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Thermoelectric viscoelastic materials with memory-dependent derivative

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Abstract. A mathematical model of electro-thermoelasticity has been constructed in the context of a new consideration of heat conduction with memory-dependent derivative. The governing coupled equations with time-delay and kernel function, which can be chosen freely according to the necessity of applications, are applied to several concrete problems. The exact solutions for all fields are obtained in the Laplace transform domain for each problem. According to the numerical results and its graphs, conclusion about the proposed model has been constructed. The predictions of the theory are discussed and compared with dynamic classical coupled theory. The result provides a motivation to investigate conducting thermoelectric viscoelastic materials as a new class of applicable materials.

Keywords: magneto-thermo-viscoelasticity; thermoelectric materials; memory-dependent derivative; time-delay; kernel function; numerical results

1. Introduction

Due to the recent large-scale development and utilization of polymers and composite materials, the linear viscoelasticity remains an important area of research.

Linear viscoelastic materials are rheological materials that exhibit time temperature rate-of- loading dependence. When their response is not only a function of the current input, but also of the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. Tschoegl (1997) has presented a general overview of time-dependent material properties. Gross (1953) investigated the mechanical-model representation of linear viscoelastic behavior results. One can refer to Atkinson and Craster (1995) for a review of fracture mechanics and generalizations to the viscoelastic materials, and Rajagopal and Saccomandi (2007) for non-linear theory. In most of these investigations the effect of the thermal state in a viscoelastic material is not considered.

The mechanical-model representation of linear viscoelastic behavior results was investigated by Staverman and Schwarzl (1956), Alfrey and Gurnee (1956), and Ferry (1970). A reciprocity theorem for the theory of viscoelasticity was derived by Fung (1980), and Pobedria (1984) derived the reciprocity theorem for the coupled thermo-viscoelasticity.

Notable works in this field were those by Gurtin and Sternberg (1962), Sternberg (1963), and Ilioushin (1968), which offered an approximation method for linear thermal viscoelastic problems. One can refer to the book of Ilioushin and Pobedria (1970) for a formulation of the mathematical theory of thermal viscoelasticity and the solutions of some boundary value problems as well as to the work of Pobedria (1984) for the coupled problems in continuum mechanics. Results of important experiments determining the mechanical properties of viscoelastic materials were involved in the book by Koltunov (1976).

The modification of the heat-conduction equation from diffusive to a wave type may be affected either by a microscopic consideration of the phenomenon of heat transport or in a phenomenological way by modifying the classical Fourier law of heat conduction. The first is due to Cattaneo (1958), who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier law.

The theory of generalized thermoelasticity has drawn attention of researchers due to its applications in various diverse fields such as earthquake engineering, nuclear reactor's design, high energy particle accelerators, etc.

Several generalizations to the coupled theory which proposed by Biot (1956) are introduced. Lord and Shulmann (1967) introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. One can refer to Ignaczak and Ostoja-Starzewski (2009) and Chandrasekharaiah (1998) for a review. Hetnarski and Ignaczak (1999) described the modern approaches to the analytical treatment of dynamical thermoelasticity.

Within the theoretical contributions to thrmo-viscoelasticity theory are the proofs of uniqueness theorems under different conditions by Ezzat and El Karamany (2002 a,b, 2003) and the boundary element formulation was done by El-Karamany and Ezzat (2002, 2004). The fundamental solutions for the cylindrical region were obtained by Ezzat

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(2004).

Ezzat *et al*. (2001) solved some problems in thermo-viscoelasticity with thermal relaxation by using state space approach (2008). Ezzat (2006) investigated the relaxation effects on the volume properties of an electrically conducting viscoelastic material. Kumar and Kumar (2013) investigated the wave propagation at the boundary surface of elastic half-space and initially stressed viscothermoelastic diffusion with voids half-space. Kumar *et al*. (2015) constructed the fundamental solution to a system of differential equations in micropolar viscothermoelastic solids with voids in case of steady oscillations in terms of elementary functions.

Thermoelectric is an old field. In 1823, Thomas Seebeck discovered that a voltage drop appears across a sample that has a temperature gradient. This phenomenon provided the basis for thermocouples used for measuring temperature and for thermoelectric power generators (See e.g., Mahan *et al*. 1997).

A direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators (See e.g., Rowe 1995). The interaction between the thermal and magnetohydrodynamic fields is a mutual one, owing to alterations in the thermal convection and to the Peltier and Thomson effects ($\Pi = S\,T$) as in Morelli (1997), where Π is a Peltier coefficient, *S* is thermoelectric power and *T* is the absolute temperature, (although these are usually small). Thermoelectric devices have many attractive features compared with the conventional fluid-based refrigerators and power generation technologies, such as long life, no moving part, no noise, easy maintenance and high reliability. However, their use has been limited by the relatively low performance of present thermoelectric materials as shown in the work of Hicks and Dresselhaus (1993). The performance of thermoelectric devices depends heavily on the material intrinsic property; *Z*, known as the figure of merit and defined by Hiroshige *et al.* (2007), $ZT = \frac{\sigma_o S^2}{k} T$ where σ_o , k and *S* are are respectively the electrical conductivity, thermal conductivity and thermoelectric power or Seebeck coefficient. Increasing of such parameter *Z* has a positive effect on the efficiency of thermoelectric device. In order to achieve a high figure of merit, one requires a high thermopower *S*, a high electrical conductivity σ_o , and a low thermal conductivity κ . However, this process is not easy as the written sentence.

The direct proportion between σ_o and k , and the inverse proportion between *S* and σ_o yields a difficulty in improving the thermoelectric efficiency.

Liquid metals are considered to be the most promising coolants for high temperature applications like nuclear fusion reactors because of the inherent high thermal diffusivity, thermal conductivity and hence excellent heat transfer characteristics. Lithium is the lightest of all metals and has the highest specific heat per unit mass. Lithium is characterized by large thermal conductivity and thermal diffusivity, low viscosity, low vapor pressure. Liquid metal in a closed container made of dissimilar metal under a magnetic field is, in general, set into motion by thermoelectric effects if the interfacial temperature is nonuniform, a situation likely to occur in fusion reactor blankets owing to the high thermoelectric power of lithium. Lithium is the most promising coolant for thermonuclear power installations. Shercliff () treats Hartmann flow and points out the relevance of thermoelectric magnetohydrodynamic (MHD) in liquid metal use, such as lithium, in nuclear reactors.

Mathematical modeling is the process of constructing mathematical objects whose behaviors or properties correspond in some way to a particular real-world system. The term real-world system could refer to a physical system, a financial system, a social system, an ecological system, or essentially any other system whose behaviors can be observed. In this description, a mathematical object could be a system of equations, a stochastic process, a geometric or algebraic structure, an algorithm or any other mathematical apparatus like a fractional derivative, integral or fractional system of equations. The fractional calculus and the fractional differential equations are served as mathematical objects describing many real-world systems.

In the last decade, considerable interest in fractional calculus has been stimulated by the applications in different areas of physics and engineering. Recently, some efforts have been done to modify the classical Fourier law of heat conduction by using the fractional calculus in Refs. Povstenko (2005), Sherief *et al*. (2010) and El-Karamany and Ezzat (2011a, b). One can refer to Podlubny (1999) for a survey of applications of fractional calculus.

