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# Statistical analysis and probabilistic modeling of WIM monitoring data of an instrumented arch bridge

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**Abstract.** Traffic load and volume is one of the most important physical quantities for bridge safety evaluation and maintenance strategies formulation. This paper aims to conduct the statistical analysis of traffic volume information and the multimodal modeling of gross vehicle weight (GVW) based on the monitoring data obtained from the weigh-in-motion (WIM) system instrumented on the arch Jiubao Bridge located in Hangzhou, China. A genetic algorithm (GA)-based mixture parameter estimation approach is developed for derivation of the unknown mixture parameters in mixed distribution models. The statistical analysis of one-year WIM data is firstly performed according to the vehicle type, single axle weight, and GVW. The probability density function (PDF) and cumulative distribution function (CDF) of the GVW data of selected vehicle types are then formulated by use of three kinds of finite mixed distributions (normal, lognormal and Weibull). The mixture parameters are determined by use of the proposed GA-based method. The results indicate that the stochastic properties of the GVW data acquired from the field-instrumented WIM sensors are effectively characterized by the method of finite mixture distributions in conjunction with the proposed GA-based mixture parameter identification algorithm. Moreover, it is revealed that the Weibull mixture distribution is relatively superior in modeling of the WIM data on the basis of the calculated Akaike's information criterion (AIC) values.

**Keywords:** structural health monitoring; weigh-in-motion; gross vehicle weight; finite mixture distributions; mixture parameter estimation; genetic algorithm

## 1. Introduction

Structural damage of in-service highway or urban bridges is mainly caused by the deterioration of structural performances as well as the increasing traffic flow, especially the overloaded trucks. There can be little argument that the traffic load is one of the most important indicators for structural behavior assessment, safety condition evaluation, maintenance strategy optimization, and life-cycle cost analysis. In the past investigations, the traffic load models were determined on the basis of the subjective and empirical decisions made by bridge engineers or very limited survey data of traffic flow (Nowak 1993). In order to acquire the vast real-time traffic information, the weigh-in-motion (WIM) system has been proposed and widely used in a structural heath monitoring (SHM) system for random vehicle load monitoring of urban road and bridge structures.

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With a field-deployed WIM monitoring system, the realistic vehicle load data can be readily obtained for further load-relevant research (Ye *et al*. 2012, 2013, 2015).

In general, three types of critical data attained from the WIM system, i.e., gross vehicle weight (GVW), axle weight, and axle spacing are related to the construction of bridge load model (Miao and Chan 2002, Chan *et al*. 2005). The bridge load models can be favorably formulated upon the condition that the probability distributions of the primary parameters are accurately achieved. Based on the WIM monitoring data from the Binzhou Yellow River Highway Bridge, Lan *et al*. (2011) developed a traffic load model by use of the GVW probabilistic distribution and the developed model was applied to conduct bridge fatigue assessment. A critical bridge load accident may be induced by a single enormous heavy vehicle or a combination of different vehicle weights simultaneously passing through the bridge (O'Brien *et al*. 2011). Thus, it is of great importance to model the statistical distribution of the GVW data with a continuous probabilistic expression aiming at sufficiently characterizing the random load state.

Traditionally, a unimodal lognormal distribution or an inverse normal distribution is utilized to model the characteristics of the GVW data because the majority of observed vehicles are light ones. However, in recent years, many researchers have perceived the multimodal distribution characteristic of collected GVW data from a WIM system due to the site-specific traffic situation which inevitably covers light, medium and heavy vehicles with different load patterns such as empty, half full, full and overloaded cases (Mei *et al*. 2004, Caprani 2008, Lan *et al*. 2011, Caprani *et al*. 2012). Therefore, a multimodal probability distribution is seemingly needed to fit the traffic data with the stochastic properties of actual vehicle loads. The finite mixture distributions are generally used to model complex probability distributions and enable the statistical modeling of random variables with multimodal behaviors where a simple parametric model fails to depict the characteristics of the observations (Timm *et al*. 2005, Isaia *et al*. 2007). To cope with the modeling of finite mixture distributions, it is a necessity to estimate the unknown parameters in the mixed distribution models, which is regarded as an issue of parameter optimization (Kwon and Frangopol 2007, Ni *et al*. 2010, Ni *et al*. 2012, Volk *et al*. 2012, Franko and Nagode 2015, Sankararaman and Mahadevan 2015).

There are a variety of estimation methods that have been employed in finite mixture modeling such as the method of moments, maximum likelihood, minimum chi-square, and least squares algorithm. Especially, the maximum likelihood estimation method and the least squares algorithm have been broadly employed to perform the unknown mixture parameter estimation (Richardson *et al*. 1997, Mei *et al*. 2004). In this study, an alternative genetic algorithm-based (GA-based) mixture parameter estimation approach is developed for effective estimation of the parameters of finite mixture distributions, which has been successfully applied to model the field-measured vehicle load data obtained from the WIM monitoring system installed on the arch Jiubao Bridge located in Hangzhou, China. Statistical analysis of one-year WIM monitoring data is firstly carried out to capture the overall characteristics of collected axle weight and GVW data. The probability density function (PDF) and cumulative distribution function (CDF) of the GVW data of selected vehicle types are then formulated by use of three kinds of finite mixed distributions (normal, lognormal and Weibull). The mixture parameters are determined by the proposed GA-based method. The performance of the proposed method is evaluated by application of the Akaike's information criterion (AIC) (Akaike 1974).

## 2. Statistical analysis of WIM monitoring data

### *2.1 SHM of Jiubao Bridge*

The Jiubao Bridge with an overall length of 1,855 m is a steel-concrete composite arch bridge located in Hangzhou, China. It was opened to the traffic in July 2012 and represents the first river-crossing bridge composed by the composite structure in full length. As shown in Fig. 1, the upper structure of the main bridge comprises a 3×210 m arch bridge with a steel-concrete composite beam structure. The bridge deck is constructed in terms of urban bridge standard of China with six traffic lanes and double-sided pavements. The design velocity of vehicle is set to be 80 km/h.

After the completion of its construction in 2012, the bridge has been instrumented with a long-term SHM system comprising nine sophisticated subsystems. The real-time monitoring data are acquired through various types of sensors, including wind velocity and direction, environmental temperature and humidity, vehicle velocity and traffic volume, structural vibration, structural temperature, structural strain, alignment, displacement of bearing, and cable force. The huge amounts of field monitoring data are collected by comprehensive data acquisition stations and then transferred to the bridge monitoring center for further analysis. In the monitoring center, the tasks of data storage and analysis are performed by means of specific hardware and software. In addition, the SHM system is connected to the internet, and administrators and authorized visitors can easily access to the server in the monitoring center for data demonstration and extraction.

