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Application of Eringen's nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams

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Abstract. In the present study, for first time the size dependent vibration behavior of a rotating functionally graded (FG) Timoshenko nanobeam based on Eringen's nonlocal theory is investigated. It is assumed that the physical and mechanical properties of the FG nanobeam are varying along the thickness based on a power law equation. The governing equations are determined using Hamilton's principle and the generalized differential quadrature method (GDQM) is used to obtain the results for cantilever boundary conditions. The accuracy and validity of the results are shown through several numerical examples. In order to display the influence of size effect on first three natural frequencies due to change of some important nanobeam parameters such as material length scale, angular velocity and gradient index of FG material, several diagrams and tables are presented. The results of this article can be used in designing and optimizing elastic and rotary type nano-electro-mechanical systems (NEMS) like nano-motors and nano-robots including rotating parts.

Keywords: bending vibration; Eringen's nonlocal theory; rotary functionally graded nanobeam; Timoshenko beam theory

1. Introduction

The experimental results have shown the disagreement between the classical continuum-based studies which proves that lacking the internal length scale parameters makes the classic theories incapable of predicting the behavior of structures in nano-scales accurately (Akgöz and Civalek 2012). Thus, a number of theories have been introduced by scientists to define nano-systems behavior. Using the vehicles of global balance laws and the second law of thermodynamics Eringen introduced the nonlocal field theory which considers the behavior of a material in point not only dependent on that point, but also dependens on the state of all other points in the body (Eringen 1972, Eringen and Edelen 1972). Since then, Eringen's theory was employed to study micro and nano scaled structures for it includes the size parameters which are of significant importance in studying the small scaled materials.

The nonlocal elasticity theory of Eringen (1983) is a common tool for analyzing structures in small scales. In this theory, unlike the classical theory, the stress at a reference point(x) in a body

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depends not only on the strain of that point but also of all points of the body. So, many researchers have studied the nano-structures such as a nanobeam based on this theory.

Reddy (2007) investigated the bending, buckling and vibration of nanobeam using Eringen's nonlocal theory. Civalek and Demir (2011) studied on the bending behavior of microtubules using Euler-Bernoulli beam model based on the Eingen theory. Nazemnezhad and Hosseini-Hashemi (2014) studied on the exact solution of nonlocal nonlinear vibration of FG nanobeams.

Civalek and Akgöz (2013) investigated the behavior of micro graphene sheet's sector in elastic matrix. Nanomachines are of great importance in the study of nanotechnology. For example, the importance of nanomachines can be clearly observed in applications such as DNA nanomachines, programmable chemical synthesis and targeted drug delivery (van Delden *et al*. 2005, Bath and Turberfield 2007, Goel and Vogel 2008, Lee *et al*. 2010, Lubbe *et al.* 2011, Tierney *et al*. 2011, Chen *et al.* 2012).

Rotating components are the main parts of most of the micro-scaled structures. Thus, the rotary effects is of great interests amongst researchers. The rotating single-walled carbon nanotube (SWCNT) with different hub radius was investigated by Murmu and Adhikari (2010). Using Euler-Bernoulli beam model, Narendar and Gopalakrishnan (2011) investigated the SWCNT with rotating effect based on Eringen nonlocal theory. Challamel and Wang (2008) studied the nonlocal effect on the bending behavior of rod. Moreover, Lim *et al*. (2009) considered the axial tortion to study nonlocal stress effect on a nanocantilever beam. Many researchers analyzed the flapwise bending vibration of rotating nanocantilever beams (Pradhan and Murmu 2010, Narendar 2012, Aranda-Ruiz *et al*. 2012). Most recently, Ghadiri and Shafiei (2015, 2015) studied the linear and nonlinear bending vibration of a rotating nanobeam with different boundary conditions by nonlocal Eringen's theory. Dehrouyeh-Semnani A (2015) studied the size effect on flapwise vibration of rotating microbeams. The rotary effects have also been studied in some researches for clamp-simply supported (propped cantilever) boundary condition (Murmu and Adhikari 2010, Narendar and Gopalakrishnan 2011). Effect of nonlocal and surface effects on vibration analysis of a rotating FG nanobeam was studied by Ghadiri *et al.* (2016). Ghadiri and Shafiei (2016) investigated the nonlocal effect on the vibrations of a rotating nanoblade that can be the basis of nano-turbine design. Kaya (2006) studied the free vibration behavior of a Timoshenko beam with rotating effect by differential transform (DTM) method.

A functionally graded material (FGM) is a composite which is made of two or more different materials. FGMs, commonly used in various industrial applications such as, mechanic, civil, aerospace, biomedicines, electronics, etc. and have excellent mechanical and physical properties like lightness, high stuffiness, elasticity and ergonomic. In a FG microbeam, the volume fractions of materials are changed from one surface to another. With this change, mechanical and physical properties vary smoothly and continuously along the axis or through the thickness. Many researchers performed the dynamic and static analyses of FG macro, micro and nanobeams. Free vibration behavior of simply supported FG beam based on the higher order shear deformation and classical beam theories was presented by Metin Aydogdu (2007). Dewey H. Hodges (1981) have studied the free vibration of rotating beams by means of a finite-element method of variable order. Şimşek (2010) presented the first and higher order shear deformation FG beam models for different boundary conditions.