Ezzat (2010, 2011a, b, c, 2012) introduced the fractional order theory of continuum mechanics, in which the heat conduction equation was assumed to be the form

$$\boldsymbol{q} + \frac{\tau^{\upsilon}}{\upsilon!}\frac{\partial^{\upsilon}\boldsymbol{q}}{\partial t^{\upsilon}} = -k\,\nabla T \tag{1}$$

Ezzat and El-Karamany (2011a,b,c) studied some problems for a perfect conducting half-space in the context of fractional magneto- thermoelasticity. Ezzat and El-Bary (2016 a,b) introduced a unified mathematical model of the equations of generalized magneto-thermoelasticty and magneto-thermo-viscoelasticty based on fractional derivative heat transfer for isotropic perfect conducting media.

Diethelm (2010) has developed Caputo (1967) derivative to be

$$D_a^{\ell} f\,(t) = \int_a^t K_{\ell}(t-\xi) f^{\,(m)}(\xi)\,\mathrm{d}\xi \tag{2}$$

with

$$K_{\ell}(t-\xi) = \frac{(t-\xi)^{m-\ell-1}}{\Gamma(m-\ell)} \tag{3}$$

where $K_\ell(t-\xi)$ is the kernel function and $f^{(m)}$ denotes the common *m*-order derivative, which has specific physical meaning.

Wang and Li (2011) introduced a memory-dependent derivative, the first order of function *f* which is simply defined in an integral form of a common derivative with a kernel function on a slipping interval, in the form

$$K_\ell(t-\xi)=\frac{(t-\xi)^{m-\ell-1}}{\Gamma(m-\ell)} \tag{4}$$

where ω is the time delay and $K(t-\omega)$ is the kernel function in which they can be chosen freely. Yu *et al*. (2014) introduced memory-dependent derivative into the Lord and Shulman (1967) generalized thermoelasticity theory. Recently, Ezzat *et al*. (2014) constructed a new generalized thermo-viscoelasticity theory with memory–dependent derivatives, to denote memory-dependence, as

$$\boldsymbol{q}+\omega D_\omega \boldsymbol{q}=-k\,\nabla T \tag{5}$$

Eq. (5) has more clear physical meaning.

Motivated by the above works, the present manuscript is an attempt to derive a new model of the linear electro-thermo-viscoelasticity by including the memory-dependent derivative and thermoelectric properties. The new model has been applied to several concrete one-dimensional problems for a conducting thermoelectric viscoelastic metal permeated by a primary uniform magnetic field. The direct approach developed in Shereif and Abd El-Latief (2015) is adopted for the solution of the problem for any set of boundary conditions. Laplace transforms techniques are used to get the solution in a closed form. The inversion of the Laplace transforms is carried out using a numerical approach proposed by Honig and Hirdes (1984).

2. Derivation heat equation with memory-dependent derivative in thermoelectric materials

The conventional electro-thermoelasticity is based on the principles of the classical theory of heat conductivity, specifically on the classical Fourier's law, in which relates the heat flux vector $\boldsymbol{q}$ and the conduction current density vector $\boldsymbol{J}$ to the temperature gradient (Kaliski and Nowacki 1963)

$$\boldsymbol{q}=-k\,\nabla T+\Pi \boldsymbol{J} \tag{6}$$

$$\boldsymbol{J}=\sigma_o\left(\boldsymbol{E}+\frac{\partial \boldsymbol{u}}{\partial t}\wedge \boldsymbol{B}-S\,\nabla T\right) \tag{7}$$

The energy equation in terms of the heat conduction vector $\boldsymbol{q}$ in the context of thermoelasticity theory is given by Biot (1956)

$$\frac{\partial}{\partial t}\big(\rho C_E\, T(\boldsymbol{x},t)+\gamma T_o\, e(\boldsymbol{x},t)\big)=-\nabla\cdot\boldsymbol{q}(\boldsymbol{x},t)+Q(\boldsymbol{x},t) \tag{8}$$

Using relation (5), we get the generalized heat conduction law for the considered new generalized theory with time-delay

$$\boldsymbol{q}(\boldsymbol{x},t+\omega)=\boldsymbol{q}(\boldsymbol{x},t)+\omega D_\omega \boldsymbol{q}(\boldsymbol{x},t) \tag{9}$$

From a mathematical viewpoint, Fourier law (6) in the theory of generalized heat conduction with time-delay, is given by

$$\boldsymbol{q}(\boldsymbol{x},t)+\omega D_\omega \boldsymbol{q}(\boldsymbol{x},t)=-k\,\nabla T(\boldsymbol{x},t)+\Pi \boldsymbol{J}(\boldsymbol{x},t) \tag{10}$$

Taking the memory-time derivative of Eq. (8) (suppressing $\boldsymbol{x}$ for convenience), we get

$$\frac{\partial}{\partial t}D_\omega\left(\rho C_E T+\gamma T_o e\right)=-\nabla\cdot D_\omega \boldsymbol{q}+D_w\,Q \tag{11}$$

Multiplying Eq. (11) by ω and adding to Eq. (8), we obtain

$$(1+\omega D_\omega)\left(\rho C_E\frac{\partial}{\partial t}T+\gamma T_o\frac{\partial e}{\partial t}\right)=-\nabla\cdot(\boldsymbol{q}+\omega D_\omega \boldsymbol{q})+(1+\omega D_\omega)Q \tag{12}$$

Substituting from Eq. (10), we get

$$(1+\omega D_\omega)\left(\rho C_E\frac{\partial}{\partial t}T+\gamma T_o\frac{\partial e}{\partial t}\right)=k\,\nabla^2 T-\nabla\cdot\Pi \boldsymbol{J}+(1+\omega D_\omega)Q \tag{13}$$

Eq. (13) is the new generalized energy equation with memory-dependent derivative, taking into account the time-delay ω for thermoelectric materials.

The dynamic coupled theory of heat conduction law follows as the limit case when $\omega\to 0$, so that $\left|D_\omega f(\boldsymbol{x},t)\right|\le\left|\frac{\partial f(\boldsymbol{x},t)}{\partial t}\right|=\left|\lim_{\omega\to 0}\frac{f(\boldsymbol{x},t+\omega)-f(\boldsymbol{x},t)}{\omega}\right|$. This model is more intuitionistic for understanding the physical meaning and the corresponding memory dependent differential equation is more expressive.

3. The mathematical model

We shall consider a conducting thermoelastic solid of finite conductivity σ_o occupying the region $x\ge 0$, where *x*-axis is taken perpendicular to the bounding plane of half-space pointing inwards. A constant magnetic field with components $(0, H_o, 0)$ is permeating the medium in the absence of an external electric field.

The governing equations for generalized magneto-thermo-visvoelasticity when the thermoelectric properties of the material are taken into account consist of (Ezzat 2011b):

1- The figure-of-merit ZT_o at some reference temperature T_o

$$ZT_o=\frac{\sigma_o k_o^{\,2}}{k}T_o \tag{14}$$

where k_o the Seebeck coefficient at T_o.