As an essential part of the long-term SHM system, a WIM system was installed on the entrance to the main span of the Jiubao Bridge, as shown in Fig. 2. The continuous monitoring data of vehicle type, vehicle velocity, axle weight, and GVW can be acquired by the instrumented WIM sensors. In particular, along with the growing traffic flow in recent years, the GVW data are playing an increasingly important role in overloading monitoring and structural safety evaluation. In this study, one-year GVW monitoring data from 1 November 2014 to 31 October 2015 are extracted to conduct the statistical analysis and probabilistic modeling of the load properties of highway traffic.

Fig. 1 Jiubao Bridge

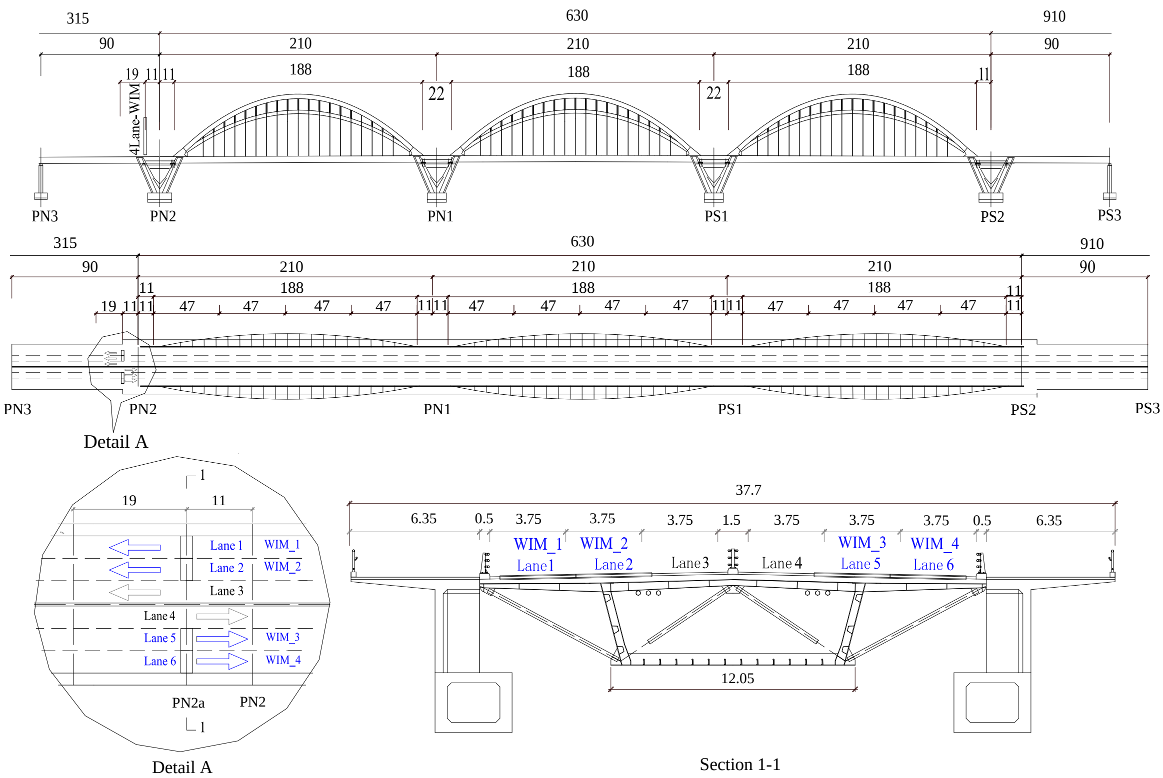


Fig. 2 Deployment of WIM system (unit: m)

### 2.2 Statistical analysis of WIM data

The availability of the traffic flow monitoring data permits the characterization of highway load information directly measured by the WIM sensors. In the statistical process, the load distribution models can be briefly divided into two groups: (i) those regarding the highway traffic as a whole, and (ii) those describing the highway traffic in different vehicle classes (O'Connor and O'Brien 2005, O'Brien and Enright 2012, Zhou *et al*. 2015).

In this study, the statistical analysis of the highway traffic as a whole is firstly carried out. As illustrated in Fig. 3, the proportion of the vehicles in the lanes 1, 2, 5 and 6 are 9.50%, 48.23%, 30.09% and 12.18%; while the average vehicle speeds in each lane are 43.49 km/h, 72.39 km/h, 49.39 km/h and 66.06 km/h, respectively. It can be found that the traffic volume from south to north (57.73%) is larger than that from north to south (42.27%) according to one-year real-time monitoring data. The maximum value of the average vehicle speed in the lane 2 is up to 72.39 km/h, which is much close to the limited design speed of 80 km/h. These statistical data provide quite useful information for bridge management department for road transportation supervision.

In addition, the vehicle type is identified and classified in accordance with the axle spacing, axle number, and axle weight by the WIM sensors. In the WIM system of the Jiubao Bridge, each passing vehicle is classified into seven types (A-G). As illustrated in Fig. 4, each type of the vehicle is described in terms of the axle number and axle type. For example, type A refers to 2-axle light and medium truck. As shown in Table 1, the proportion and average axle weight of each vehicle type are addressed. Obviously, the light and medium trucks account for the majority of the passing vehicle volume, which is up to 98.12%.

With the collected WIM monitoring data, the daily, monthly and yearly vehicle load properties can be derived. The total number of the vehicle traffic volume is 4,199,668 attained by the WIM system during one year period (1 November 2014 to 31 October 2015). As shown in Fig. 5, the peak traffic flow is occurred in May 2015, and the traffic volume in February 2015 is at the lowest level which is reflecting the less vehicle passing through the bridge due to the Spring Festival in February. Likewise, in terms of the typical daily traffic of one week, the weekend traffic decreases sharply in comparison with the weekday data, as shown in Fig. 6. In addition, in one day, the rush hour in the morning and evening is well exhibited in Fig. 7.

