Smart Structures and Systems, *Vol. 20, No. 3 (2017) 369-383*
DOI: https://doi.org/10.12989/sss.2017.20.3.369

Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory

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(Received June 6, 2016, Revised January 11, 2017, Accepted January 12, 2017)

Abstract. In this work, the effects of moisture and temperature on free vibration characteristics of functionally graded (FG) nanobeams resting on elastic foundation is studied by proposing a novel simple trigonometric shear deformation theory. The main advantage of this theory is that, in addition to including the shear deformation influence, the displacement field is modeled with only 2 unknowns as the case of the classical beam theory (CBT) and which is even less than the Timoshenko beam theory (TBT). Three types of environmental condition namely uniform, linear, and sinusoidal hygrothermal loading are studied. Material properties of FG beams are assumed to vary according to a power law distribution of the volume fraction of the constituents. Equations of motion are derived from Hamilton's principle. Numerical examples are presented to show the validity and accuracy of present shear deformation theories. The effects of hygro-thermal environments, power law index, nonlocality and elastic foundation on the free vibration responses of FG beams under hygro-thermal effect are investigated.

Keywords: free vibration; trigonometric shear deformation beam theory; functionally graded nanobeam; nonlocal elasticity theory; hygrothermal effect

1. Introduction

Applications of the functionally graded structures have been increasing in the field of aerospace, aircrafts, automotive industry and other engineering domain (Kar and Panda 2015, Akbaş 2015, Akavci 2015, Ait Atmane *et al*. 2015, Arefi, 2015a, b, Arefi and Allam 2015, Bakora and Tounsi 2015, Aizikovich *et al*. 2016, Benferhat *et al*. 2016, Mehar and Panda 2016a, b, c, Mehar *et al*. 2016, Mehar and Panda 2017a, b, c, El-Haina *et al*. 2017). FGMs structures with the continuous variation of properties of materials possess advantages to have the reduction of residual and thermal stresses. Advanced structural components composed of functionally graded materials (FGMs) are exposed to environmental conditions such as high temperature and moisture effect that are detrimental to the resistance and rigidity of the advanced composite structures even at nanoscale components. Hence the analysis of such composite nanoscale structures under hygrothermal loading has been of considerable interest to researchers. Recently, considerable interests have been devoted to experimental and theoretical works of the hygrothermal response of graded structures. Since, controlling the experimental conditions is not evident for nanoscale structures, theoretical models become necessary (Benguediab *et al*. 2014, Zemri *et al*. 2015, Adda Bedia *et al*. 2015, Belkorissat *et al*. 2015, Larbi Chaht *et al*. 2015, Ahouel *et al*. 2016, Bounouara *et al*. 2016, Barati and Shahverdi 2016, Bouafia *et al*. 2017, Besseghier *et al*. 2017, Bellifa *et al*. 2017). Lee and Kim (2013) studied the thermal postbuckling behavior of the FGM plates considering hygrothermal as well as moisture effects based on the first-order shear deformation plate theory. Akbarzadeh and Chen (2013) presented an analytical solutions for hygrothermal stresses in one-dimensional functionally graded piezoelectric subjected to an external constant magnetic field and resting on a Winkler-type elastic foundation. Zidi *et al*. (2014) studied the bending analysis of FGM plates under hygro-thermomechanical loading using a four-variable refined plate theory. Ebrahimi and Barati (2016a) investigated the influence of the environments on the damping vibration of FG nanobeams based on nonlocal strain gradient elasticity theory. Sobhy (2016) proposed an analytical approach to illustrate the hygrothermal vibration and buckling of FGM sandwich plates resting on Winkler–Pasternak elastic foundations based using new accurate four-variable shear

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http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

deformation plate theory. Also, recently Laoufi *et al.* (2016) presented an analytical method to determine the deflection and stress distributions in functionally graded plates subjected to a mechanical load, a temperature and moisture fields. Most recently Beldjelili *et al.* (2016) investigated bending behavior of sigmoid functionally graded material (S-FGM) plate resting on variable two-parameter elastic foundations with consideration hygro-thermo-mechanical of based on a four-variable refined plate theory. Due to the inability of classical beam theory or Euler–Bernoulli beam theory (EBT) to take into account the transverse shear deformations, as well as a shear correction factor is required to compensate for the difference between the actual stress state and the constant stress state in the case of Timoshenko beam theory (TBT), a number of higher-order shear deformation theories are proposed and developed based on the assumption of the higher-order variation of axial displacement through the height of the beam and applied in analysis of FG structures. For thick and moderately deep FG beams, the CBT overrated natural frequency due to ignoring the transverse shear deformation effect (Şimşek 2009, Eltaher *et al.* 2012, Kaci *et al*. 2012). In TBT (Şimşek and Yurtçu 2012, bouremana *et al*. 2013), the distribution of the transverse shear stress is assumed constant with respect to the thickness coordinate. However, influences of the transverse shear stress may become more considerable for moderately short beams. In fact, to keep away the use of a shear correction factor, many higher order shear deformation theories have been proposed, notable among them are the parabolic theory deformation beam theory (PSDBT) of Reddy (1984), the trigonometric shear deformation beam theory (TSDBT) of Touratier (1991), the hyperbolic shear deformation beam theory (HSDBT) of Soldatos (1992), the exponential shear deformation beam theory (ESDBT) of Karama *et al*. (2003).

This work aims to develop a new trigonometric shear deformation theory to investigate the influences of moisture and temperature rise due to various hygro-thermal loads on vibration of nanosize FGM beams resting on elastic foundation. The proposed theory contain fewer unknowns and equations of motion than the first-order shear deformation theory, but satisfy the zero traction boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not required. In addition, unlike the previous mentioned theories, the number of variables in the present theory is same as that in the CPT. Three types of environmental condition namely uniform, linear, and sinusoidal hygrothermal loading are studied. Material properties of FG beams are assumed to vary according to a power law distribution of the volume fraction of the constituents. Equations of motion are derived from Hamilton's principle. Numerical examples are presented to show the validity and accuracy of present shear deformation theories. The effects of hygro-thermal environments, power law index, nonlocality and elastic foundation on the free vibration responses of FG beams under hygro-thermal effect are investigated.

2. Mathematical formulation

2.1 Nonlocal power-low FG nanobeam model

According to power-law form the material properties of the FG nanobeam (Fig. 1) such as Young's modulus (E), Poisson's ratio (ν), the mass density (ρ),the thermal expansion (α), moisture expansion coefficient (β) and shear modulus (G) can be calculated (Tounsi *et al*. 2013, Bouderba *et al.* 2013, Attia *et al*. 2015, Bellifa *et al*. 2016, Bouderba *et al.* 2016, Bousahla *et al.* 2016, Fahsi *et al*. 2017, Meksi *et al*. 2017)

$$P(z) = P_c V_c + P_m V_m \tag{1}$$

where, V_i ($i = c, m$) is the volume fraction of the phase material. The subscripts $'c'$ and $'m'$ represent the ceramic and metal phases, respectively. The volume fractions of the ceramic and metal phases are related by

$$V_c + V_m = 1 \tag{2}$$

and V_c is written as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^p, \quad p \geq 0 \tag{3}$$

