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Free vibration of functionally graded plates resting on elastic foundations based on quasi-3D hybrid-type higher order shear deformation theory

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Abstract. In this article, a free vibration analysis of functionally graded (FG) plates resting on elastic foundations is presented using a quasi-3D hybrid-type higher order shear deformation theory. Undetermined integral terms are employed in the proposed displacement field and modeled based on a hybrid-type (sinusoidal and parabolic) quasi-3D HSDT with five unknowns in which the stretching effect is taken into account. Thus, it can be said that the significant feature of this theory is that it deals with only 5 unknowns as the first order shear deformation theory (FSDT). The elastic foundation parameters are introduced in the present formulation by following the Pasternak (two-parameter) mathematical model. Equations of motion are obtained via the Hamilton's principles and solved using Navier's method. Accuracy of the proposed theory is confirmed by comparing the results of numerical examples with the ones available in literature.

Keywords: vibration; functionally graded plate; elastic foundation; shear deformation theory; stretching effect

1. Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composites often fabricated from a mixture of metals and ceramics. The mechanical characteristics of FGM vary gradually and continuously within the thickness direction in the material depending on a function. Because of this feature, the FGMs have some advantages such as avoiding the material discontinuity and decreasing the delamination failure, diminishing the stress levels and deflections. Combination of these properties attracts practical application of FGMs in many engineering areas such as aircraft, aerospace, naval/marine, construction and mechanical engineering (Bourada *et al*. 2012, Bessaim *et al*. 2013, Bouderba *et al.* 2013, Ait Amar Meziane *et al*. 2014, Akbaş 2015, Arefi 2015a, b, Arefi and Allam 2015, Attia *et al*. 2015, Bouguenina *et al*. 2015, Bennai *et al*. 2015, Bakora and Tounsi 2015, Ait Atmane *et al*. 2015, Barati and Shahverdi 2016, Boukhari *et al.* 2016, Barka *et al*. 2016, Akbarov *et al*. 2016, Aizikovich *et al*. 2016, Abdelbari *et al.* 2016, Abdelhak *et al.* 2016, Ahouel *et al*. 2016, Benferhat *et al*. 2016, Celebi *et al*. 2016, Darabi and Vosoughi 2016, Ebrahimi and Jafari 2016, Ebrahimi and Shafiei 2016, Trinh *et al.* 2016, Turan *et al.* 2016).

Given the widespread employ of engineering structures including FGM, many computational models have been proposed to assess its structural response. Reddy and Chin (1998) performed a thermo-mechanical investigation of FGM cylinders and plates. Kashtalyan (2004) presented a three dimensional elasticity solution for a simply supported FG plate under transverse loading. Birman and Bird (2007) proposed a system of FGM-structural modeling. Matsunaga (2009) used a two-dimensional higher-order theory for analyzing the displacement and stresses in FG plates under to thermal and mechanical loadings. Zhao *et al*. (2009) investigated the free vibration behavior of FG plates that utilizes the element-free kp-Ritz method. The FSDT is employed to consider the transverse shear strain and rotary inertia, and mesh-free kernel particle functions are employed to approximate the two-dimensional displacement fields. Other works where we can find the use of FSDT can be consulted in references of Meksi *et al*. (2015), Adda Bedia *et al*. (2015), Hadji *et al.* (2016), Bouderba *et al.* (2016) and Bellifa *et al*. (2016). Baferani *et al.* (2011) discussed the vibration response of FG rectangular plate resting on two parameter elastic foundation by employing the third-order shear deformation plate theory. Tounsi *et al*. (2013) proposed a refined trigonometric shear deformation theory for thermoelastic bending of FG sandwich plates. Taj and Chakrabarti (2013) studied FG skew plates subjected to static and dynamic loadings. Zhang *et al.* (2014) presented a 3D elasticity solution for static bending of thick FG plates using a hybrid semi-analytical approach-the state-pace based differential quadrature method. Hosseini-Hashemi *et al.* (2011)

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proposed an exact analytical solution for transverse vibration investigation of Lévy-type rectangular plates based on the Reddy's third-order shear deformation plate model. Hasani Baferani *et al.* (2011) studied the vibration behavior of a FG rectangular plate resting on elastic foundation by employing the third-order shear deformation plate theory. Sheikholeslami and Saidi (2013) analyzed the free vibration response of FG plates resting on two-parameter elastic foundation by utilizing a higher-order shear and normal deformable plate theory. The authors expanded the displacement components in the thickness direction via the Legendre polynomials. Taibi *et al.* (2015) proposed a simple shear deformation theory for thermo-mechanical behaviour of FG sandwich plates on elastic foundations. Ait Yahia *et al.* (2015) studied the wave propagation in FG plates with porosities applying various higher-order shear deformation plate theories of four unknowns. Mahmoud *et al.* (2015) examined the problem of wave propagation in magneto-rotating orthotropic non-homogeneous medium. Kar and Panda (2015) studied the free vibration responses of temperature dependent FG curved panels under thermal environment. Bounouara *et al.* (2016) presented a nonlocal zeroth-order shear deformation theory for free vibration of FG nano-plates resting on elastic foundation. Many investigations are reported in literature to present HSDTs for composite structures as well as graded CNT structures such as (Mehar *et al*. 2017a, b, c, Mehar and Panda 2017a, b, Hirwani *et al*. 2017a, b, Kar and Panda 2017, Kar *et al.* 2017, Sahoo *et al*. 2017, Kar *et al*. 2016, Singh *et al.* 2016, Houari *et al*. 2016, Bousahla *et al*. 2016, Kar and Panda 2016a, b, Mahi *et al*. 2016, Katariya and Panda 2016, Sahoo *et al*. 2016, Mehar *et al*. 2016, Singh and Panda 2015, Belkorissat *et al*. 2015, Larbi Chaht *et al*. 2015, Bourada *et al*. 2015, Panda and Katariya 2015, Nguyen *et al*. 2015, Zemri *et al*. 2015, Zidi *et al*. 2014, Merazi *et al*. 2015, Mouaici *et al*. 2016, Laoufi *et al*. 2016, Beldjelili *et al.* 2016, Raminnea *et al*. 2016, Saidi *et al*. 2016, El-Hassar *et al*. 2016, Ghorbanpour Arani *et al*. 2016, Bellifa *et al*. 2017, Bouafia *et al*. 2017, Benahmed *et al*. 2017, Zidi *et al*. 2017, El-Haina *et al*. 2017, Mouffoki *et al*. 2017, Klouche *et al*. 2017, Sekkal *et al*. 2017).