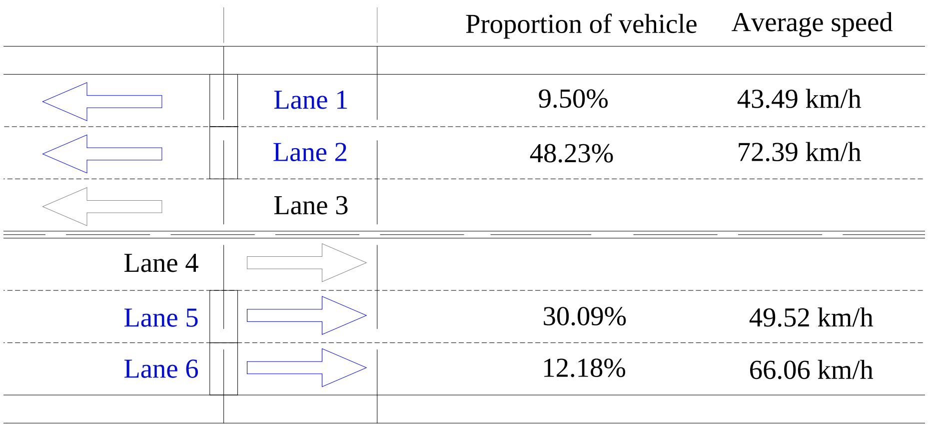



Fig. 3 Proportion and average speed of vehicle in each lane

Table 1 Statistics of axle weight

| Vehicle | | | Average value of axle weight (t) | | | | | |
|---|---|---|---|---|---|---|---|---|
| Type | Number | Ratio | Axle 1 | Axle 2 | Axle 3 | Axle 4 | Axle 5 | Axle 6 |
| A | 4,120,820 | 98.12% | 2.89 | 6.78 | -- | -- | -- | -- |
| B | 32,506 | 0.77% | 6.52 | 10.33 | 12.50 | -- | -- | -- |
| C | 23,711 | 0.56% | 5.76 | 9.26 | 12.70 | 14.55 | -- | -- |
| D | 2,681 | 0.06% | 5.87 | 12.04 | 12.58 | 11.75 | 10.83 | |
| E | 19,680 | 0.47% | 5.02 | 7.63 | 9.87 | 8.93 | 9.52 | 10.23 |
| F | 99 | <0.01% | 2.75 | 6.51 | -- | -- | -- | -- |
| G | 98 | <0.01% | 4.87 | 10.21 | -- | -- | -- | -- |

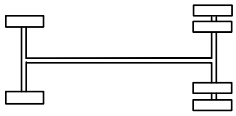
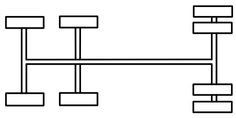
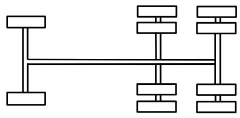
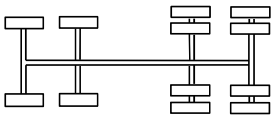
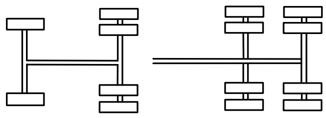
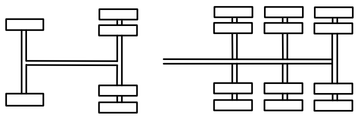
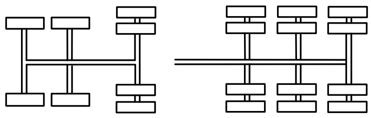
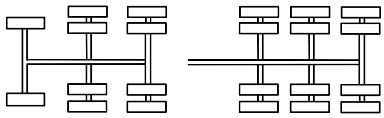
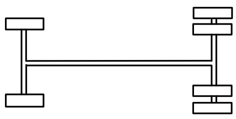
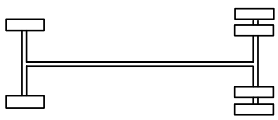
| Type | Description | Graphic symbol |
|---|---|---|
| A | 2-axle light and medium truck |  |
| B | 3-axle heavy truck |   |
| C | 4-axle heavy truck |   |
| D | 5-axle heavy truck |  |
| E | 6-axle heavy truck |   |
| F | 2-axle medium passenger car |  |
| G | 2-axle heavy passenger car |  |

Fig. 4 Vehicle type classification

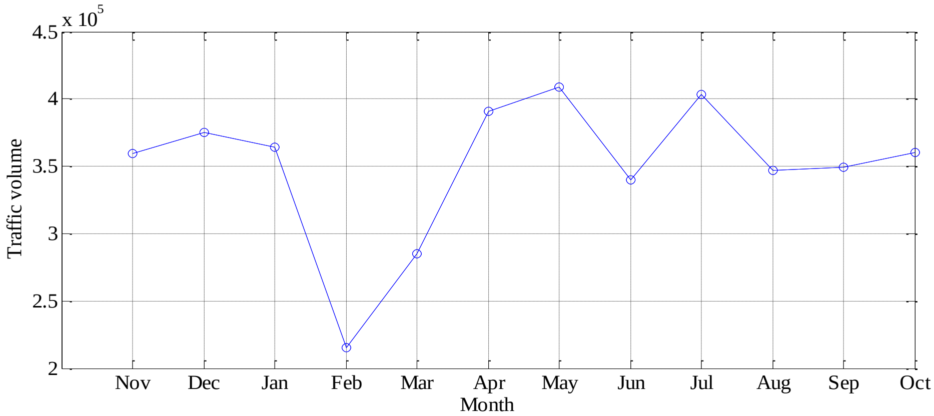



Fig. 5 Monthly traffic in one year (from November 2014 to October 2015)

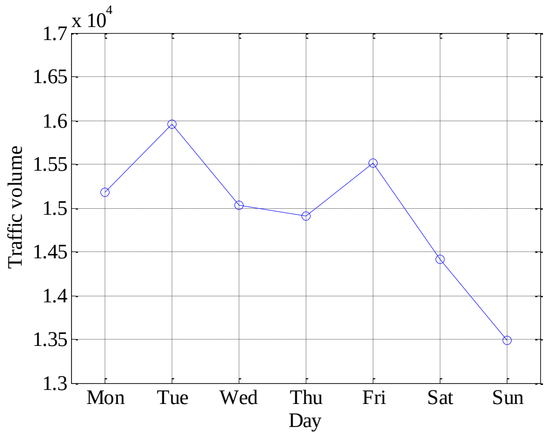



Fig. 6 Typical daily traffic of one week (from 12 October 2015 to 18 October 2015)