With $'p'$ is the gradient index which determines the material distribution through the thickness of the beam and z is the distance from the mid-surface of the FG nanobeam, the effective material properties of the non-local FG beam including Young's modulus (E), shear modulus (G), mass density (ρ), thermal expansion (α), and moisture expansion coefficient (β) can be expressed in the following form

$$E(z) = (E_c - E_m)\left(\frac{2z+h}{2h}\right)^p + E_m \tag{4a}$$

$$G(z) = (G_c - G_m)\left(\frac{2z+h}{2h}\right)^p + G_m \tag{4b}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^p + \rho_m \tag{4c}$$

$$\alpha(z) = (\alpha_c - \alpha_m)\left(\frac{2z+h}{2h}\right)^p + \alpha_m \tag{4d}$$

$$\beta(z) = (\beta_c - \beta_m)\left(\frac{2z+h}{2h}\right)^p + \beta_m \tag{4e}$$

The non-linear FG nanobeam material can be computed as the following equation

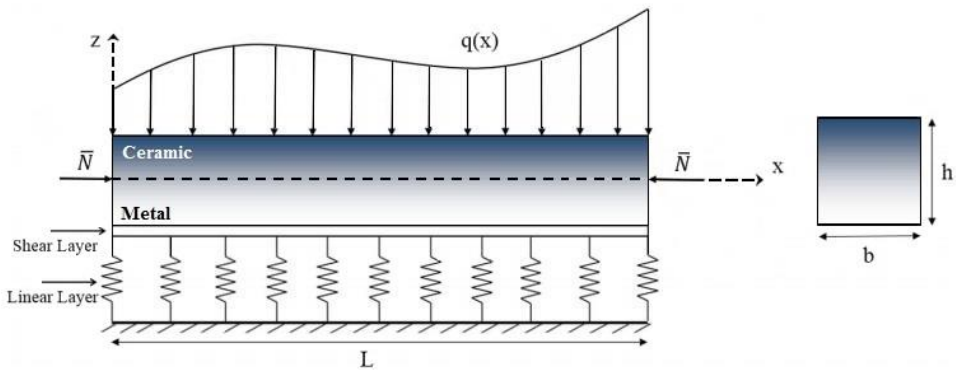

Fig. 1 Geometry of functionally graded nanobeam resting on elastic foundation

Table 1 Temperature-dependent material properties of FGM constituents (Ebrahimi and Salari 2015).

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E(Pa)	3.4843e+11	0	-3.070e-4	2.160e-7	-8.946e-11
	α(K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ(Kg/m^3)	2370	0	0	0	0
	v	0.24	0	0	0	0
SUS 304	E(Pa)	2.0104e+11	0	3.079e-4	-6.534e-7	0
	α(K^{-1})	12.33e-6	0	8.086e-4	0	0
	ρ(Kg/m^3)	8166	0	0	0	0
	v	0.3262	0	-2.002e-4	3.797e-7	0

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T^1 + P_2T^2 + P_3T^3) \tag{5}$$

where, T is the environmental temperature; P indicates material property; P_{-1}, P_0, P_1, P_2 and P_3 are the coefficients of temperature-dependent material properties unique to the constituent materials (Table 1).

2.2 Kinematic relations

Consider a simply supported FG beam with the length L and rectangular cross-section $b \times h$ with b being the width and h being the height (Fig. 1). The beam is made of isotropic material with material properties varying smoothly in the thickness direction. Unlike the previous mentioned theories, the number of unknown functions involved in the present theory is only two as in EBT.

The displacement field of the proposed two unknowns shear deformation theory is built upon the Euler-Bernoulli beam theory (EBT) including the trigonometric function in terms of thickness coordinate to represent shear deformation and is assumed as follows (Tounsi *et al*. 2016, Houari *et al.* 2016, Klouche *et al*. 2017)

$$\begin{aligned} u(x,z,t) &= u_0(x,t) - z\frac{\partial w_0}{\partial x} - \beta\, f(z)\frac{\partial^3 w_0}{\partial x^3} \\ w(x,z,t) &= w_0(x,t) \end{aligned} \tag{6}$$

where u_0 and w_0 are two unknown displacement functions of mid-axis of the beam. $f(z)$ is a shape function representing the variation of the transverse shear strains and shear stresses through the thickness of the beam and is given as

$$f(z) = \frac{z\left(\pi + 2\cos\left(\frac{\pi z}{h}\right)\right)}{(2+\pi)} \tag{7}$$

The nonzero linear strains related to displacement field in Eq. (1) are

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x + \beta\, f(z)\eta_x, \quad \gamma_{xz} = \beta\, g(z)\gamma_{xz}^0 \tag{8}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x = -\frac{\partial^2 w_0}{\partial x^2}, \quad \eta_x = -\frac{\partial^4 w_0}{\partial x^4}, \quad \gamma_{xz}^0 = -\frac{\partial^3 w_0}{\partial x^3} \tag{9}$$

and

$$g(z) = f'(z) \tag{10}$$

where β is defined in Eq. (27).

2.3 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in the following form (Ait Amar Meziane *et al*. 2014, Bennai *et al*. 2015, Taibi *et al.* 2015, Ait Yahia *et al*. 2015, Mahi *et al*. 2015, Meksi *et al.* 2015, Becheri *et al*. 2016, Boukhari *et al*. 2016, Merdaci *et al*. 2016, Chikh *et al*. 2016)

$$0 = \int_0^t \delta(U + V - K)\, dt \tag{11}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; U is strain energy, V is work done by external forces, and K is the variation of kinetic energy of the beam.

The variation of the strain energy of the beam can be stated as

$$\delta U = \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})\, dxdz = \int_0^L \left(N_x \delta\varepsilon_x^0 + M_x \delta k_x + \beta(S_x \delta\eta_x + Q_{xz}\delta\gamma_{xz}^0)\right) dx \tag{12}$$

In which the stress resultants N_x, M_x, S_x and Q_{xz} are defined by

$$(N_x, M_x, S_x) = \int_{-h/2}^{h/2} \sigma_x \,(1, z, \beta\, f(z))\, dz, \quad Q_{xz} = \int_{-h/2}^{h/2} \tau_{xz}\, \beta\, g(z)\, dz \tag{13}$$

The first variation of the work done by applied forces can be written in the form

$$\delta V = \int_0^L \left[\left(N^T + N^H\right)\left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\right) - k_w w\delta w + k_s \left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\right)\right] dx \tag{14}$$

where k_w and k_s are linear and shear coefficient of elastic foundation, respectively, and N^T and N^H are applied forces due to temperature and moisture change as

$$N^T = \int_{-h/2}^{h/2} E(z)\alpha(z,T)(T - T_0)\, dz, \quad N^H = \int_{-h/2}^{h/2} E(z)\beta(z,T)(C - C_0)\, dz \tag{15}$$

where T_0 and C_0 are the reference temperature and moisture concentrations, respectively. The variation of kinetic energy is written as

$$\begin{aligned}\delta K &= \int_0^L \int_{-h/2}^{h/2} [\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}]\rho(z)\, dxdz \\ &= \int_0^L \int_{-h/2}^{h/2} \Big\{ I_0(\dot{u}_0\delta\dot{u}_0 + \dot{w}_0\delta\dot{w}_0) - I_1\left(\dot{u}_0 \frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0\right) - J_1\beta\left(\dot{u}_0\frac{\partial^3\delta\dot{w}_0}{\partial x^3} + \frac{\partial^3\dot{w}_0}{\partial x^3}\delta\dot{u}_0\right) \\ &\quad + I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + K_2\beta^2\left(\frac{\partial^3\dot{w}_0}{\partial x^3}\frac{\partial^3\delta\dot{w}_0}{\partial x^3}\right) + J_2\beta\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial^3\delta\dot{w}_0}{\partial x^3} + \frac{\partial^3\dot{w}_0}{\partial x^3}\frac{\partial\delta\dot{w}_0}{\partial x}\right)\Big\} dxdz\end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-h/2}^{h/2} (1, z, f, z^2, z\,f, f^2)\rho(z)\, dz \tag{17}$$

