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Dynamics of silicon nanobeams with axial motion subjected to transverse and longitudinal loads considering nonlocal and surface effects

J.P. Shen[1], C. Li[*1,2], X.L. Fan[1] and C.M. Jung[2]

[1]*School of Urban Rail Transportation, Soochow University, 8 Jixue Road, Xiangcheng District, Suzhou 215131, China*
[2]*Department of Railroad Civil Engineering, College of Railroad and Logistics, Woosong University, 171 Dongdaejeon-ro, Dong-gu, Daejeon, Republic of Korea*

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Abstract. A microstructure-dependent dynamic model for silicon nanobeams with axial motion is developed by considering the effects of nonlocal elasticity and surface energy. The nanobeam is considered to subject to both transverse and longitudinal loads arising from nanostructural surface effect and all positive directions of physical quantities are defined clearly prior to modeling so as to clarify the confusions of sign in governing equations of previous work. The nonlocal and surface effects are taken into consideration in the dynamic behaviors of silicon nanobeams with axial motion including circular natural frequency, vibration mode, transverse displacement and critical speed. Various supporting conditions are presented to investigate the circular frequencies by a numerical method and the effects of many variables such as nonlocal nanoscale, axial velocity and external loads on non-dimensional circular frequencies are addressed. It is found that both nonlocal and surface effects play remarkable roles on the dynamics of nanobeams with axial motion and cause the frequencies and critical speed to decrease compared with the classical continuum results. The comparisons of the non-dimensional calculation values by present and previous studies validate the correctness of the present work. Additionally, numerical examples for silicon nanobeams with axial motion are addressed to show the nonlocal and surface effects on circular frequencies intuitively. Results obtained in this paper are helpful for the design and optimization of nanobeam-like microstructures based sensors and oscillators at nanoscale with desired dynamic mechanical properties.

Keywords: silicon nanobeam; nonlocal elasticity; surface effect; circular frequency; critical speed; axial motion

1. Introduction

Nowadays, nanomaterials and nanostructures have increasingly received considerable interests as sensors and oscillators at nanoscale due to their excellent application in nanotechnology during the past decades. For example, resonant sensors can be utilized to detect the nanoparticles. A subject of current intensive technological interest is that of nanostructure with axial motion modeling the subminiature belt at nanoscale, which is a common component in nano-engineering devices such as medical nanorobotics, molecular motor and nano-electro-mechanical system (NEMS). The existence of subminiature belt in nano-engineering device involves vibration of nanobeam model with axial motion. The crystalline silicon is most frequently used for nanobeam structure in nanotechnology and hence the dynamic behaviors of nanoscaled subminiature belts can be treated as the transverse vibration of silicon nanobeams with axial motion.

There are a number of studies focusing on the enduring topic of macro-beams with axial motion based on the classical (local) continuum theory (Skutch 1897, Ashley and Haviland 1950, Mote 1965, Pellicano and Zirilli 1998, Marynowski and Kapitaniak 2002, Pakdemirli and Oz 2008, Yang, Zhang *et al.* 2013, Farokhi, Ghayesh *et al.* 2016, Kurki, Jeronen *et al.* 2016). Such topic can even be traced back to the late 19th Century. Skutch (1897) addressed the transverse vibration of a stretched filament under both fixed end supporting conditions with a constant axial velocity. Subsequently, investigations on this field were extended by Ashley and Haviland (1950), who were interested in vibration of oil pipeline and started modeling the engineering problem of lateral bending vibration for a pipe line with flowing fluid. Mote (1965) constructed the mathematical model of macro-beams with axial motion firstly based on Hamilton principle and determined the first three mode frequencies and vibration modes, which were verified by experiments later. Pellicano and Zirilli (1998) analyzed nonlinear oscillations of an axial-motion beam with vanishing flexural stiffness and weak nonlinearities and they treated it as a boundary layer problem. Marynowski and Kapitaniak (2002) investigated dynamic behavior of beams with axial motion through Galerkin method. Pakdemirli and Oz (2008) studied the transverse vibration of simply supported Euler-Bernoulli beams with axial motion and the beam was subjected to time-varying axial velocity with viscous damping. A recent work by Farokhi, Ghayesh *et al.* (2016) presented the three-dimensional nonlinear global dynamics of a viscoelastic

∗Corresponding author, Associate Professor
E-mail: licheng@suda.edu.cn

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beam with axial motion numerically. Although these articles aforementioned are useful for the macro-beams, they cannot direct the structures in small size, such as the nanoscale. In recent years, there has been a less quantity of research on nanobeams with axial motion. For example, Lim, Li *et al*. (2010) addressed the transverse vibration of a nanobeam with axial motion based on the nonlocal theory and the natural frequencies of simply supported and clamped boundary constraints were determined. Li (2013) studied the size and thermal effects of axial-motion nanobeams using the gradient type of nonlocal theory. However, the effect of loads from surface elasticity of nanostructures was not considered in the previous literature concerning with nanobeams with axial motion. The nanoscaled subminiature belt subjected to both the transverse and longitudinal loads related to surface energies are considered for the first time in this study.