Jin *et al.* (2014) proposed a 3D exact solution for the free vibrations of thick FG plates with general boundary conditions. Akavci (2014) studied the free vibration response of FG plates on elastic foundation using a non-polynomial HSDT and an optimization procedure. Alijani and Amabili (2014) studied the nonlinear forced vibrations of moderately thick FG rectangular plates by using higher-order shear deformation theories that consider the thickness deformation effect. Belabed *et al.* (2014) proposed an efficient and simple higher order shear and normal deformation theory for FG plates. Hebali *et al.* (2014) presented a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of FG plates. Fekrar *et al*. (2014) developed a new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates. Bousahla *et al*. (2014) studied the bending of advanced composite plates using a novel higher order shear and normal deformation theory based on neutral surface position. Akavci and Tanrikulu (2015) presented 2D and quasi-3D shear deformation theories for bending and free vibration analysis of single-layer FG plates using a new hyperbolic shape function. Hamidi *et al.* (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermo-mechanical bending of FG sandwich plates. Meradjah *et al*. (2015) proposed a new higher order shear and normal deformation theory for FG beams. Akavci (2016) presented a new hyperbolic shear and normal deformation plate theory to study the static, free vibration and buckling analysis of the simply supported FG sandwich plates on elastic foundation. Draiche *et al.* (2016) presented a refined theory with stretching effect for the flexure analysis of laminated composite plates. Bennoun *et al.* (2016) presented a novel five variable quasi-3D plate theory for vibration analysis of FG sandwich plates.

The present article presents a generalized quasi-3D hybrid-type higher order shear deformation theory for the vibration analysis of FG plates on elastic foundation. The highlight of this model is that, in addition to introducing the thickness stretching effect ($\varepsilon_z \neq 0$), the displacement field is modeled with only 5 unknowns by considering undetermined integral terms. Thus the number of unknowns is even less than the FSDT and do not need shear correction factor. The displacement field is modeled based on a hybrid-type (sinusoidal and parabolic) shear strain shape functions. The mechanical properties of the plates are supposed to vary in the thickness direction according to a power law variation in terms of the volume fractions of the constituents. The equations of motion of FG plates resting on elastic foundation are derived by using the Hamilton's principle. These governing equations are then solved via Navier method. As a result, fundamental frequencies are obtained by solving eigenvalue problem. The accuracy of the present theory is verified by comparing the obtained results with those of HSDT's solutions available in literature.

2. Analytical modeling

2.1 Functionally graded plates

We consider in this work, a rectangular plate of uniform thickness "h", length "a", and the width "b", fabricated from FGM and supported by an elastic foundation. The rectangular Cartesian coordinate system *x*, *y*, *z*, has the surface $z = 0$, coinciding with the mid-plane of the plate.

The material characteristics change across the thickness according to a power law distribution, which is defined below

$$P(z) = P_b + \left(P_t - P_b\right)\left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{1}$$

where P represents the effective material property, P_t and P_b represent the property of the top and bottom faces of the plate, respectively, and "k" is the exponent that specifies the material distribution profile within the thickness. The effective material characteristics of the plate, including Young's modulus, E, and shear modulus, G, vary according to Eq. (1), and Poisson ratio, "ν" is considered to be constant (Qian *et al*., 2004).

2.2 Kinematic relations and constitutive relations

In this work, the conventional quasi-3D HSDT is modified by considering some simplifying suppositions so that the number of unknowns is reduced. The displacement field of the conventional quasi-3D HSDT is defined by

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_x(x,y,t) \quad (2a)$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\varphi_y(x,y,t) \quad (2b)$$

$$w(x,y,z,t)=w_0(x,y,t)+g(z)\varphi_z(x,y,t) \quad (2c)$$

where u_0, v_0, w_0, φ_x, φ_y and φ_z are six unknown displacements of the mid-plane of the plate, $f(z)$ represents shape function defining the variation of the transverse shear strains and stresses within the thickness.

By considering that $\varphi_x=\int\theta(x,y)dx$ and $\varphi_y=\int\theta(x,y)dy$ (Merdaci *et al.* 2016, Benbakhti *et al.* 2016, Bourada *et al*. 2016, Hebali *et al.* 2016, Chikh *et al.* 2016, Benchohra *et al*. 2017, Chikh *et al.* 2017, Fahsi *et al.* 2017, Meksi *et al*. 2017, Khetir *et al*. 2017, Besseghier *et al.* 2017, Menasria *et al*. 2017), the Kinematic of the proposed theory can be expressed in a simpler form as

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1 f(z)\int\theta(x,y,t) \quad (3a)$$

$$v(x,y,z)=v_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_2 f(z)\int\theta(x,y,t)dy \quad (3b)$$

$$w(x,y,z,t)=w_0(x,y,t)+g(z)\varphi_z(x,y,t) \quad (3c)$$

In this study, the hybrid type shear strain shape functions are

$$f(z)=\frac{h}{\pi}\sin\frac{\pi z}{h},\ \text{and}\ g(z)=1-4\left(\frac{z}{h}\right)^2 \quad (4)$$

The necessary equations are obtained by assuming small strains are assumed (i.e., displacements and rotations are small, and obey Hooke's law). The linear strain relations determined from the kinematic of Eqs. (3(a)- 3(c)), valid for thin, moderately thick and thick plate under consideration are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \quad (5)$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix},\quad \varepsilon_z=g'(z)\varepsilon_z^0$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},$$

$$\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \quad (6a)$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy+\dfrac{\partial\varphi_z}{\partial y}\\ k_1\int\theta\,dx+\dfrac{\partial\varphi_z}{\partial x}\end{Bmatrix}, \quad (6b)$$

$$\varepsilon_z^0=\varphi_z\ \text{and}\ g'(z)=\frac{\partial g(z)}{\partial z}$$

The integrals appearing in the above expressions shall be resolved by a Navier type solution and can be expressed as follows

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y}, \quad (7)$$

$$\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\ \int\theta\,dy=B'\frac{\partial\theta}{\partial y}$$

where the coefficients A' and B' are defined according to the type of solution adopted, in this case via Navier. Therefore, A' and B' are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{8}$$

where α and β are defined in expression (24).

For the FG plates, the stress–strain relationships for plane-stress state can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{9}$$

in which, (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{xz}) are the stresses and the strain vectors with respect to the plate coordinate system. The Q_{ij} expressions in terms of engineering constants are given below

$$Q_{11} = Q_{22} = Q_{33} = \frac{E(z)(1-\nu)}{(1-2\nu)(1+\nu)}, \tag{10a}$$

$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu\, E(z)}{(1-2\nu)(1+\nu)}, \tag{10b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10c}$$

2.3 Hamilton's principle

Hamilton's principle is employed herein to obtain the equations of motion appropriate to the displacement field and the constitutive equations. The principle can be stated in analytical form as

$$0 = \int_0^t (\delta U + \delta V_e - \delta K)\, dt \tag{11}$$

where δU is the variation of strain energy; δV_e is the variation of the potential energy of elastic foundation; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x\, \delta\varepsilon_x + \sigma_y\, \delta\varepsilon_y + \sigma_z\, \delta\varepsilon_z + \tau_{xz}\, \delta\gamma_{xz} + \tau_{xz}\, \delta\gamma_{xz} + \tau_{xz}\, \delta\gamma_{xz}\right] dV \\ &= \int_A \left[N_x\, \delta\varepsilon_x^0 + N_y\, \delta\varepsilon_y^0 + N_z\, \delta\varepsilon_z^0 + N_{xy}\, \delta\gamma_{xy}^0 + M_x^b\, \delta k_x^b + M_y^b\, \delta k_y^b + M_{xy}^b\, \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s\, \delta k_x^s + M_y^s\, \delta k_y^s + M_{xy}^s\, \delta k_{xy}^s + S_{yz}^s\, \delta\gamma_{yz}^0 + S_{yz}^s\, \delta\gamma_{xz}^0\right] dA \end{aligned} \tag{12}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz, \quad (i = x, y, xy), \tag{13}$$

$$N_z = \int_{-h/2}^{h/2} g'(z)\, \sigma_z\, dz \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz$$

The variation of the potential energy of elastic foundation can be expressed by

$$\delta V_e = \int_A f_e\, \delta w_0\, dA \tag{14}$$

where f_e is the density of reaction force of foundation. For the Pasternak foundation model:

$$f_e = K_w w - K_s \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{15}$$

in which K_w and K_s are the Winkler foundation stiffness and the shear stiffness of the elastic foundatio n.