Using the expressions for δU, δV, and δK from Eqs. (14) and (16) into Eq. (11) and integrating by parts, and collecting the coefficients of δu_0 and δw_0, the following equations of motion of the beam are obtained

$$\begin{aligned}\delta u_0 &: \frac{\partial N_x}{\partial x} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} - \beta J_1\frac{\partial^3\ddot{w}_0}{\partial x^3} \\ \delta w_0 &: \frac{\partial^2 M_x}{\partial x^2} + \beta\left(\frac{\partial^4 S_x}{\partial x^4} - \frac{\partial^3 Q_{xz}}{\partial x^3}\right) + \left(N^T + N^H\right)\frac{\partial^2 w}{\partial x^2} + k_w w - k_s\frac{\partial^2 w}{\partial x^2} = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x}\right) - I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}\right) \\ &\quad + \beta J_1\left(\frac{\partial^3\ddot{u}_0}{\partial x^3}\right) - 2\beta J_2\left(\frac{\partial^4\ddot{w}_0}{\partial x^4}\right) - \beta^2 K_2\left(\frac{\partial^6\ddot{w}_0}{\partial x^6}\right)\end{aligned} \tag{18}$$

2.4 The nonlocal elasticity model for FG nanobeam

According to nonlocal elasticity theory (Eringen and Edelen 1972, Eringen 1983), the stress state at a point inside a body is regarded to be a function of strains of all points in the neighbor regions. For homogeneous elastic solids, the nonlocal stress-tensor components σ_{ij} at each point x in the solid can be expressed as

$$\sigma_{ij}(x) = \int_\Omega \alpha(|x' - x|, \tau)\, t_{ij}(x')\, d\Omega(x') \tag{19}$$

where $t_{ij}(x')$ are the components available in local stress tensor at point x which are associated to the strain tensor components ε_{kl} as

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{20}$$

The concept of Eq. (19) is that the nonlocal stress at any point is a weighting average of the local stress of all near points, and the nonlocal kernel $\alpha(|x' - x|, \tau)$ considers the influence of the strain at the point x' on the stress at the point x in the elastic body. The parameter α is an internal characteristic length (e.g., lattice parameter, granular distance, the length of C–C bonds). Also $|x' - x|$ is Euclidean distance and τ is a constant value as follows

$$\tau = \frac{e_0 a}{l} \tag{21}$$

which presents the relation of a characteristic internal length, and a characteristic external length, l (e.g., crack length and wavelength) using a constant, e_0, dependent on each material. The value of e_0 is experimentally evaluated by comparing the scattering curves of plane waves with those of atomistic dynamics. In the nonlocal model of elasticity, the points undergo translational motion as in the classical case, but the stress at a point depends on the strain in a region near that point. As for physical interpretation, the nonlocal model introduces long range interactions between points in a continuum model. Such long range interactions occur between charged atoms or molecules in a solid. Eringen (Eringen 1972, Eringen 1983) numerically determined the functional form of the kernel. By appropriate selection of the kernel function, Eringen shown that the nonlocal constitutive equation given in integral form (see Eq. (19)) can be represented in an equivalent differential form as

$$\left(1-(e_0 a)\nabla^2\right)\sigma_{kl} = t_{kl} \tag{22}$$

In which ∇^2 is the Laplacian operator. Hence, the scale length $e_0 a$ considers the effects of small size on the behavior of nanostructures. Thus, the constitutive relations of nonlocal theory for a FG nanobeam can be written as

$$\left(1-\mu\nabla^2\right)\begin{Bmatrix}\sigma_{xx}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}Q_{11} & 0\\ 0 & Q_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \gamma_{xz}\end{Bmatrix} \tag{23}$$

In which $\mu = (e_0 a)^2$ and the stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = E(z),\ Q_{44} = G(z). \tag{24}$$

Integrating Eq. (23) over the beam's cross-section area yields the force–strain and the moment–strain of the nonlocal refined FG beams as follows

$$\left(1-\mu\nabla^2\right)\begin{Bmatrix}N_x\\ M_x\\ S_x\end{Bmatrix} = \begin{bmatrix}A_{11} & B_{11} & \beta B_{11}^s\\ B_{11} & D_{11} & \beta D_{11}^s\\ \beta B_{11}^s & \beta D_{11}^s & \beta^2 H_{11}^s\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ k_x\\ \eta_x\end{Bmatrix} \tag{25a}$$

$$\left(1-\mu\nabla^2\right)Q_{xz} = \beta^2 A_{44}^s\gamma_{xz}^0 \tag{25b}$$

where

$$\left\{A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right\} = \int_{-h/2}^{h/2} Q_{11}(z)\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right)dz \tag{26a}$$

$$A_{55}^s = \int_{-h/2}^{h/2} Q_{55}(z)[g(z)]^2\,dz, \tag{26b}$$

with

$$\beta = -\frac{\left(D_{11}^s A_{11} - B_{11}B_{11}^s\right)}{\left(A_{11}A_{44}^s + A_{11}\lambda^2 H_{11}^s - B_{11}^s\lambda^2\right)}, \tag{27a}$$

for an isotropic beam β takes the following expression

$$\beta = -\frac{D_{11}^s}{\left(A_{44}^s + \lambda^2 H_{11}^s\right)}, \tag{27b}$$

By substituting Eq. (25) into Eq. (18), the governing equations can be written in terms of generalized displacements (u_0 and w_0) as

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \beta B_{11}^s\frac{\partial^5 w_0}{\partial x^5} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} - \beta J_1\frac{\partial^3\ddot{w}_0}{\partial x^3}, \tag{28a}$$

$$\begin{aligned}&B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} + \beta\left(B_{11}^s\frac{\partial^5 u_0}{\partial x^5} - 2D_{11}^s\frac{\partial^6 w_0}{\partial x^6}\right) - \beta^2\left(H_{11}^s\frac{\partial^8 w_0}{\partial x^8} - A_{44}^s\frac{\partial^6 w_0}{\partial x^6}\right) + \left(N^T + N^H\right)\frac{\partial^2 w}{\partial x^2}\\ &+ k_w w - k_s\frac{\partial^2 w}{\partial x^2} = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x}\right) - I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}\right) + \beta J_1\left(\frac{\partial^3\ddot{u}_0}{\partial x^3}\right) - 2\beta J_2\left(\frac{\partial^4\ddot{w}_0}{\partial x^4}\right) - \beta^2 K_2\left(\frac{\partial^6\ddot{w}_0}{\partial x^6}\right)\end{aligned} \tag{28b}$$