2- The first Thomson relation at T_o

$$\pi_o = k_o\, T_o \tag{15}$$

where π_o is the Peltier coefficient at T_o .

3-The equation of motion the absence of body forces

$$\sigma_{ji,j} + \mu_o \varepsilon_{ijk} J_k H_j = \rho u_{i,tt}\ , \tag{16}$$

where $\boldsymbol{B}$ magnetic induction vector given by

$$B_i = \mu_o H_i \tag{17}$$

and modified Ohm's law is defined

$$J_i = \sigma_0\left(E_i + \mu_o \varepsilon_{ijk} u_{k,t} H_j - k_o T_{,i}\right) \tag{18}$$

4-The constitutive Ea. (54)

$$S_{ij} = \int_0^t R(t-\tau)\,\frac{\partial e_{ij}(x,\tau)}{\partial\, \tau}\, d\tau = \breve{R}\left(e_{ij}\right) \tag{19}$$

where

$$S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij} \tag{20}$$

and $R(t)$ is relaxation function given by

$$\breve{R}(t) = 2\mu\left[1 - A^*\int_0^t e^{-\beta^* t} t^{\alpha^*-1}\, dt\right] \tag{21}$$

where α^*, β^* and A^* are non-dimensional empirical constants and $\Gamma(\alpha^*)$ is the Gamma function, $0<\alpha^*<1,\ \beta^*>0,\ 0\le A^*<\dfrac{\beta^*}{\Gamma(\alpha^*)},\ \breve{R}(t)>0,\ \dfrac{d}{dt}\breve{R}(t)<0\cdot$

5- The kinematic relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij}, \quad e = \varepsilon_{kk} \tag{22}$$

6- The stress-strain temperature relation

$$\sigma = K_o\left[e - 3\alpha_T\left(T - T_o\right)\right] \tag{23}$$

where

$$\sigma = \frac{\sigma_{kk}}{3}, \quad \sigma_{ij} = \sigma_{ji}$$

Substituting from (23) into (19) we obtain

$$\sigma_{ij} = \breve{R}\left(\varepsilon_{ij} - \frac{e}{3}\delta_{ij}\right) + K_o\, e\, \delta_{ij} - \gamma\left(T - T_o\right)\delta_{ij} \tag{24}$$

7- The heat equation with memory-dependent derivative in the absence of heat sources

$$\begin{aligned} k\, T_{,ii} - \pi_o J_{i,i} &= \rho C_E \frac{\partial T(x,t)}{\partial t} + \gamma T_o \frac{\partial e(x,t)}{\partial t} \\ &+ \int_{t-\omega}^{t} K(t-\xi)\left(\rho C_E \frac{\partial^2 T(x,\xi)}{\partial \xi^2} + \gamma T_o \frac{\partial^2 e(x,\xi)}{\partial \xi^2}\right) d\xi\ , \end{aligned} \tag{25}$$

In the above equations a comma denotes material derivatives and the summation convention are used. Now f or the one-dimensional problems, all the considered fun ctions will depend only on the space variables x and t and the displacement vector has components ($u(x, t)$, 0, 0). Since no external electric field is applied, and the effect of polarization of the ionized medium can be ne glected, it follows that the total electric field E vanishe s identically inside the medium.

The components of the electromagnetic induction vector are given by

$B_x = B_z = 0,\quad B_y = \mu_o H_o = B_o\,(\text{constant})$, while the components of the Lorentz force appearing in Eq. (16) are given by

$$F_x = -\sigma B_o^2 \frac{\partial u}{\partial t},\quad F_y = F_z = 0.$$

Let us introduce the following non-dimensional variables

$$x^* = c_o \eta_o x,\quad u^* = c_o \eta_o u,\quad t^* = c_o^2 \eta_o t,\quad \theta^* = \frac{\gamma(T - T_o)}{\rho c_o^2},\quad \sigma_{ij}^* = \frac{1}{K_o}\sigma_{ij},\ \theta^* = \frac{\gamma}{\rho c_o^2}\theta,$$

$$q_i^* = \frac{\gamma}{k\rho c_o^3 \eta_0} q_i,\ \breve{R}^* = \frac{2}{3K_o}\breve{R},\ \varepsilon = \frac{\gamma}{\rho C_E},\ M = \frac{\sigma_o B_o^2}{\rho c_o^2 \eta_o},\ T_o = \frac{\rho c_o^2}{\gamma}.$$

Using the above values, Eqs. (16), (24) and (25) reduce to (dropping the asterisks for convenience)

$$\left(1 + \breve{R}\right)\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + M\frac{\partial u}{\partial t} + \frac{\partial \theta}{\partial x} \tag{26}$$

$$\left(1 + ZT_o\right)\frac{\partial^2 \theta}{\partial x^2} = \left(1 + \omega D_\omega\right)\left(\frac{\partial \theta(x,t)}{\partial t} + \varepsilon\frac{\partial^2 u(x,t)}{\partial x\, \partial t}\right) \tag{27}$$

$$\sigma = \left(1 + \breve{R}\right)\frac{\partial u}{\partial x} - \theta \tag{28}$$

$$\breve{R}(t) = \frac{4\mu}{3K_o}\left[1 - A^*\int_0^t e^{-\beta^* t} t^{\alpha^*-1}\, dt\right] \tag{29}$$

From now on, the kernel function form $K(t-\xi)$ can be chosen freely as

$$K(t-\xi) = 1 - \frac{2b}{\omega}(t-\xi) + \frac{a^2(t-\xi)^2}{\omega^2} = \begin{cases} 1 & if\ \ a = b = 0 \\ 1 - \dfrac{(t-\xi)}{\omega} & if\ \ a = 0, b = \dfrac{1}{2} \\ 1 - (t-\xi) & if\ \ a = 0\ \ b = \omega/2 \\ (1 - \dfrac{t-\xi}{\omega})^2 & if\ \ a = b = 1, \end{cases} \tag{30}$$

where a and $b$are constants.

4. The Analytical solutions in the Laplace-transform domain

Performing the Laplace transform defined by the relation

$$\bar{g}(s)=\int_0^\infty e^{-st}g(t)\,dt$$

of both sides Eqs. (26)-(29), with the homogeneous initial conditions

$$\left[D^2-\alpha s(s+M)\right]\bar{u}=\alpha D\theta, \tag{31}$$

$$D^2\bar{\theta}=s\left(\frac{1+G}{1+ZT_o}\right)(\bar{\theta}+\varepsilon D\bar{u}), \tag{32}$$

$$\bar{\sigma}=\frac{1}{\alpha}\frac{\partial\bar{u}}{\partial x}-\bar{\theta}, \tag{33}$$

where

$$D=\frac{\partial}{\partial x}\quad,\qquad L\left\{\breve{R}\frac{\partial^2 u}{\partial x^2}\right\}=s\bar{R}(s)\frac{\partial^2\bar{u}}{\partial x^2}\quad,$$

$$\alpha=1/\left(1+s\bar{R}\right),\quad \bar{R}(s)=\frac{4\mu}{3sK_o}\left[1-\frac{A^*\Gamma(\alpha^*)}{(s+\beta^*)^{\alpha^*}}\right],$$