Asghari *et al.* (2011) studied the bending vibration of the FG Euler–Bernoulli and Timoshenko microbeams based on the modified couple stress theory. The behavior of FG and axially FG beam was investigated by Alshorbagy *et al*. (2011). The small-scale effect on dynamic behavior of FG microbeams based on Timoshenko model was studied by Ke and Wang (2011). The effect of the value of gradient index on vibration behavior of FG Timoshenko microbeam based on the strain gradient theory was studied by Ansari *et al*. (2011). Ke *et al*. (2012) presented the effect of von-Kármán geometric nonlinearity on nonlinear vibration of FG microbeam using modified couple stress theory. The free vibration behavior of FG Euler-Bernoulli nanobeam using finite element method and nonlocal elasticity theory was presented by Eltaher *et al*. (2012). Morever, Eltaher *et al*. (2013) based on nonlocal elasticity model presented the buckling analysis of FG nanobeam. Also, Şimşek and Yurtcu (2013) studied the static buckling and bending of FG nonlocal beam with analytical method. Rahmani and Pedram (2014) using nonlocal and Timoshenko model presented the modal analysis of FG nanobeam. A new shear correction factors are presented for FG Timoshenko microbeams by Akgöz and Civalek (2014a). Also, they studied on buckling of FG microbeam based on the modified couple stress and sinusoidal shear deformation model of beam (Akgöz and Civalek 2014b). Morever, effects of rotary inertia shear deformation of FG beam was investigated by Avcar (2015). Recently, Ebrahimi and Salari (2015a, b) studied the thermal analyses of FG nanobeam with exact solution in thermal environment.

It is found that most of the previous studies on vibration analysis of FG nanobeams assumed to be stationary and ignored the rotary effects. As it is seen in the literature, investigation of vibration behavior of a rotating FG nanobeam based on the nonlocal elasticity theory is completely a new research and it hasn't been done before. Therefore in this study, the size dependent vibration behavior of a rotating FG Timoshenko nanobeam is presented. Also, the natural frequencies of cantilever nanobeams under changes of some parameters such as angular velocity, material length scale and FG index of Timoshenko shear deformation theories are verified. It is assumed that the FG nanobeam is made of ceramic and metal and the properties of FG materials vary through the thickness (z direction) according to the power law. Also to investigate material behaviors at micro scale, the nonlocal theory is applied. Hamilton's principle and the GDQM are utilized to derive cantilever boundary conditions and to solve governing equations, respectively. The effects of changes of some important parameters such as material length scale, angular velocity and FG index on the values of frequencies are studied. Moreover, the accuracy of the results is shown through several numerical examples.

The results of this article can be refered for designing and optimizing the elastic and rotatin nono-electro-mechanical systems.

2. Theory and formulation

2.1 Nonlocal power-law FG nanobeam equations based

The problem of interest is a rotating nanobeam whose length 'L', height 'h' and width 'b' are located along x, z and y directions, respectively as shown in Fig.1. Considering FG Timoshenko nanobeams made of composing two different materials (in this paper metal and ceramic), the material and geometrical properties of the nanobeams are assumed to vary through the thickness (along z axis). This variation in properties can be defined as the power law which is presented as following below

$$P_f = P_c V_c + P_m V_m \tag{1}$$

where P_m, P_c, V_m and V_c are the material properties and the volume fractions of the metal and the ceramic constituents related by

$$V_c + V_m = 1 \tag{2a}$$

The volume fraction of the ceramic constituent of the beam is assumed to be given by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{2b}$$

Here *n* is the power-law exponent which determines the material distribution through the thickness of the beam and z is the distance from the mid-plane of the FG beam. By using the Eqs. (1) and (2), Young's modulus (E), mass density (ρ) and Poisson's ratio (ν) can be described respectively as

$$\begin{aligned} E(z) &= (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + E_m \\ \rho(z) &= (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \rho_m \\ \nu(z) &= (\nu_c - \nu_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \nu_m \end{aligned} \tag{3}$$

The top surface of the functionally graded nanobeam (at z = h/2) is pure ceramic (Si_3N_4), where the bottom surface of the FG nanobeam (z = -h/2) is pure metal (S).

2.2 Kinematic relations

The equations of motion are derived based on the Timoshenko beam theory according to the displacement field at any point of the beam written as

$$u_x(x,z,t) = u(x,t) + z\varphi(x,t) \tag{4a}$$

$$u_z(x,z,t) = w(x,t) \tag{4b}$$

where t is time, φ is the total bending rotation of the cross-section, u and w are displacement components of the mid-plane along x and z directions, respectively. Therefore, strain tensor, curvature tensor and rotation vector can be determined as follows

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \varphi}{\partial x} \tag{5}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \varphi \tag{6}$$

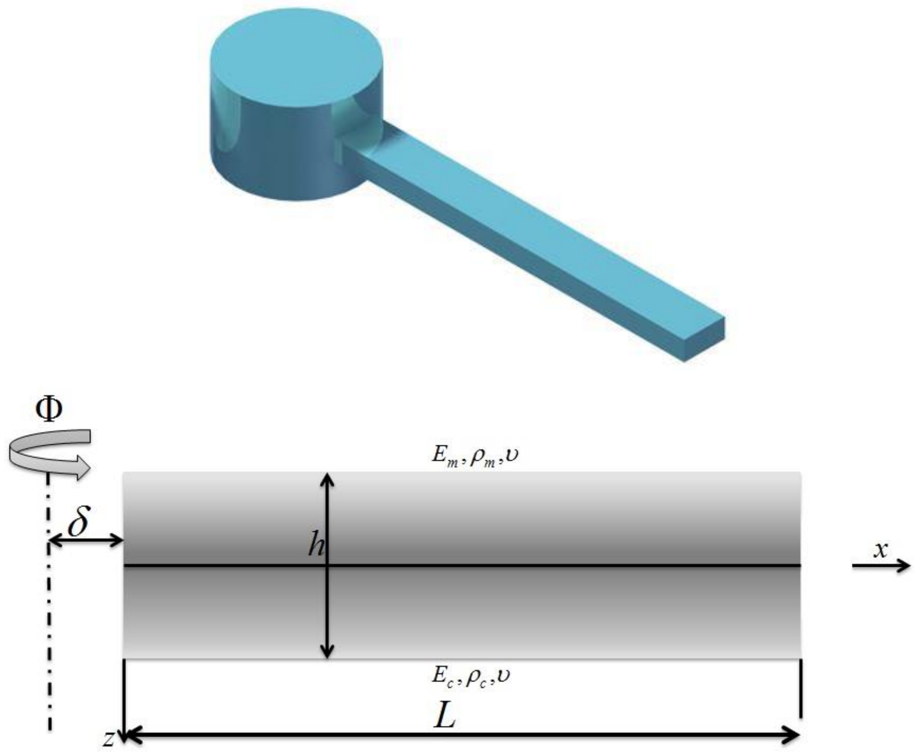

Fig. 1 Schematic of rotating cantilever nanobeam and material distribution over z plane and geometrical properties of nanobeam