The variation of kinetic energy of the plate can be written as

$$\begin{aligned} \delta K &= \int_V [\dot u\, \delta\dot u + \dot v\, \delta\dot v + \dot w\, \delta\dot w]\rho(z)\, dV \\ &= \int_A \Big\{ I_0(\dot u_0\, \delta\dot u_0 + \dot v_0\, \delta\dot v_0 + \dot w_0\, \delta\dot w_0) + J_0(\dot w_0\, \delta\dot\varphi_z + \dot\varphi_z\, \delta\dot w_0) \\ &\quad - I_1\left(\dot u_0 \frac{\partial\delta\dot w_0}{\partial x} + \frac{\partial\dot w_0}{\partial x}\delta\dot u_0 + \dot v_0 \frac{\partial\delta\dot w_0}{\partial y} + \frac{\partial\dot w_0}{\partial y}\delta\dot v_0\right) \\ &\quad + J_1\left(k_1 A'\left((\dot u_0 \frac{\partial\delta\dot\theta}{\partial x} + \frac{\partial\dot\theta}{\partial x}\delta\dot u_0\right) + k_2 B'\left(\dot v_0 \frac{\partial\delta\dot\theta}{\partial y} + \frac{\partial\dot\theta}{\partial y}\delta\dot v_0\right)\right) \\ &\quad + I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x} + \frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right) + K_2\left((k_1 A')^2 \frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x} + (k_2 B')^2 \frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}\right) \\ &\quad - J_2\left(k_1 A'\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot\theta}{\partial x} + \frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}\right) + k_2 B'\left(\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot\theta}{\partial y} + \frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right)\right) + K_0\, \dot\varphi_z\, \delta\dot\varphi_z \Big\} dA \end{aligned} \tag{16}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, K_i) are mass inertias expressed by

$$(I_0, I_1, I_2, J_1, J_2, J_0, K_0, K_2) = \int_{-h/2}^{h/2} \left(1, z, z^2, f, zf, g, g^2, f^2\right)\rho(z)\, dz \tag{17}$$

Employing the generalized displacement–strain expressions (Eqs. (5) and (6)) and stress–strain relations (9), and applying integrating by parts and the fundamental lemma of variational calculus and collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$ and $\delta\varphi_z$ in Eq. (11), the equations of motion are obtained as

$$\begin{aligned} &\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot u_0 - I_1\frac{\partial\ddot w_0}{\partial x} + J_1 k_1 A'\frac{\partial\ddot\theta}{\partial x} \\ &\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0\ddot v_0 - I_1\frac{\partial\ddot w_0}{\partial y} + J_1 k_2 B'\frac{\partial\ddot\theta}{\partial y} \\ &\delta w_0 : \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} - f_e = I_0\ddot w_0 + J_0\ddot\varphi + I_1\left(\frac{\partial\ddot u_0}{\partial x} + \frac{\partial\ddot v_0}{\partial y}\right) - I_2\nabla\ddot w_0 + J_2\left(k_1 A'\frac{\partial^2\ddot\theta}{\partial x^2} + k_2 B'\frac{\partial^2\ddot\theta}{\partial y^2}\right) \\ &\delta\theta : -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial x} = -J_1\left(k_1 A'\frac{\partial\ddot u_0}{\partial x} + k_2 B'\frac{\partial\ddot v_0}{\partial y}\right) \\ &\qquad + J_2\left(k_1 A'\frac{\partial^2\ddot w_0}{\partial x^2} + k_2 B'\frac{\partial^2\ddot w_0}{\partial y^2}\right) - K_2\left((k_1 A')^2\frac{\partial^2\ddot\theta}{\partial x^2} + (k_2 B')^2\frac{\partial^2\ddot\theta}{\partial y^2}\right) \\ &\delta\varphi_z : \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial x} - N_z = J_0\ddot w_0 + K_0\ddot\varphi_z \end{aligned} \tag{18}$$

Substituting Eq. (5) into Eq. (9) and the subsequent results into Eq. (13), the stress resultants can be expressed in terms of generalized displacements ($u_0, v_0, w_0, \theta, \varphi_z$) as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23} \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23} \\ 0 & 0 & B_{66} & 0 & 0 & D_{11} & 0 & 0 & D_{66}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x} \\ -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \\ k_1\theta \\ k_2\theta \\ (k_1A'+k_2B')\frac{\partial^2\theta}{\partial x\partial y} \\ \varphi_z \end{Bmatrix} \quad (19a)$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2 B' \frac{\partial \theta}{\partial y} + \frac{\partial \varphi_z}{\partial y} \\ k_1 A' \frac{\partial \theta}{\partial x} + \frac{\partial \varphi_z}{\partial x} \end{Bmatrix} \quad (19b)$$

where

$$\left(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, g^2(z), z, z^2, f(z), z\, f(z), f^2(z)\right) dz \quad (20a)$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), g'(z)\right) g'(z) Q_{ij}\, dz \quad (20b)$$

Substituting Eqs. (19) into Eqs. (18), the equations of motion of the proposed quasi-3D hybrid-type HSDT can be expressed in terms of displacements ($u_0, v_0, w_0, \theta, \varphi_z$) as

$$\begin{aligned} &A_{11} d_{11}u_0 + A_{66} d_{22}u_0 + (A_{12}+A_{66}) d_{12}v_0 - B_{11} d_{111}w_0 - (B_{12}+2B_{66}) d_{122}w_0 + (k_1B_{11}^s + k_2B_{12}^s) d_1\theta \\ &\quad + (k_1A' + k_2B')B_{66}^s d_{122}\theta + X_{13} d_1\varphi_z = I_0\ddot{u}_0 - I_1 d_1\ddot{w}_0 + (k_1A')J_1 d_1\ddot{\theta} \end{aligned} \quad (21a)$$

$$\begin{aligned} &(A_{12}+A_{66}) d_{12}u_0 + A_{66} d_{11}v_0 + A_{22} d_{22}v_0 - (B_{12}+2B_{66}) d_{112}w_0 - B_{22} d_{222}w_0 + (k_1B_{12}^s + k_2B_{22}^s) d_2\theta \\ &\quad + (k_1A' + k_2B')B_{66}^s d_{112}\theta + X_{23} d_2\varphi_z = I_0\ddot{v}_0 - I_1 d_2\ddot{w}_0 + J_1 k_2 B' d_2\ddot{\theta} \end{aligned} \quad (21b)$$