3. Analytical solution

The above governing equations are analytically solved for bending problems of a simply supported beam. Based on Navier solution procedure, the displacements are assumed as follows

$$\begin{Bmatrix}u_0(x)\\ w_0(x)\end{Bmatrix} = \sum_{m=1}^{\infty}\begin{Bmatrix}U_m\cos(\lambda x)e^{i\omega t}\\ W_m\sin(\lambda x)e^{i\omega t}\end{Bmatrix} \tag{29}$$

where $\lambda = m\pi/a$, (U_m, W_m) are arbitrary parameters to be determined, ω is the eigenfrequency associated with m-th eigenmode

Substituting the expansions of u_0 and w_0 from Eqs. (29) into the equations of motion, Eq. (28), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix}a_{11} & a_{12}\\ a_{12} & a_{22}\end{bmatrix} - \xi\,\omega^2\begin{bmatrix}m_{11} & m_{12}\\ m_{12} & m_{22}\end{bmatrix}\right)\begin{Bmatrix}U_m\\ W_m\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{30}$$

where

$$a_{11} = -A_{11}\lambda^2$$

$$a_{12} = B_{11}\lambda^3 - \beta B_{11}^s\lambda^5 \tag{31}$$

$$a_{22} = -D_{11}\lambda^4 - 2\beta D_{11}^s\lambda^6 - \beta^2\left(H_{11}^s\lambda^8 + A_{55}^s\lambda^6\right) + \left(N^T + N^H\right)\lambda^2 - k_w - k_s\lambda^2$$

$$m_{11} = -I_0$$

$$m_{12} = I_1\lambda + \beta J_1\lambda^3$$

$$m_{22} = -I_0 - I_2\lambda^2 - 2\beta J_2\lambda^4 + \beta^2 K_2\lambda^6$$
$$\lambda = 1 + \mu\,\lambda^2$$

4. Various hygro-thermal environments

4.1 Uniform moisture and temperature rise

For a FG nanobeam at reference moisture concentration C_0 and reference temperature T_0, the moisture and temperature are uniformly raised to a final value C and T, respectively, in which the moisture and temperature change are $\Delta T = T - T_0$ and $\Delta C = C - C_0$.

4.2 Linear moisture and temperature rise

For a FG nanobeam for which the plate thickness is thin enough, the moisture and temperature distributions are linearly variable through the thickness as follows

$$T = T_m + \Delta T\left(\frac{z}{h} + \frac{1}{2}\right), C = C_m + \Delta C\left(\frac{z}{h} + \frac{1}{2}\right) \tag{32}$$

The $(\Delta T, \Delta C)$ in Eq. (32) could be defined $\Delta T = T - T_0$, $\Delta C = C - C_0$.

4.3 Sinusoidal moisture and temperature rise

The moisture and temperature fields when FG nanobeam is exposed to sinusoidal moisture/temperature rise across the thickness can be defined as (Na and Kim 2004, Ebrahimi and Barati 2016b)

$$T = T_m + \Delta T\left(1 - \cos\left(\frac{\pi}{2}\right)\left(\frac{z}{h} + \frac{1}{2}\right)\right) \tag{33a}$$

$$C = C_m + \Delta C\left(1 - \cos\left(\frac{\pi}{2}\right)\left(\frac{z}{h} + \frac{1}{2}\right)\right) \tag{33b}$$

where $\Delta T = T - T_0$ and $\Delta C = C - C_0$ are temperature and moisture change.

5. Numerical results and discussion

The general approach outlined in the previous sections for hygro-thermomechanical vibration analysis of a simply supported nanoscale FGM beam has been investigated through many numerical examples to verify the accuracy of the proposed new nonlocal trigonometric shear deformation beam theory. The material properties of the FG nanobeam as Young's modulus, Poisson ratio, thermal, and moisture expansion coefficients vary within the thickness direction according to power-law homogenization model. In thermal environment, high temperature makes a important change in mechanical properties of the constituent materials.

Therefore, it is necessary to take into account the temperature-dependent material property to predict the behavior of FGMs under high temperature more accurately. Temperature-dependent material properties of nonlocal P-FGM beam which is made from steel (SUS 304) with $\beta_m = 0.0005$ and silicon nitride (Si3N4) with $\beta_c = 0$ are given in Table 1. Verification is carried out by assuming the values of different quantities in the ceramic and metal as: $L(\text{length}) = 10\,\text{nm}$, $b(\text{width}) = 1\,\text{nm}$, and $h(\text{thickness}) = \text{varied}$. The temperature rise in fully metal surface of FG nanobeam to reference temperature T_0 is $T_m - T_0 = 5\,\text{K}$. To indicate the exactness of the validity of present model, a comparison study is provided.

Tables 2 show the natural frequency of presented sinusoidal FG nanobeam under linear temperature rise. The obtained results are compared with those computed independently based on the Euler-Bernoulli beam theory (CBT) and the parabolic shear deformation beam theory (PSBT), and those reported by Ebrahimi and Salari (2015) for Timoshenko beam theory (TBT), when nonlocal parameter changes from 0 to 4 nm^2. It can be observed that the present model can evaluate the vibrational behavior of FG nanobeams with excellent agreement. The shear correction factor is taken as 5/6 for Timoshenko beam theory. For better presentation of the results, the following nondimensionalizations are used

$$\hat{\omega} = \omega L^2\sqrt{\frac{\rho_c A}{E_c I}},\quad K_w = k_w\frac{L^4}{E_c I}, K_p = k_p\frac{L^2}{E_c I}$$

Tables 3-5 show the variations of the dimensionless frequencies of FG nanobeam resting on elastic foundation for various beam theories and three environmental conditions called uniform, linear, and sinusoidal hygro-thermal loadings at L/h = 20. It can be seen that all of the proposed beam theories provide excellent agreement as values of non-dimensional fundamental frequency are consistent with presented analytical solution. Also, it is observed that frequency results of shear deformation beam theories are lower than classical beam theory due to the cause that CBT is unable to capture shear deformation effect. Note that CBT disregards the shear deformation effect so that predicted dimensionless frequencies through CBT are overestimated. The dimensionless frequency diminishes as nonlocal parameter grows, for any type of hygro-thermal loading. The reason is the lower rigidity of the nanobeam when its size reduces.

Therefore, the variations of moisture or temperature have a substantial impact on the rigidity and vibration responses of size-dependent FG nanobeams and natural frequencies regardless of hygrothermal loading type. Moreover, it is observable that sinusoidal distribution of temperature and moisture provides higher natural frequency than other hygro-thermal loads; the uniform hygro-thermal loading has the lowest one.