The internal length scales are essential in modeling the mechanical properties of nanostructures but the classical continuum theory is inherently size-independent due to the lack of characterizing the internal length scales. The results of experiments and atomistic simulations have shown significant size effects in mechanical properties when the dimensions of structures diminish to nanoscale. Hence some modified continuum theories were developed to reveal the size effects in nanostructures, one of which the nonlocal theory was initially addressed by Eringen and Edelen (Eringen and Edelen 1972, Eringen 1983). It has been proved that the nonlocal continuum theory plays an indispensable role in theoretical analyses for nanotechnology applications. In the constitution of nonlocal theory, the stress at a reference point x is considered to be a function of the stain fields at every point x' in the continuum body and such theories contain long range inter-molecular/atomic information (Eringen and Edelen 1972, Eringen 1983). The theory of nonlocal continuum mechanics has been employed extensively in nano-mechanics and investigations currently are focused on mechanical characteristics of nanobeams, carbon nanotubes and graphene sheets (Peddieson, Buchanan *et al*. 2003, Zhang, Liu *et al*. 2004, Wang and Varadan 2006, Lu, Lee *et al*. 2006, Duan and Wang 2007, Heireche, Tounsi *et al*. 2008, Lim 2010, Ke, Wang *et al*. 2012, Li 2014, Lim, Zhang *et al*. 2015, Li, Li *et al*. 2015, Farajpour, Yazdi *et al*. 2016, Liu, Chen *et al*. 2016). For example, Wang and Varadan (2006) applied the nonlocal continuum model to the free vibrations of single-walled and double-walled carbon nanotubes, respectively to demonstrate the small-scale effect on dynamic behaviors of carbon nanotubes. Ke, Wang *et al*. (2012) presented the nonlinear dynamics of piezoelectric nanobeams via the nonlocal Timoshenko beam theory and they revealed the effects of the nonlocal parameter, temperature change and external electric voltage on the size-dependent nonlinear behaviors of the piezoelectric nanobeams. Lim, Zhang *et al*. (2015) constructed a higher-order nonlocal elasticity and strain gradient theoretical model considering both the higher-order stress gradients and strain gradient nonlocality by extending the Eringen's nonlocal theory. Recently, the free vibrations of magneto-electro-elastic nanoplates subjected to external electric and magnetic potentials were presented by Farajpour, Yazdi *et al*. (2016) and size-dependent nonlinear behaviors were observed by employing the nonlocal theory and von Kármán's nonlinearity.

The free vibration at nanoscale is quite different from that at macro-scale by the theory of classical continuum mechanics. It is necessary to understand the dynamic responses of nanobeams with axial motion subjected to different kinds of loads emerging from surface effect so that one can control and optimize the subminiature belt in nano-engineering. On the other hand, it is strange to find the sign in core equations is confused in different literatures (e.g., Peddieson, Buchanan *et al*. 2003, Wang and Varadan 2006, Heireche, Tounsi *et al*. 2008, Lim 2010), which results in different conclusions and puzzles engineers and technicians for years (see Eqs. (1)-(9) in Peddieson, Buchanan *et al*. 2003, Eqs. (3)-(7) in Wang and Varadan 2006, Eqs. (7)-(13) in Heireche, Tounsi *et al*. 2008, and Eqs. (4)-(14) in Lim 2010). Another issue is that only the nonlocal effect was taken into consideration in the previous studies. In fact, it has been verified that surface effect plays a significant role in the buckling and vibration of nanostructures (He and Lim, 2006, He and Lilley 2008, Wang and Feng 2007, 2009a, 2009b). For instance, Wang and Feng (2007) introduced a thin surface layer to the upper and lower surfaces of microstructure and investigated how the surface elasticity and residual surface tension play remarkable roles on the natural frequencies of microbeams. He and Lilley (2008) examined the effect of surface stress on natural frequency of nanowires by employing the modified Euler-Bernoulli beam model incorporating the Young-Laplace relation. The comparison of the results of nonlocal continuum models with the available molecular dynamics simulation data from the literature has shown that both surface and nonlocal effects should be taken into consideration in order to accurately predict the mechanical behaviors of nanostructures. Hence, the surface elasticity should also be taken into account in the nonlocal elasticity model to reveal the effect of surface energy. As a matter of fact, Eringen (1983) pointed out that "nonlocal theory accounts for surface physics, an important assert not included in classical theories". Consequently, the surface effect is considered in modeling the transverse and longitudinal loads and an effective flexural rigidity is also adopted since the nonlocal theory is related to surface physics to some extent.

In the present work, the nonlocal and surface effects are considered in the governing equation of motion for transverse vibration of silicon nanobeams with axial motion and the positive directions of all physical quantities and coordinate system are defined clearly prior to modeling. Subsequently, some typical boundary conditions are employed and the circular natural frequency and critical speed are determined and discussed in detail. Besides the boundary conditions usually considered in the previous literatures, such as fully simple supports, double-fixed, and cantilever, this paper also presents a mixed boundary with elastic clamped supports which is between the simple supports and double-fixed, and it exists widely in engineering devices. It is found that the dynamic behaviors