$$\begin{aligned} &B_{11} d_{111}u_0 + (B_{12}+2B_{66}) d_{122}u_0 + (B_{12}+2B_{66}) d_{112}v_0 + B_{22} d_{222}v_0 - D_{11} d_{1111}w_0 - D_{22} d_{2222}w_0 \\ &-2(D_{12}+2D_{66}) d_{1122}w_0 + (k_1D_{11}^s + k_2D_{12}^s) d_{11}\theta + (k_1D_{12}^s + k_2D_{22}^s) d_{22}\theta + 2(k_1A' + k_2B') D_{66}^s d_{1122}\theta \\ &+Y_{13} d_{11}\varphi_z + Y_{23} d_{22}\varphi_z - f_e = I_0\ddot{w}_0 + J_0\ddot{\varphi}_z + I_1\left(d_1\ddot{u}_0 + d_2\ddot{v}_0\right) - I_2\nabla^2\ddot{w}_0 + J_2\left(k_1A' d_{11}\ddot{\theta} + k_2B' d_{22}\ddot{\theta}\right) \end{aligned} \quad (21c)$$

$$\begin{aligned} &-(k_1B_{11}^s + k_2B_{12}^s) d_1u_0 - (k_1B_{12}^s + k_2B_{22}^s) d_2v_0 - B_{66}^s(k_1A' + k_2B')(d_{122}u_0 + d_{112}v_0) \\ &+(k_1D_{11}^s + k_2D_{12}^s) d_{11}w_0 + (k_1D_{12}^s + k_2D_{22}^s) d_{22}w_0 + 2D_{66}^s(k_1A' + k_2B') d_{1122}w_0 \\ &-({k_1}^2H_{11}^s + {k_2}^2H_{22}^s + 2k_1k_2H_{12}^s)\theta + (k_1A')^2 A_{55}^s d_{11}\theta + (k_2B')^2 A_{44}^s d_{22}\theta - H_{66}^s(k_1A' + k_2B')^2 d_{1122}\theta \\ &-(k_1Y_{13}^s + k_2Y_{23}^s)\varphi_z + k_1A'A_{55}^s d_{11}\varphi_z + k_2B'A_{44}^s d_{22}\varphi_z = -J_1\left(k_1A' d_1\ddot{u}_0 + k_2B' d_2\ddot{v}_0\right) \\ &\quad + J_2\left(k_1A' d_{11}\ddot{w}_0 + k_2B' d_{22}\ddot{w}_0\right) - K_2\left((k_1A')^2 d_{11}\ddot{\theta} + (k_2B')^2 d_{22}\ddot{\theta}\right) \end{aligned} \quad (21d)$$

$$\begin{aligned} &-X_{13} d_1u_0 - X_{23} d_2v_0 + Y_{13} d_{11}w_0 + Y_{23} d_{22}w_0 + (k_1(A_{55}^s - Y_{13}^s) + k_2(A_{44}^s - Y_{23}^s))\theta \\ &\quad + A_{55}^s d_{11}\varphi_z + A_{44}^s d_{22}\varphi_z - Z_{33}\varphi_z = J_0\ddot{w}_0 + K_0\ddot{\varphi}_z \end{aligned} \quad (21e)$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned} d_{ij} &= \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \\ d_{ijlm} &= \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i,j,l,m = 1,2). \end{aligned} \quad (22)$$

3. Solution procedure

For the analytical solution of the partial differential Eqs. (21(a)-21(e)), the Navier method, based on double Fourier series, is employed under the specified boundary conditions. Using Navier's procedure, the solution of the displacement variables satisfying the above boundary conditions can be expressed in the following Fourier series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha x)\sin(\beta y)e^{i\omega t} \\ V_{mn}\sin(\alpha x)\cos(\beta y)e^{i\omega t} \\ W_{mn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ X_{mn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ \Phi_{mn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \end{Bmatrix} \quad (23)$$

where (U_{mn}, V_{mn}, W_{mn}, X_{mn}, Φ_{mn}) are unknown functions to be determined and ω is the natural frequency. α and β are expressed as

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \quad (24)$$

Substituting Eq. (23) into equations of motion (21) we get below eigenvalue equation for any fixed value of m and n, for free vibration problem

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{12} & s_{22} & s_{23} & s_{24} & s_{25} \\ s_{13} & s_{23} & s_{33} & s_{34} & s_{35} \\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45} \\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{12} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{13} & m_{23} & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & m_{45} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_{55} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (25)$$

where

$$\begin{aligned} &s_{11} = \alpha^2 A_{11} + \beta^2 A_{66}, \quad s_{12} = \alpha\beta(A_{12} + A_{66}), \\ &s_{13} = -\alpha^3 B_{11} - \alpha\beta^2(B_{12} + 2B_{66}) \\ &s_{14} = -\alpha(k_1B_{11}^s + k_2B_{12}^s) + \alpha\beta^2 B_{66}^s(k_1A' + k_2B'), \\ &s_{15} = \alpha X_{13}, \quad s_{22} = \alpha^2 A_{66} + \beta^2 A_{22} \\ &s_{23} = -\alpha^2\beta(B_{12} + 2B_{66}) - \beta^3 B_{22}, \\ &s_{24} = -\beta(k_1B_{12}^s + k_2B_{22}^s) + \alpha^2\beta(k_1A' + k_2B')B_{66}^s \\ &s_{25} = -\beta X_{23}, \\ &s_{33} = \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2(D_{12} + 2D_{66}) + K_w + K_s(\alpha^2 + \beta^2) \\ &s_{34} = \alpha^2 k_1 D_{11}^s + (k_2\alpha^2 + k_1\beta^2)D_{12}^s + \beta^2 k_2 D_{22}^s - 2\alpha^2\beta^2(k_1A' + k_2B')D_{66}^s \\ &s_{35} = \alpha^2 Y_{13} + \beta^2 Y_{23} \\ &s_{44} = k_1^2 H_{11}^s + k_2^2 H_{22}^s + 2k_1k_2 H_{12}^s + \alpha^2\beta^2(k_1A' + k_2B')^2 \\ &\quad H_{66}^s + \alpha^2(k_1A')^2 A_{55}^s + \beta^2(k_2B')^2 A_{44}^s \end{aligned} \quad (26)$$

Table 1 Material properties used in the FG plates

Material	Properties		
	Young's modulus (GPa)	Poisson's ratio	Mass density kg/m^3
Aluminium (Al)	70	0.3	2702
Alumina (Al_2O_3)	380	0.3	3800
Zirconia (ZrO_2)	200	0.3	5700