$$L\{\omega D_\omega f(t)\}=F(s)\begin{cases}[(1-e^{-s\omega})], & a=b=0\\ [1-\frac{1}{\omega s}(1-e^{-s\omega})], & a=0,b=\frac{1}{2}\\ [(1-e^{-s\omega})-\frac{1}{s}(1-e^{-s\omega})+\omega e^{-s\omega}], & a=0,b=\frac{\omega}{2}\\ [(1-\frac{2}{\omega s})+\frac{2}{\omega^2 s^2}(1-e^{-s\omega})], & a=b=1\end{cases},$$

$$G(s)=(1-e^{-s\omega})(1-\frac{2b}{\omega s}+\frac{2a^2}{\omega^2 s^2})-(a^2-2b+\frac{2a^2}{\omega s})e^{-s\omega} \tag{34}$$

and

$$F(s)=L\{(\frac{\partial\theta}{\partial t}+\varepsilon\frac{\partial^2 u}{\partial x\partial t})\}=s(\bar{\theta}+\varepsilon D\bar{u})$$

Eliminating $\bar{u}$ between Eqs. (25) and (26), we obtain

$$\left\{D^4-\left[\alpha s(s+M)+s\left(\frac{1+G}{1+ZT_o}\right)(1+\alpha\varepsilon)\right]D^2+\alpha s^2(s+M)\left(\frac{1+G}{1+ZT_o}\right)\right\}\bar{\theta}=0 \tag{35}$$

In a similar manner, we can show that $\bar{u}$ satisfies the equation

$$\left\{D^4-\left[\alpha s(s+M)+s\left(\frac{1+G}{1+ZT_o}\right)(1+\alpha\varepsilon)\right]D^2+\alpha s^2(s+M)\left(\frac{1+G}{1+ZT_o}\right)\right\}\bar{u}=0 \tag{36}$$

The solutions of Eqs. (35) and (36) which are bounded for x ≥ 0 have the form

$$\bar{\theta}(x,s)=C_1e^{-k_1x}+C_2e^{-k_2x} \tag{37}$$

$$\bar{u}(x,s)=C_3e^{-k_1x}+C_4e^{-k_2x} \tag{38}$$

where k_1 and k_2 are the roots with positive real parts of the characteristic equation

$$k^4-\left[\alpha s(s+M)+s\left(\frac{1+G}{1+ZT_o}\right)(1+\alpha\varepsilon)\right]k^2+\alpha s^2(s+M)\left(\frac{1+G}{1+ZT_o}\right)=0$$

satisfying the relations

$$\begin{aligned}k_1^2+k_2^2&=\alpha s(s+M)+s\left(\frac{1+G}{1+ZT_o}\right)(1+\alpha\varepsilon)\\ k_1^2k_2^2&=\alpha s^2(s+M)\left(\frac{1+G}{1+ZT_o}\right)\end{aligned} \tag{39}$$

and $C_i,\ i=1,2,3,4$ are parameters depending on s to be determined from the boundary conditions of the considered problem.

Substitution from Eqs. (37) and (38) into Eq. (31), we obtain the following relations

$$C_3=-\frac{\alpha k_1}{k_1^2-s(s+M)}C_1,\quad C_4=-\frac{\alpha k_2}{k_2^2-s(s+M)}C_2 \tag{40}$$

Substitution from Eq. (40) into Eq. (38), we have

$$\bar{u}(x,s)=-\frac{\alpha k_1}{k_1^2-\alpha s(s+M)}C_1e^{-k_1x}-\frac{\alpha k_2}{k_2^2-\alpha s(s+M)}C_2e^{-k_2x} \tag{41}$$

Substitution from Eqs. (37) and (41) into Eq. (33), we have

$$\bar{\sigma}(x,s)=\alpha s(s+M)\left(\frac{C_1}{k_1^2-\alpha s(s+M)}e^{-k_1x}+\frac{C_2}{k_2^2-\alpha s(s+M)}e^{-k_2x}\right) \tag{42}$$

5. Applications

(i) *A problem of time-dependent thermal shock*

The problem considered is that of producing combined viscoelastic and electro-magnetic waves in an elastic half-space by means of thermal shock acting on the boundary of the half-space. It is assumed that the elastic half-space is adjacent to a vacuum.

(1) Thermal boundary condition:

A thermal shock is applied to the boundary plane x = 0 in the form

$$\theta(0,t)=f(t),\qquad\text{or}\qquad \bar{\theta}(0,s)=\bar{f}(s) \tag{43}$$

(2) Mechanical boundary condition:

The bounding plane x = 0 is taken to be traction-free, i.e.

$$\sigma(0,t)=0,\qquad\text{or}\qquad \bar{\sigma}(0,s)=0 \tag{44}$$

In order to determine the C_1 and C_2, we shall use the boundary conditions (43) and (44), we obtain

$$C_1 = \frac{k_1^2 - \alpha s(s+M)}{k_1^2 - k_2^2}\bar{f}(s), \quad C_2 = -\frac{k_2^2 - \alpha s(s+M)}{k_1^2 - k_2^2}\bar{f}(s) \quad (45a)$$

$$C_3 = -\frac{\alpha k_1}{k_1^2 - k_2^2}\bar{f}(s), \qquad C_4 = \frac{\alpha k_2}{k_1^2 - k_2^2}\bar{f}(s) \quad (45b)$$

Hence, we can use the conditions on (45) into Eqs. (37), (38) and (42) to get the exact solution in the Laplace transform domain in the following forms

$$\bar{\theta}(x,s) = \frac{1}{k_1^2 - k_2^2}\left(\left[k_1^2 - \alpha s(s+M)\right]e^{-k_1 x} - \left[k_2^2 - \alpha s(s+M)\right]e^{-k_2 x}\right)\bar{f}(s) \quad (46)$$

$$\bar{u}(x,s) = -\frac{\alpha}{k_1^2 - k_2^2}\left(k_1 e^{-k_1 x} - k_2 e^{-k_2 x}\right)\bar{f}(s), \quad (47)$$

$$\bar{\sigma}(x,s) = \frac{\alpha s(s+M)}{k_1^2 - k_2^2}\left(e^{-k_1 x} - e^{-k_2 x}\right)\bar{f}(s) \quad (48)$$

(ii) ***A problem for a half-space subjected to ramp-type heating***

Consider a half-space of homogeneous elastic medium occupying the region $x \geq 0$.