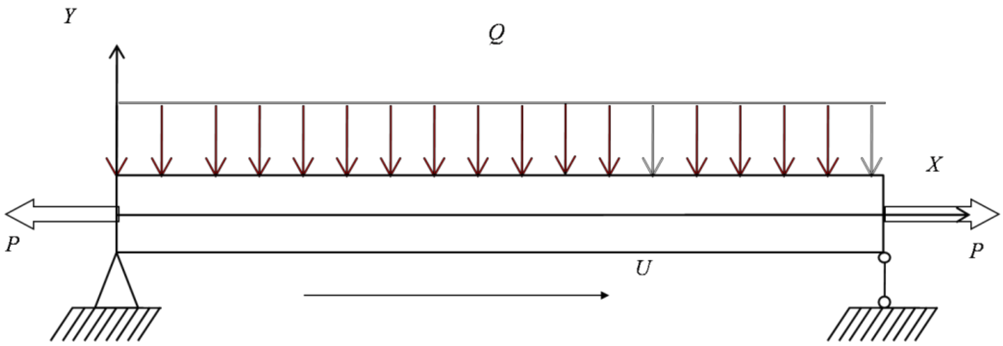


Fig. 1 Sketch of a nanobeam with axial motion subjected to transverse and longitudinal loads

of a nanobeam with axial motion differ considerably from the classical vibration theory for a beam with axial motion.

2. Theoretical model and governing equations

Firstly we construct a Cartesian coordinate system for a nanobeam with axial motion as Fig. 1 where X axis is located at horizontal direction measuring from left end of the silicon nanobeam (taking right as the positive direction) while Y axis is located at vertical direction on the cross section of the left end measuring from the neutral layer of the silicon nanobeam (taking upward as the positive direction). In Fig. 1 P is longitudinal tension, Q is transverse uniformly distributed load per length and its direction is downward, and U is axial transport speed.

Note that both the nonlocal and surface elasticity theories are utilized to take into account the effects of nonlocal constitutive and surface energy of nanostructures. For surface effect, we consider the transverse and longitudinal loads are related to surface energy. Therefore, for a nanobeam with rectangular cross section one has (Wang and Feng 2007, He and Lilley 2008)

$$P=\sigma^{\mathrm{S}}A=\left(\tau^{0}+E^{\mathrm{S}}\varepsilon\right)A \tag{1}$$

$$Q=HW,_{XX}=2\tau^{0}BW,_{XX} \tag{2}$$

where σ^{S} is the one-dimensional surface stress, τ^{0} and E^{S} are the residual surface tension under unstrained condition and the surface elastic modulus, respectively, H is a constant depending on the residual surface tension and for rectangular cross section $H=2\tau^{0}B$, A is the cross sectional area of the rectangle, ε is the strain, W is the transverse bending deflection, B is width of the rectangular cross section, and $W,_{XX}=d^{2}W/dX^{2}$.

For a uniform, elastic subminiature belt at nanoscale subjected to the longitudinal tension and transverse uniformly distributed load which is modeled as a silicon nanobeam with axial motion, its governing equation of motion can be deduced from Wickert (1992) and Oz and Pakdemirli (1999) and shown as

$$PW,_{XX}-M,_{XX}-\rho\left(W,_{TT}+2UW,_{XT}+U^{2}W,_{XX}\right)=Q \tag{3}$$

where M is bending moment and its positive direction is the one making the top section hollows of the nanobeam, ρ is line density, and T is time. It is noted that the suffix X or T denotes the corresponding derivative with respect to X or T, e.g., $W,_{XT}=\partial^{2}W/\partial X\partial T$.

Then the nonlocal elasticity theory is considered to the model. According to Eringen (1983), the nonlocal differential constitutive relation is expressed by

$$\left[1-\left(e_{0}a\right)^{2}\nabla^{2}\right]\boldsymbol{\sigma}_{\mathbf{kl}}=\lambda\boldsymbol{\varepsilon}_{\mathbf{rr}}\boldsymbol{\delta}_{\mathbf{kl}}+\mu\boldsymbol{\varepsilon}_{\mathbf{kl}} \tag{4}$$

where $\boldsymbol{\sigma}_{\mathbf{kl}}$ is the nonlocal stress tensor, $\boldsymbol{\varepsilon}_{\mathbf{kl}}$ is the strain tensor, $\boldsymbol{\delta}_{\mathbf{kl}}$ is the Kronecker delta function, λ and μ are material constants (Lame Constants), e_{0} is a constant for adjusting the model in matching some reliable results by experiments or other models and a is an internal characteristic length (e.g., for lattice parameter, C-C bond length, granular distance, etc.). For an Euler beam-like structure, the scales in thickness and width are much smaller than that in length (axial or longitudinal direction). Hence the nonlocal constitutive relations (4) can be approximated to one-dimensional form as

$$\sigma-\left(e_{0}a\right)^{2}\sigma_{,XX}=E^{*}\varepsilon \tag{5}$$

where $\varepsilon=-YW,_{XX}$ is the beam strain-displacement relation according to the regulation of positive directions in Cartesian coordinate and E^{*} is the effective Young's modulus of the nanobeam including surface effect.

Multiplying Eq. (5) by YdA and integrating the result over the cross-sectional area yields

$$M-\left(e_{0}a\right)^{2}M,_{XX}=\left(EI\right)^{*}W,_{XX} \tag{6}$$

where $M=-\iint_{A}\sigma YdA$ because σ and Y are always opposite in the constructed coordinate (see Fig. 1) due to the transverse distributed load towards down, and $\left(EI\right)^{*}$ is the effective flexural rigidity for nanobeams with rectangular cross section (Wang and Feng 2007, He and Lilley 2008), given by

$$\left(EI\right)^{*}=\frac{EBK^{3}}{12}+\frac{E^{S}BK^{2}}{2}+\frac{E^{S}K^{3}}{6} \quad (7)$$

where E is the corresponding classical Young's modulus of bulk material, K is thickness of the nanobeam.