Table 2 Comparison of the nondimensional fundamental frequency for a FG nanobeam under linear temperature rise without elastic foundation with various gradient indexes (*L*/*h* =20)

μ	*P*=0				*P*=0.2				*P*=1				*P*=5			
(nm^2)	CBT	TBT[(a)]	PSBT	Present	CBT	TBT[(a)]	PSBT	Present	CBT	TBT[(a)]	PSBT	Present	CBT	TBT[(a)]	PSBT	Present
0	9.1796	9.1475	9.1475	9.1477	7.3681	7.3420	7.3422	7.3423	5.3740	5.3537	5.3537	5.3538	4.3059	4.2875	4.2868	4.2869
1	8.6910	8.6601	8.6601	8.6603	6.9670	6.9419	6.9421	6.9422	5.0676	5.0480	5.0480	5.0481	4.0496	4.0317	4.0311	4.0311
2	8.2608	8.2310	8.2310	8.2311	6.6135	6.5892	6.5894	6.5895	4.7967	4.7777	4.7777	4.7778	3.8223	3.8049	3.8043	3.8043
3	7.8777	7.8488	7.8488	7.8490	6.2983	6.2747	6.2748	6.2750	4.5545	4.5360	4.5360	4.5361	3.6185	3.6015	3.6009	3.6009
4	7.5334	7.5053	7.5053	7.5054	6.0145	5.9916	5.9917	5.9918	4.3357	4.3177	4.3177	4.3178	3.4338	3.4172	3.4166	3.4167

[(a)] Ebrahimi and Salari (2015)

Table 3 Variation of the fundamental nondimensional frequencies of S-S FG nanobeam under uniform hygro-thermal loading for various beam theories ($K_w = 0$, $K_p = 0$, $L/h = 20$)

μ	Beam theory	$(\Delta T, \Delta C) = (0, 0)$			$(\Delta T, \Delta C) = (20, 1)$			$(\Delta T, \Delta C) = (40, 2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7.9923	5.9506	4.8629	7.4706	5.2423	4.0328	6.9006	4.4134	2.9692
	TBT	7.9683	5.9324	4.8466	7.4449	5.2216	4.0132	6.8728	4.3887	2.9423
	PSBT	7.9684	5.9324	4.8460	7.4451	5.2214	4.0124	6.8729	4.3887	2.9414
	Present	7.9686	5.9325	4.8461	7.4452	5.2215	4.0125	6.8731	4.3888	2.9414
1	CBT	7.6249	5.6770	4.6393	7.0759	4.9289	3.7594	6.4710	4.0354	2.5845
	TBT	7.6020	5.6597	4.6238	7.0512	4.9089	3.7402	6.4439	4.0108	2.5564
	PSBT	7.6021	5.6597	4.6233	7.0513	4.9089	3.7395	6.4441	4.0108	2.5554
	Present	7.6022	5.6598	4.6233	7.0515	4.9090	3.7395	6.4442	4.0110	2.5554
2	CBT	7.3039	5.4380	4.4440	6.7285	4.6511	3.5148	6.0889	3.6902	2.2125
	TBT	7.2819	5.4214	4.4292	6.7047	4.6316	3.4960	6.0625	3.6655	2.1824
	PSBT	7.2820	5.4214	4.4286	6.7048	4.6316	3.4953	6.0627	3.6655	2.1813
	Present	7.2822	5.4215	4.4287	6.7050	4.6317	3.4953	6.0628	3.6657	2.1814
3	CBT	7.0203	5.2269	4.2714	6.4193	4.4018	3.2933	5.7452	3.3700	1.8393
	TBT	6.9992	5.2109	4.2572	6.3963	4.3828	3.2747	5.7193	3.3450	1.8057
	PSBT	6.9993	5.2109	4.2567	6.3964	4.3828	3.2740	5.7195	3.3450	1.8045
	Present	6.9994	5.2110	4.2567	6.3965	4.3829	3.2741	5.7196	3.3452	1.8045
4	CBT	6.7673	5.0385	4.1175	6.1415	4.1760	3.0904	5.4328	3.0686	1.4438
	TBT	6.7470	5.0231	4.1038	6.1191	4.1574	3.0721	5.4074	3.0431	1.4039
	PSBT	6.7471	5.0231	4.1033	6.1192	4.1574	3.0714	5.4076	3.0431	1.4024
	Present	6.7472	5.0232	4.1033	6.1194	4.1575	3.0714	5.4077	3.0432	1.4025

Fig. 2 illustrate the variation of dimensionless natural frequency of simply supported sinusoidal FG nanobeam versus uniform, linear, and sinusoidal moisture concentration rise in prebuckling domain for different nonlocal parameters at $p = 1$, $L/h = 20$, $K_w = K_p = 0$ and $\Delta T = 40(K)$. It is observable that at a specified environmental condition, the nonlocal beam model creates lower natural frequency than local beam model. Also, it is shown that for all environmental conditions, the dimensionless natural frequency declines with the moisture increment. Therefore, moisture concentration and nonlocality have an important softening impact and desirable characteristic in beam structure and should be considered in the analysis of size-dependent FG nanobeams.

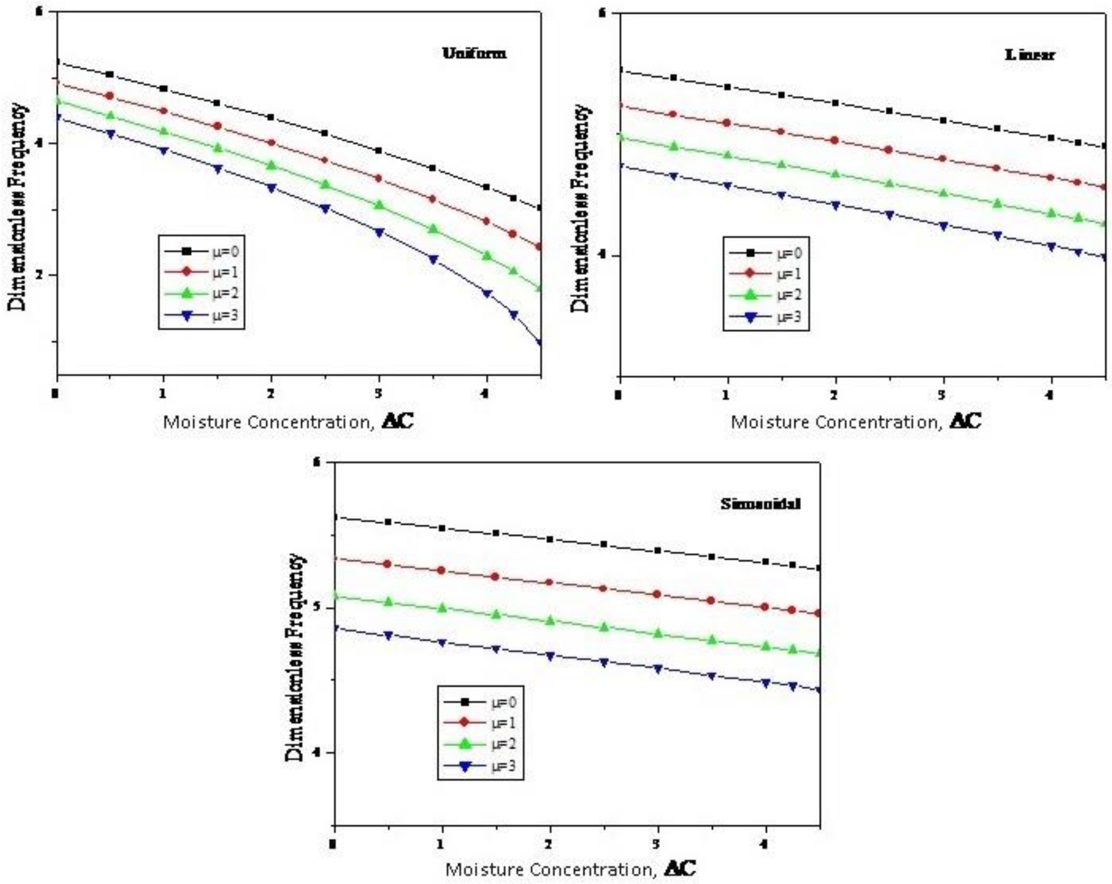

Fig. 2 Influence of moisture and nonlocal parameter on the dimensionless frequency of the S–S FG beam for various hygro-thermal loadings ($p=1$, $L/h=20$, $K_w=0$, $K_p=0$, $\Delta T = 40\,(\mathrm{K})$)

The variations of the dimensionless natural frequencies of the simply supported sinusoidal FG nanobeams as function of various temperature rises for different values of moisture concentration at $p=1$, $L/h=20$, $K_w=K_p=0$ and $l=2(\mathrm{nm})^2$ are depicted in Figure 3. For any type of hygro-thermal loadings with the temperature increment, the dimensionless natural frequencies of FG nanobeam reaches to zero nearby the critical temperature point. This particularly refers to stiffness degradation of nanobeam when the temperature increases. After the branching point, the grown in temperature yields larger values of natural frequency. Also, it is seen that moisture concentration has an important influence on the prebuckling and postbuckling configuration of FG nanobeam under hygro-thermal loads.