where ε_{xx} and γ_{xz} are the normal and shear strains, respectively. Based on Hamilton's principle, which states that the motion of an elastic structure during the time interval $t_1 < t < t_2$ is such that the time integral of the total dynamics potential is extremum (Tauchert 1974) we have

$$\int_0^1 \delta\left(U - T + V\right)dt = 0 \tag{7}$$

Here U is strain energy, T is kinetic energy and V is work done by external forces. The virtual strain energy can be calculated as

$$\delta U = \int_V \sigma_{ij}\,\delta\varepsilon_{ij}\,dV = \int_V \left(\sigma_{xx}\,\delta\varepsilon_{xx} + \sigma_{xz}\,\delta\gamma_{xz}\right)dV \tag{8}$$

Substituting Eqs. (5) and (6) into Eq. (8) yields

$$\delta U = \int_0^L \left(N\left(\delta\frac{\partial u}{\partial x}\right) + M\left(\delta\frac{\partial \varphi}{\partial x}\right) + Q\left(\delta\frac{\partial w}{\partial x} + \delta\varphi\right)\right)dx \tag{9}$$

In which N is the axial force, M is the bending moment and Q is the shear force. These stress resultants used in Eq. (9) are defined as

$$N = \int_A \sigma_{xx} dA, \quad M = \int_A \sigma_{xx} z dA, \quad Q = \int_A K_S \sigma_{xz} dA \tag{10}$$

The kinetic energy of Timoshenko nanobeam can be written as

$$T = \frac{1}{2} \int_0^L \int_A \rho(z) \left(\left(\frac{\partial u_x}{\partial t} \right)^2 + \left(\frac{\partial u_z}{\partial t} \right)^2 \right) dAdx \tag{11}$$

Also the virtual kinetic energy can be expressed as

$$\delta T = \int_0^L \left[\begin{array}{l} I_0 \left(\frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \right) + I_1 \left(\frac{\partial \varphi}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial u}{\partial t} \frac{\partial \delta \varphi}{\partial t} \right) \\ + I_2 \left(\Omega^2 \varphi \delta \varphi + \frac{\partial \varphi}{\partial t} \frac{\partial \delta \varphi}{\partial t} \right) \end{array} \right] dx \tag{12}$$

where (I_0, I_1, I_2) are the mass moment of inertias, defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z) (1, z, z^2) dA \tag{13}$$

Rotary effect on the governing equation is evaluated. The variation of the work done corresponding to angular velocity that can be obtained by

$$\delta V = \int_0^L N^R \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} dx \tag{14}$$

Where N is obtained by

$$N^R = \int_x^L \int_A \rho(z) dA \, \Omega^2 (r + \zeta) dA \, d\zeta \tag{15}$$

where ρ, Ω, r and x are beam density at certain point z, angular velocity, hub radius, beam length variation along x direction. By Substituting Eqs. (9), (12) and (14) into Eq. (7) and setting the coefficients of $\delta u, \delta w$ and $\delta \varphi$ to zero, the following Euler–Lagrange equation can be obtained

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi}{\partial t^2} \tag{16a}$$

$$\frac{\partial Q}{\partial x} - N^R \frac{\partial^2 w}{\partial x^2} = I_0 \frac{\partial^2 w}{\partial t^2} \tag{16b}$$

$$\frac{\partial M}{\partial x} - Q = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \varphi}{\partial t^2} \tag{16c}$$

Under the following boundary conditions

$$N = 0 \ or \quad u = 0 \ at \quad x = 0 \ and \quad x = L \tag{17a}$$

$$Q = 0 \ or \quad w = 0 \ at \quad x = 0 \ and \quad x = L \tag{17b}$$

$$M = 0 \ or \quad \varphi = 0 \ at \quad x = 0 \ and \quad x = L \tag{17c}$$

2.3 Nonlocal elasticity model for FG nanobeam

In nonlocal elasticity, the stress at point x is considered to be a function of the strain field at every point in the body. For a homogeneous and isotropic elastic solid the nonlocal stress-tensor components σ_{ij} at any point *x* in the body can be expressed as

$$\sigma_{ij}(x) = \int_{\Gamma} \alpha\left(|x' - x|, \tau\right) t_{ij}(x') \, d\Gamma(x') \tag{18}$$

where $t_{ij}(x')$ are the classical, macroscopic second Piola-Kirchhoff stress tensor at point x represent to the components of the linear strain tensor ε_{kl}

$$t_{ij} = C_{ijkl} \varepsilon_{kl} \tag{19}$$

In nonlocal elasticity, the stress at a point x is considered to be a function of the strain field at every point in the body. But, in the local model of elasticity, the effect of strain at other points is neglected. $\alpha(|\acute{x} - x|)$ and $|x' - x|$ are respectively nonlocal kernel and the Euclidean distance and τ is a constant given by

$$\tau = \frac{e_0 a}{l} \tag{20}$$

where e_0 indicates a material constant,which represents the ratio between a characteristic internal length, a (such as lattice parameter, C–C bond length and granular distance) and a characteristic external one, l (e.g., crack length, wavelength) trough an adjusting constant. When the local stress tensor is expressed in terms of the displacement gradients through the generalized Hooke's law, the displacements appearing on the right-hand side of Eq. (21) are assumed to be the nonlocal displacements. However, it is possible to represent the integral constitutive relations in an equivalent differential form as

$$\left(1 - (e_0 a)^2 \nabla^2\right) \sigma_{kl} = t_{kl} \tag{21}$$

where ∇^2 is the Laplacian operator. Thus, the scale length e_0a takes into account the size effect on the response of nanostructures. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\,\varepsilon_{xx} \tag{22}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G\,\gamma_{xz} \tag{23}$$

where σ and ε are the nonlocal stress and strain, respectively. E is the Young's modulus, $G = E \,/\, 2(1+\upsilon)$ is the shear modulus (where υ is the poisson's ratio). For Timoshenko nonlocal FG nanobeam, Eqs. (22) and (23) can be rewritten as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\,(z,T)\,\varepsilon_{xx} \tag{24}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G\,(z,T)\,\gamma_{xz} \tag{25}$$