From Eqs. (3) and (6), governing equation, nonlocal bending moment and shear force for a silicon nanobeam with axial motion subjected to longitudinal tensile and transverse uniformly distributed loads are thus found to be

$$\begin{aligned}&PW,_{XX}-\rho\left(W,_{TT}+2UW,_{XT}+U^{2}W,_{XX}\right)-Q-\left(e_{0}a\right)^{2}\\&\times\left[PW,_{XXXX}-\rho\left(W,_{XXTT}+2UW,_{XXXT}+U^{2}W,_{XXXX}\right)\right]=\left(EI\right)^{*}W,_{XXXX}\end{aligned} \quad (8)$$

$$M=\left(e_{0}a\right)^{2}\left[PW,_{XX}-\rho\left(W,_{TT}+2UW,_{XT}+U^{2}W,_{XX}\right)-Q\right]+\left(EI\right)^{*}W,_{XX} \quad (9)$$

$$\begin{aligned}V&=-M,_{X}+PW,_{X}\\&=-\left(e_{0}a\right)^{2}\left[PW,_{XXX}-\rho\left(W,_{XTT}+2UW,_{XXT}+U^{2}W,_{XXX}\right)\right]-\left(EI\right)^{*}W,_{XXX}+PW,_{X}\end{aligned} \quad (10)$$

where V is shear force. Note that the nonlocal shear force consists of not only the first derivative of nonlocal bending moment but also the contribution of longitudinal tension. Since expressions of the longitudinal and transverse loads contains surface effect, shown in Eqs. (1) and (2), the nonlocal shear force is also associated with the surface elasticity.

It is seen from Eq. (8) that the classical Euler-Bernoulli beam model is recovered when the parameters e_0 and a are set identically to zero. The non-dimensional formulation of the above Eqs. (8)-(10) are given by

$$w,_{tt}+\left(u^{2}-1\right)w,_{xx}+2uw,_{xt}+q-\tau^{2}w,_{xxtt}-2u\tau^{2}w,_{xxxt}-\left(u^{2}\tau^{2}-\tau^{2}-\nu_{f}^{2}\right)w,_{xxxx}=0 \quad (11)$$

$$m=\left(\tau^{2}-u^{2}\tau^{2}+\nu_{f}^{2}\right)w,_{xx}-\tau^{2}w,_{tt}-2u\tau^{2}w,_{xt}-\tau^{2}q \quad (12)$$

$$v=\left(u^{2}\tau^{2}-\tau^{2}-\nu_{f}^{2}\right)w,_{xxx}+\tau^{2}w,_{xtt}+2u\tau^{2}w,_{xxt}+w,_{x} \quad (13)$$

where the dimensionless axial coordinate, transverse displacement, time, transverse distributed load, axial velocity, nonlocal nanoscale and bending rigidity are, respectively as

$$x=\frac{X}{L},\ w=\frac{W}{L},\ t=T\sqrt{\frac{P}{\rho L^{2}}},\ q=\frac{QL}{P},\ u=U\sqrt{\frac{\rho}{P}},\ \tau=\frac{e_{0}a}{L},\ \nu_{f}=\sqrt{\frac{\left(EI\right)^{*}}{PL^{2}}} \quad (14)$$

For linear free vibration, the nth-mode transverse displacement of the silicon nanobeam can be represented by

$$w_{n}\left(x,t\right)=\phi_{n}\left(x\right)e^{i\omega_{n}t} \quad (15)$$

where $\phi_n\left(x\right)$ is nth vibration mode, ω_n is the nth dimensionless circular natural frequency with n=1,2,3…, and i is the imaginary number unit. The correlation between physical and normalized circular frequency is

$$w_{n}\left(x,t\right)=\phi_{n}\left(x\right)e^{i\omega_{n}t} \quad (16)$$

Substituting Eq. (15) into (11) one yields the frequency domain equation as

$$w_{n}\left(x,t\right)=\phi_{n}\left(x\right)e^{i\omega_{n}t} \quad (17)$$

Since Eq. (17) is a fourth-order ordinary differential equation, one can suppose that

$$\phi_{n}\left(x\right)=C_{1n}e^{i\lambda_{1n}x}+C_{2n}e^{i\lambda_{2n}x}+C_{3n}e^{i\lambda_{3n}x}+C_{4n}e^{i\lambda_{4n}x} \quad (18)$$

where C_{jn} are four constants of integration and λ_{jn} are the four roots of the characteristic equation of Eq. (17), in which j=1,2,3,4.

3. Methodology and solutions

In this section, various common boundary conditions are considered to reveal the effects of dimensionless parameters including the nonlocal and surface elasticity, axial velocity, bending rigidity and transverse distributed load on dynamic behaviors of the silicon nanobeams with axial motion. The non-dimensional circular natural frequency and critical speed are determined, and the dimensionless results are compared with those by classical mechanics theory and previous nonlocal theory, respectively. In the end, a numerical example is addressed to reveal the nonlocal and surface effects directly.