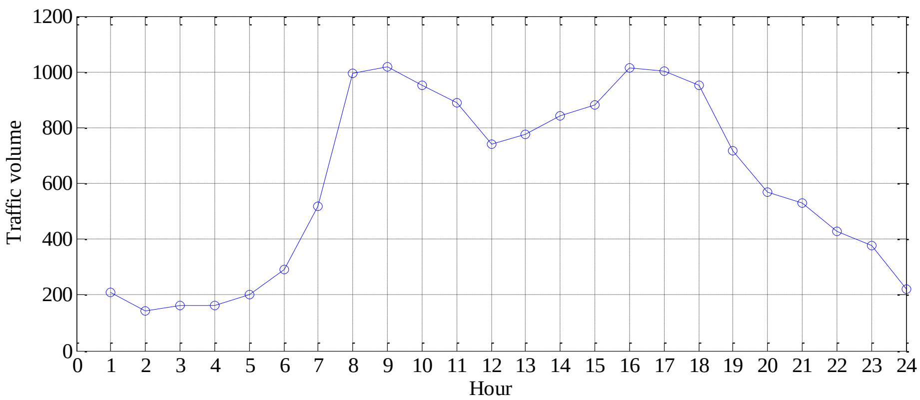
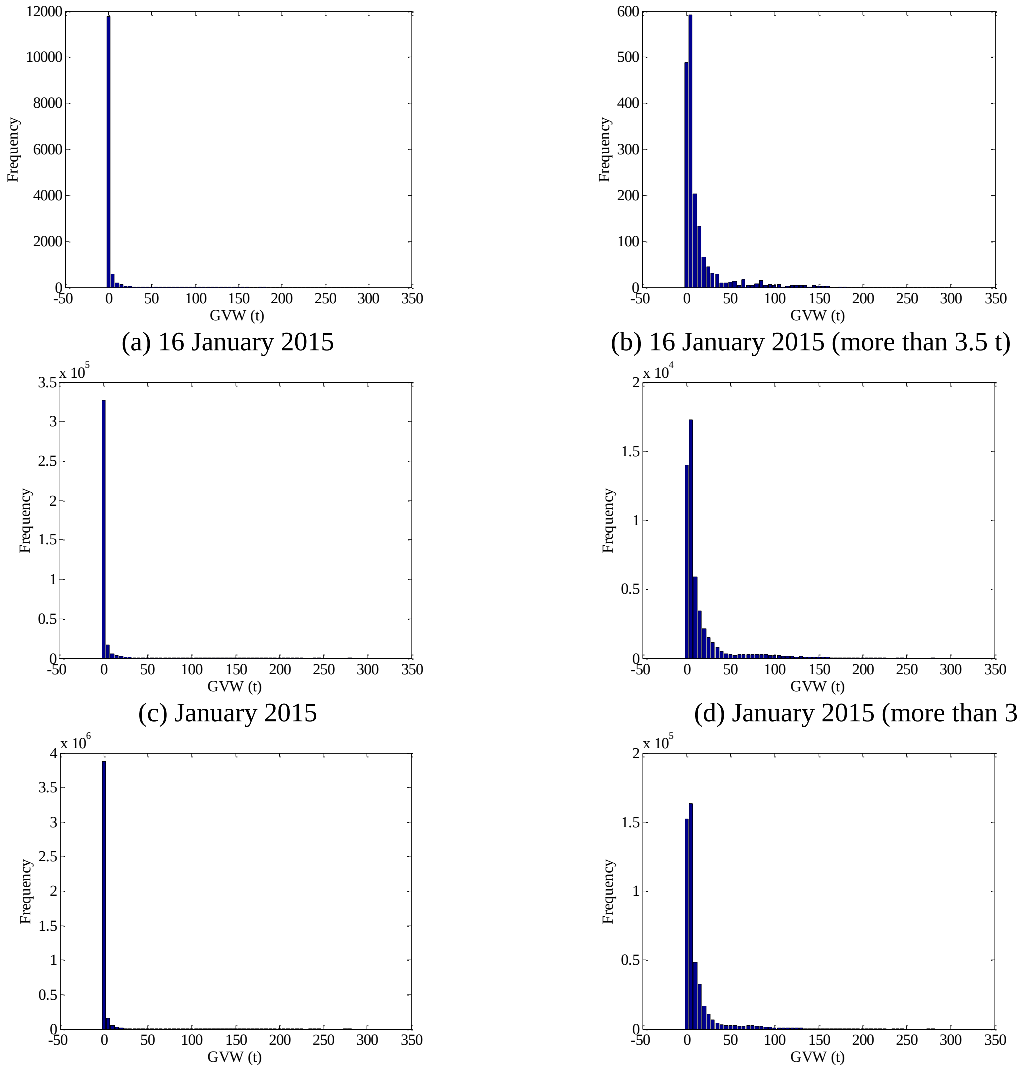



Fig. 7 Typical hourly traffic of one day (20 October 2015)

### 2.3 Stochastic characteristics of GVW data

In order to examine the stochastic characteristics of the GVW data, the histogram analysis of the typical daily, monthly and yearly monitoring data is performed. At the beginning of the statistical analysis, the raw GVW data are preprocessed by the data cleaning rule as reported in the literature, i.e., the GVW data less than 3.5 t are excluded (O'Brien and Enright 2012). It should be noted that, for the sake of showing the difference of the two distributions, the histograms of the raw GVW data and the GVW data more than 3.5 t are separately described, as illustrated in Fig. 8.




(a) 16 January 2015 (b) 16 January 2015 (more than 3.5 t)

(c) January 2015 (d) January 2015 (more than 3.5 t)

(e) one-year GVW data (f) one-year GVW data (more than 3.5 t)

Fig. 8 Histograms of daily, monthly and yearly GVW data

It is seen from Fig. 8 that both the raw GVW data and processed GVW data are monotonically decreased with one peak. In general, the monotonically decreasing data can be well modeled by a simple distribution. Meanwhile, the GVW data of each type are analyzed and represented in Fig. 9. It is noteworthy that the vehicle type F and G are not considered in this study due to the excessively little data collected by the WIM sensors.

