damage induced and those of the DEA.

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