3.1 Fully simple supports

Considering a nanobeam with the constraints of simple supports at both ends, the boundary conditions are described as

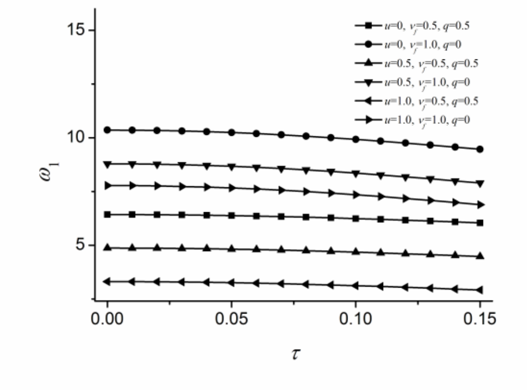
$$w(0,t)=0,\ w(1,t)=0,\ m(0,t)=0,\ m(1,t)=0 \quad (19)$$

Substituting Eqs. (12) and (15) into (19) and further substitution of Eq. (18) into the results, one can rewrite the algebraic equations using a matrix form as

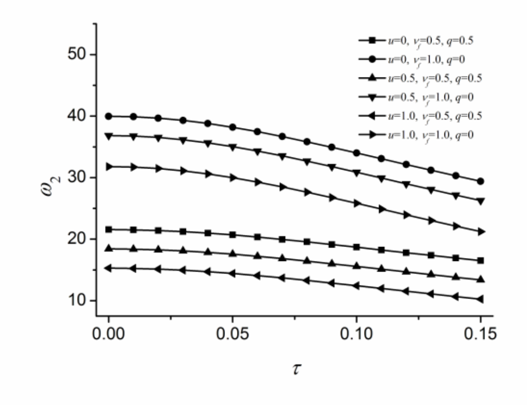
$$\begin{pmatrix}1&1&1&1\\e^{i\lambda_{1n}}&e^{i\lambda_{2n}}&e^{i\lambda_{3n}}&e^{i\lambda_{4n}}\\l_{1}&l_{2}&l_{3}&l_{4}\\l_{1}e^{i\lambda_{1n}}&l_{2}e^{i\lambda_{2n}}&l_{3}e^{i\lambda_{3n}}&l_{4}e^{i\lambda_{4n}}\end{pmatrix}\begin{pmatrix}C_{1n}\\C_{2n}\\C_{3n}\\C_{4n}\end{pmatrix}=\begin{pmatrix}0\\0\\\tau^{2}q\\\tau^{2}q\end{pmatrix} \quad (20)$$

where $l_{j}=-\left(\tau^{2}-u^{2}\tau^{2}+\nu_{f}^{2}\right)\lambda_{jn}^{2}+\tau^{2}\omega_{n}^{2}+2u\tau^{2}\omega_{n}\lambda_{jn}$ in which j=1,2,3,4.

It is seen easily that the four coefficients C_{jn} (j =1,2,3,4) contain nonlocal nanoscale parameter τ, namely, the nonlocal nanoscale has significant effect on vibration modes. In the meanwhile, for nontrivial solutions of the four coefficients in Eq. (20), the approach is the rank of the coefficient matrix should be equal to the rank of augmented matrix shown in Eq. (20). Therefore, following such analysis framework and solution methodology we can obtain the correlation between the circular natural frequency and nonlocal nanoscale under different values of non-dimensional axial velocity, surface energy based transverse load and stiffness according to Eqs. (17) and (20). The relations are plotted in Figs. 2(a)-2(c).



(a)



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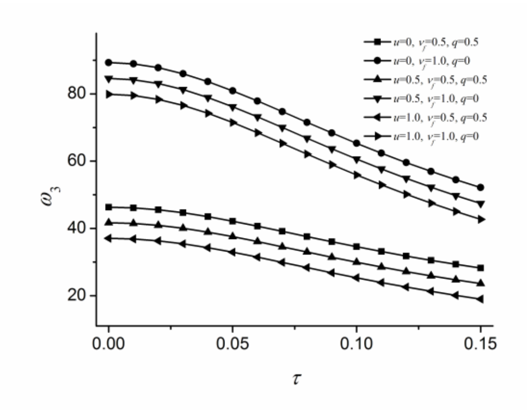


(c)

Fig. 2 Variations of the first three mode circular frequencies for the simple supports

The results show that, compared with the results by classical vibration theory (i.e., the values without nonlocal effect or τ=0), circular frequencies decrease with the increase of nonlocal nanoscale. Compared with the nanobeam without axial motion (i.e., u=0), circular frequencies of a nanobeam with axial motion are lower, and the higher the velocity is, the lower circular frequencies are. To a certain silicon nanobeam under the same values of nonlocal nanoscale and axial velocity, an increase in dimensionless stiffness causes frequencies to increase while an increase in dimensionless transverse loads induce frequencies to decrease. This is because the existence of transverse loads reduces the bending rigidity of silicon nanobeams. Therefore, the nonlocal nanoscale, axial velocity and external loads have remarkable effects on circular frequencies. According to Eq. (15), these factors also influence transverse displacement with the framework of nonlocal elasticity.

In order to validate the presented theoretical results, the calculations are compared with those from literature (Oz and Pakdemirli 1999, Rezaee and Lotfan 2015). For instance, when v_f=1.0, τ=0, u=0 and q=0, the first two mode circular frequencies are 10.3691 and 39.9925, respectively (Oz and Pakdemirli 1999), while the present results are 10.3576 and 39.9753, respectively. Moreover, the other detailed comparison is shown in Table 1. It indicates good agreement for the comparisons between the results calculated here and previous references. Consequently, the validity of the present model and computation is thus confirmed.