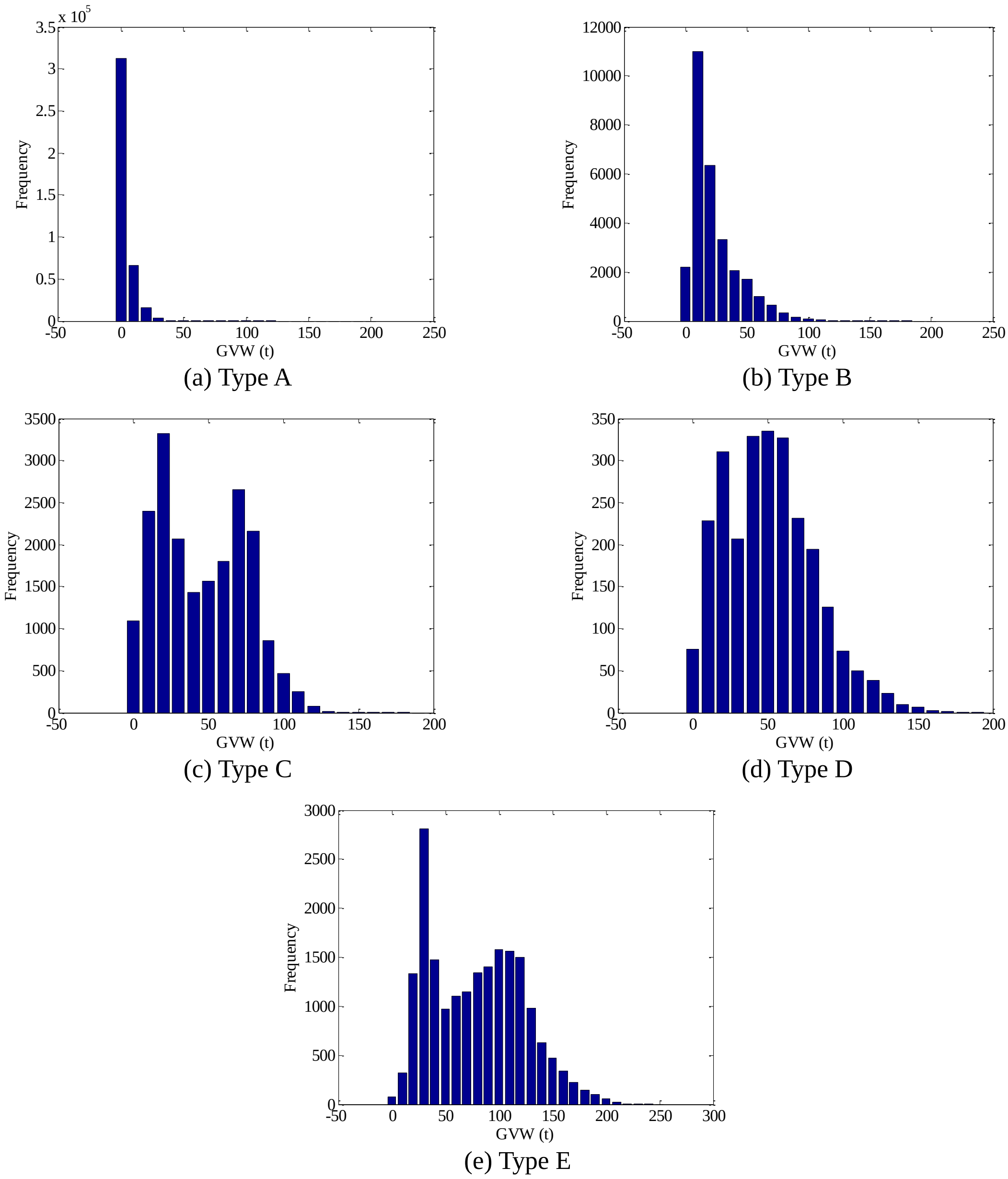



Fig. 9 Histograms of each vehicle type GVW data

## 3. Probabilistic modeling of GVW data

### 3.1 Finite mixture distributions

The method of finite mixture distributions has been increasingly employed to model the probability distributions of a variety of random phenomena. A finite mixture model is a convex combination of two or more PDFs. Generally, the basic structure of finite mixture distributions for independent scalar or vector observations **x** can be defined as a weighted sum of component distribution (McLachlan and Peel 2000)

$$f(\mathbf{x}|c,\mathbf{w},\boldsymbol{\theta}) = \sum_{l=1}^{c} w_l f_l(\mathbf{x}|\boldsymbol{\theta}_l) \tag{1}$$

$$\sum_{l=1}^{c} w_l = 1 \quad \text{and} \quad w_l \geq 0 \tag{2}$$

where $f(\mathbf{x}|c,\mathbf{w},\boldsymbol{\theta})$ is a predictive mixture density, $f_l(\mathbf{x}|\boldsymbol{\theta}_l)$ is a given parametric family of predictive component densities indexed by the scalar or vector parameters $\boldsymbol{\theta}_l$. The objective of the analysis inference about the unknowns which include the number of components or groups, $c$, the component weights, $w_l$, summing to 1, and the component parameters $\boldsymbol{\theta}_l$.

Thus, for instance, the finite mixed normal distributions, the finite mixed lognormal distributions, and the finite mixed Weibull distributions can be expressed respectively by

$$f(\mathbf{x}|c,\mathrm{w},\theta) = \sum_{l=1}^{c} w_l \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu_l)^2}{\sigma_l^2}\right\} \tag{3}$$

$$f(\mathbf{x}|c,\mathbf{w},\boldsymbol{\theta}) = \sum_{l=1}^{c} w_l \frac{1}{\sqrt{2\pi}\sigma_l x} \exp\left\{-\frac{1}{2}\frac{(\ln(x)-\mu_l)^2}{\sigma_l^2}\right\} \tag{4}$$

$$f(\mathbf{x}|c,\mathrm{w},\theta) = \sum_{l=1}^{c} w_l \frac{\gamma_l}{\eta_l}\left(\frac{x}{\eta_l}\right)^{\gamma_l-1} \exp\left\{-\left(\frac{x}{\eta_l}\right)^{\gamma_l}\right\} \tag{5}$$

where $\mu_l$ and $\sigma_l$ are the mean values and standard deviations of normal mixture parameters; $\gamma_l$ and $\eta_l$ are the shape parameter and scale parameter of Weibull mixture parameters. The parameter estimation of mixture distribution models is deemed as a problem of parameter optimization. In this study, these unknown parameters of each distribution are determined by GA-based method as discussed in the following section.

### 3.2 GA-based mixture parameter estimation approach

Genetic algorithm (GA), proposed by Holland in 1975 (Holland 1975), is a stochastic algorithm inspired by Darwin’s theory of evolution. With the renewal and advance of computational intelligence, GA has been successfully applied to solve a variety of optimization problems in the area of science, biology and engineering. In this section, a GA-based mixture parameter estimation approach is introduced. As shown in Fig. 10, the principle of GA mainly consists of fitness function, selection, crossover and mutation. The fitness function can reflect the quality of fitting result. In this study, the fitness function is established based on the law of large numbers.

Assuming that $f(\mathbf{x}|c,\mathbf{w},\boldsymbol{\theta})$ is continuous and does not vary appreciably over region $R_v$, of the $v$th bin volume $\xi_v$. The probability that a scalar or vector observation $\mathbf{x}$ will fall inside $R_v$ is given by

$$\int_{R_v} f(\mathbf{x}|c,\mathbf{w},\boldsymbol{\theta}) \approx f(\mathbf{x}_v|c,\mathbf{w},\boldsymbol{\theta})\xi_v \tag{6}$$

Region $R_v$ is taken to be a hypersquare with the sides of length $Z_v=[z_{1v},\ldots,z_{dv}]^T$ centered on $x_v$. Its volume is expressed as

$$\xi_v = \prod_{j=1}^{d} z_{jv} \tag{7}$$

The relative frequencies $p_v$ of obvious data falling in $R_v$ are given by

$$p_v = \frac{N_v}{N} \approx f(\mathbf{x}_v|c,\mathrm{w},\theta)\xi_v \tag{8}$$

where $N$ stands for the total number of independent scalar or vector observations; and $N_v$ is the fraction of observations falling inside $R_v$.