Integrating Eqs. (24) and (25) over the nanobeam's cross-section area, the force-strain and the moment-strain of the nonlocal Timoshenko FG nanobeam theory can be obtained as follows

$$N - (e_0 a)^2 \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} + B_{xx} \frac{\partial \varphi}{\partial x} \tag{26}$$

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} + D_{xx} \frac{\partial \varphi}{\partial x} \tag{27}$$

$$Q - (e_0 a)^2 \frac{\partial^2 Q}{\partial x^2} = C_{xz} \left(\frac{\partial w}{\partial x} + \varphi \right) \tag{28}$$

In which the cross-sectional rigidities are defined as follows

$$(A_{xx}, B_{xx}, D_{xx}) = \int_A E\,(z)\,(1, z, z^2)\,dA \tag{29}$$

$$C_{xz} = K_S \int_A G\,(z)\,dA \tag{30}$$

where $K_S = 5/6$ is the shear correction factor. The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq. (16(a)) into Eq. (26) as follow

$$N = A_{xx} \frac{\partial u}{\partial x} + B_{xx} \frac{\partial \varphi}{\partial x} + (e_0 a)^2 \left(I_0 \frac{\partial^3 u}{\partial x\, \partial t^2} + I_1 \frac{\partial^3 \varphi}{\partial x\, \partial t^2} \right) \tag{31}$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of M from Eq. (16.c) into Eq. (27) as follows

$$M = B_{xx} \frac{\partial u}{\partial x} + D_{xx} \frac{\partial \varphi}{\partial x} + \left(e_0 a\right)^2 \left(I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 I_1 \frac{\partial^3 \varphi}{\partial x \partial t^2} + N^R \frac{\partial^2 w}{\partial x^2} \right) \tag{32}$$

By substituting for the second derivative of Q from Eq. (16(b)) into Eq. (28), the following expression for the nonlocal shear force is derived

$$Q = C_{xz} \left(\frac{\partial w}{\partial x} + \varphi \right) + \left(e_0 a\right)^2 \left(I_0 \frac{\partial^3 w}{\partial x \partial t^2} + N^R \frac{\partial^3 w}{\partial x^3} \right) \tag{33}$$

The nonlocal governing equations of Timoshenko FG nanobeam in terms of the displacement can be derived by substituting N, M and Q from Eqs. (31)-(33), respectively, into Eq. (16) as follows

$$A_{xx} \frac{\partial^2 u}{\partial x^2} + B_{xx} \frac{\partial^2 \varphi}{\partial x^2} + \left(e_0 a\right)^2 \left(I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} + I_1 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} \right) - I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^2 \varphi}{\partial t^2} = 0 \tag{34}$$

$$C_{xz} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x} \right) + \left(e_0 a\right)^2 \left(N^R \frac{\partial^4 w}{\partial x^4} + I_0 \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) - \frac{\partial}{\partial x} \left(N^R \frac{\partial w}{\partial x} \right) - I_0 \frac{\partial^2 w}{\partial t^2} = 0 \tag{35}$$

$$\begin{aligned} & B_{xx} \frac{\partial^2 u}{\partial x^2} + D_{xx} \frac{\partial^2 \varphi}{\partial x^2} - C_{xz} \left(\frac{\partial w}{\partial x} + \varphi \right) + I_2 \Omega^2 \varphi + \left(e_0 a\right)^2 \left(I_1 \frac{\partial^4 u}{\partial x^2 \partial t^2} + I_2 \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} \right) \\ & -I_1 \frac{\partial^2 u}{\partial t^2} - I_2 \frac{\partial^2 \varphi}{\partial t^2} = 0 \end{aligned} \tag{36}$$

3. Solution procedure (GDQM)

Bellman *et al*. introduced differential quadrature method (DQM) in the early 1970s (Bellman and Casti 1971, Bellman *et al*. 1972) as an efficient and accurate method. The number of grid points determines the accuracy of the weight coefficients which affects the accuracy of DQM. In the preliminary formulations of DQM, weight coefficients were calculated by an algebraic equation system which limits the number of grid points. Shu (2000) presented simple explicit formula for the weight coefficients with infinite number of grid points leading to GDQM. Early applications of GDQ were limited to regular domain problems. Shu and Richards (1992) developed a domain decomposition technique to study the multi-domain problems. According to this method, the domain of the problem is divided into a number of sub-domains or elements, before discretizing each subdomain using GDQ. For this purpose, the following solution procedure is considered. The r-th order derivative of function $f(x_i)$ can be defined as (Shu 2000)

$$\frac{\partial^r f(x)}{\partial x^r}\bigg|_{x=x_p} = \sum_{j=1}^{n} C_{ij}^{(r)} f(x_i) \tag{37}$$

From the above equation it can be understood that the main components of this method are weighting coefficients. In this method, the most important step is to find the weight coefficients. Where C_{ij} , $M(x)$ and $C^{(r)}$ are defined as

$$C_{ij}^{(1)} = \frac{M(x_i)}{(x_i - x_j)M(x_j)} \qquad i,\ j = 1,\ 2,\ ..n \text{ and } i \neq j \tag{38a}$$

$$C_{ij}^{(1)} = -\sum_{j=1, i\neq j}^{n} C_{ij}^{(1)} \qquad i = j \tag{38b}$$

$$M(x_i) = \prod_{j=1, i\neq j}^{n} (x_i - x_j) \tag{38c}$$

$$C_{ij}^{(r)} = r\left[C_{ij}^{(r-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(r-1)}}{(x_i - x_j)}\right] \qquad i,\ j = 1,\ 2,\ ..n. i \neq j \text{ , and } \quad 2 \leq r \leq n-1 \tag{39a}$$

$$C_{ii}^{(r)} = -\sum_{j=1, i\neq j}^{n} C_{ij}^{(r)} \qquad i,\ j = 1,\ 2,\ ..n \text{ and } \quad 1 \leq r \leq n-1 \tag{39b}$$

where n is the number of grid points along x direction and superscript r is the order of the derivative and $C^{(r)}$ is the weighing coefficient along x direction. In order to increase convergence speed, Chebyshev-Gauss-Lobatto technique has been defined as follows

$$\zeta_i = \frac{1}{2}\left(1 - \cos\left(\frac{(i-1)}{(N-1)}\pi\right)\right) \qquad i = 1, 2, 3, \ldots, n \qquad i = 1\ ,\ 2\ ,\ 3\ ,n. \tag{40}$$