(1) Thermal boundary condition:

A thermal shock is applied to the boundary plane $x = 0$ in the form

$$\theta(0,t) = g(t) \quad \text{or} \quad \bar{\theta}(0,s) = \bar{g}(s) \quad (49)$$

(2) Mechanical boundary condition:

The bounding plane $x = 0$ has a constant displacement, that is

$$u'(0,t) = 0 \quad \text{or} \quad \bar{u}'(0,s) = 0 \quad (50)$$

The parameters C_1, C_2 can obtain by using the boundary conditions (49) and (50), hence

$$C_1 = \frac{k_2^2\left[k_1^2 - \alpha s(s+M)\right]}{\alpha s(s+M)\left(k_1^2 - k_2^2\right)}\bar{g}(s), \qquad C_2 = -\frac{k_1^2\left[k_2^2 - \alpha s(s+M)\right]}{\alpha s(s+M)\left(k_1^2 - k_2^2\right)}\bar{g}(s) \quad (51)$$

Substituting from Eq. (51) into (37), (41) and (42), we have

$$\bar{\theta}(x,s) = \frac{1}{\alpha s(s+M)\left(k_1^2 - k_2^2\right)}\left(k_2^2\left[k_1^2 - \alpha s(s+M)\right]e^{-k_1 x} - k_1^2\left[k_2^2 - \alpha s(s+M)\right]e^{-k_2 x}\right)\bar{g}(s) \quad (52)$$

$$\bar{u}(x,s) = -\frac{k_1 k_2}{s(s+M)\left(k_1^2 - k_2^2\right)}\left(k_2 e^{-k_1 x} - k_1 e^{-k_2 x}\right) \quad (53)$$

$$\bar{\sigma}(x,s) = \frac{1}{k_1^2 - k_2^2}\left(k_2^2 e^{-k_1 x} - k_1^2 e^{-k_2 x}\right)\bar{g}(s) \quad (54)$$

(iii) ***A problem for a layered medium***

Consider a layer of thickness X whose lower surface rests on a rigid base, while its upper surface is traction free. We choose the coordinate axes such that the upper plane lies at $x = 0$ and the x-axis pointing downwards. The mechanical boundary conditions can be written as

$$(1) \quad \sigma(0,t) = 0 \quad \text{or} \quad \bar{\sigma}(0,s) = 0 \quad (55)$$

$$(2) \quad u(X,t) = 0 \quad \text{or} \quad \bar{u}(X,s) = 0 \quad (56)$$

The thermal boundary conditions are assumed to be

$$(3) \quad \theta(0,t) = h(t) \quad \text{or} \quad \bar{\theta}(0,s) = \bar{h}(s) \quad (57)$$

$$(4) \quad q(X,t) = 0 \quad \text{or} \quad \bar{q}(X,s) = 0 \quad (58)$$

where q denotes the component of the heat flux vector perpendicular to the surface of the layer. Condition (57) means that the upper surface is acted on by a constant thermal shock at time $t = 0$, while condition (58) signifies that the lower rigid surface is thermally insulated. This problem is somewhat similar to a one treated by Ezzat (1997) for generalized thermoelasticity theory.

Using the Fourier's law of heat conduction, which is valid for generalized thermoelasticity theory (Ezzat 2006), Eq. (58) reduces to

$$\bar{\theta}'(X,s) = 0 \quad (59)$$

Eqs. (33), (55) and (57) can be combined to give

$$\bar{u}'(0,s) = \alpha\bar{h}(s) \quad (60)$$

The general solution of the Eq. (36) for a bounded region is assumed to be

$$\bar{u}(x,s) = A\cosh k_1 x + B\sinh k_1 x + C\cosh k_2 x + D\sinh k_2 x \quad (61)$$

where A, B, C and D are some parameters depending on s and X.

From Eqs. (31) and (61), we get

$$\bar{\theta}(x,s) = \left[\frac{A\left(k_1^2 - \alpha s(s+M)\right)}{k_1}\sinh k_1 x + \frac{B\left(k_1^2 - \alpha s(s+M)\right)}{k_1}\cosh k_1 x + \frac{C\left(k_2^2 - \alpha s(s+M)\right)}{k_2}\sinh k_2 x + \frac{D\left(k_2^2 - \alpha s(s+M)\right)}{k_2}\cosh k_2 x\right] \quad (62)$$

Using the boundary conditions (55)-(60) in Eqs. (61) and (62), the parameters A, B, C and D can be obtained as

$$A = -\frac{k_1\bar{h}}{k_1^2 - k_2^2}\tanh k_1 X, \quad B = \frac{k_1}{k_1^2 - k_2^2}\bar{h}, \quad C = \frac{k_2\bar{h}}{k_1^2 - k_2^2}\tanh k_2 X, \quad D = -\frac{k_2}{k_1^2 - k_2^2}\bar{h} \quad (63)$$

Substituting from Eq. (63) into (61) and (62), we have

$$\bar{u}(x,s) = -\frac{\alpha}{k_1^2 - k_2^2}\left[k_1 \frac{\sinh k_1(X-x)}{\cosh k_1 X} - k_2 \frac{\sinh k_2(X-x)}{\cosh k_2 X}\right]\bar{h}(s) \quad (64)$$

$$\bar{\theta}(x,s) = \frac{1}{k_1^2 - k_2^2}\left\{\left[k_1^2 - \alpha s\,(s+M)\right]\frac{\cosh k_1(X-x)}{\cosh k_1 X} - \left[k_2^2 - \alpha s\,(s+M)\right]\frac{\cosh k_2(X-x)}{\cosh k_2 X}\right\}\bar{h}(s) \quad (65)$$

Substituting Eqs. (64) and (65) into Eq. (33), one obtains

$$\bar{\sigma}(x,s) = \frac{s(s+M)}{k_1^2 - k_2^2}\left[\frac{\cosh k_1(X-x)}{\cosh k_1 X} - \frac{\cosh k_2(X-x)}{\cosh k_2 X}\right]\bar{h}(s). \quad (66)$$

6. Inversion of Laplace transforms

In order to invert the Laplace transform in the above equations, we adopt a numerical inversion method based on a Fourier series expansion proposed by Honig and Hirdes (1984). In this method, the inverse g (t) of the Laplace transform $\bar{g}\,(s)$ is approximated by the relation

$$g\,(t) = \frac{e^{ct}}{t_1}\,[\,\frac{1}{2}\bar{g}\,(c) + \mathrm{Re}(\sum_{k=1}^{\infty} e^{i\,k\,\pi t/t_1}\,\bar{g}\,(c+i\,k\,\pi/t_1))\,], \qquad 0 \le t \le 2t_1, \quad (67)$$

where N is a sufficiently large integer representing the number of terms in the truncated infinite Fourier series. N must chose such that

$$e^{ct}\,\mathrm{Re}[\,e^{i\,N\,\pi t/t_1}\,\bar{g}\,(c+i\,N\,\pi/t_1)\,] \le \varepsilon_1$$

where ε_1 is a persecuted small positive number that corresponds to the degree of accuracy to be achieved. The parameter c is a positive free parameter that must be greater than the real parts of all singularities of $\bar{g}\,(s)$. The optimal choice of c was obtained according to the criteria described (See Ref. Honig and Hirdes 1984).

7. Numerical results

The method based on a Fourier series expansion proposed by Honig and Hirdes (1984) and is developed in detail in many texts such as Ezzat *et al*. (1996, $1997_{a,b}$) and Sherief and Abd El-Latief (2013) is adopted to invert the Laplace transform in Eqs. (46)-(48), (52)-(54) and (64)-(66). The numerical code has been prepared using Fortran 77 programming language. The accuracy maintained was five digits for the numerical program.

The analysis is conducted for a Polymethyl Methacrylate (Plexiglas) material. Following the values of physical constants are shown in Table 1 (Ezzat *et al*. 2014).