3.2 Other boundary conditions

The problem of the silicon nanobeam with axial motion under other boundary conditions such as double-fixed, elastic clamped supports and cantilever can be also solved using the same numerical approach. Note that beams are always fastened up by elastic objects at both ends and it is a mixed supporting condition called elastic clamped, which is between the simple supports and double-fixed conditions. In fact, complete simple supports and double-fixed ends are seldom in engineering, and elastic clamped mixed condition is closer to the real engineering constraint. The boundary conditions for double-fixed, elastic clamped supports and cantilever are given by, respectively

$$w(0,t)=0,\ w(1,t)=0,\ w,_x(0,t)=0,\ w,_x(1,t)=0 \quad (21)$$

$$w(0,t)=0,\ w(1,t)=0,\ m(0,t)-kw,_x(0,t)=0,\ m(1,t)+kw,_x(1,t)=0 \quad (22)$$

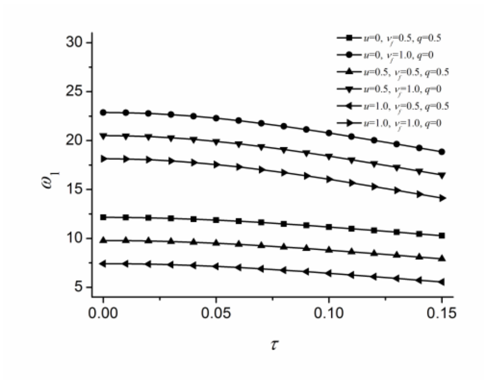
$$w(0,t)=0,\ w,_x(0,t)=0,\ m(1,t)=0,\ v(1,t)=0 \quad (23)$$

where $k=K/PL$ is the dimensionless flexible coefficient of elastic clamped ends.

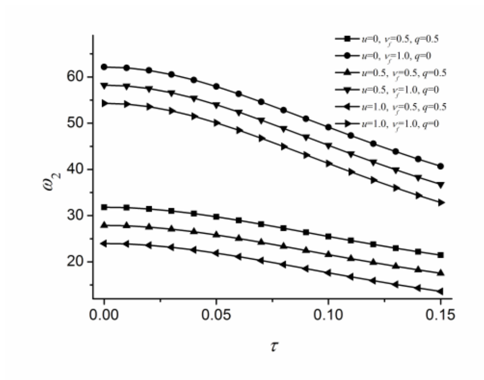
Table 1 Comparisons of the present results and reference (Rezaee and Lotfan, 2015) results, where u=1.0 and q=0

ω_1						ω_2					
τ=0, v_f=0.6		τ=0, v_f=0.8		τ=0.025, v_f=0.8		τ=0, v_f=0.6		τ=0, v_f=0.8		τ=0.025, v_f=0.8	
Ref.	Here	Ref.	Here	Ref.	Here	Ref.	Here	Ref.	Here	Ref.	Here
5.7665	5.7728	7.7771	7.7682	7.6129	7.6258	23.8264	23.8398	31.6914	31.7155	31.1682	31.1869

According to the similar matrix method, each mode circular natural frequency associated with nonlocal nanoscale τ, axial velocity u, stiffness v_f and transverse load q can be obtained. These relationships are drawn in Figs. 3-6, respectively.



(a)



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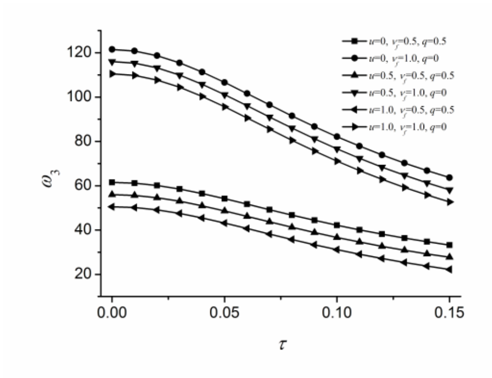


(c)

Fig. 3 Variations of the first three mode circular frequencies for the double-fixed supports

Observations from Figs. 3-6 are similar with the results of fully simple supports nanobeams with axial motion. An increase in nonlocal nanoscale results in decreasing the circular natural frequencies and the latter gets prominent as the nonlocal nanoscale increases continuously. The vibration mode and transverse displacement are also influenced by nonlocal nanoscale effect since they are related to circular frequencies via Eq. (15). The circular frequencies of double-fixed are higher than those of the simple supports ones while the circular frequencies of cantilever supports are lower. In addition, for elastic clamped supports the larger flexible coefficient k is, the higher circular natural frequencies are.

3.3 Critical speed and local-nonlocal comparison

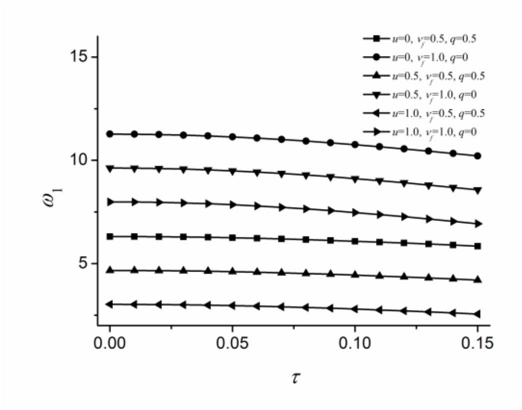
Firstly, we consider the critical speed representing the certain axial velocity that induces a minimum value for the first mode circular frequency. Taking the fully simple supports nanobeam as an example, we show the relation of critical speed and nonlocal nanoscale in Fig. 7. It demonstrates that the critical speed also reveals microstructure-dependence and it decreases with increasing the nonlocal nanoscale.

