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Damage detection in plate structures using frequency response function and 2D-PCA

Faramarz Khoshnoudian* and Vahid Bokaeian

Faculty of Civil Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran

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Abstract. One of the suitable structural damage detection methods using vibrational characteristics are damage-index-based methods. In this study, a damage index for identifying damages in plate structures using frequency response function (FRF) data has been provided. One of the significant challenges of identifying the damages in plate structures is high number of degrees of freedom resulting in decreased damage identifying accuracy. On the other hand, FRF data are of high volume and this dramatically decreases the computing speed and increases the memory necessary to store the data, which makes the use of this method difficult. In this study, FRF data are compressed using two-dimensional principal component analysis (2D-PCA), and then converted into damage index vectors. The damage indices, each of which represents a specific condition of intact or damaged structures are stored in a database. After computing damage index of structure with unknown damage and using algorithm of lookup tables, the structural damage including the severity and location of the damage will be identified. In this study, damage detection accuracy using the proposed damage index in square-shaped structural plates with dimensions of 3, 7 and 10 meters and with boundary conditions of four simply supported edges (4S), three clamped edges (3C), and four clamped edges (4C) under various single and multiple-element damage scenarios have been studied. Furthermore, in order to model uncertainties of measurement, insensitivity of this method to noises in the data measured by applying values of 5, 10, 15 and 20 percent of normal Gaussian noise to FRF values is discussed.

Keywords: structural health monitoring; damage index; frequency response function; two-dimensional principal component analysis; lookup tables; plate

1. Introduction

In the science of structural engineering, structural damage is defined as a series of changes in the material or geometric characteristics of the structure, which influences the performance of the structure presently or in the future. These changes may include a change in the dynamic characteristics of the structure, changes in boundary conditions, changes in the overall system integrity and various other items. In this definition, structural damage can be determined by comparing structural characteristics of the intact and the existing (probably damaged) structures (Moaveni *et al.* 2011).

In this regard, studies that assess the causes of damage in structures include behavioral research of materials (such as fatigue and corrosion) and structural analysis (maximum stress and maximum strain). But trying to answer the questions such as how to identify damage in the structure and progress in structural damage results in the emergence of new cluster entitled structural health monitoring (Balageas *et al.* 2006).

The need for more comprehensive systems which do not require visual inspections for detecting structural damage, led to the focus of numerous studies and research on the methods based on the changes of vibrational properties of structures. The principal objective of vibration-based structural monitoring methods is that, structural modal parameters (such as frequencies, mode shapes and modal damping) are function of the physical properties of the structure such as mass, stiffness and damping. Thus, change in these physical properties results in the change in the structural modal parameters.

One of the dynamic characteristics that can be employed for detecting structural damage is the frequency response function of the structure. Simplicity of practical measurement and also the need for less equipment and tools can be seen as the advantages of using FRF (Fang *et al.* 2005). Since the FRF data mining is done via seismic tests conducted on structures and there is no need for experimental modal analysis on the structures, this information is more reliable and contains detail information concerning the status of the structures (Huynh *et al.* 2005).

Numerous studies have been conducted on damage detection techniques using FRF data directly (Choudhury 1996) or its derivatives such as FRF curvature (Sampaio *et al.* 1999) and FRF difference (Trendafilova and Heylen 2003). Esfandiari *et al.* (2013) in a study presented a method for identifying damages in any number that would reduce the structural stiffness using the natural frequencies and FRF. In this study, the sensitivity of natural frequencies were regarded as a second order function of the stiffness reduction of elements. Numerical and experimental results given in this paper indicate excellent performance of the

*Corresponding author, Professor
E-mail: khoshnud@aut.ac.ir

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proposed method without the need for an accurate model of the damaged structure. Shadan *et al.* (2016a) in a study using FRF data, presented a method for identifying the location and extent of damage in truss structures by means of a sensitivity equation. In fact, this sensitivity equation provided a relationship between FRF changes and changes in structural parameters and by solving this equation by least squares method, the parameters of the damaged structure were obtained. Shadan *et al.* (2016b) also experimentally validated their proposed method.

FRF is appreciably sensitive to noise and environmental effects and that is why there is the likelihood of inaccurate or wrong detection of damage. On the other hand, the FRF data directly have high volume and this dramatically decreases the computing speed and increases the memory needed to store the information, which makes the use of this method difficult. Data compression techniques such as principal component analysis (PCA) decreases for all observations based on a feature index and categorization of similar observations. Zang and Imregun (2001) proposed a method for slight damage detection using FRF data, artificial neural networks (ANNs) and a data compression technique based on PCA. This method required sufficient sensor distribution along the monitored structure. Furthermore, the measurement points had to be selected carefully to have reliable measured data. With the extension of this method, the researchers invented two-dimensional principal component analysis (2D-PCA). In 2D-PCA, data compression and pattern extraction techniques are used directly. Unlike traditional PCA method, 2D-PCA method performs based on two-dimensional input data instead of one-dimensional vectors. 2D-PCA has two major advantages compared to the traditional PCA approach: Firstly, in this method, there is the likelihood of calculating the precise covariance matrix, secondly, this method requires less time to calculate the eigenvectors. Yang *et al.* (2004) indicated that 2D-PCA, has better speed and performance than the traditional PCA in extracting patterns and decreasing data dimensions especially when data dimensions are not high. Khoshnoudian and Talaei (2016) proposed a method of damage identification using FRF data and 2D-PCA technique. They also used ANNs and Lookup tables (LUT) as pattern recognition techniques. These researchers numerically evaluated their method using 2D truss and 2D frame structures with 13 and 36 degrees of freedom, respectively. They reported that the LUT method was more effective in filtering the influence of the measured data noises than the ANNs. Khoshnoudian *et al.* (2016) assessed 2D-PCA method for compressing FRF data and identifying damage as a suitable method. These methods have not been applied on structures with high number of degrees of freedom, such as plates yet. Plate structures usually required to be modeled with significantly higher number of DOFs. Therefore, damage detection in plate structures can be a serious challenge.

So far, numerous studies have been conducted on the identification of damages in the plates. Fan and Qiao (2012), using the strain energy information presented a method for identifying damages in plate structures. The fundamental stages of the method proposed by these researchers include choosing modes sensitive to damage, damage location and damage severity detection. As other structural response-based damage detection techniques, this method required modal information of the structure such as natural frequencies, mode shapes curvature prior to and after the damage. This method has excellent accuracy in the damages with moderate and low severity. Increased amount of noise has decreasing effect on the performance of this damage recognition method.

Zhang *et al.* (2013) presented a method for detecting damage in plate structures using the frequency shift surface curvature. The numerical results of this study propose that if the damage takes place near the nodal points, the detection using this method is impossible.

Xiang and Liang (2012) in a study offered a two-stage method for detecting damages in thin plates. In the first stage, the location of damage was identified by applying two-dimensional wavelet transform to mode shapes and recognizing singularities. After identifying the damage location on the plate and at the next step, particle swarm optimization algorithm (PSO) was used to detect the severity of damages. In this study, the sinusoidal noise effect up to 3% was studied.

Vo-Duy *et al.* (2016) identified the damage in the composite plates using modal strain energy and improved differential evaluation algorithm (IDEA). In the first phase of the study, elements with high damage potential were recognized using modal strain energy. Subsequently, using an improved differential assessment algorithm, they tried to reduce the error in mode shapes and determine the extent of damage in elements. Results of this study revealed that the proposed approach was relatively acceptable in identifying damages in composite plate structures, and modal data with or without noise pollution (up to 3 percent). The effect of higher levels of noise has not been examined in this study.

Yam *et al.* (2002) investigated the sensitivity of damage recognition parameters using dynamic and static methods in the plate structures and finally they offered their damage indices in static and dynamic modes. These researchers introduced the displacement curvature as the best index of structural damage in static condition. In dynamic mode, the use of a combination of these two indices had the best accuracy in identifying the damage.

In another study, Xu *et al.* (2015) identified damages in the plates using two-dimensional mode shape curvature obtained via the wavelet and Teager energy transform. The results obtained from the use of this method in recognizing the damage in numerical and experimental models of an aluminum plate showed that this method has better performance in identifying little damages and in dealing with noise-polluted data compared to traditional methods of using two-dimensional curvature of mode shape.