The calculations were carried out for different functions f (t), g(t) and h(t). We have chosen the following cases:

Problem 1: *A problem of time-dependent thermal shock* (Ezzat *et al*. 2015)

$$f\,(t) = \begin{cases} \sin\left(\dfrac{\pi t}{\ell_o}\right) & 0 \le t \le \ell_o \\ 0 & \text{otherwise} \end{cases} \qquad \text{or} \qquad \bar{f}\,(s) = \frac{\pi \ell_o\left(s + e^{-\ell_o s}\right)}{\ell_o^2 s^2 + \pi^2}$$

Problem II: *A problem for a half-space subjected to ramp-type heating* (Ezzat and Youssef 2014)

$$g\,(t) = \begin{cases} 0 & 0 \le t \\ \theta_1 \dfrac{t}{t_o} & 0 \le t \le t_o \\ \theta_1 & t > 0 \end{cases} \qquad \text{or} \qquad \bar{g}\,(s) = \frac{\theta_1\left(1 - e^{-st_o}\right)}{t_o s^2}$$

Problem III: *A problem for a layered medium* (Ezzat 2001)

$$h(t) = H\,(t) \qquad \text{or} \qquad \bar{f}\,(s) = \frac{1}{s}$$

For each problem, we apply the following procedure:

The computations were carried out for one value of time, namely t = 0.1 and different values of time-delay, namely, ω= 0.0, Biot (1956), DCT and ω > 0 (new theory) as well as the kernel function forms $K(t-\xi)$ can also be chosen freely, such as 1, $1-(t-\xi)$, $1-(t-\xi)/\omega$ $\left[1-\frac{(t-\xi)}{\omega}\right]^2$. The temperature, stress and displacement distributions are obtained and plotted. Problem I is shown in Figs. 1-7 and problem II is shown in Figs. 8-11, while problem III is shown in Fig. 12-14. In these figures, solid lines represent the solution obtained in the frame of Biot theory and other lines represent the new theory.

From these figures, we observe the following:

- The important phenomenon observed in these figures that the solution to all fields considered vanishes identically outside a bounded region of space surrounding the heat source at a distance from it equal to $x^*(t)$, say $x^*(t)$ is a particular value of x depending only on the choice of t and is the location of the wave front. This demonstrates clearly the difference between the solution corresponding to the classical use of the Fourier heat equation and to the use of new generalized case ($\omega > 0$). In the first and older theory, the waves propagate with infinite speeds, so the value of any of the functions is not identically zero (though it may be very small) for any large value of x. This result is very important since the new theory may preserves the advantage of the generalized theory, i.e. the response to the thermal and mechanical effects does not reach infinity instantaneously but remains in the bounded region of space that expands with the passing of time.
- The temperature fields have been affected by the time-delay ω , where the increasing of the value of the parameter ω causes decreasing in temperature fields. The thermal waves are continuous functions, smooth and reach to steady state depending on the value of time-delay ω, which means that the particles transport the heat to the other particles easily and this makes the decreasing rate of the temperature greater than the other ones. Also, the thermal waves cut x-axis more rapidly when ω increases.

Table 1 Values of the Constants

$\rho = 1.2\times10^3\ kg/m$	$k = 0.55\ J/m.\sec.K$	$E = 525x10^7\ N/m$
$C_E = 1.4x10^3\ J/kg.K$	$\lambda = 453.7\times10^7\ N/m^2$	$\mu = 194\times10^7\ N/m^2$
$\gamma = 210\times10^4\ N/m^2K$	$\eta_0 = 3.36\times10^6\ \sec/m^2$	$c_0 = 2200\ m/\sec$
$\varepsilon = 0.12$	$\ell_0 = 0.35$	$\alpha_T = 13\times10^{-5}$

- The stress and displacement fields have the same behavior as the temperature at and the absolute value of the maximum stress and displacement decrease.
- The magnetic field acts to decrease the displacement field. This is mainly due to the fact that the magnetic field corresponds to a term signifying a positive
- The temperature and stress fields increase when the value of the ramping parameter t_o decreases.
- The efficiency of a thermoelectric figure-of-merit is proportional to the temperature of the material particles (See e.g., Mahan *et al*. 1997 and Ezzat and Youssef 2010).
- The time-delay ω and ramping parameter t_o have significant effect on thermoelectric figure-of-merit. As they increase the thermoelectric figure-of-merit decrease.

8. Conclusions

- The main goal of this work is to introduce a new mathematical model for Fourier law of heat conduction with memory-dependent derivative and includes the thermoelectric figure-of-merit. According to this new theory, we have to construct a new classification for materials according to a time-delay and kernel function where these variables become new indicator of its ability to conduct heat in conducting medium. This model enables us to improve the efficiency of a thermoelectric viscoelastic material figure-of-merit. The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric viscoelastic materials (Ezzat and El-Karamany 2012).
- Owing to the complicated nature of the governing equations for the generalized thermo-viscoelasticity, few attempts have made to solve different problems in this field. These attempts utilized approximate method valid for only a specific range of some parameters (See e.g., Tschoegl 1997).
- In this work, a simply method is introduced in the field of generalized thermo-viscoelasticity with memory-dependent derivative and applied to three different problems. This method gives exact solutions in the Laplace transform domain without any assumed restrictions on either the temperature or the displacement distributions. A numerical method based on a Fourier- series expansion has used for the inversion process (Ezzat *et al.* 2014).
- The method used in the present work is applicable to a wide range of thermo-viscoelasticity problems. It can be applied to problems, which are described by the linearized Navier-Stokes equations for thermoelectric fluid, were the governing equations are coupled (See e.g., Ezzat and El-Bary 2016).
- Representative results for the all functions for generalized theory are distinctly different from those obtained for the coupled theory. This due to the fact that thermal waves in the coupled theory travel with an infinite speed of propagation as opposed to finite speed in the generalized case. It is clear that for small values of time the solution is localized in a finite region. This region grows with increasing time and its edge is the location of the wave front. This region is determined by the values of time t and time-delay. The predictions of the new theory are discussed and compared with dynamic classical coupled theory.