Table 2 Non-dimensional fundamental frequencies $\hat{\omega}=\omega a^2\sqrt{\rho h/D_0}$ for simply supported isotropic square plates

a/h	Theory	Mode (1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(3,1)	(2,3)	(3,2)
1000	Leissa (1973)	19.7392	49.3480	49.3480	78.9568	98.6960	98.6960	128.3021	128.3021
	Zhou *et al.* (2002)	19.7115	49.3470	49.3470	78.9528	98.6911	98.6911	128.3048	128.3048
	Akavci (2014)	19.7391	49.3476	49.3476	78.9557	98.6943	98.6943	128.3020	128.3020
	Mantari (2015)	19.7395	49.3483	49.3483	78.9568	98.6957	98.6957	128.3037	128.3037
	Present	19.7391	49.3476	49.3476	78.9557	98.6943	98.6943	128.3019	128.3019
100	Liu and Liew (1999)	19.7319	49.3027	49.3027	78.8410	98.5150	98.5150	127.9993	127.9993
	Nagino *et al.* (2008)	19.7320	49.3050	49.3050	78.8460	98.5250	98.5250	128.0100	128.0100
	Akavci (2014)	19.7322	49.3045	49.3045	78.8456	98.5223	98.5223	128.0346	128.0346
	Mantari (2015)	19.7326	49.3056	49.3056	78.8477	98.5253	98.5253	128.0160	128.0160
	Present	19.7323	49.3049	49.3049	78.8466	98.5239	98.5239	128.0143	128.0143
10	Liu and Liew (1999)	19.0584	45.4478	45.4478	69.7167	84.9264	84.9264	106.5154	106.5154
	Nagino *et al.* (2008)	19.0653	45.4869	45.4869	69.8093	85.0646	85.0646	106.7350	106.7350
	Akavci (2014)	19.0850	45.5957	45.5957	70.0595	85.4315	85.4315	107.3040	107.3040
	Mantari (2015)	19.0909	45.6242	45.6242	70.1176	85.5096	85.5096	107.4092	107.4092
	Present	19.0908	45.6251	45.6251	70.1214	85.5164	85.5164	107.4222	107.4222
5	Shufrin *et al.* (2005)	17.4524	38.1884	38.1884	55.2539	65.3130	65.3130	78.9864	78.9864
	Hosseini *et al.* (2011)	17.4523	38.1883	38.1883	55.2543	65.3135	65.3135	78.9865	78.9865
	Akavci (2014)	17.5149	38.4722	38.4722	55.8358	66.1207	66.1207	80.1637	80.1637
	Mantari (2015)	17.5294	38.5079	38.5079	55.8561	66.1060	66.1060	80.0589	80.0589
	Present	17.5303	38.5169	38.5169	55.8817	66.1471	66.1471	80.1296	80.1296

$$s_{45}=k_1Y_{13}^s+k_2Y_{23}^s+\alpha^2k_1A^{'}A_{55}^s+\beta^2k_2B^{'}A_{44}^s$$
$$s_{55}=\alpha^2A_{55}^s+\beta^2A_{44}^s+Z_{33}$$
$$m_{11}=m_{22}=I_0,\quad m_{12}=m_{15}=m_{25}=m_{45}=0$$
$$m_{13}=-\alpha I_1,\quad m_{14}=\alpha k_1A^{'}J_1,$$
$$m_{15}=0,\quad m_{23}=-\beta I_1$$
$$m_{24}=\beta k_2B^{'}J_1,\quad m_{33}=I_0+I_2(\alpha^2+\beta^2),$$
$$m_{34}=-J_2(k_1A^{'}\alpha^2+k_2B^{'}\beta^2)$$
$$m_{44}=K_2((k_1A^{'})^2\alpha^2+(k_2B^{'})^2\beta^2),$$
$$m_{35}=J_0,\quad m_{55}=K_0$$

4. Numerical results and discussions

The results of various numerical analyses are presented in this section for vibration analysis of a simply supported FG plates with various indexes that specify the material distribution profile within the thickness and several values of the thickness ratio a/h and aspect ratio a/b. Typical mechanical characteristics for metal and ceramics employed in the FG plates are given in Table 1. For the validation of the proposed quasi-3D hybrid-type HSDT, both, homogeneous isotropic plates and FG plates are investigated.

4.1 Investigation of homogeneous isotropic plates

In this part of study, homogeneous isotropic material is investigated. Unless otherwise stated, the following expressions to compute the non-dimensional natural frequencies and foundation parameters were used

$$\hat{\omega}=\omega a^2\sqrt{\rho h/D_0},\quad k_w=K_wa^4/D_0,$$
$$k_s=K_sa^2/D_0,\quad D_0=Eh^3/\left[12(1-\nu^2)\right] \tag{27}$$

Table 2 shows the first eight non-dimensional fundamental frequencies. These values are compared with the solutions of different researchers: 3D exact solutions by Leissa (1973), Zhou *et al.* (2002), Nagino *et al.* (2008), FSDT results obtained using differential quadrature element method (DQM) by Liu and Liew (1999), and HSDTs by Shufrin and Eisenberger (2005), Hosseini-Hashemi *et al.*

(2011) Akavci (2014) and Mantari (2015). From these results it can be deduced that for the thickness ratio "$a/h = 1000$", the computed results are very close to the results reported by Leissa (1973) and have proximity with the ones given by Akavci (2014), for the first eight modes of free vibration. For the thickness ratio "$a/h = 100$", the obtained results have also proximity with those computed by Akavci (2014) and Nagino *et al*. (2008). By reducing the thickness ratio "a/h" the obtained results demonstrate good agreement with the other models presented in Table 2.

4.2 Investigation of FG Plates

In this part of study, FG plates are investigated. Two types of FG plates (Al/Al_2O_3 and Al/ZrO_2) are employed (see mechanical properties in Table 1). Unless otherwise has been stated, for this section, the following relations of non-dimensional natural frequencies and foundation parameters was used

$$\bar{\omega} = \omega h \sqrt{\rho_m / E_m}\,, \quad \beta = \omega h \sqrt{\rho_c / E_c}\,, \tag{28a}$$

$$\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_m / E_m}\,, \quad \bar{\beta} = (\omega a^2 / h)\sqrt{\rho_c / E_c}\,,$$

$$k_w = K_w a^4 / \bar{D}, \quad k_s = K_s a^2 / \bar{D}$$

where

$$\bar{D} = h^3 / 12(1-v^2)\left[p(8+3p+p^2)E_m + 3(2+p+p^2)E_c\right] / \left[(1+p)(2+p)(3+p)\right] \tag{28b}$$

In Table 3, non-dimensional fundamental frequencies of simply supported plate are calculated for four different gradient indexes and compared with the 3D exact solution developed by Jin *et al.* (2014) and the theory proposed by and Mantari (2015). The results obtained demonstrate good accuracy for square plates. In rectangular plates, the results are close to the referential value in the cases when the thickness ratio $a/h \geq 5$.

In Table 4, non-dimensional natural frequencies for different gradient indexes are calculated and compared with the 3D exact solution by Vel and Batra (2004), the quasi-3D sinusoidal and hyperbolic HSDTs by Neves *et al.* (2012a,b); and the HSDTs by Akavci (2014), Hosseini-Hashemi et al. (2011), Mantari (2015) and Matsunaga (2008). It can be observed that the results computed by the proposed theory agree with the HSDTs, quasi-3D and 3D exact results.