Huh *et al.* (2015) in a study assessed vibratory power and then identified the damage in plate structures using the measured accelerations from different parts of the structure. In this study, the vibratory power is defined as energy transfer rate through the cross section with unit width in a vibrating structure. Numerical and experimental results of this study demonstrated acceptable accuracy of this method in identifying the location and severity of damage. Although this method has been less accurate in identifying angled cracks.

Damage index-based methods are a branch of damage detection techniques that by applying several types of data such as shift changes, modal changes and changes in natural frequencies, anticipate damages (Gomes and Silva 2008). In general, damage index-based methods examine and compare data extracted from the existing structure (a structure in which there is a likelihood of damage) and intact structures, using mathematical methods (Chen and Xu 2007). One of the disadvantages of most of the methods suggested so far is the impossibility of identifying damages in the presence of noise above 10 percent. The majority of structural health monitoring techniques based on pattern recognition have controversial performance in the recognition of multiple damages in structures. In this study, a damage index has been presented to identify structural damage in plate structures. The major characteristic of these structures is the high number of degrees of freedom (DOFs) that can cause difficulty in identifying the damage correctly. Many of these DOFs are rotational DOFs that exciting and measuring them are virtually impossible. In this study, FRF data using two-dimensional principal component analysis (2D-PCA) are compressed and converted into damage index vectors. The performance of the suggested damage index in plate structures with various dimensions and boundary conditions and several damage scenarios (damage in one element, damage in two and three elements simultaneously) have been studied. FRF data are polluted with random noise values from zero to 20 percent in order to simulate experimental conditions.

2. Methodology

The overall process employed in this study for identifying the damage in plate structures is that, the considered structure is modeled with finite elements method and using mass and stiffness matrix of the model, FRF of each structure in certain DOFs entitled as measurement and excitation DOFs are obtained. After choosing the appropriate frequency ranges (frequency ranges with reasonable distance from modal peaks, and less affected by the damping of the structure, in order to avoid damping modeling errors (Shadan *et al.* 2016a) to study the condition of the structure, the FRF data are compressed by 2D-PCA and converted into vectors. At the next step, components of these vectors that have satisfactory sensitivity to the damage and are also less sensitive to noise in the data are chosen as the structural damage indices (DIs). This process is carried out for each structure with single and multiple element damage cases (each damage case contains several damage scenarios) and a database including different amounts of DIs in various states of damage in studied structures is created. After creating the DI database, the DI for the real structure is calculated and by examining the database using a lookup table, the closest DI to existing damage indices is identified. Thus, the location and severity of damage are measured. This method has been previously evaluated for structures with low number of DOFs (Khoshnoudian and Talaei 2016).

2.1 Frequency Response Function (FRF)

FRF of a structural system is a transfer function in the frequency domain. This function is defined as the ratio of displacement formed in the structure to the force applied to the structure. In fact, it specifies the displacement response of the structure under the forces acting on it in terms of the frequency as Eq. (1).

$$[M][\ddot{x}(t)]+[C][\dot{x}(t)]+K[x(t)]=[f(t)] \tag{1}$$

In the above equation M, C, and K, are respectively matrices of mass, damping and stiffness of the structure and $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ are respectively vectors of acceleration, velocity and displacement of the structure at the time t. In addition, f(t) is the vector of the force applied at DOFs of the structure.

If the force applied on the structure is considered as a harmonic force, the force and the displacement of the structure at the moment t is written as follows using Fourier transform (Chopra 1995) as Eqs. (1) and (2).

$$[f(t)]=[F(\omega)]e^{i\omega t} \tag{2}$$

$$[x(t)]=[X(\omega)]e^{i\omega t} \tag{3}$$

In the above equations, ω is the excitation frequency and X(ω) and F(ω) are displacement and force applied on the structure in the frequency domain, respectively. Substituting Eqs. (2) and (3) in Eq. (1), Eq. (4) is obtained

$$[X(\omega)]=[H(\omega)].[F(\omega)] \tag{4}$$

In the above equation, H(ω) is the structural response in the frequency domain and is defined as the FRF. As is evident, FRF is derived as the ratio of the structural response function to excitation function, and it can be obtained in practical applications using this equation. In general, H(ω) will be a square N×N matrix where N is the number of free DOFs of the structure considered. By excitement of the structure in DOF *i* and measuring the structural response in DOF *j*, element $H_{ij}(\omega)$ of FRF matrix can be obtained. Thus obtaining the full matrix of FRF requires excitation in all DOFs and measurement in all DOFs.

In order to deal with dimensionless FRF data, matrix of FRF ratio is defined as Eq. (5).

$$\overline{H}=H_{Damaged}/H_{Intact} \tag{5}$$

In this equation, $H_{Damaged}$ is obtained from the structure with a certain damage severity and location (damage scenario), and H_{Intact} is the FRF of the intact structure. The division is done element by element.

In this method, due to the use of FRF, it is essential to define and choose excitation and measurement DOFs. Since excitation and measurement in the vertical transitional DOFs is practically simpler than in-plate rotational DOFs, thus in numerical examples, six transitional DOFs have been chosen as excitation DOFs and six transitional DOFs as measurement DOFs.

2.2 Two-Dimensional Principal Components Analysis (2D-PCA)

The FRF matrix of damaged structures that is reduced by choosing a limited number of excitation and measurement points, can be used itself for detecting the structural state. However, it is preferable that FRF data dimensions be reduced further by the principal component analysis for the following reasons:

Firstly, the storage of such a high volume of FRF data in lookup tables will require a lot of time and memory. Because of the limited capacity of CPUs and the significance of time in damage detection, compressed data are more desirable.

Secondly, if the data received from the sensors are noisy, noise will influence all data and elimination of the effect of noise will be impossible. Whereas the use of principal component analysis, using concepts of eigenvalues and eigenvectors, will filter the effect of noise on some PCs.

In order to reduce dimension of the FRF matrix by 2D-PCA method, it is assumed that X is a j-dimensional column vector. The reason for the calculations presented in this section is to image the matrix of FRF ratio $\overline{\mathrm{H}}$ using a linear transformation on the vector X as in Eq. (6).

$$Y = \overline{H}X \tag{6}$$

Therefore, the vector Y, which is named the characteristic projected vector of matrix of FRF ratio$\overline{\mathrm{H}}$, will be obtained.

The overall dispersion of samples shows a good measure of the strength of distinctive of vector X. This dispersion can be stated as the sum of the main diagonal elements of covariance matrix of projected characteristic vectors, as presented in Eq. (7).

$$J(X) = tr(S_X) \tag{7}$$

S_x represents the covariance of projected characteristic vectors of all samples and $tr(S_x)$ is the sum of the main diagonal elements. S_x can be explained as Eqs. (8) and (9)

$$\begin{aligned} S_X &= E(Y-EY)(Y-EY)^T = E[\overline{H}X - E(\overline{H}X)][\overline{H}X - E(\overline{H}X)]^T \\ &= E[(\overline{H} - E\overline{H})X][(\overline{H} - E\overline{H})X]^T \end{aligned} \tag{8}$$

Therefore

$$tr(S_X) = X^T[E(\overline{H} - E\overline{H})^T(\overline{H} - E\overline{H})] \tag{9}$$

Then in identifying the main concept, the following matrix is defined as presented in Eq. (10)

$$G_t = E[(\overline{H} - E\overline{H})^T(\overline{H} - E\overline{H})] \tag{10}$$

G_t is called the dispersion matrix while its dimensions are j×j and all the elements are non-negative. G_t can be calculated directly from the sample matrix of FRF ratio.

Assuming that generally there are M samples of the frequency response matrix with dimensions i×j and the *n*-th sample of that is $\overline{\mathrm{H}}_{\mathrm{n}}, \{n = 1,2,\dots,M\}$ and the mean of all the samples is $\overline{\overline{\mathrm{H}}}$, G_t then may be represented in the form of Eq. (11).

$$G_t = \frac{1}{M}\sum_{j=1}^{M}(\overline{H}_j - \overline{\overline{H}})^T(\overline{H}_j - \overline{\overline{H}}) \tag{11}$$

Thus, the concept presented in Eq. (7) can be rewritten in the form of Eq. (12).

$$J(X) = X^T G_t X \tag{12}$$

Where X is a column vector. The concept of J(X) is called the overall dispersion index. X vectors that cause this index attained its maximum are called the optimal mapping axes. Optimized mapping axes $X_1, X_2, \dots, X_d$ are the vectors that maximize J(X) and therefore maximize the overall diffusion of projected samples as Eq. (13).

$$\begin{cases} [X_1, X_2, \dots, X_d] = \arg\max J(X) \\ X_i^T X_j = 0, i \neq j \end{cases} \tag{13}$$

In fact, these vectors are the largest number of eigenvalues of this matrix. These vectors are used for extracting features of mapping matrices.