Finally, by using Eqs. (34)-(36) with boundary conditions and using Eigenvalue equation in the form of Eq. (41) the overall problem will be solved.

$$\begin{bmatrix} [K_{ww}] & [K_{w\varphi}] \\ [K_{\varphi w}] & [K_{\varphi\varphi}] \end{bmatrix} \begin{Bmatrix} \{w_i\} \\ \{\varphi_i\} \end{Bmatrix} = \Psi^4 \begin{bmatrix} [M_{ww}] & 0 \\ 0 & [M_{\varphi\varphi}] \end{bmatrix} \begin{Bmatrix} \{w_i\} \\ \{\varphi_i\} \end{Bmatrix} \tag{41}$$

4. Result and discussions

In this section, numerical results are presented for vibration behaviour of cantilever nanobeams with investigating angular velocity, small-scale parameter and power law index of functionally graded nanobeam composition effects. The Timoshenko beam model based on Eringen's nonlocal theory is utilized. Validations have been done by modifying utilized DQM and comparing with available literatures. The beam geometry has the following dimensions: *L* (length) = 10 nm, *b* (width) =2 nm and h (thickness) = 1nm. By considering $W = we^{i\omega t}$, relations described in Eqs. (42)-(44) are presented in the form of dimensionless parameters in order to have better judgment on results

$$\Psi = \sqrt{\omega^2 L^4 \frac{m_{10}}{D_{11}}} \tag{42}$$

$$\Phi^2 = \Omega^2 L^4 \frac{m_{10}}{D_{11}} \tag{43}$$

$$\mu = \frac{e_0 a}{L} \tag{44}$$

where $m_{10} = \int_A \rho_c dA$ *and* $D_{11} = \int_A E_c z^2 dA$ and μ is nondimensional nonlocal parameter. To verify the accuracy of results, nondimensional frequency of non-rotating cantilever nanobeams are compared with results obtained by Wang *et al*. (2007) in Table 1.

The verification of the results are shown in Table 2, where the first two frequencies of the Timoshenko nanobeam are compared with the results of Dehrouyeh-Semnani A (2015) and Shafiei *et al.* (2015, 2016) for a different range of angular velocities. As it can be seen, the results of the presented DQM have excellent agreement with the results of other papers. Finally, the results are presented for the parametric study considering different effects such as small scale, power law index of functionally graded nanobeam composition and nondimensional angular velocity. The FG nanobeam is composed of ceramic and steel and their properties are given in Table 3 (at $\mathrm{T} = 300$ k).

4.1 Cantilever FG-nanobeam

4.1.1 Fundamental frequency

Fig.2 shows the nondimensional fundamental frequency variation of cantilever nanobeam with respect to gradient index (n) for various angular velocities and nonlocal effects. It's observed that increasing n, means that the nanobeam composition contains more metal. It is seen that by increasing power law index of functionally graded nanobeam material distribution (n), nondimensional frequency for each small-scale and angular velocity decreases. Increasing the metallic volume fraction which reduces the total stiffness of the nanobeam is happened with gradient index. In both cases, the decreaseing the fundamental frequency is according to degradation of stiffness of the nanobeam. Also, in this boundary condition (cantilever) of the FG nanobeam, unlike other types of boundary conditions, it can be seen that the fundamental frequency increases with nonlocal parameter which is clearly shown in Fig.3. By comparing sections (a), (b), (c) and (d) from Fig. 2, it is observed that fundamental frequency increases with angular velocity, also, it is more clearly shown in Fig. 4.

Fundamental frequencies obtained for various small-scale values of the nanobeam have been plotted in Fig. 3. It can be observed that higher frequency is obtained at higher angular velocities. Furthermore it can be observed that as nonlocal parameter increases, frequency also increases. By comparing sections (a), (b), (c) and (d) from Fig. 3, it is observed that fundamental frequency decreases with gradient index.. In order to study the effect of angular velocity on behavior of FG nanobeam, the backbone curves of a cantilever nanobeam are depicted in Fig. 4 for various values of nonlocal and gradient index parameters. It can be seen that the angular velocity has a considerable effect on the bending vibration behavior of FG nanobeam. The fundamental frequency increases with angular velocity.

Table 1 Comparison of results for nondimensional frequency, Ψ of cantilever nanobeam

Nonlocal parameter	$\Phi=0, \Psi_1$		$\Phi=0, \Psi_2$	
(μ)	Present	(Wang *et al*. 2007)	Present	(Wang *et al*. 2007)
0	1.86102	1.8610	4.47341	4.4733
0.1	1.86509	1.8650	4.35059	4.3506
0.3	1.89999	1.8999	3.65938	3.6594
0.5	2.00239	2.0024	2.89025	2.8903