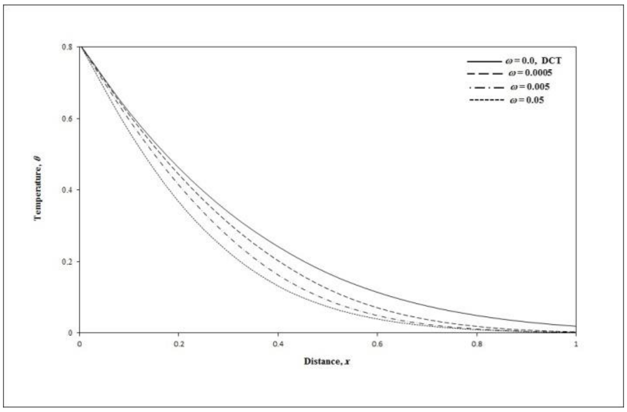

Fig. 1 The variation of temperature for different values of time-delay ω and kernel function $K(t, \xi)$=1

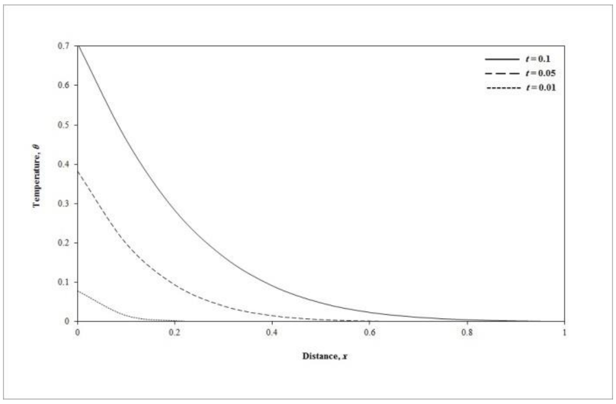

Fig. 2 The variation of temperature for time-delay ω =0.1 and kernel function $K(t, \xi)=1-(t-\xi)/\omega$

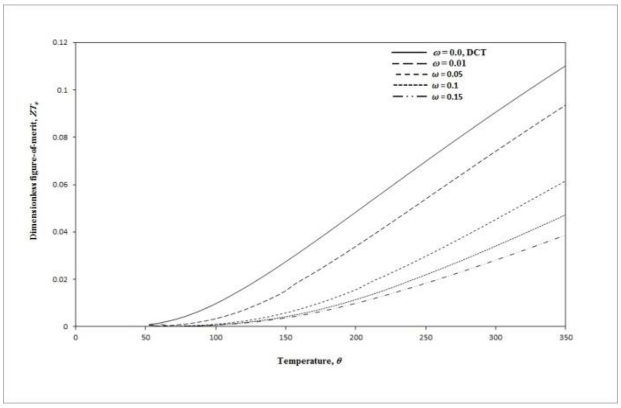

Fig. 3 The dimensionless figure of merit ZT is plotted as a function of temperature for several values of time-delay and kernel function $K(t, \xi)$=1-(t-ξ)

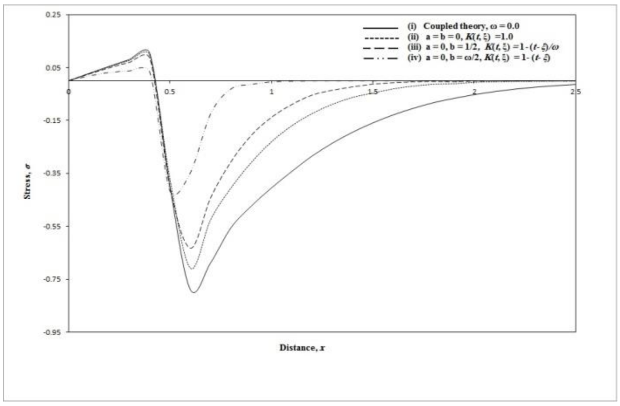

Fig. 4 The variation of stress for different forms of kernel function $K(t, \xi)$ and time-delay ω = 0.001

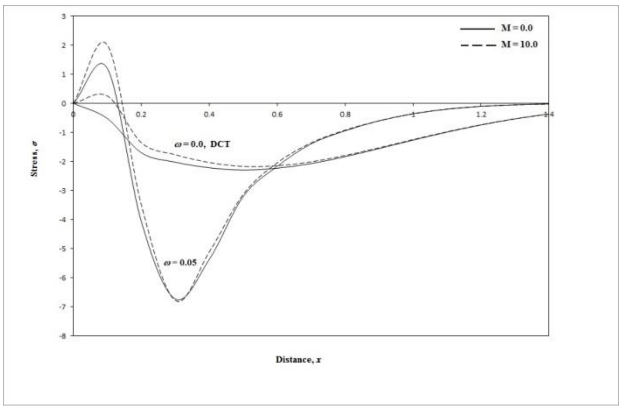

Fig. 5 The variation of stress for different value of time-delay ω and kernel function $K(t, \xi)$ = 1- (t, ξ)

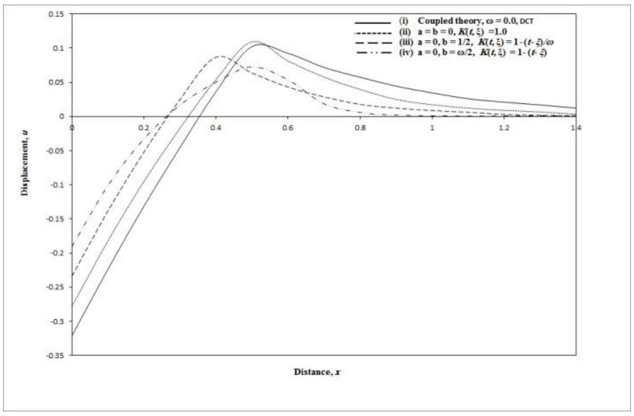

Fig. 6 The variation of temperature for different form of kernel function and time-delay ω = 0.001

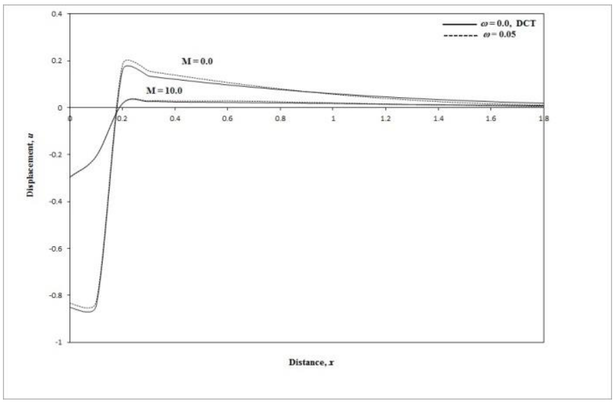

Fig. 7 The variation of displacement for different value of time-delay ω and Kernal function $K(t, \xi)$=1-(t-ξ)

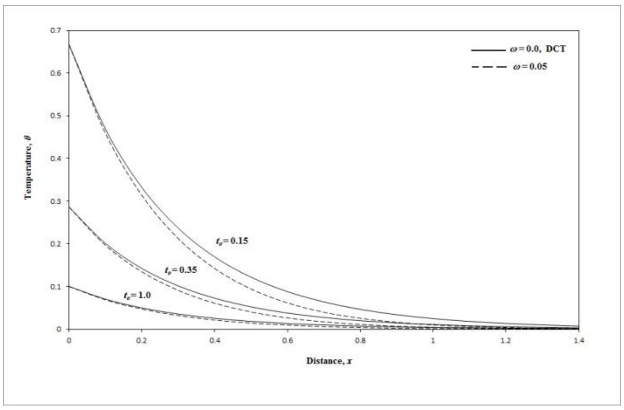

Fig. 8 The variation of temperature for different value of time-delay ω and ramping parameter t_0 for $K(t, \xi)$=[1-(t-ξ)/ω]^2