For FRF ratio matrix of the sample $\overline{\mathrm{H}}$, it can be said ain Eq. (14) that

$$Y_k = \overline{H}X_k, k = 1,2,\dots,d \tag{14}$$

Feature vectors $Y_1, Y_2, \dots, Y_d$ achieved from this relationship are the principal component vectors of FRF ratio matrix $\overline{\mathrm{H}}$.

Embedding these vectors together with the matrix $B = [Y_1, Y_2, \dots, Y_d]$, the dimensions of i×d will be obtained which is named property matrix of FRF$(\overline{\mathrm{H}})$.

Thus far, one of the dimensions of the FRF matrix is reduced. By performing this operation once more on transpose matrix B, the other dimension of the matrix will also decrease and ultimately will result in matrix $\overline{\mathrm{B}}$ while dimensions of c×d will be achieved (d <j and c <i).

The matrix $\overline{\mathrm{B}}$ can rearrange in the form of a vector. M-dimensional vector resulting from this rearrangement (m = c×d) in this study, is regarded as the principal components that may be employed as a model for identifying structural state. Among the various elements of output vector of 2D-PCA process, elements that are less sensitive to different amounts of noise probably have better performance in filtering noise effects in the data and thus such elements are chosen as damage index vectors.

In this study, FRF data are compressed by applying 2D-PCA method and damage indices are calculated for various damage scenarios. The effects of noise on damage indices as a result of the use of the concept of eigenvalues is such that all elements of each index are not similarly affected by noise of the data, and thus some of the elements of damage index that are less sensitive to noise are used to evaluate the damage. For this aim, FRF data are polluted with normal Gaussian noise with zero mean and standard deviations of

0.25, 0.5, 0.75 and 1 representing 5%, 10%, 15% and 20% noise respectively. Since damage index vectors in fact represented different structural states and are used to identify the severity and location of damage in the structure, it seems that the principal components that are less sensitive to different amounts of noise causes higher likelihood of accurate damage detection in the presence of noises. It is evident that by eliminating noise-sensitive outputs, a part of structural status information is actually disregarded. Thus, the damage detection process may face some troubles if the damage indices are not chosen carefully.

2.3 Forming database of various damage indices

Assuming a reasonable number of possible structural damage cases (including single, double and triple damaged elements simultaneously, with various severities) and calculating DI for all damage scenarios of the damage cases, a database including a variety of damage scenarios and related damage indices can be obtained. In the process of structural health monitoring using input data received from the sensors, FRF of the real structure will be obtained and the above process will calculate the DI of the structure. Subsequently, the location and the severity of damages can be determined by searching the database mentioned and finding the closest DI in the database to DI of the present structure. In this study, lookup table method have been used to search through the damage indices.

To use lookup tables in order to search the created database and discover the closest DI in the database to DI of the existing structure, Eq. (15) is used.

$$\arg\min(\left|[DI]_{current} - [DI]_i\right|), i = 1,2,...,m \qquad (15)$$

In this case, m is the total number of damage indices available in the database, $[\mathrm{DI}]_{\mathrm{current}}$ is the damage index for the current structure and $[\mathrm{DI}]_{\mathrm{i}}$ is the *i*-th damage index in the database. In other words, the distance of each DI vector in database is calculated with DI of the existing structures.

It should be noted that the distance between two vectors A and B is calculated by Eq. (16).

$$\left|[A] - [B]\right| = \sum_{j=1}^{n} (A_j - B_j)^2 \qquad (16)$$

Where n is the number of components of vectors A and B. Since each of the damage indices in the database corresponds to the damage scenario stated with known location and severity, by finding the closest damage index to the damage index of the existing structure, the location and severity of the current damage can be evaluated.

At this stage, different states of damage are applied to the FE model of structures, and accuracy of the damage detection method is examined. For example, in the case of single-element damage, damages of 1 to 60% are applied in any single structural element as decrease in the stiffness matrix of that element in the structural model. Damage indices of all damage scenarios are stored in the database.

In order to investigate the ability of the proposed damage index in detecting multiple damages, various two and three simultaneous element damage cases are also studied. It should be noted that in multiple damage cases, because of the large number of permutations of the elements and the impossibility of evaluation of all likely scenarios, a number of scenarios are chosen for investigation.

Damage indices calculated are stored in the database and afterwards, various damage scenarios are applied on the structure (as unknown damages).In each case, the current damage index is calculated and by searching the database, the closest damage index to the current damage index is specified and the severity and location of the structural damage are determined. The lookup table method has been employed to search the database.

2.4 The process of the damage detection in the reference example

In this section, the entire process of detection of single-element damage scenarios in 7-meter plate with boundary conditions four clamped edges (4C) is given as an example. Meshing of this structure is illustrated in Fig. 1.

Red and blue dots highlighted on the figures are the nodes and their vertical DOFs have been chosen respectively as excitation and measurement DOFs. The thickness of this structure is equal to 5cm. The structure is made of steel with elastic modulus of 200 GPa, density of 7850 kg/m^3 and Poisson's ratio of 0.3.

In the first step, FRF of intact structure and damaged structure under all studied damage scenarios is measured in excitation and measurement DOFs. An example of structural FRFs obtained from FE model for intact and damaged structures under a specific damage scenario has been demonstrated in Fig. 2. Khoshnoudian and Talaei (2016) showed that damping has an essential effect on the FRF data near the resonance zones (natural Frequencies) and the anti-resonance zones (which are the local minimums of the FRF data). Thus, frequency ranges far from these zones that are less affected by damping seem to be appropriate for the damage detection process. In this study, appropriate FRF frequency domains are chosen between 5th and 12th mode shapes for each structure by trial and error. Although higher frequencies are more sensitive to damages, there are practical limitations for high frequency excitation.

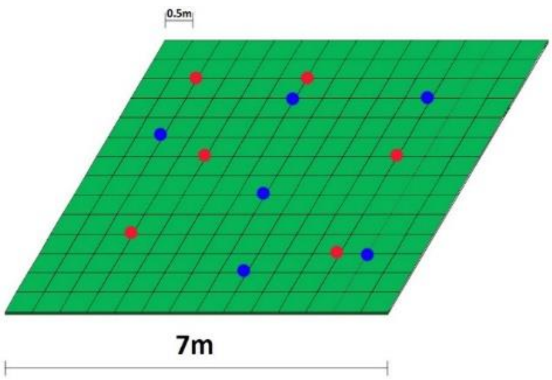


Fig. 1 Meshing of the reference example

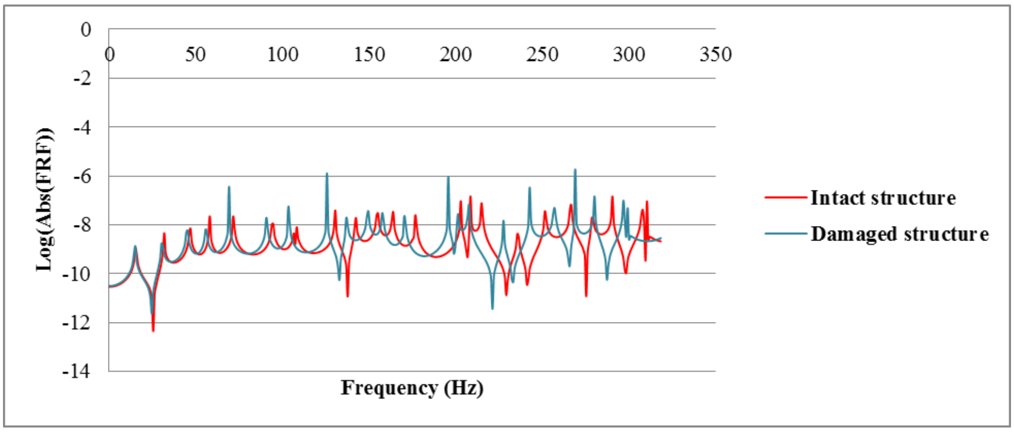


Fig. 2 An example of structural FRFs obtained from FE model for intact and damaged structures under a specific damage scenario