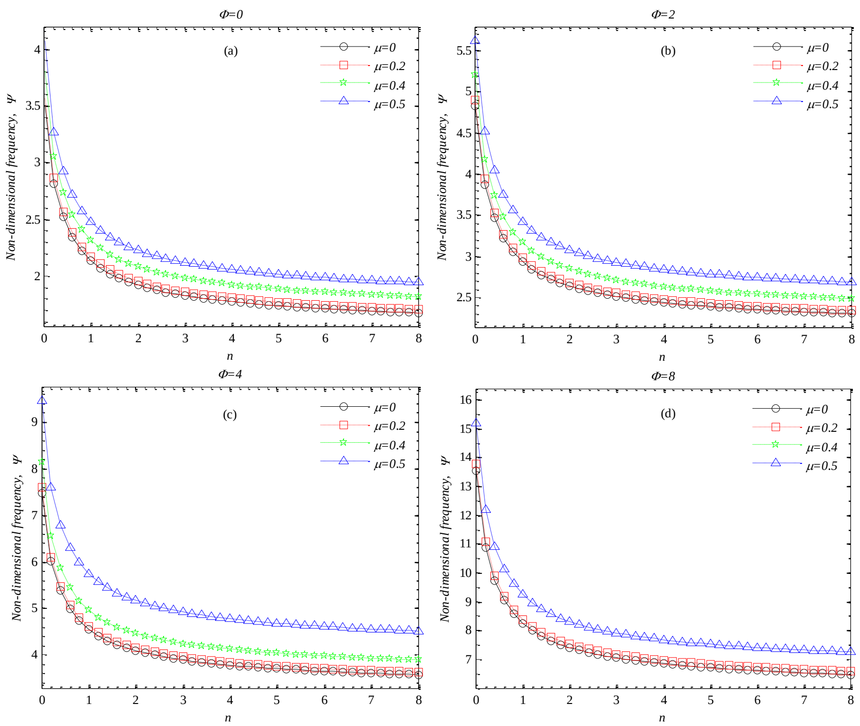

Fig. 2 Fundamental nondimensional frequency variation of cantilever nanobeam with respect to FG indexes nondimensional angular velocity for different nondimensional nonlocal parameter

Table 2 Comparison of results for nondimensional frequency, Ψ of rotating cantilever beam

Φ	Fundamental Frequency: Present	Fundamental Frequency: Dehrouyeh-Semnani (2015) and Shafiei *et al.*(2015, 2016)	Second Frequency: Present	Second Frequency: Dehrouyeh-Semnani (2015) and Shafiei *et al.*(2015, 2016)
0	**3.516024127**	**3.516**	**22.03437512**	**22.035**
1	**3.681657949**	**3.6816**	**22.18089419**	**22.181**
2	**4.137299432**	**4.1373**	**22.61480281**	**22.615**
3	**4.797278856**	**4.7973**	**23.32014566**	**23.32**
4	**5.584996142**	**5.585**	**24.27323227**	**24.273**
5	**6.449523977**	**6.4495**	**25.44596417**	**25.446**
6	**7.360344878**	**7.3604**	**26.80896705**	**26.809**
7	**8.299605386**	**8.2996**	**28.33396811**	**28.334**
8	**9.256792429**	**9.2568**	**29.99526949**	**29.995**
9	**10.22567529**	**10.226**	**31.77038743**	**31.771**
10	**11.20232093**	**11.202**	**33.64023661**	**33.64**

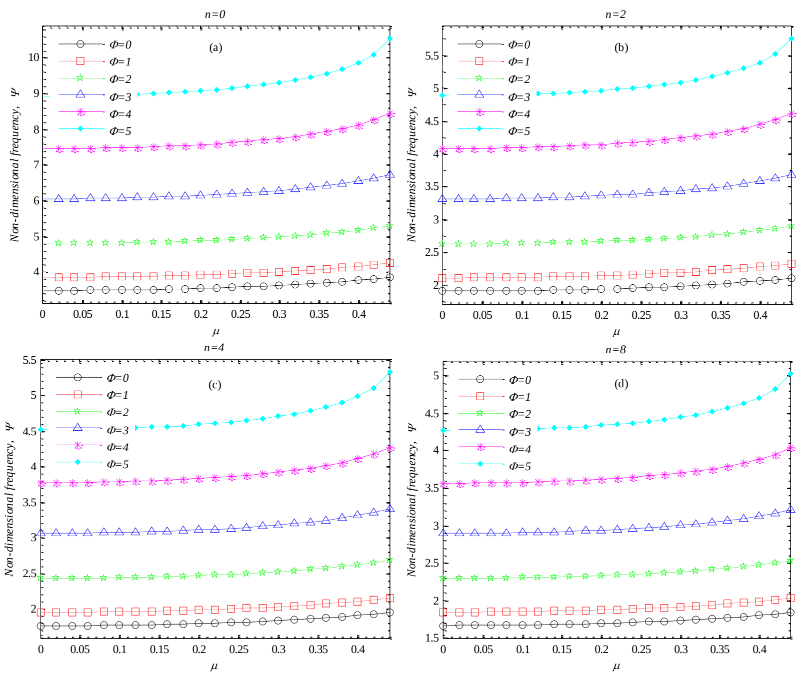

Fig. 3 Fundamental nondimensional frequency variation of cantilever nanobeam with respect to FG indexes and nonlocal parameter for different nondimensional angular velocity

Table 3 Properties of utilized materials (at T=300(k))

Properties material	Young's modulus $E(Gpa)$	Density ρ (kg/m^3)	Poisson's ratio υ
Si3N4	322.27	2370	0.24
SUS304	207.79	8166	0.3178