Secondly, it is known that dynamic behaviors of macro-beams with axial motion based on classical continuum theory were investigated systematically by Oz and Pakdemirli (1999) and Pakdemirli and Oz (2008). The circular natural frequencies obtained herein agree with those in Oz and Pakdemirli (1999) and Pakdemirli and Oz (2008) well in case of vanishing nonlocal nanoscale, i.e., τ=0. Comparisons of the fundamental frequencies derived by the local and nonlocal models for the simple supports end conditions are illustrated in Fig. 8. It is demonstrated that the significant nonlocal and surface effects result in a decline of natural frequencies. The differences between classical local and nonlocal models are larger with a higher value of nanostructural stiffness. It is obvious that both nonlocal and surface elasticity theories are essential to study the nanoscale mechanics.

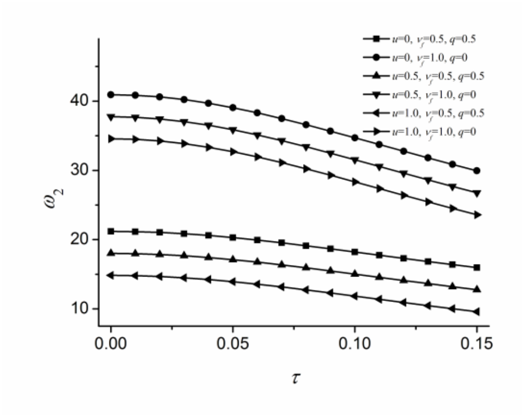
3.4 Numerical results and discussion

A numerical example is calculated to directly and numerically reveal the nonlocal and surface effects on vibration of the silicon nanobeam with axial motion because the specific values of nonlocal and surface effects are not involved in the non-dimensional analyses above. The following parameters for silicon material are adopted: bulk elastic modulus E=210GPa, mass density ρ_m=2370 kg/m^3, surface elastic modulus E^S=−10.6543 N/m, residual surface tension τ^0=0.6048 N/m (Miller and Shenoy 2000, Sharabiani and Yazdi 2013), length L=20 nm, width B=5 nm and thickness K=2 nm.

As a result, the effective flexural rigidity shown in Eq. (7) can be determined. Further, we can calculate the physical natural frequencies with respect to the axial velocity as Fig. 9 according to the transformational relations in Eqs. (14) and (16) as well as the correlation between line density and mass density $\rho=\rho_m A$.



(a)



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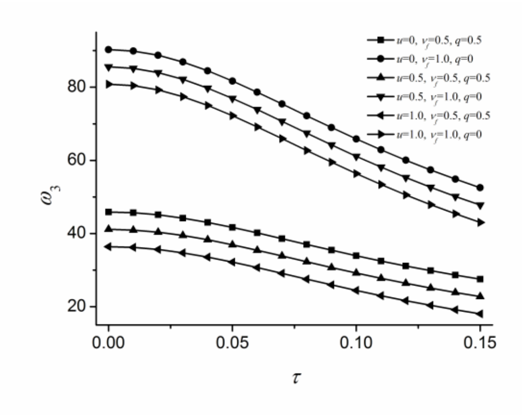
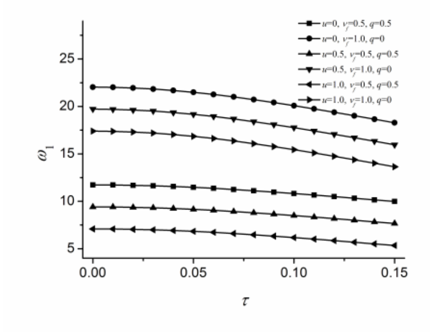


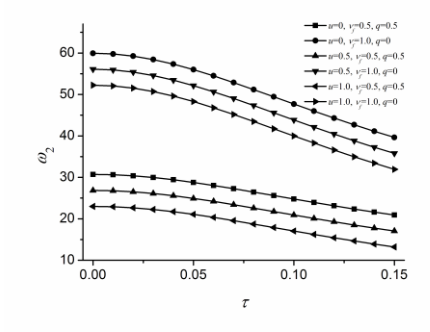
(c)

Fig. 4 Variations of the first three mode circular freque ncies for the elastic clamped supports (k=0.5)

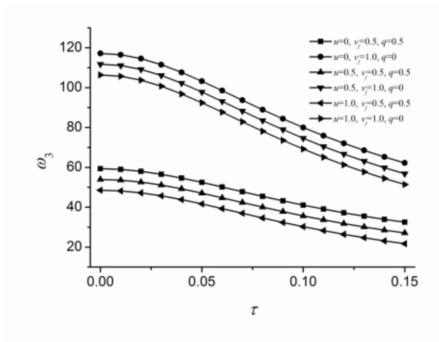
(k=100)



(a)



(b)