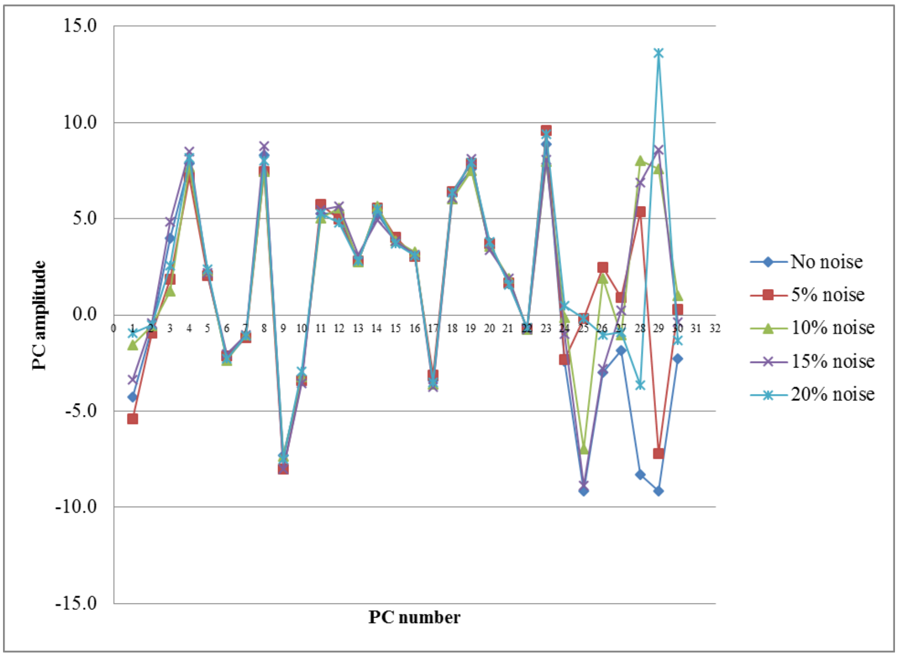


Fig. 3 An example of principal components changes in a certain damage scenario for different amounts of noises

In the next step, after choosing the appropriate frequency ranges from available FRFs, FRF data are compressed by 2D-PCA method. Principal components (PCs) are the outputs of 2D-PCA process, while the inputs are the FRF data, which are affected by many factors such as damage properties and amount of noise. Thus, some PCs can be found less sensitive to different noise levels, and selecting them as damage indices can lead to reduction of the noise effect and uncertainties involved in the problem. An example of these output vectors (principal components) in a certain damage scenario for different amounts of noises is shown in Fig. 3.

It should be noted that in practical cases, data compression techniques are applied at extend experimental datasets of repeated measurements which vary due to numerous factors, e.g. noise. So, results of 2D-PCA (PCs) per noise level will not be provided in practical cases.

However, in order to establish an efficient damage detection platform, some PCs that are less noise sensitive must be selected to form damage index (DI) vectors. Each DI vector represents a certain damage scenario, and will be saved into a database. The DI of the real structure condition can be obtained and be looked up among the DIs database to detect the nearest probable scenario and estimate the location and severity of the probable damage. Considering Fig. 3 (as a sample), it is observed that the effect of noise (even at 20%) on the 4th to 23rd elements of the PC matrix is low. Thus, by selecting such PCs as damage index, the effect of noise by the investigated algorithm on FRF data can be filtered and the sensitivity of the proposed damage index to noise decreased. Hence, these elements have been used to create the database of the DIs and other elements are excluded from the process of structural health monitoring.

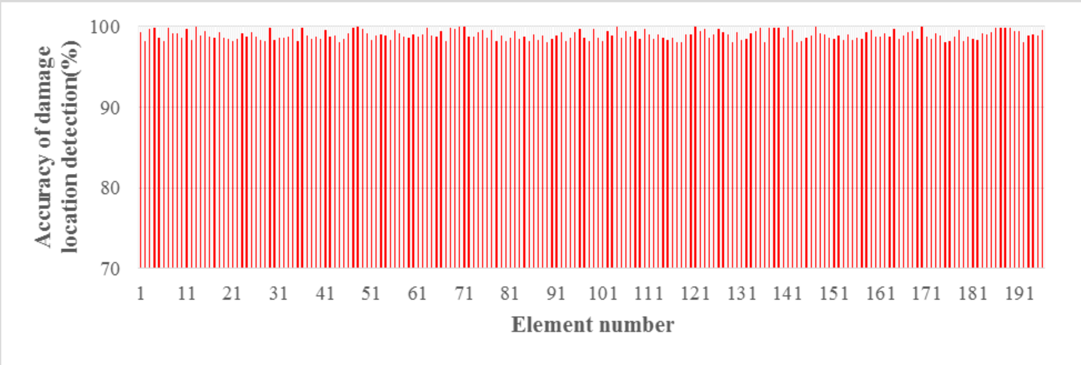


Fig. 4 An example of accuracy of damage location detection vector (single-element damage case- 7m plate)

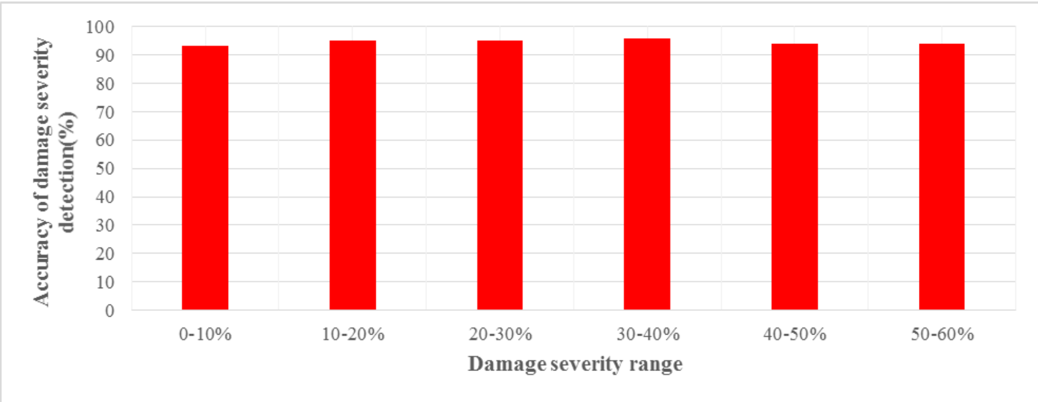


Fig. 5 An example of accuracy of damage severity detection vector (single-element damage case- 7m plate)

After saving the damage indices for different damage cases in the database, it is time to identify the damage in a real structure. For this reason, by calculating the DI for real damaged structure and using lookup tables, the closest DI vector in the program database to real DI is and given that, the severity and position of damage are assessed.

At the end of the damage detection process, in each structure and for each damage scenario and for each level of noise pollution in the data, a vector of accuracy of damage location detection and a vector of accuracy of damage severity detection are obtained. An example of these vectors for a damage condition is presented in Figs. 4 and 5.

In these bar graphs, as an example, accuracy of damage location detection for single-element damage scenarios in 7m plate structure (196 elements) can be observed in percent. Given that providing information in this way has resulted to high number of diagrams and made it difficult to interpret and analyze the results, results for each graph will be reported as three numbers (minimum, maximum and average accuracy).

3. Numerical results

In this study, to assess the performance of the proposed damage index provided to identify the damage in plate structures, different structures are modeled with different dimensions and boundary conditions. The geometry of these structures are square in plan with dimensions of 3, 7 and 10 meters. Meshing of these structures are illustrated in Fig. 6.

In order to have better comparison of the results, six excitation and measurement DOFs (transitional) are determined for each structure. These DOFs are arbitrarily selected, but some practical limitations of exciting and measuring the response of a plate structure have been considered. It should also be noted that in practical cases, the choice of adjacent input and output stations could introduce erroneous results. Moreover, accuracy of the damage detection method would probably increase, if more excitation and measurement DOFs were chose. The FRF data can be obtained from each measurement point and each excitation point by vibration tests. Thus, 36 sets of the FRF data will be obtained from six excitation and six measurement DOFs which represent the plate vibration properties. Although in practice, the type of excitation directly affects the quality of the obtained FRF data, it is assumed in numerical simulations that these data finely represent the response of the actual structure. Furthermore, for each of these structures, three different boundary conditions are considered and nine different structures have been studied.