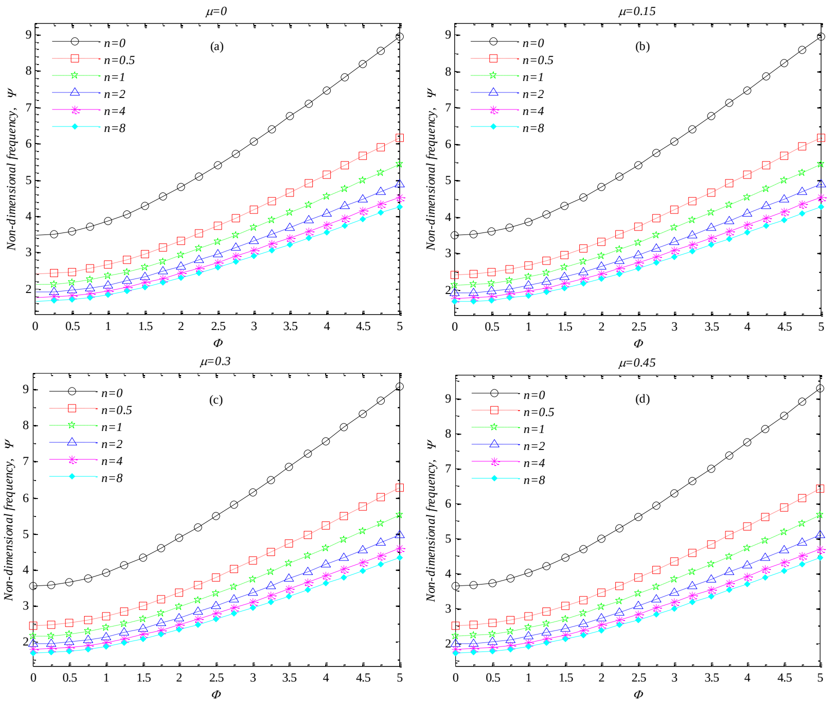

Fig. 4 Fundamental nondimensional frequency variation of cantilever nanobeam with respect to nondimensional velocity and nonlocal parameter for different nondimensional FG indexes

4.1.2 Second and third frequency

Variations of the dimensionless second and third frequencies of the cantilever FG nanobeam with respect to angular velocity, gradient indexes (n) and nonlocal parameters are depicted in Figs. 5-7 and Figs. 8-10, respectively. It's observed that the natural frequencies increase with Φ. Unlike fundamental frequency, if nonlocal parameter increases, the second and third frequency decrease. Also, it can be stated that angular velocity has a significant effect on the dimensionless natural frequencies, especially for lower mode numbers. On the other hand, it is revealed that the frequencies decrease with an increase in material gradient index due to decreasing in total stiffness of the nanobeam.

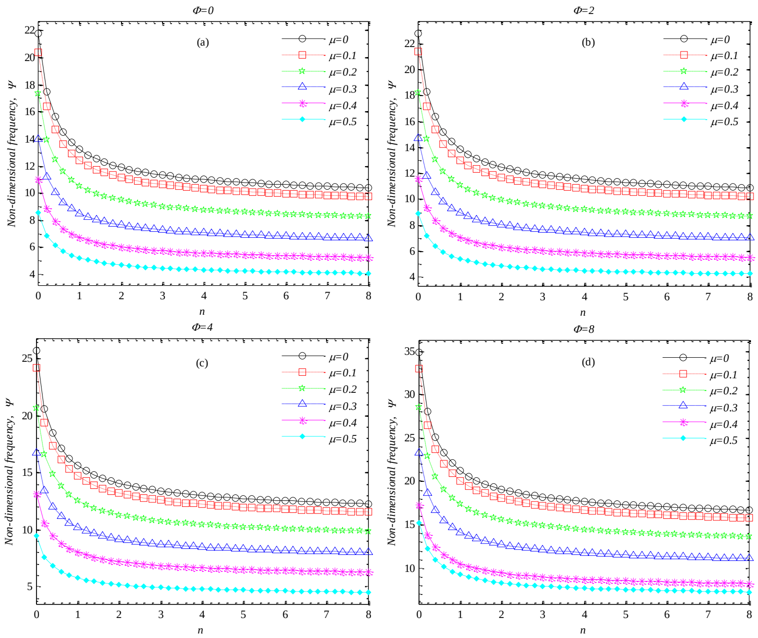

Fig. 5 second nondimensional frequency variation of cantilever nanobeam with respect to FG indexes nondimensional angular velocity for different nondimensional nonlocal parameter

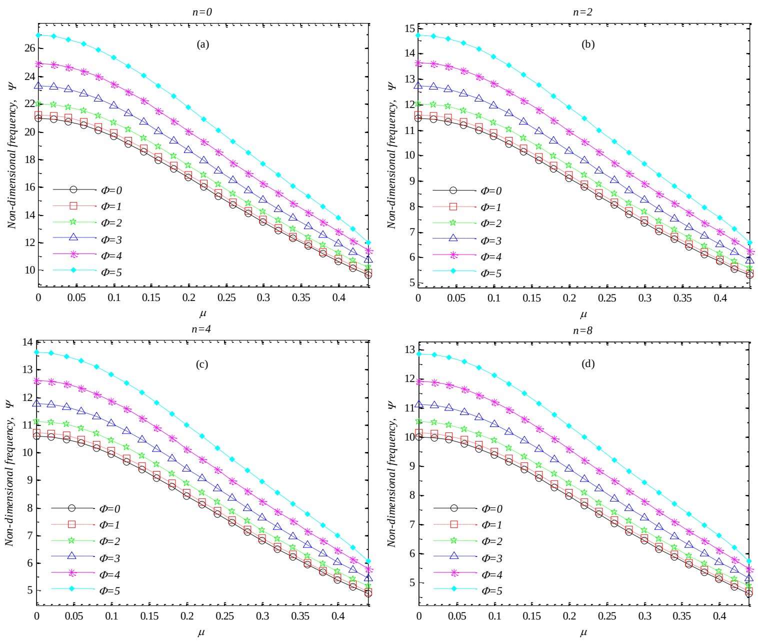

Fig. 6 Second nondimensional frequency variation of cantilever nanobeam with respect to FG indexes and nonlocal parameter for different nondimensional angular velocity

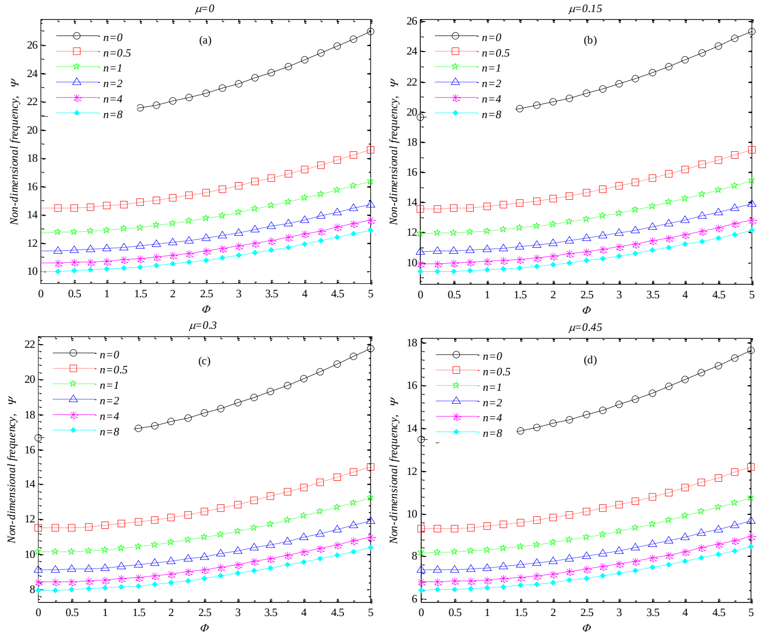

Fig. 7 Second nondimensional frequency variation of cantilever nanobeam with respect to nondimensional velocity and nonlocal parameter for different nondimensional FG indexes