Table 5 shows the non-dimensional fundamental frequencies of FG plates resting on elastic foundations for different values of the thickness ratio a/h. The results computed using the present model, are compared with the FSDT's results by Hosseini-Hashemi *et al*. (2010), and the HSDTs by Akavci (2014) and Mantari (2015). From this table can be observed that the results of the proposed theory are closer to the results reported by Akavci (2014) (optimized shear deformation theory for the dynamic analysis of FG plates).

Table 3 Comparison of non-dimensional fundamental frequencies $\bar{\omega} = \omega h \sqrt{\rho_m / E_m}$ of Al/Al_2O_3 FG plates

b/a	a/h	p	Theory: Jin *et al.* (2014)	Mantari (2015)	Present
1	10	0	0.1135	0.1135	0.1135
		1	0.0870	0.0882	0.0882
		2	0.0789	0.0806	0.0806
		5	0.0741	0.0755	0.0755
	5	0	0.4169	0.4169	0.4196
		1	0.3222	0.3261	0.3261
		2	0.2905	0.2962	0.2961
		5	0.2676	0.2722	0.2720
	2	0	1.8470	1.8510	1.8526
		1	1.4687	1.4778	1.4789
		2	1.3095	1.3223	1.3230
		5	1.1450	1.1557	1.1547
2	10	0	0.0719	0.0718	0.0718
		1	0.0550	0.0557	0.0557
		2	0.0499	0.0510	0.0509
		5	0.0471	0.0479	0.0479
	5	0	0.2713	0.2713	0.2713
		1	0.2088	0.2115	0.2115
		2	0.1888	0.1926	0.1926
		5	0.1754	0.1786	0.1785
	2	0	0.9570	1.3044	1.3049
		1	0.7937	1.0348	1.0352
		2	0.7149	0.9296	0.9297
		5	0.6168	0.8241	0.8231

Table 4 Comparison of non-dimensional fundamental frequencies $\bar{\omega} = \omega h \sqrt{\rho_m / E_m}$ of Al/ZrO_2 FG square plates (a/h = 5)

Theory	$p=2$	$p=3$	$p=5$
Vel and Batra (2004)	0.2197	0.2211	0.2225
Neves *et al.* (2012a) $\varepsilon_z = 0$	0.2189	0.2202	0.2215
Neves *et al.* (2012a) $\varepsilon_z \neq 0$	0.2198	0.2212	0.2225
Neves *et al.* (2012b) $\varepsilon_z = 0$	0.2191	0.2205	0.2220
Neves *et al.* (2012b) $\varepsilon_z \neq 0$	0.2201	0.2216	0.2230
Matsunaga (2008)	0.2264	0.2270	0.2280
Hosseini-Hashemi *et al.* (2011)	0.2264	0.2276	0.2291
Akavci (2014)	0.2264	0.2269	0.2278
Mantari (2015)	0.2285	0.2290	0.2295
Present	0.2285	0.2290	0.2295

4.3 Parameter studies

Fig. 1 presents the variation of non-dimensional natural frequency of a simply supported FG plate versus the gradient index "k" for different values of the thickness ratio "a/h". It can be noticed from this figurethat for a given value of "k", as the thickness ratio increase, the natural frequency increase,

Table 5 Comparison of non-dimensional fundamental frequencies $\beta = \omega h\sqrt{\rho_C / E_C}$ of Al/ZrO_2 FG rectangular plate ($a/b = 1.5$)

(k_w, k_s)	a/h	p	Theory: Akavci (2014)	Hosseini-Hashemi *et al.* (2010)	Mantari (2014)	Present
(0, 0)	20	0	0.02393	0.02392	0.02393	0.02397
		0.25	0.02309	0.02269	0.02312	0.02315
		1	0.02202	0.02156	0.02217	0.02220
		5	0.02244	0.02180	0.02260	0.02262
		∞	0.02056	0.02046	0.02057	0.02060
	10	0	0.09203	0.09188	0.09207	0.09224
		0.25	0.08895	0.08603	0.08909	0.08925
		1	0.08489	0.08155	0.08549	0.08564
		5	0.08576	0.08171	0.08638	0.08651
		∞	0.07908	0.07895	0.07911	0.07927
	5	0	0.32471	0.32284	0.32498	0.32583
		0.25	0.31531	0.31003	0.31591	0.31670
		1	0.30152	0.29399	0.30349	0.30425
		5	0.31860	0.29099	0.29990	0.30053
		∞	0.27902	0.27788	0.27925	0.28001
(250, 25)	20	0	0.03422	0.03421	0.03417	0.03419
		0.25	0.03312	0.03285	0.03309	0.03311
		1	0.03213	0.03184	0.03220	0.03222
		5	0.03277	0.03235	0.03283	0.03285
		∞	0.02940	0.02937	0.02936	0.02939
	10	0	0.13375	0.13365	0.13302	0.13315
		0.25	0.12959	0.12771	0.12895	0.12907
		1	0.12585	0.12381	0.12557	0.12568
		5	0.12778	0.12533	0.12755	0.12764
		∞	0.11492	0.11484	0.11430	0.11443
	5	0	0.50044	0.49945	0.48945	0.49020
		0.25	0.48594	0.48327	0.47535	0.47610
		1	0.47298	0.46997	0.46401	0.46468
		5	0.47637	0.47400	0.46838	0.46880
		∞	0.43000	0.43001	0.42057	0.42129

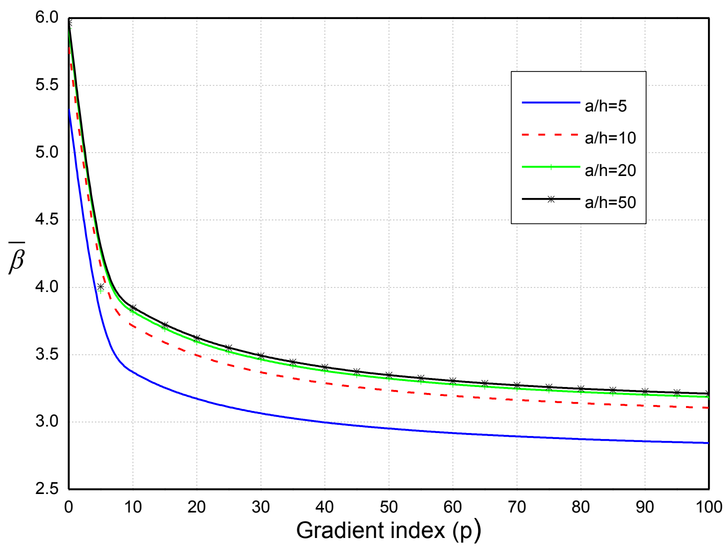


Fig. 1 Variation of non-dimensional fundamental frequency $\overline{\beta} = (\omega a^2 / h)\sqrt{\rho_c / E_c}$ of Al/Al_2O_3 FG square plates with gradient index