The information of the models used and damage cases investigated in this paper are presented in Tables 1 and 2.

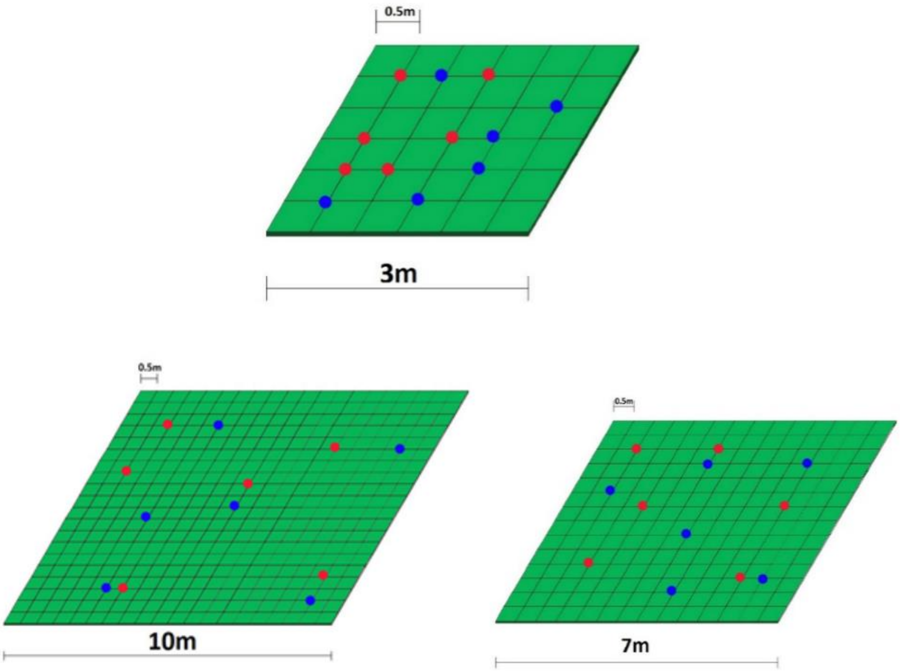


Fig. 6 Dimensions and meshing of the plate structures modeled in this study

Table 1 Models, damage cases and noise levels investigated in this paper

Plate dimensions	3, 7, 10 m (square)
Boundary conditions	Four clamped edges (4C) Three clamped edges with a free side (3C) Four simply-supported edges (4S)
Damage cases	One element Two elements Three elements
Noises applied to FRF data	0%, 5%, 10%, 15%, 20%

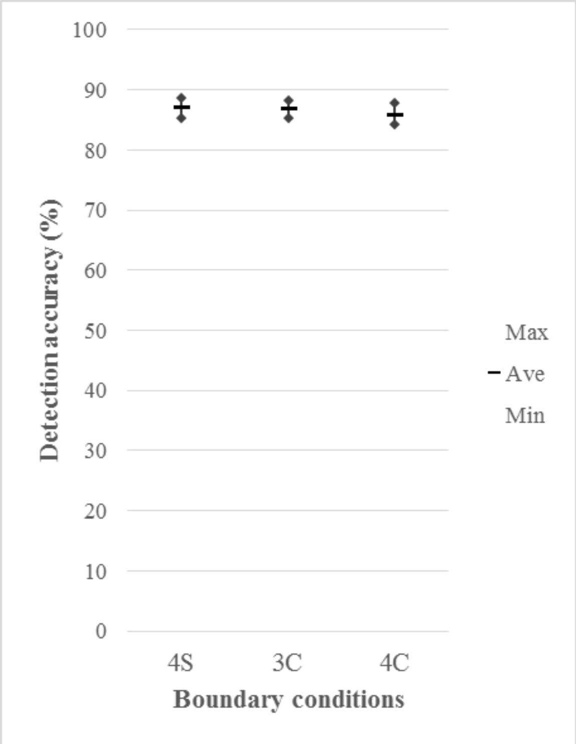
Table 2 Number of damage scenarios in each damage case

Damage Case / Structure	1-element*	2-element	3-element
3 m	36	36	36
7 m	196	196	196
10 m	400	400	400

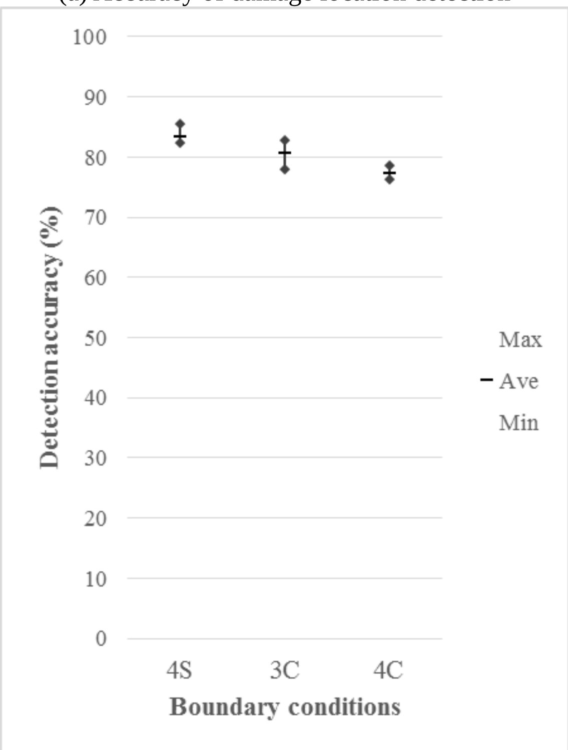
* Damage in every single element of the structures is considered as a 1-element damage scenario; each 2-element damage scenario is formed by adding one randomly selected middle element to 1-element scenarios; each 3-element damage scenario is formed by adding two randomly selected middle elements to 1-element scenarios

3.1 Effect of different boundary conditions on the performance of damage detection using the proposed damage index

Since change in the boundary conditions of a plate structure results in change in stiffness matrix and consequently change in its FRF, investigation of how this method works in detecting the location and severity of damage of the structures with different boundary conditions (BCs) appears necessary. For this purpose, structures with three types of boundary conditions with different clamping (four simply supported edges, three clamped edges, four clamped edges) have been considered. In this section, the results of damage location and severity detection accuracy in each of these BCs are presented. The results concerning the range of damage severity and location detection in the 10 m plate are shown in Figs. 7(a) and 7(b).



(a) Accuracy of damage location detection



(b) Accuracy of damage severity detection

Fig. 7 Effect of different boundary conditions on damage detection accuracy of the 10 m plate

By investigating these figures, it is observed that on the plate with dimension of 10 meters, the boundary conditions of four simply supported edges had the highest average damage location and severity accuracy and four clamped edges had the lowest value. The overall difference is not very obvious.

For instance, the 10 m plate with four simply supported edges (4S) has the average damage location and severity detection accuracy of respectively 87 and 83 percent. These values for the plate with three clamped edges are respectively 86 and 81, and for the plate with four clamped edges, they are 85 and 77 percent.

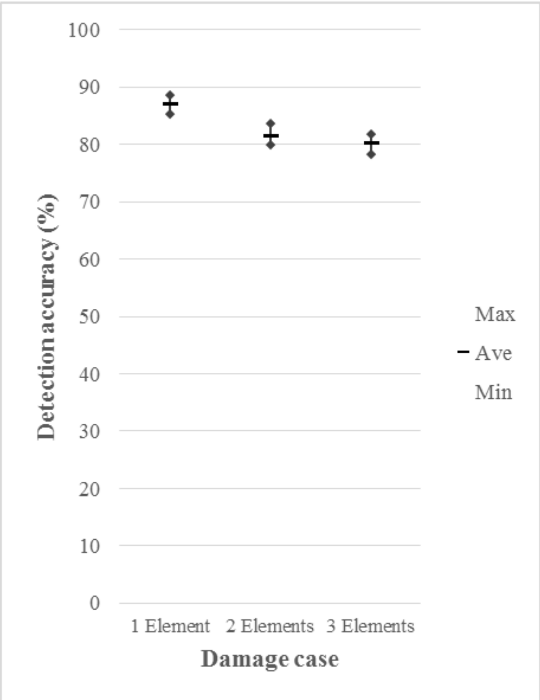
This general trend of the reduction of accuracy that is established in the other damage scenarios (other damage scenarios will be discussed in the next section), states that with the existing meshing by increasing the fixedness of the structure, the damage detection accuracy is relatively reduced. Since in the side and corner elements of the FE model a number of DOFs are constrained and these constrained DOFs are excluded from the stiffness matrix of the entire structure, hence damage in these boundary elements compared to middle elements of the structure have less participation in the structural stiffness matrix in free DOFs. Therefore, detection of damage scenarios that comprise these corner and side elements are conducted with less accuracy. By increasing the clamping in boundary conditions of the structure, edge elements will have less number of free DOFs. Therefore damage in these elements has less effect in structural stiffness matrix in free DOFs, and damage detection in scenarios including these elements and thus damage detection in the entire structure is done with slightly less accuracy. It should be mentioned that this difference between detection accuracies with different BCs may vary due to change in the dimension of the structure. In other words, although the detection accuracy generally decreases as the structure becomes more fixed, the absolute decrease value probably differs in structures of different dimensions. Results of the other structures are shown and discussed in section 3.3.