Fig. 4 indicates the influence of elastic foundation parameters on pre-buckling and post-buckling vibrational behavior of FG nanobeam versus various temperature rises under thermal $\Delta C=0$ and hygro-thermal $\Delta C=2$ loadings when $p=1$, $L/h=20$ and $l=2(\mathrm{nm})^2$. It can be seen that existence of elastic foundation enhances the beam structure in hygrothermal environment and increases the dimensionless critical buckling temperature. At a specified environmental of moisture or humidity, the inclusion of the shear layer or Pasternak foundation (K_p) parameter yields higher magnitude results for on the dimensionless frequency than those with the inclusion of Winkler foundation parameters (K_w). The variation of natural frequency with respect to volume fraction index for various thermal and hygro-thermal loadings with and without elastic foundation at $L/h=20$ and $l=2(\mathrm{nm})^2$ are depicted in Fig. 5. It can be seen that under any type of environmental conditions, the natural frequency reduces with the rise in gradient index, significantly for lower volume fraction indexes. It can be seen that existence of elastic foundation enhances the beam structure in hygrothermal environment and increases the dimensionless frequency. Also, it is observed that the effect of moisture concentration on the dimensionless natural frequencies responses of functionally graded nanobeams is more important for larger values of volume fraction index. This is due to the fact that lower values of nonhomogeneity index are correspond to more portion of the ceramic phase which has a moisture expansion coefficient equal to zero ($\beta_c=0$).

Table 4 Variation of the fundamental nondimensional frequencies of S-S FG nanobeam under linear hygro-thermal loading for various beam theories ($K_w = 0$, $K_p = 0$, $L/h = 20$)

μ	Beam theory	$(\Delta T, \Delta C) = (0, 0)$			$(\Delta T, \Delta C) = (20, 1)$			$(\Delta T, \Delta C) = (40, 2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7.9053	5.8680	4.7844	7.6880	5.5817	4.4153	7.4600	5.2763	4.0085
	TBT	7.8810	5.8496	4.7679	7.6631	5.5623	4.3974	7.4343	5.2557	3.9887
	PSBT	7.8812	5.8496	4.7673	7.6632	5.5623	4.3967	7.4344	5.2557	3.9880
	Present	7.8813	5.8497	4.7674	7.6634	5.5624	4.3967	7.4345	5.2558	3.9881
1	CBT	7.5336	5.5904	4.5570	7.3051	5.2885	4.1670	7.0644	4.9645	3.7326
	TBT	7.5105	5.5728	4.5413	7.2812	5.2698	4.1498	7.0397	4.9445	3.7132
	PSBT	7.5106	5.5728	4.5407	7.2813	5.2698	4.1491	7.0398	4.9445	3.7125
	Present	7.5107	5.5729	4.5407	7.2814	5.2699	4.1492	7.0400	4.9446	3.7125
2	CBT	7.2086	5.3476	4.3580	6.9691	5.0306	3.9478	6.7162	4.6882	3.4854
	TBT	7.1864	5.3307	4.3429	6.9461	5.0126	3.9311	6.6923	4.6688	3.4663
	PSBT	7.1865	5.3307	4.3423	6.9463	5.0126	3.9304	6.6925	4.6688	3.4656
	Present	7.1866	5.3308	4.3424	6.9464	5.0127	3.9305	6.6926	4.6689	3.4657
3	CBT	6.9211	5.1327	4.1819	6.6711	4.8011	3.7519	6.4063	4.4405	3.2612
	TBT	6.8997	5.1164	4.1674	6.6489	4.7836	3.7357	6.3831	4.4216	3.2424
	PSBT	6.8998	5.1164	4.1668	6.6491	4.7836	3.7350	6.3833	4.4216	3.2417
	Present	6.8999	5.1165	4.1668	6.6492	4.7837	3.7351	6.3834	4.4217	3.2418
4	CBT	6.6644	4.9408	4.0246	6.4042	4.5949	3.5752	6.1277	4.2163	3.0556
	TBT	6.6437	4.9251	4.0105	6.3827	4.5780	3.5593	6.1052	4.1978	3.0370
	PSBT	6.6439	4.9251	4.0100	6.3829	4.5780	3.5588	6.1053	4.1978	3.0363
	Present	6.6440	4.9252	4.0100	6.3830	4.5781	3.5588	6.1054	4.1979	3.0364

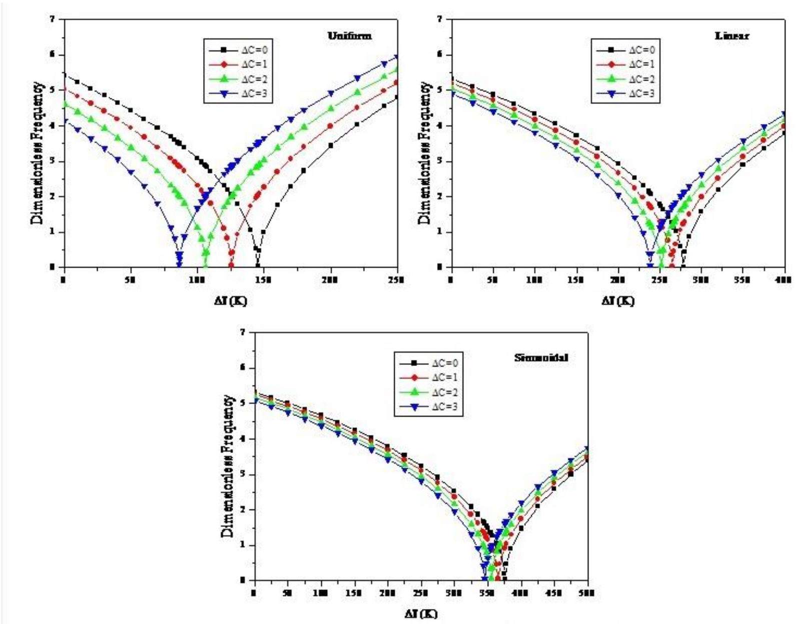

Fig. 3 Influence of moisture concentration on the dimensionless frequency of the S–S FG nanobeam with respect to various temperature rises ($p = 1$, $L/h = 20$, $K_w = 0$, $K_p = 0$)

Table 5 Variation of the fundamental nondimensional frequencies of S-S FG nanobeam under sinusoidal hygro-thermal loading for various beam theories ($K_w = 0$, $K_p = 0$, $L/h = 20$)