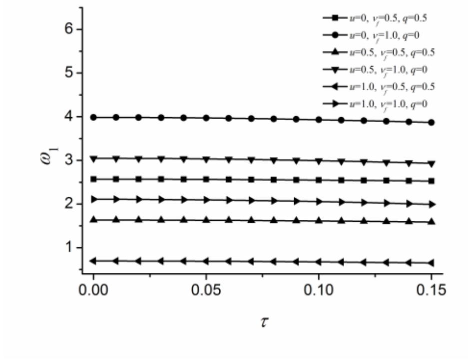


(c)

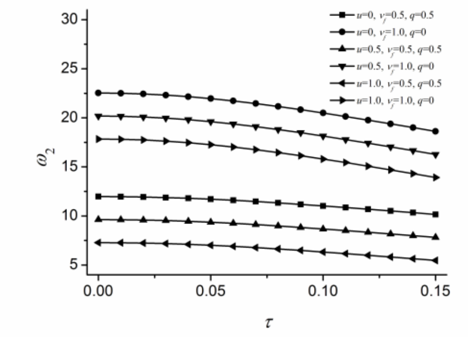
Fig. 5 Variations of the first three mode circular freque ncies for the elastic clamped supports

The non-dimensional results in Sections 3.1 and 3.2 hide the surface effect but it does exist because the surface elasticity quantities just disappear during the procedure of dimensionless treatment, while, the numerical example for dimensional results shows both the nonlocal and surface effects intuitively because the magnitude and unit of surface quantities are utilized and of course, the dimensional numerical example makes the results more practical and meaningful.

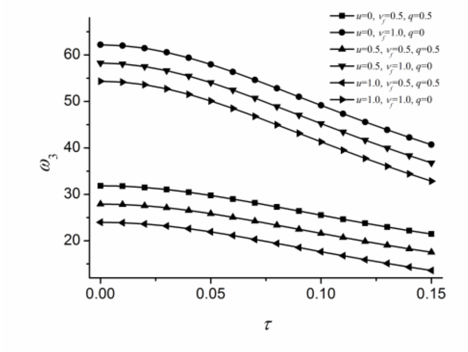
It determines a circular frequency incorporating the nonlocal elasticity and surface energy for engineers under a certain value of axial velocity for silicon nanobeam in Fig. 9. Consequently, the numerical results include both the nonlocal and surface effects and may provide some reference for designing and optimizing the NEMS where the nanobeam with axial motion acting as a basic component.



(a)



(b)



(c)

Fig. 6 Variations of the first three mode circular freque ncies for the cantilever supports

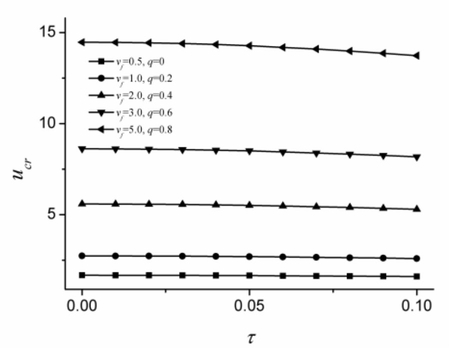


Fig. 7 Critical speed versus nonlocal nanoscale for simple supports

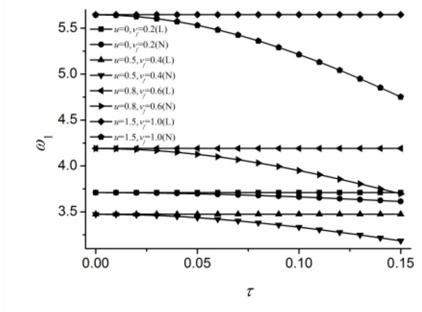


Fig. 8 The comparisons between fundamental frequencies of the simple supports by local model (L) and nonlocal model (N) with q=0

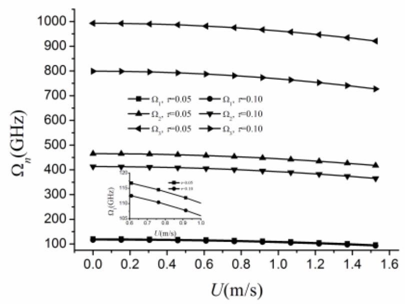


Fig. 9 Numerical example for the effect of axial velocity on circular natural frequencies including nonlocal and surface factors

4. Conclusions

The transverse vibrations of the silicon nanobeams with axial motion subjected to both longitudinal tension and transverse distributed loads are presented based on the nonlocal and surface elasticity theories. It shows that the nonlocal nanoscale is a key factor from classical continuum theory to nonlocal theory and it has remarkable effect on dynamic behaviors of such a nanobeam with axial motion. For four end supporting conditions including simple supports, double-fixed, elastic clamped supports and cantilever, three mode circular natural frequencies are determined and they decrease with increasing nonlocal nanoscale and axial velocity. Meanwhile, a larger stiffness or a lower transverse load causes the frequencies to increase. The critical speed is calculated to reveal the significant small scale effect and it decreases with an increase in nonlocal nanoscale. The present work is only concerned with the longitudinal tensile force, and for a longitudinal compression the conclusion of dynamics versus longitudinal load is reversed. Besides, the numerical example for silicon nanobeam with axial motion is addressed to reveal the complete microstructure-dependent effects including both nonlocal and surface effects. Investigation of nonlocal and surface effects on free vibration of silicon nanobeams with axial motion may be beneficial for the application of subminiature belt component in nano-electronic drive devices and nano-sensors whose basic elements can be modeled as the axially moving nanobeams.

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