3.2 Effect of complexity of damage cases on the performance of damage detection method

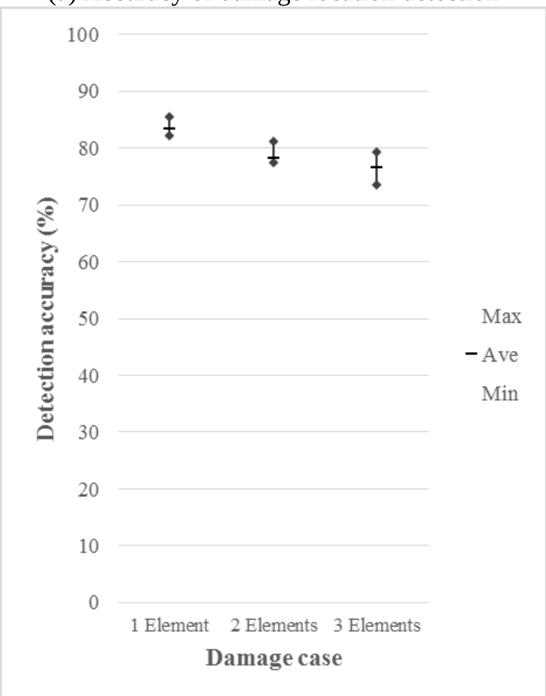
In this section, performance of the proposed damage index in the detection of damage cases with different complexities is examined. For this aim, as stated in the previous sections, the single-element damage cases (as the simplest form of structural damage), two-element damage cases, and finally three-element damage cases (as complex damage cases) are concurrently applied to the structures and the damage detection accuracy is investigated.

Graphs related to damage detection accuracy in damage cases of plate structures with dimension of 10m are indicated in Figs. 8(a) and 8(b).

By evaluating these figures, it is obvious that generally in all boundary conditions of the structures with dimension of 10m, the range of changes in average accuracies in various cases was low and even it can be said that damage detection in different damage cases is virtually carried out with the same accuracy. However, single-element damage case has the highest damage severity and location detection accuracy and the lowest values of accuracy is observed in damage cases containing three damaged elements simultaneously. For instance, in the 10m structure with the boundary conditions of 4S, the average accuracy of detecting the location and severity of damage in the 1-element damage case are respectively 87 and 83 percent, 82 and 78 percent in 2-element damage case, and 80 and 77 percent in 3-element damage case.



(a) Accuracy of damage location detection

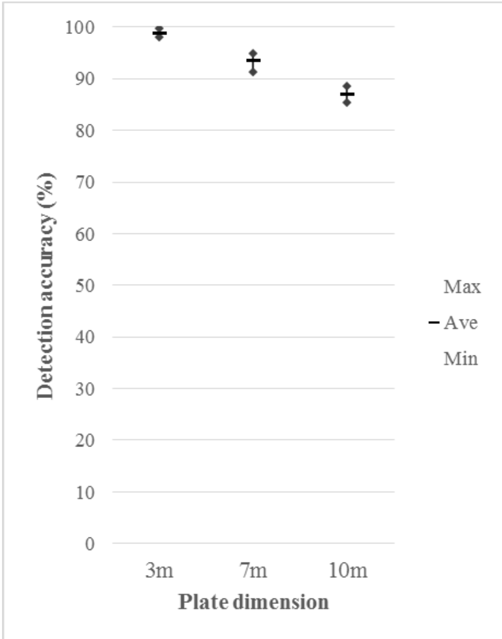


(b) Accuracy of damage severity detection

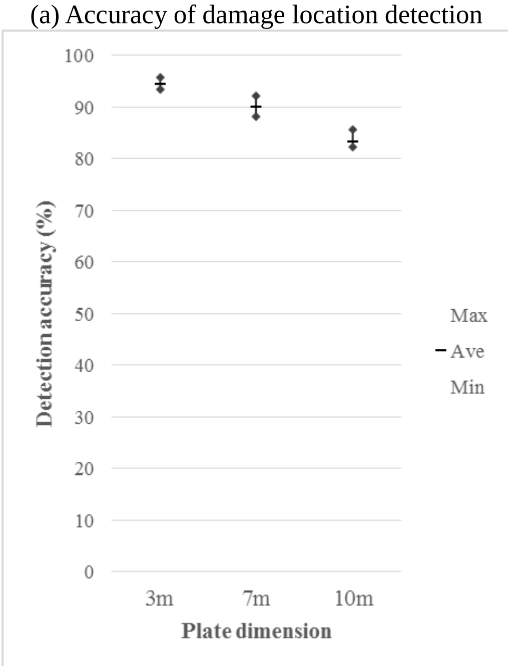
Fig. 8 Detection accuracy of different damage cases – 10 m 4S plate

This implies that the more complex damage scenario in the structure reduces the accuracy of the current damage detection method. This could be because of the disruption of selected DIs, and interference of these indices and also large changes in FRF matrix as a result of the complexity of the damage scenario. However, this reduction in the accuracy is insignificant and it can be said that this method has a good performance in identifying more complex damage scenarios.

3.3 Effect of change in plate dimensions on damage detection accuracy



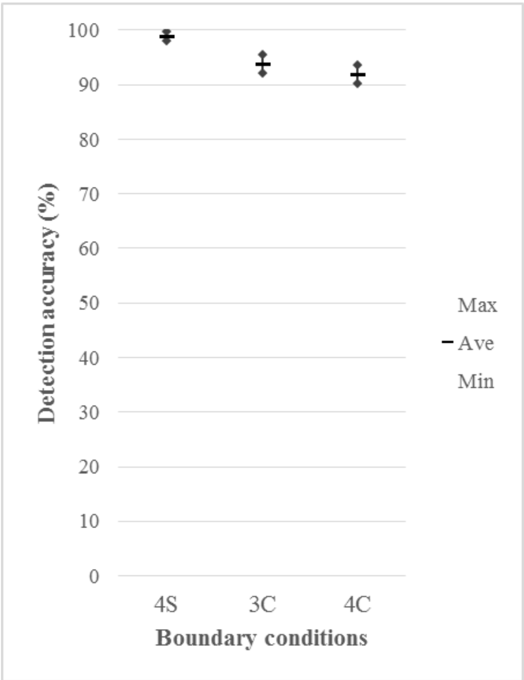
(a) Accuracy of damage location detection



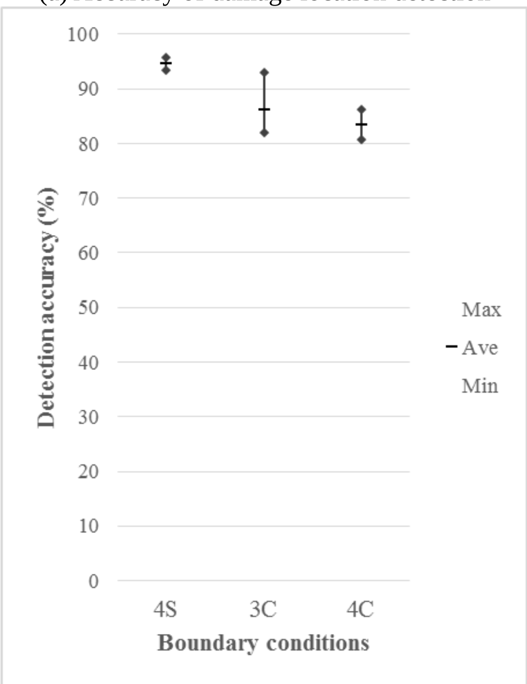
(b) Accuracy of damage severity detection

Fig. 9 Damage detection accuracy in plates with different dimensions - 4S plates- 1 Element damage

As stated earlier, change in dimensions of plate structures is one of the parameters investigated in this study. Thus, the results of damage detection accuracy for different dimensions of the plate are given below. Because the results of the various boundary conditions have been stated and examined in other ways in the previous sections, in this section diagrams of the boundary conditions of 4S under various damage cases are illustrated in Figs. 9(a) and 9(b). Evidently, the general trend of the results in other boundary conditions is similar to this mode.