μ	Beam theory	$(\Delta T, \Delta C) = (0, 0)$			$(\Delta T, \Delta C) = (20, 1)$			$(\Delta T, \Delta C) = (40, 2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7.9053	5.8680	4.7844	7.7592	5.6833	4.5349	7.6070	5.4897	4.2683
	TBT	7.8810	5.8496	4.7679	7.7344	5.6642	4.5174	7.5818	5.4699	4.2497
	PSBT	7.8812	5.8496	4.7673	7.7346	5.6642	4.5168	7.5819	5.4699	4.2490
	Present	7.8813	5.8497	4.7674	7.7347	5.6643	4.5168	7.5820	5.4700	4.2490
1	CBT	7.5336	5.5904	4.5570	7.3799	5.3956	4.2936	7.2194	5.1907	4.0102
	TBT	7.5105	5.5728	4.5413	7.3562	5.3773	4.2768	7.1953	5.1717	3.9922
	PSBT	7.5106	5.5728	4.5407	7.3564	5.3773	4.2762	7.1954	5.1717	3.9915
	Present	7.5107	5.5729	4.5407	7.3565	5.3774	4.2762	7.1955	5.1718	3.9916
2	CBT	7.2086	5.3476	4.3580	7.0475	5.1430	4.0811	6.8791	4.9272	3.7812
	TBT	7.1864	5.3307	4.3429	7.0248	5.1254	4.0649	6.8558	4.9087	3.7637
	PSBT	7.1865	5.3307	4.3423	7.0249	5.1254	4.0643	6.8559	4.9087	3.7630
	Present	7.1866	5.3308	4.3424	7.0250	5.1255	4.0644	6.8560	4.9088	3.7631
3	CBT	6.9211	5.1327	4.1819	6.7530	4.9188	3.8920	6.5768	4.6921	3.5756
	TBT	6.8997	5.1164	4.1674	6.7310	4.9018	3.8763	6.5543	4.6742	3.5585
	PSBT	6.8998	5.1164	4.1668	6.7312	4.9018	3.8757	6.5544	4.6742	3.5579
	Present	6.8999	5.1165	4.1668	6.7313	4.9019	3.8757	6.5545	4.6743	3.5579
4	CBT	6.6644	4.9408	4.0246	6.4894	4.7178	3.7219	6.3058	4.4806	3.3892
	TBT	6.6437	4.9251	4.0105	6.4682	4.7013	3.7067	6.2840	4.4631	3.3724
	PSBT	6.6439	4.9251	4.0100	6.4684	4.7013	3.7061	6.2840	4.4631	3.3718
	Present	6.6440	4.9252	4.0100	6.4685	4.7014	3.7062	6.2842	4.4632	3.3718

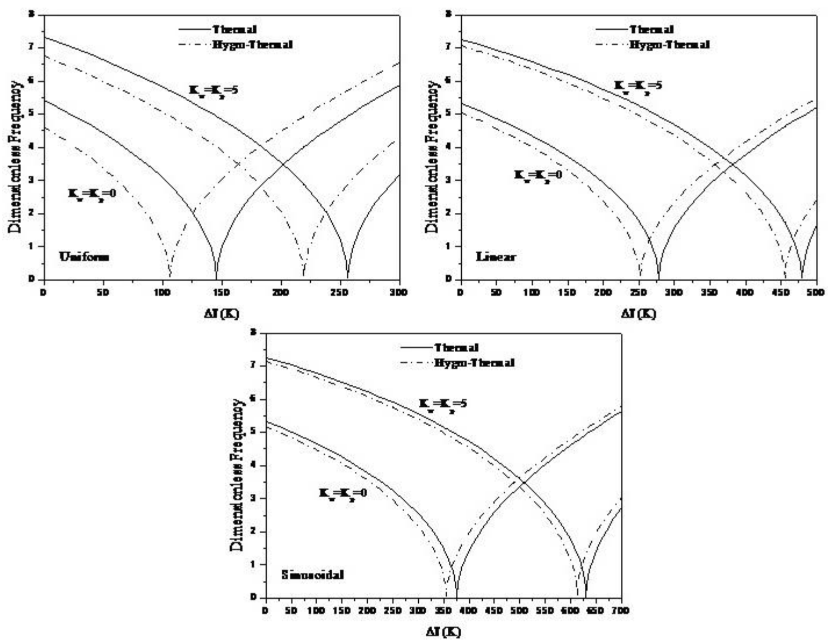

Fig. 4 Influence of elastic foundation on the dimensionless frequency of the S–S FG nanobeam with respect to temperature change for thermal, $\Delta C = 0$ and hygro-thermal, $\Delta C = 2$ environments ($p = 1$, $L/h = 20$, $\mu = 2\,(\mathrm{nm})^2$)

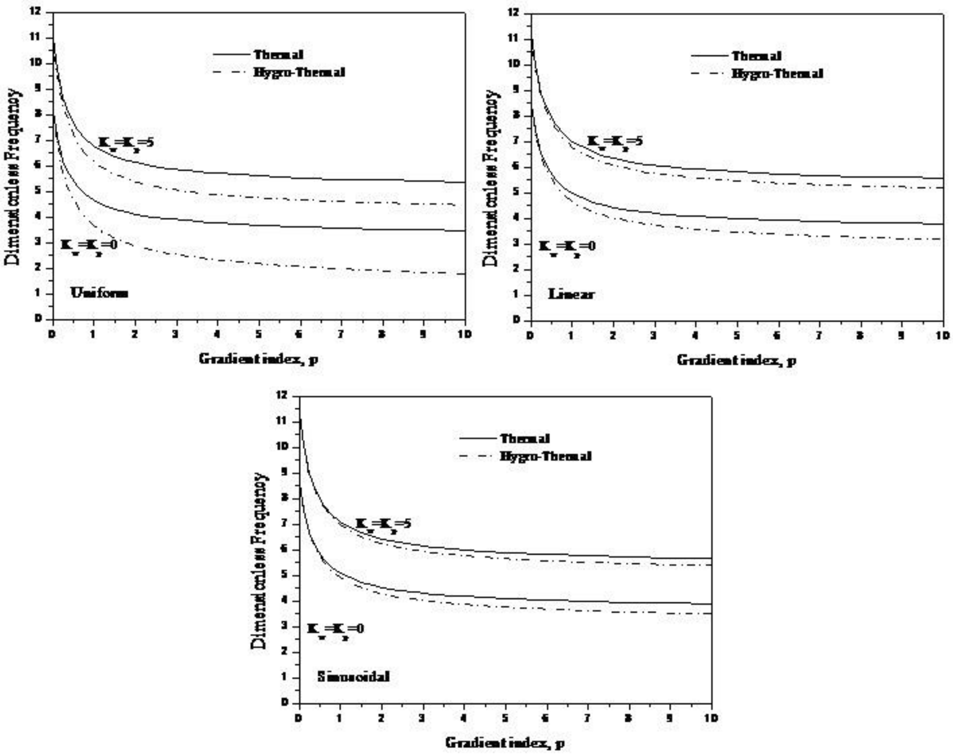

Fig. 5 Influence of material graduation on the dimensionless frequency of the S–S FG nanobeam for thermal, $\Delta C = 0$ and hygro-thermal, $\Delta C = 2$ environments ($L/h = 20$, $\mu = 2\,(\mathrm{nm})^2$, $\Delta T = 40\,(\mathrm{K})$)

Fig. 6 indicates the influence of slenderness ratio on the natural frequencies of FG nanobeams for various types of thermal and hygro-thermal loadings with and without elastic foundation at $p = 1$ and $l = 2(\mathrm{nm})^2$. For all environmental conditions, the dimensionless frequency increases for lower slenderness ratios and then reduces for higher slenderness ratios which indicates the significance of shear deformation when the beam thickness is large. Moreover, it is seen that the hygrothermal effect is not significant for lower values of slenderness ratio. Therefore, as the slenderness ratio increases, the effect of hygrothermal loading becomes more remarkable.