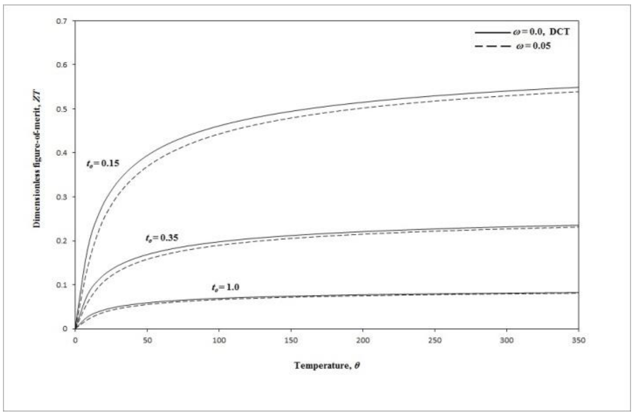

Fig. 9 The dimensionless figure of merit ZT is plotted as a function of temperature for several values of time-delay and kernel function $K(t, \xi)=[1-(t-\xi)/\omega]^2$

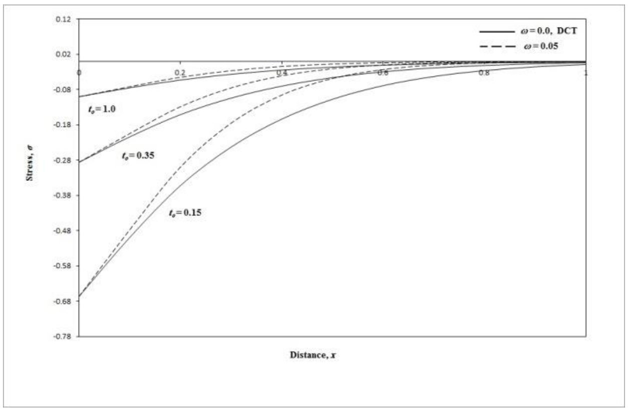

Fig. 10 The variation of stress for different values of time-delay ω and ramping parameter t_0 for $K(t, \xi)=1.0$

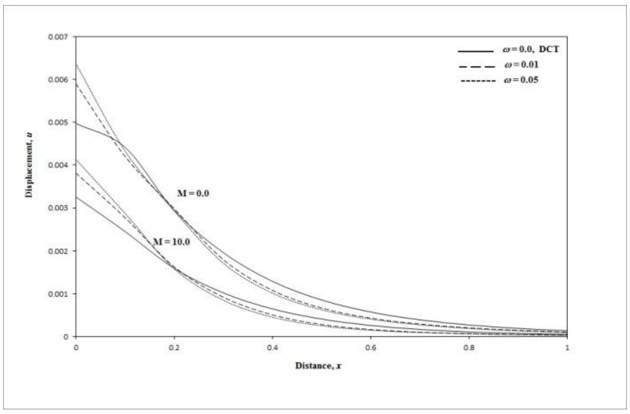

Fig. 11 The variation of displacement for different values of time-delay ω and kernel function $K(t, \xi)=1$

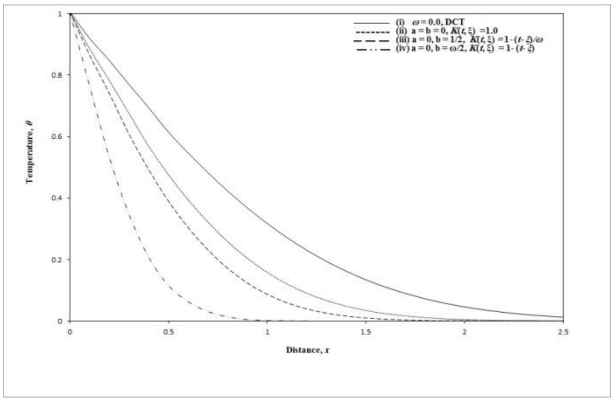

Fig. 12 The variation of temperature for different forms of kernel function $K(t, \xi)$ and time-delay ω =0.01

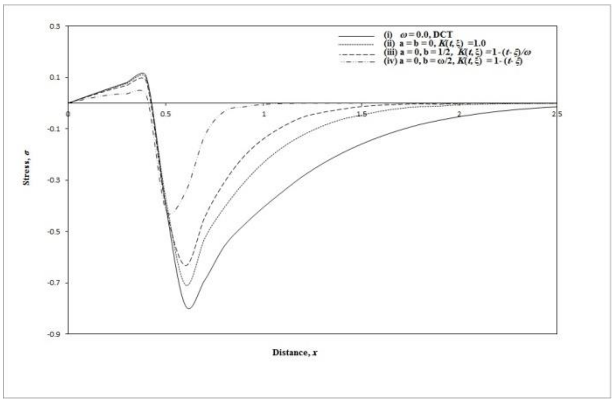

Fig. 13 The variation of stress for different forms of kernel function $K(t, \xi)$ and time-delay ω =0.01

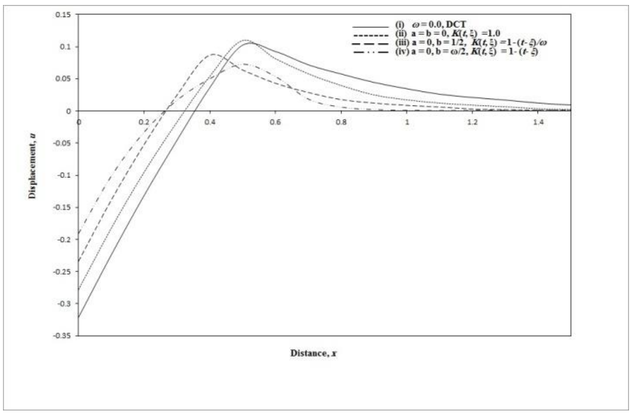

Fig. 14 The variation of displacement for different forms of kernel function $K(t, \xi)$ and time-delay ω =0.01

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CC

Nomenclature

λ, μ	Lame's constants
ρ	density
t	time
C_E	specific heat at constant strain
K_o	$= \lambda + (2/3)\,\mu$, bulk modulus
k	thermal conductivity
T	temperature
T_o	reference temperature
μ_o	magnetic permeability
ε_o	electric permittivity
σ_o	electric conductivity
S_{ij}	components of stress deviator tensor
σ_{ij}	components of stress tensor
e_{ij}	components of strain deviator tensor
e_{ij}	components of strain deviator tensor
ε_{ij}	components of strain tensor
u_i	components of displacement vector
c_o	$=\left[(\lambda+2\mu)/\rho\right]^{1/2}$, speed of propagation of isothermal elastic waves
η_o	$= \rho C_E / \kappa$
θ	$= T - T_0$, such that $\left\|\theta / T_0\right\| << 1,$
q_i	components of heat flux vector
B_i	components of magnetic field strength
E_i	components of electric field vector
J_i	components electric density vector
H_i	magnetic field intensity
S, k_o	Seebeck coefficient
Π, π_o	Peltier coefficient
e	$= \varepsilon_{ii}$, dilatation
δ_o	non-dimensional constant for adjusting the reference
Q	the intensity of applied heat source per unit mass
α_T	coefficient of linear thermal expansion
M	magnetic field parameter
ε	thermoelastic coupling parameter
τ	relaxation time
υ	fractional order
γ	$= 3K_o \alpha_T$
δ_{ij}	Kronecker delta
$\Gamma(.)$	Gamma function