(a) Accuracy of damage location detection

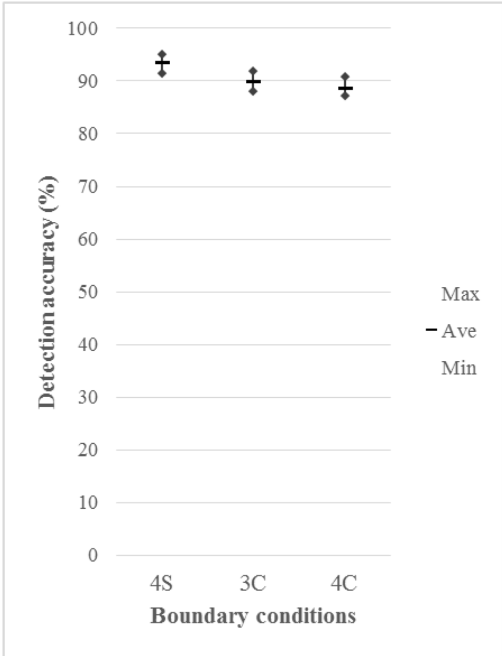


(b) Accuracy of damage severity detection

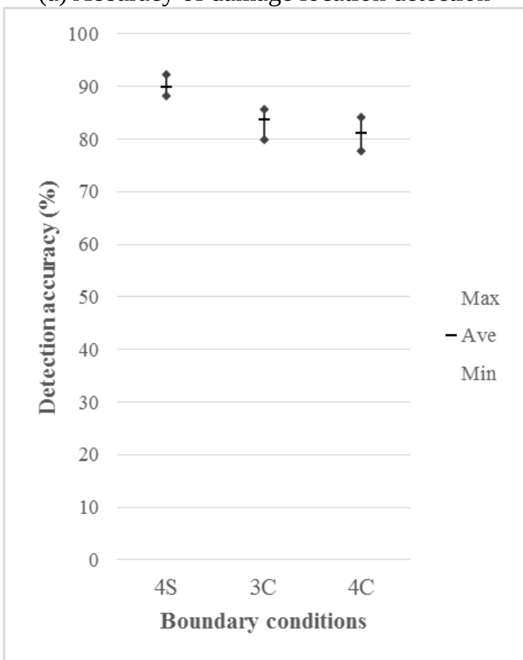
Fig. 10 Effect of different boundary conditions on damage detection accuracy of the 3 m plate

By evaluating these figures, it can be observed that the 3m structure has had the highest accuracy and 10m structure has been associated with the lowest accuracy. In fact, for a given damage scenario applied to a structure with a given boundary condition, the more plate dimension result in less damage detection accuracy.

For instance, in single-element damage condition of the plate with the 3 m dimension, the average accuracy for the location and severity of damage is respectively 99 and 95 percent. For 7 m plate, it is 94 and 90 percent and for the 10m plate, it is 87 and 83 percent.



(a) Accuracy of damage location detection



(b) Accuracy of damage severity detection

Fig. 11 Effect of different boundary conditions on damage detection accuracy of the 7m plate

Table 3 Models, damage cases and noise levels investigated in this paper

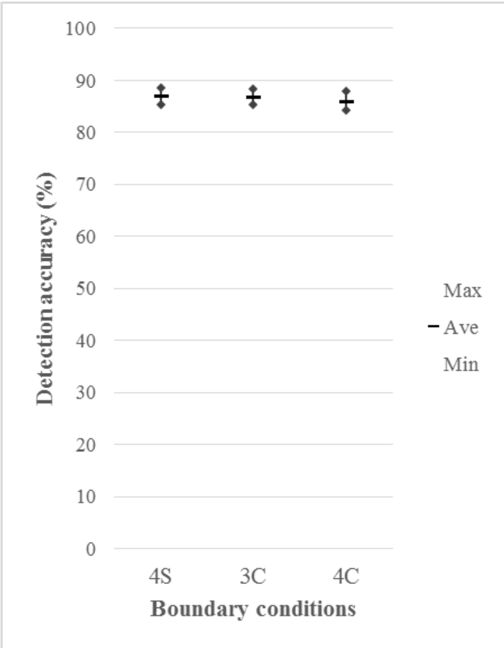
B.C.	Dim.	Average detection accuracy (%)	No noise	5%	10%	15%	20%
4S	3 m	Location	98.9	97.4	96.5	92.8	91.5
		Severity	94.6	93.2	91.3	90.6	86.3
	7 m	Location	93.6	92.1	90.7	87.5	85.6
		Severity	90.0	88.1	86.3	84.2	81.8
	10 m	Location	87.1	85.5	84.2	81.5	79.3
		Severity	83.4	81.6	80.3	76.8	76.3
3C	3 m	Location	93.8	92.0	91.9	88.0	86.4
		Severity	86.3	85.4	84.6	79.2	78.3
	7 m	Location	90.0	88.5	86.2	83.5	82.3
		Severity	83.8	82.4	81.1	78.1	75.6
	10 m	Location	86.9	85.8	83.8	81.9	79.3
		Severity	80.7	79.1	76.9	75.7	73.4
4C	3 m	Location	92.0	90.3	89.0	86.4	84.3
		Severity	83.5	82.5	79.9	74.9	75.4
	7 m	Location	88.8	87.6	85.2	82.8	81.8
		Severity	81.1	80.0	78.0	74.3	72.8
	10 m	Location	85.9	84.6	83.0	80.3	78.4
		Severity	77.4	76.1	74.6	71.0	69.9

Since fixed-dimension meshes of 0.5x0.5 meters are used for modeling the structures examined in this research, increased dimension signify an increase in the number of elements used in the model and thus increased number of DOFs of the structure. Evidently, the greater number of DOFs resulted in less effect of a given damage scenario (which includes damage in a specific number of DOFs) on the dynamic response of the structure and thus the proper recognition of hypothetical scenarios have less accuracy.

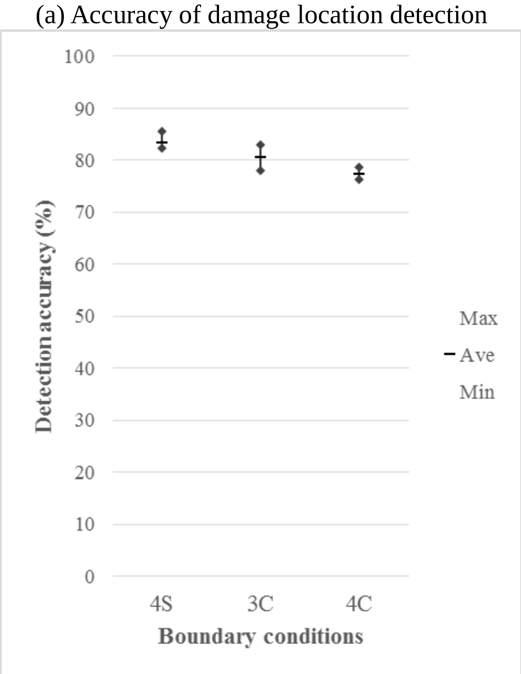
An example of the diagrams of damage detection accuracy changes in different boundary conditions on the plates with various sizes are given in Figs. 10-12. Since different damage scenarios are discussed comprehensively in the previous sections, only the results of the single-element damage case will be provided here. Evidently, the general trend of changes discussed in other damage cases is similar. It should also be considered that results of detection accuracy for 10 m structures with different boundary conditions are previously discussed in section 3.1. However, in order to compare the effect of change in boundary conditions in structures of different dimensions, complementary results are given in this section.