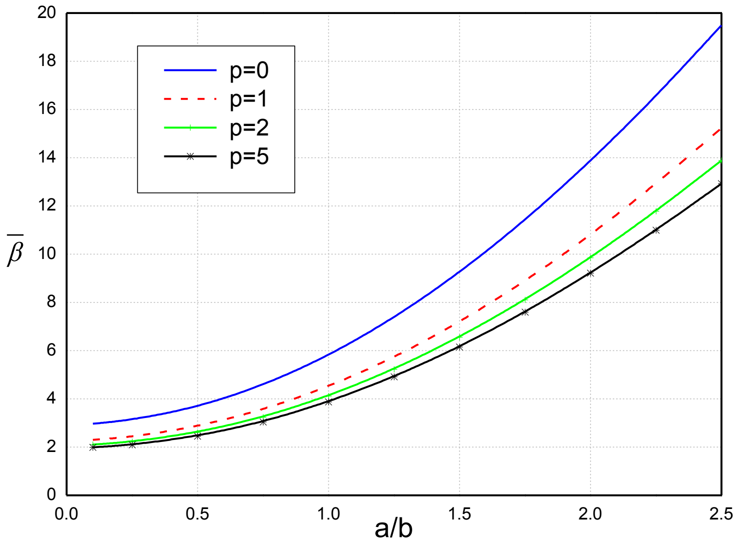


Fig. 2 Variation of non-dimensional fundamental frequency $\overline{\beta} = (\omega a^2 / h)\sqrt{\rho_c / E_c}$ of Al/Al_2O_3 FG square plates versus aspect ratio ($a/h = 10$)

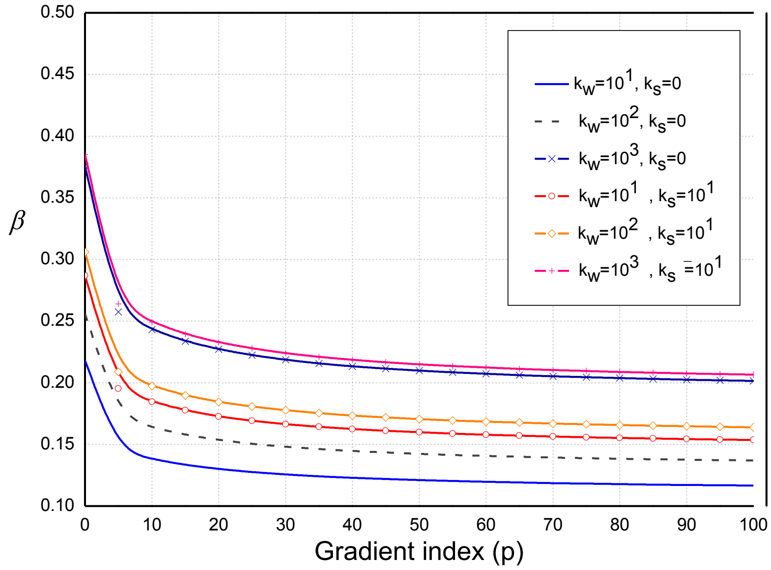


Fig. 3 Variation of non-dimensional fundamental frequency $\beta = \omega h\sqrt{\rho_c / E_c}$ of Al/Al_2O_3 FG square plates resting on elastic foundation with gradient index ($a/h = 5$)

increment ratio decreases for high values of “ k ”. For but the high values of the gradient index “ k ” and thesame thickness ratio “ a/h ”, the natural frequency does not change too much. Fig. 2 presents the variation of natural frequency of FG plates versus the aspect ratio “ a/b ” for different values of gradient index “ k ”. From these results, it can be observed that for a given value of “ a/b ” as the

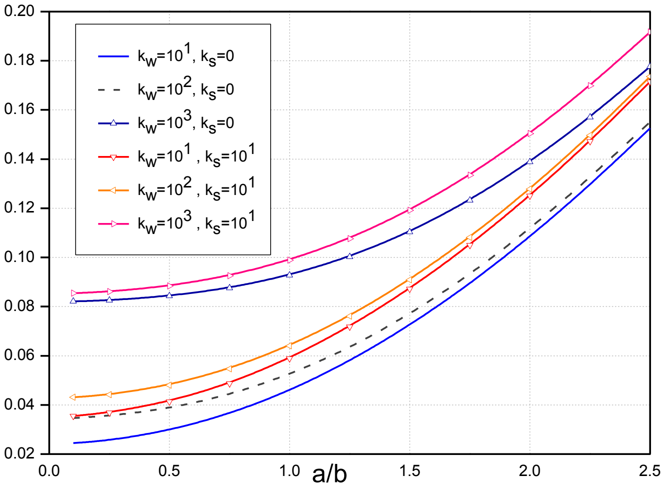


Fig. 4 Variation of non-dimensional fundamental frequency $\beta = \omega h\sqrt{\rho_c / E_c}$ of Al/Al_2O_3 FG square plates resting on elastic foundation versus aspect ratio ($a/h = 10$, $p = 1$)

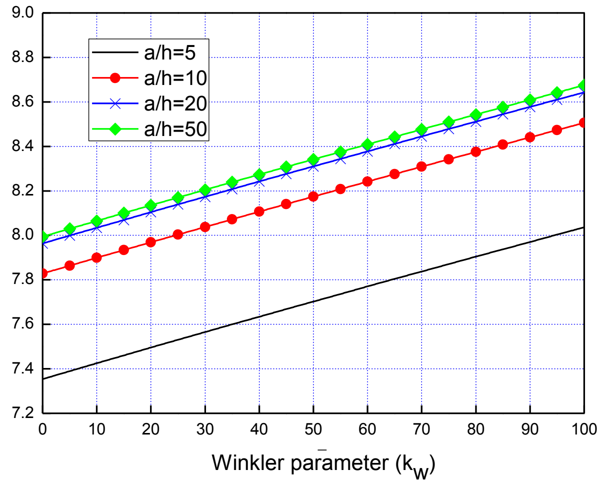


Fig. 5 Variation of non-dimensional fundamental frequency $\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_m / E_m}$ of Al/ZrO_2 FG square plates resting on elastic foundation versus the Winkler parameter ($k_s = 10$, $p = 1$)

gradient index " k " increases, the natural frequency decrease. It can be also seen that for a fixed value of the gradient index " k ", as the aspect ratio " a/b " increases, the nominal frequency increases.