GA is used to find the optimal solution of the unknown parameters $\Theta=\{\mathrm{w},\theta\}$ by developing the following fitness function

$$T = \frac{1}{\sum_{v=1}^{V}\left(\frac{p_v - f(\mathbf{x}_v|c,\mathbf{w},\boldsymbol{\theta})\xi_v}{p_v}\right)^2} \tag{9}$$

It is found that the closer between $p_v$ and $f(\mathbf{x}_v|c,\mathbf{w},\boldsymbol{\theta})\xi$, the larger the $T$ is.

The values of AIC for each type of mixed distribution are calculated to select both the optimal number of components and the parametric family. The best mixed distribution is selected at the lowest values of AIC (Akaike 1974).

In the general case, the AIC is defined as

$$\mathrm{AIC} = 2M - 2\ln(L) \tag{10}$$

where $M$ is the number of unknown parameters in the mixed distribution; and $L$ is the maximized value of the likelihood function for the estimated mixed distribution.

### *3.3 Multimodal probabilistic modeling*

As described in Section 2, the GVW monitoring data of types C, D and E obtained from the WIM system exhibit a two-peak characteristic. In this study, the GVW monitoring data of these three types are selected to perform the probabilistic modeling analysis. Three types of finite mixture distributions (normal, lognormal and Weibull) are employed to model the probability distribution of the GVW monitoring data, and the unknown parameters (weight, mean value, and standard deviation of normal distribution and lognormal distribution, and weight, scale parameter and shape parameter of Weibull distribution) of the mixture distribution models are calculated by the proposed GA-based approach. The optimal fitting model is determined according to the AIC value.

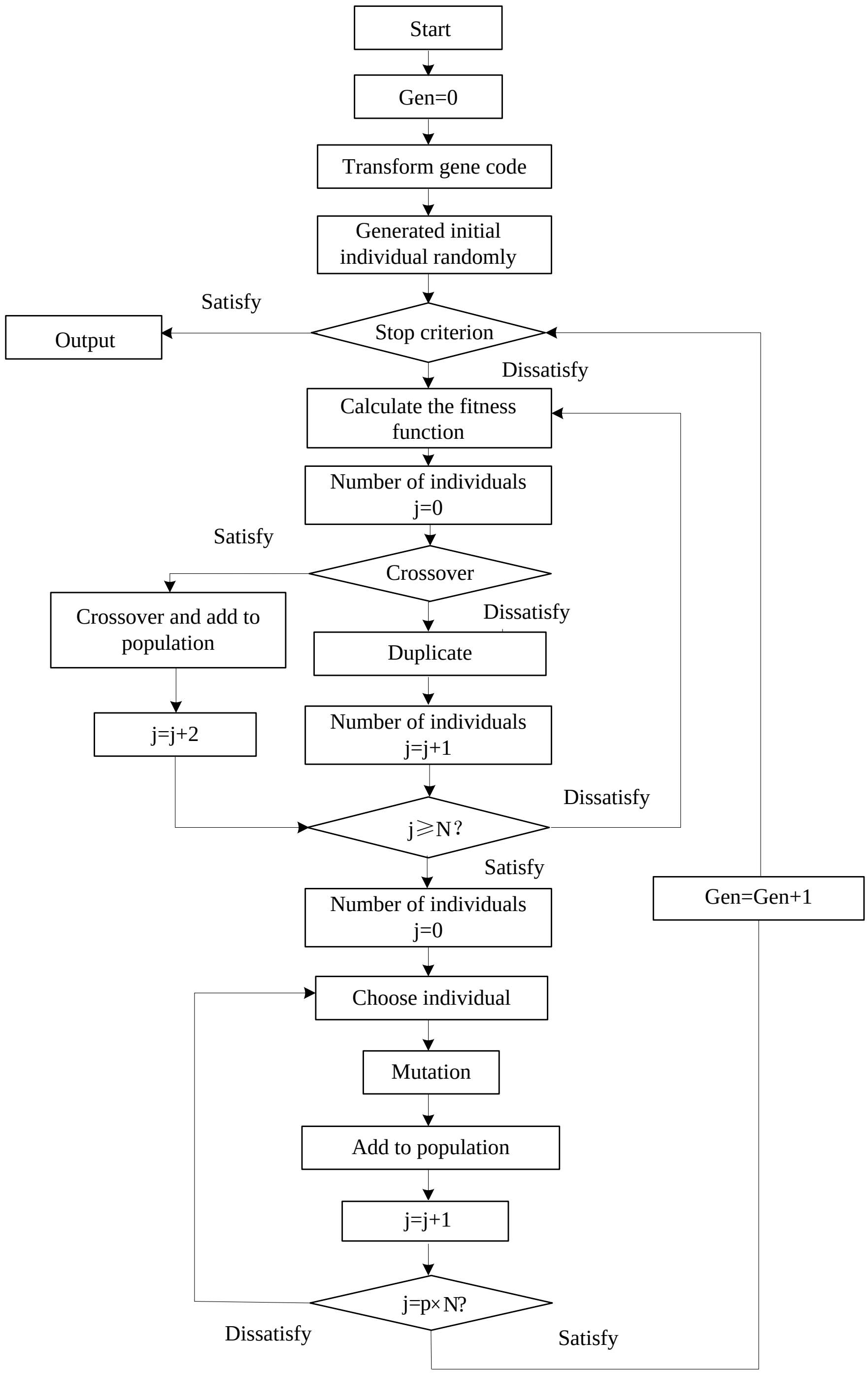



Fig. 10 Flowchart of GA calculation process

Fig. 11 shows the calculated AIC values with different numbers of the component for type C. It is revealed that the AIC value is almost stable when the number of component exceeds 3 for normal mixture, 4 for lognormal mixture, and 6 for Weibull distribution. The Weibull distribution gives the minimum AIC value, and therefore it is rational to choose the Weibull mixture with 6 components as the optimal probability distribution of the GVW monitoring data of type C. Figs. 12 and 13 illustrate the PDFs and CDFs of the GVW monitoring data of type C. The estimated mixture parameters of each distribution are listed in Table 2. The GA-based finite mixture distribution provides an effective and alternative tool for modeling the multimodal distribution. The achieved analytical formulations of the concerned GVW data can be employed for further investigations, e.g., establishment of live load model, structural reliability assessment, etc.