Observing the above figures and investigating the general trend of changes in damage detection accuracy in terms of changes in plate dimension, it is clear that by increasing the plate dimension, the diffusion of these indices because of the changes in structural boundary decreases. In other words, the higher length of the plate and the difference between the accuracies in the boundary conditions decreases. In fact, as the plate dimension grows, change in the boundary conditions has less effect on damage detection accuracy. As discussed in the previous sections, changes in the boundary conditions of structures will change the number of free DOFs in the edge elements of the model, and thus damage detection in scenarios including these elements is done with different accuracies in the structures with different boundary conditions. But since the dimensions of an element is fixed in all models, if the number of elements increases, the ratio of edge elements to the total number of elements decreases and consequently the effect of the edge elements in damage detection in the entire structure decreases. Thus with the existing modeling, virtually the higher structure length decreases the impact of structural clamping in recognizing various damages.

By investigating the data in Table 3, it is observed that by increasing the amount of noise in the data, the accuracy of the proposed damage index in determining the damage severity decreases. For instance, for the 3 m plate with boundary conditions of 4S, the detection accuracy of damage location and severity in single-element damage scenarios in noise-free data is respectively 99 and 95 percent. By applying 10% of noise to the measured data, these values will be respectively 96 and 91 percent and by applying 20% of noise, they reach 91 and 86 percent. It should be noted that 20 percent of noise in the data is in excess and thus uncommon. However, in spite of such a strong noise in the data, there is less than 10% reduction in performance of damage detection, which shows the proper functioning of this method, facing noisy experimental data.



(a) Accuracy of damage location detection



(b) Accuracy of damage severity detection

Fig. 12 Effect of different boundary conditions on damage detection accuracy of the 10 m plate(repetitious)

4. Conclusions

In this paper, the performance of damage index-based health monitoring approach was examined using FRF data and data compression technique 2D-PCA using algorithm of lookup tables in plate structures.

For this aim, various plates with dimensions of 3, 7 and 10m and boundary conditions of 4S, 3C, and 4C were modeled by finite element method and these models were used for measuring the performance of health monitoring in the detection of various single-element and multiple-element damages. The impact of different amounts of noises from zero to 20% noise in the data on the proposed damage index performance was assessed.

By examining the results of different scenarios, the following items can be presented as the conclusions:

- By increasing the clamping in the structural boundary conditions, proper location and severity detection accuracy decreases. This could be because of the impact of the side and corner elements of the model on FRF. Conversely, by increasing clamping of structures, structural stiffness and thus the modal frequencies increases. This increase results in higher broadening of the natural frequencies corresponding to the FRF of the structure, which can reduce damage detection accuracy.
- Generally, in all the structures studied, single-element damage detection is performed with the highest accuracy and three-element damage detection with the lowest accuracy. However, the tolerance range in accuracy is insignificant. This could be due to the disruption of the structure of the selected damage indices and the interaction of these indices and changes of FRF matrix because of the complexity of the damage scenario.

- In all the cases studied, for a certain damage scenario applied to a structure with a given boundary condition, the higher plate dimension results in less accuracy of damage detection. This could be due to the increased number of DOFs of the structure.

- By increasing the dimension of the plate structure, the difference between accuracy indicators in different boundary conditions decreased and in fact, the change in the boundary conditions will have little effect on the accuracy in identifying structural damage properly as the structure enlarges. In the case of structures with higher dimensions, the ratio of the number of edge elements to the total number of elements decreases and therefore the effect of edge elements on damage detection in the entire structure decreases. Thus with the existing modeling, the effect of clamping in detecting different damages decreases by increasing the length of the structure.
- Increased amount of noise in the data reduces the accuracy in detecting structural damage. However, the

results reveal satisfactory performance of the proposed damage even in high noise-contaminated data.

References

Balageas, D., Fritzen, C.-P. and Güemes, A. (2006), *Structural Health Monitoring*, Wiley Online Library, London, UK.

Chen, B. and Xu, Y. (2007), "A new damage index for detecting sudden change of structural stiffness", *Struct. Eng. Mech.*, **26**(3), 315-341.

Chopra, A.K. (1995), *Dynamics of Structures*, Prentice Hall, New Jersey, USA.

Choudhury, A.R. (1996), "Damage detection in structures using measured frequency response function data", Ph.D. Dissertation; Victoria University of Technology, Melbourne, Australia.

Esfandiari, A., Bakhtiari-Nejad, F. and Rahai, A. (2013), "Theoretical and experimental structural damage diagnosis method using natural frequencies through an improved sensitivity equation", *Int. J. Mech. Sci*., **70** 79-89.

Fan, W. and Qiao, P. (2012), "A strain energy-based damage severity correction factor method for damage identification in plate-type structures", *Mech. Syst. Signal Pr.*, **28** 660-678.

Fang, X., Luo, H. and Tang, J. (2005), "Structural damage detection using neural network with learning rate improvement", *Comput. Struct.*, **83**(25), 2150-2161.

Gomes, H. and Silva, N. (2008), "Some comparisons for damage detection on structures using genetic algorithms and modal sensitivity method", *Appl. Math. Model*., **32**(11), 2216-2232.

Huh, Y.C., Chung, T.Y., Lee, J.W. and Kim, J.K. (2015), "Damage identification in plates using vibratory power estimated from measured accelerations", *J. Sound Vib*., **336** 106-131.

Huynh, D., He, J. and Tran, D. (2005), "Damage location vector: A non-destructive structural damage detection technique", *Comput. Struct*., **83**(28), 2353-2367.

Khoshnoudian, F. and Talaei, S. (2016), "A new damage index using FRF data, 2D-PCA method and pattern recognition techniques", *Int. J. Struct. Stab. Dy*., 1750090.

Khoshnoudian, F., Talaei, S. and Fallahian, M. (2016), "Structural damage detection using FRF data, 2D-PCA, artificial neural networks and imperialist competitive algorithm simultaneously", *Int. J. Struct. Stab. Dy*., 1750073.

Moaveni, B., Lombaert, G., Stavridis, A., Conte, J.P. and Shing, P. B. (2011), "Damage identification of a three-story infilled RC frame tested on the UCSD-NEES shake table", *Dynam. Civil Struct*., **4**, Springer**:** 145-154.

Sampaio, R., Maia, N. and Silva, J. (1999), "Damage detection using the frequency-response-function curvature method", *J. Sound Vib*., **226**(5), 1029-1042.

Shadan, F., Khoshnoudian, F. and Esfandiari, A. (2016a), "A frequency response-based structural damage identification using model updating method", *Struct. Control Health*., **23**(2), 286-302.

Shadan, F., Khoshnoudian, F., Inman, D.J. and Esfandiari, A. (2016b), "Experimental validation of a FRF-based model updating method", *J. Vib. Control*, 1077546316664675.

Trendafilova, I. and Heylen, W. (2003), "Categorisation and pattern recognition methods for damage localisation from vibration measurements", *Mech. Syst. Signal Pr*., **17**(4), 825-836.

Vo-Duy, T., Ho-Huu, V., Dang-Trung, H., Dinh-Cong, D. and Nguyen-Thoi, T. (2016), "Damage Detection in Laminated Composite Plates Using Modal Strain Energy and Improved Differential Evolution Algorithm", *Procedia Eng*., **142**, 181-188.

Xiang, J. and Liang, M. (2012), "A two-step approach to multi-damage detection for plate structures", *Eng. Fract. Mech*., **91** 73-86.

Xu, W., Cao, M., Ostachowicz, W., Radzieński, M. and Xia, N. (2015), "Two-dimensional curvature mode shape method based on wavelets and Teager energy for damage detection in plates", *J. Sound Vib*., **347** 266-278.

Yam, L., Li, Y. and Wong, W. (2002), "Sensitivity studies of parameters for damage detection of plate-like structures using static and dynamic approaches", *Eng. Struct*., **24**(11), 1465-1475.

Yang, J., Zhang, D., Frangi, A.F. and Yang, J.Y. (2004), "Two-dimensional PCA: a new approach to appearance-based face representation and recognition", *IEEE T. Pattern Anal*., **26**(1), 131-137.

Zang, C. and Imregun, M. (2001), "Combined neural network and reduced FRF techniques for slight damage detection using measured response data", *Arch. Appl. Mech*., **71**(8), 525-536.

Zhang, Y., Wang, L., Lie, S.T. and Xiang, Z. (2013), "Damage detection in plates structures based on frequency shift surface curvature", *J. Sound Vib*., **332**(25), 6665-6684.

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