Fig. 7 show the variation of the dimensionless frequency of sinusoidal FG nanobeam with respect to Winkler and Pasternak parameters for different uniform and linear moisture concentrations, at $p = 1$, $L/h = 20$ and $l = 2(\mathrm{nm})^2$. The dimensionless frequency of FG nanobeam increases with increase in the Winkler and Pasternak parameters for all values of moisture concentration. Also, it is found that the inclusion of the shear layer or Pasternak foundation (K_p) parameter yields higher magnitude results for dimensionless frequency than those with the inclusion of Winkler foundation parameters (K_w).

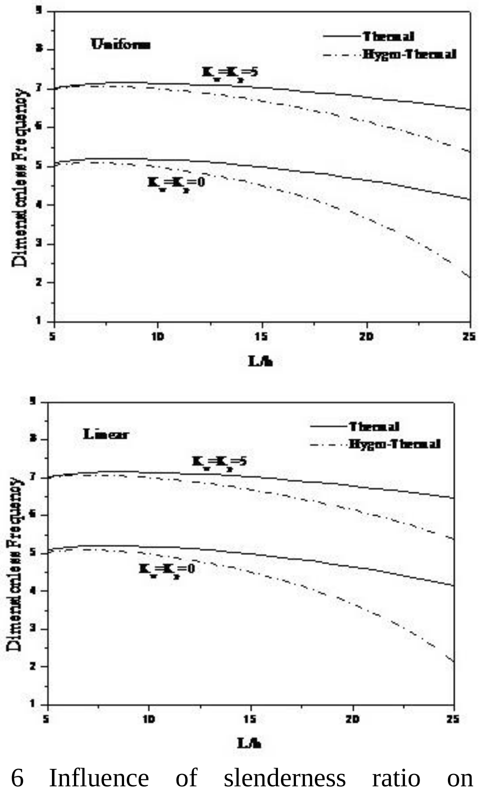

Fig. 6 Influence of slenderness ratio on the dimensionless frequency of the FG nanobeam for various moisture rises ($p = 1, \mu = 2\,(\mathrm{nm})^2$)

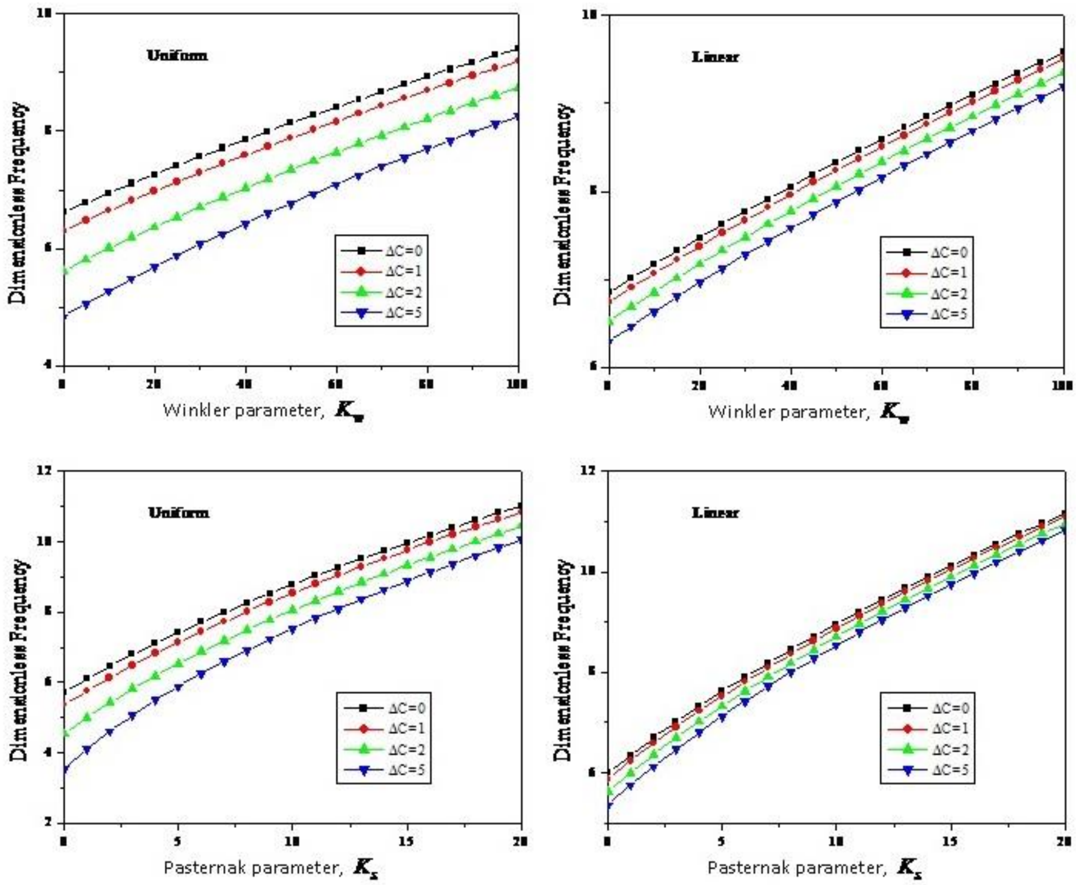

Fig. 7 Influence of elastic foundation parameters on the dimensionless frequency of the FG nanobeam for various moisture rises ($p = 1$, $L/h = 20$, $\mu = 2\,(\mathrm{nm})^2$, $\Delta T = 40\,(\mathrm{K})$)

6. Conclusions

In this paper, hygro-thermo-mechanical free vibration analysis of functionally graded nanobeams resting of elastic foundation is performed by using new trigonometric shear deformation beam theory with Navier's analytical method. Three types of environmental conditions namely, uniform, linear, and sinusoidal hygro-thermal loadings are investigated. The model uses two parameters to capture the size-scale effect of nanobeam much accurately using nonlocal elasticity theory of Eringen. Spatially graded material properties according to power-law model are supposed to be temperature-dependent. Equations of motion are derived from Hamilton's principle. The effects of hygro-thermal environments, power law index, nonlocality and elastic foundation on the free vibration responses of FG beams under hygro-thermal effect are explored. The influence of moisture or humidity is significant for higher values of gradient index and slenderness ratio. Also, it is found that at a prescribed environmental condition, nonlocality and gradient index have a notable decreasing effect on the natural frequency of FG nanobeams. It is deduced that Pasternak foundation parameter has a more prominent effect on increasing rigidity and dimensionless natural frequency of FG nanobeam than Winkler foundation parameter. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Hamidi et al. 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al*. 2016, Benahmed *et al*. 2017).

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