Likewise, for type D, it is seen from Fig. 14 that the AIC value is almost stable when the number of component exceeds 4 for normal mixture, 4 for lognormal mixture, and 6 for Weibull distribution. Figs. 15 and 16 illustrate the PDFs and CDFs of the GVW monitoring data of type D. Thus, it is rational to choose the Weibull mixture with 6 components as the optimal probability distribution of the GVW monitoring data of type D. The estimated mixture parameters of each distribution for Type D are listed in Table 3.

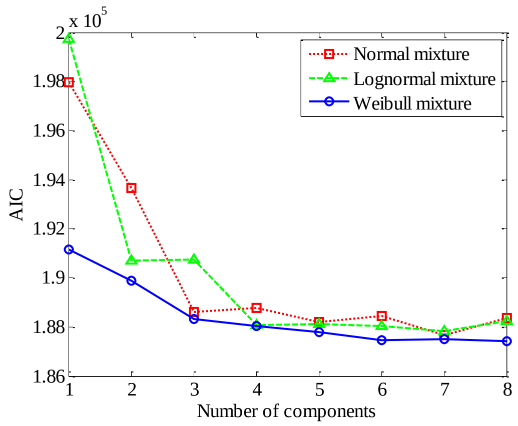



Fig. 11 AIC values (Type C)

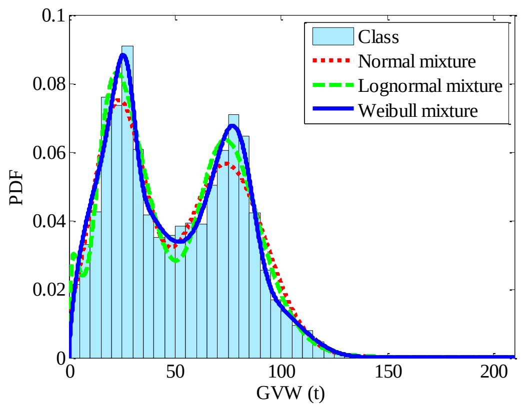



Fig. 12 PDFs (Type C)

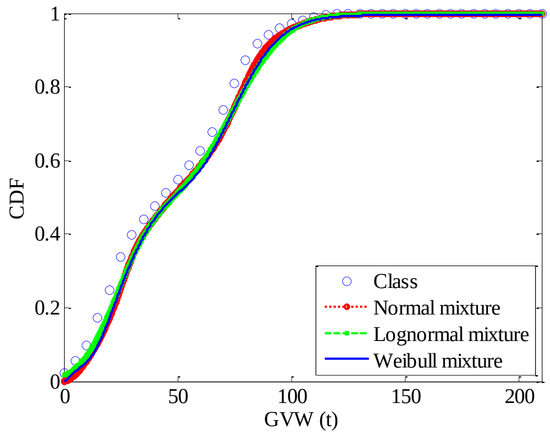



Fig. 13 CDFs (Type C)

Table 2 Estimated mixture parameters (Type C)

| Distribution | Parameters | | |
|---|---|---|---|
| | Weight ($w_l$) | Mean value ($\mu_l$) | Standard deviation ($\sigma_l$) |
| Normal | 0.0069 | 106.7257 | 49.9721 |
| | 0.5280 | 73.9460 | 18.7192 |
| | 0.4651 | 23.1658 | 12.5983 |
| Lognormal | 0.4801 | 3.3697 | 0.4768 |
| | 0.0032 | 19.1959 | 20.4447 |
| | 0.0958 | 2.3953 | 1.2382 |
| | 0.4210 | 4.3493 | 0.1870 |
| Weibull | Weight ($w_l$) | Scale parameter ($\eta_l$) | Shape parameter ($\gamma_l$) |
| | 0.5020 | 36.4334 | 1.6777 |
| | 0.0134 | 78.0971 | 1.1359 |
| | 0.0117 | 113.1449 | 9.1976 |
| | 0.1398 | 91.7222 | 5.3845 |
| | 0.0889 | 26.8694 | 5.6875 |
| | 0.2441 | 78.3981 | 8.4090 |

For type E, it can be found from Fig. 17 that the AIC value is almost stable when the number of component exceeds 3 for normal mixture, 4 for lognormal mixture, and 3 for Weibull distribution. Figs. 18 and 19 illustrate the PDFs and CDFs of the GVW monitoring data of type E. The Weibull mixture with 3 components is chosen as the optimal probability distribution of the GVW monitoring data of type E. The estimated mixture parameters of each distribution for type E are listed in Table 4.

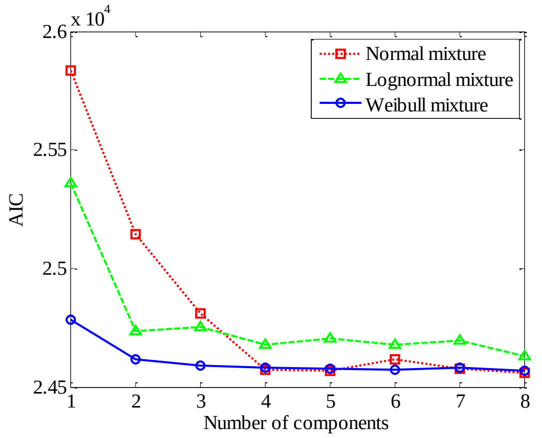



Fig. 14 AIC values (Type D)

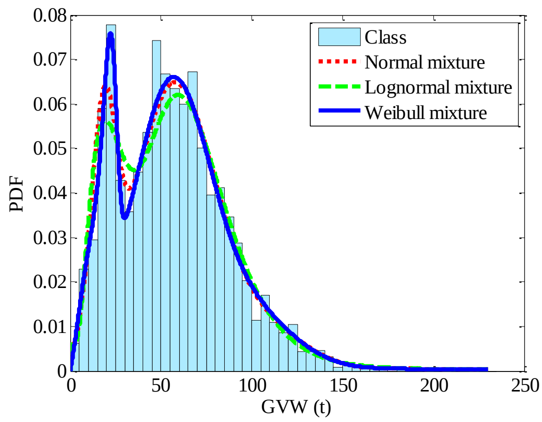



Fig. 15 PDFs (Type D)