Fig. 3 indicates the variation of non-dimensional natural frequency of FG plates resting on elastic foundation versus the gradient index " k " for different values of " k_w " and " k_s ". From this figure, it can be observed that for a given value of " k " and one coefficient of Pasternak, as the other coefficient increase, the natural frequency increase. Again, it can be noticed that for high values of the gradient index " k " the natural frequency does not change too much. Fig. 4 presents the variation of the natural frequency versus the aspect ratio " a/b " of FG plate ($k = 1$) for different values of foundation parameters " k_w, k_s ". It can be observed that for a given value of aspect ratio " a/b " and Winkler coefficient " k_w ", as the Parameter coefficient

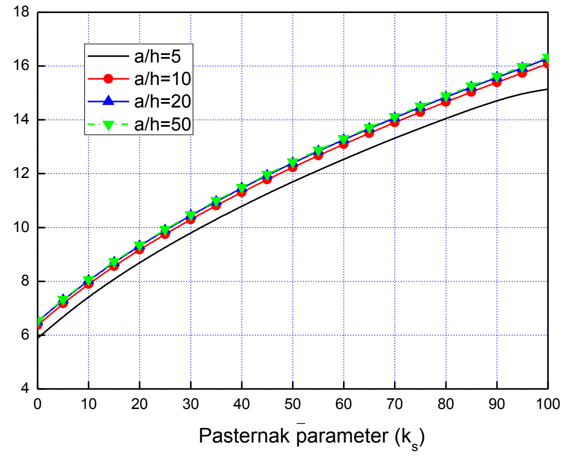
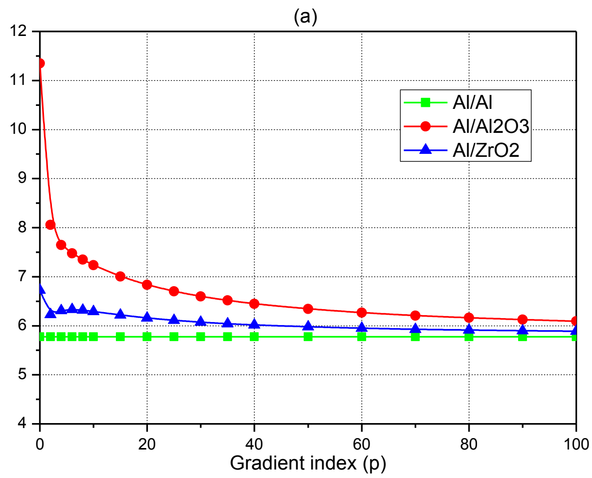
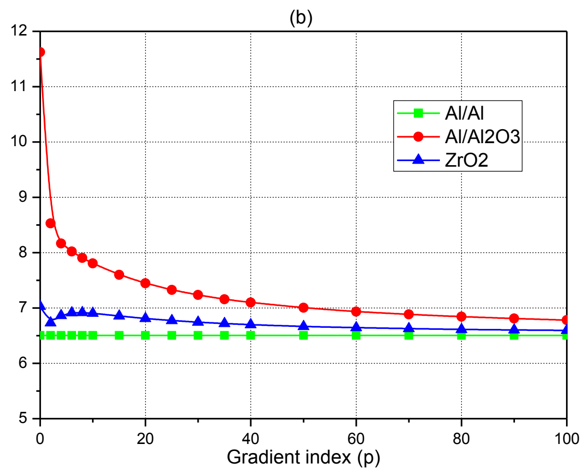


Fig. 6 Variation of non-dimensional fundamental frequency $\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_m / E_m}$ of Al/ZrO_2 FG square plates resting on elastic foundation versus the Pasternak parameter ($k_w = 10$, $p = 1$)





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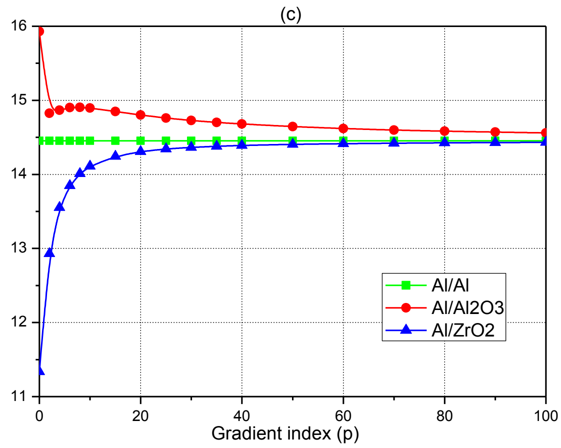

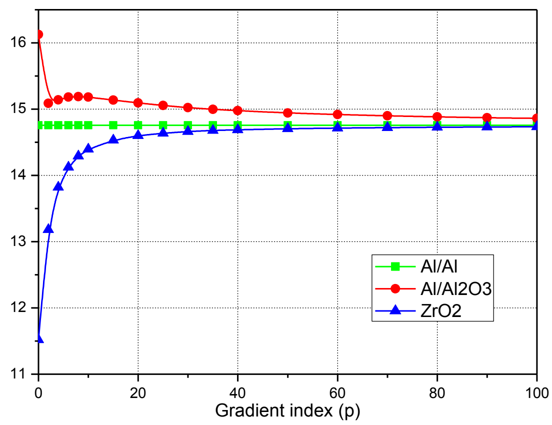

Fig. 7 Variation of non-dimensional fundamental frequency $\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_m / E_m}$ of different functionally graded square plates versus the gradient index ($a/h = 10$, $p = 1$): (a) $k_w = k_s = 0$, (b) $k_w = 100$, $k_s = 0$, (c) $k_w = 0$, $k_s = 100$ and (d) $k_w = 100$, $k_s = 100$

" k_s " increases, the natural frequency increases. Also it is observed that for small values of " a/b " and " k_w " constant, the curves tend to approach to the same value, this can be seen more clearly for large values of " k_w ". When " k_s " is constant for a given value of " a/b ", as the value of the Winkler coefficient " k_w " increases, the value of natural frequency increases. It is also remarked that the curves approach each other as the ratio " a/b " increases.

Figs. 5 and 6 present the variation of the natural frequency versus the Winkler parameter " k_w " and the Pasternak parameter " k_s ", respectively. In Fig. 5 can be observed that the natural frequency vary linearly with the Winkler parameter " k_w ". The curves presented in Fig. 6 have a greater slope than the curves in Fig. 5, i.e., the Pasternak parameter " k_s " has greater effect in the natural frequency than the Winkler parameter " k_w ".

The non-dimensional frequency-gradient index plots of FG plates are shown in Figs. 7(a)-(d) for different FGMs and different values of foundation parameters. It can be seen the foundation coefficients have a great effect on vibration response of FG plate. Also,

4. Conclusions

This work presents a dynamic analysis for FG plates resting on elastic foundation by employing a new quasi-3D hybrid type HSDT. The theory is developed by making further simplifying assumptions to the existing HSDTs, with the incorporation of an undetermined integral term. The number of variables and equations of motion of the proposed quasi-3D hybrid type HSDT are reduced by one, and hence, make this theory simple and efficient to use. The equations of motion are obtained through the Hamilton's principle. These equations are solved by utilizing Navier's procedure, subsequently the fundamental frequencies are found by solving the corresponding after eigenvalue problem. The results were compared with the solutions of several theories. The results determined by the proposed theory can be summarized as follows:

- It has been noticed that the proposed formulation can accurately predict fundamental frequencies of FG plates resting on two-layer elastic foundations.
- The fundamental frequencies of FG plate decrease with the increase of gradient index.
- In the presence of elastic foundation, increasing value of Winkler and Pasternak coefficients causes to increase in the fundamental frequency of FG plate.
- The Pasternak modulus coefficient of foundation has more significant effect on increasing natural frequency of FG plate than the Winkler modulus coefficient.
- Increasing value of gradient index increases the effect of elastic foundation on natural frequency.

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