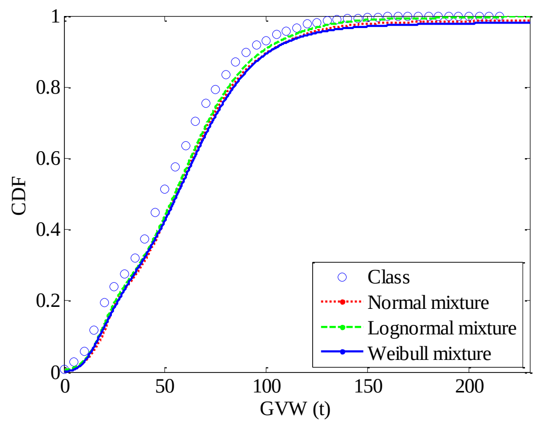



Fig. 16 CDFs (Type D)

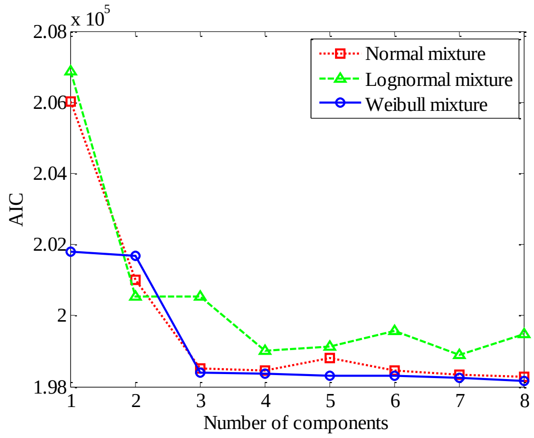



Fig. 17 AIC values (Type E)

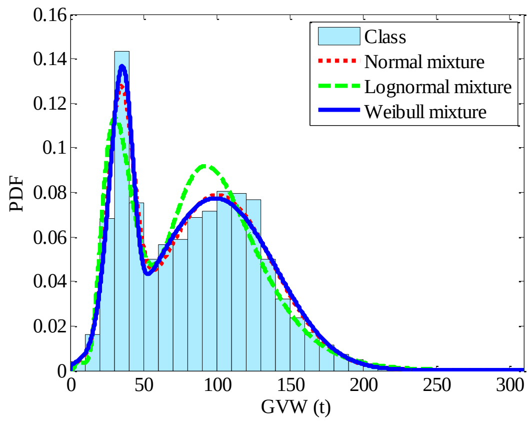



Fig. 18 PDFs (Type E)

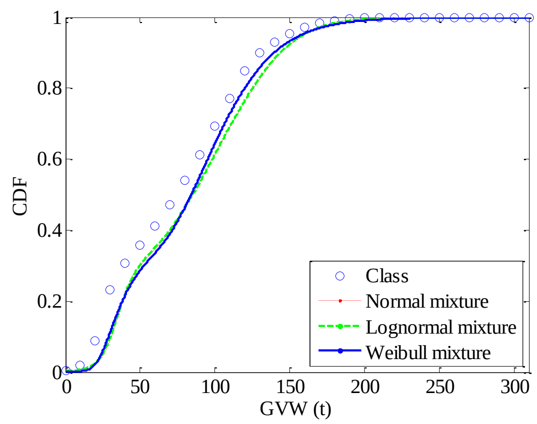



Fig. 19 CDFs (Type E)

Table 3 Estimated mixture parameters (Type D)

| Distribution | Parameters | | |
|---|---|---|---|
| | Weight ($w_l$) | Mean value ($\mu_l$) | Standard deviation ($\sigma_l$) |
| Normal | 0.7323 | 57.3749 | 22.6992 |
| | 0.1659 | 18.4347 | 6.7860 |
| | 0.0116 | 184.6148 | 49.6978 |
| | 0.0901 | 109.0656 | 21.6881 |
| Lognormal | 0.5332 | 4.2486 | 0.3141 |
| | 0.0655 | 4.3020 | 1.9464 |
| | 0.0006 | 31.0491 | 26.4164 |
| | 0.4007 | 3.3688 | 0.6262 |
| Weibull | Weight ($w_l$) | Scale parameter ($\eta_l$) | Shape parameter ($\gamma_l$) |
| | 0.2304 | 95.9636 | 3.4687 |
| | 0.0862 | 16.7313 | 2.2526 |
| | 0.5457 | 61.4150 | 3.1978 |
| | 0.0182 | 73.8373 | 1.4372 |
| | 0.0832 | 22.7769 | 6.7938 |
| | 0.0364 | 220.2186 | 1.1147 |

Table 4 Estimated mixture parameters (Type E)

| Distribution | Parameters | | |
|---|---|---|---|
| | Weight ($w_l$) | Mean value ($\mu_l$) | Standard deviation ($\sigma_l$) |
| Normal | 0.0069 | 106.7257 | 49.9721 |
| | 0.5280 | 73.9460 | 18.7192 |
| | 0.4651 | 23.1658 | 12.5983 |
| Lognormal | 0.4801 | 3.3697 | 0.4768 |
| | 0.0032 | 19.1959 | 20.4447 |
| | 0.0958 | 2.3953 | 1.2382 |
| | 0.4210 | 4.3493 | 0.1870 |
| Weibull | Weight ($w_l$) | Scale parameter ($\eta_l$) | Shape parameter ($\gamma_l$) |
| | 0.5154 | 36.4334 | 1.6777 |
| | 0.1516 | 113.1449 | 9.1976 |
| | 0.3330 | 26.8694 | 5.6875 |

## 4. Conclusions

In this study, the statistical analysis of traffic loads and the probability modeling of the GVW data measured by the instrumented WIM system by use of the proposed GA-based finite mixture distributions have been addressed. The GA-based unknown mixture parameter estimation approach is developed for estimation of the parameters in mixture distribution models, and applied to model the vehicle load monitoring data attained from the WIM system instrumented on the arch Jiubao Bridge located in Hangzhou, China. The statistical distributions of the GVW data are derived by use of three kinds of finite mixed distributions (normal, lognormal and Weibull), and the mixture parameters are determined by the proposed GA-based method. The results show that the stochastic properties of the vehicle load data measured by the field-installed WIM sensors are effectively characterized using the method of finite mixture distributions together with the developed GA-based mixture parameter identification approach. The multimodal characteristic of the GVW data can be well modeled by the proposed method which provides an effective technique for parameter identification of finite mixture distributions in modeling real-time GVW data. Moreover, the Weibull mixture distribution exhibits a superior performance in modeling assignments in accordance with the calculated AIC values. The stochastic properties of the GVW data derived from the field-instrumented WIM sensors are effectively characterized by the method of Weibull mixture distribution with the proposed GA-based mixture parameter identification approach. Meanwhile, the results of statistical analysis and probabilistic modeling of the site-specific traffic data from the WIM system will serve as a useful reference for bridge management department to conduct prompt structural condition assessment and effective transportation arrangement.

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