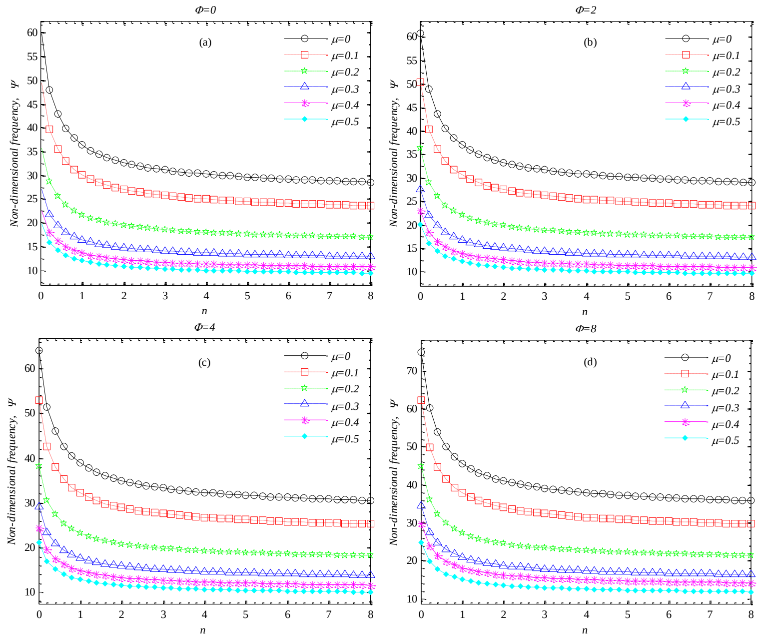

Fig. 8 Third nondimensional frequency variation of cantilever nanobeam with respect to FG indexes nondimensional angular velocity for different nondimensional nonlocal parameter

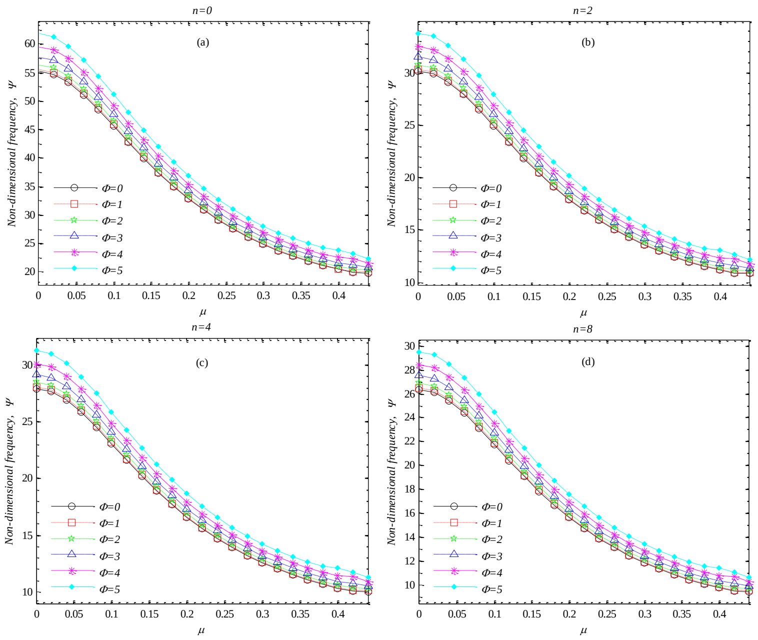

Fig. 9 Third nondimensional frequency variation of cantilever nanobeam with respect to FG indexes and nonlocal parameter for different nondimensional angular velocity

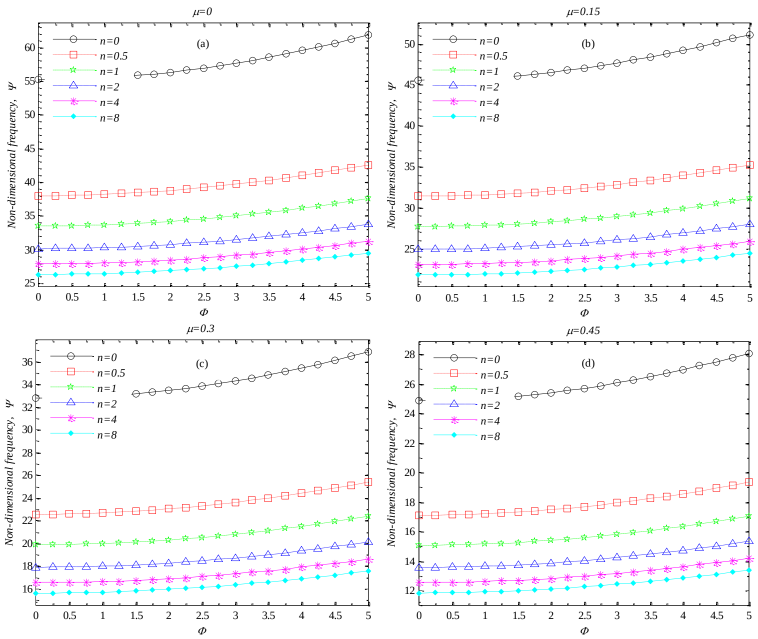

Fig. 10 Third nondimensional frequency variation of cantilever nanobeam with respect to nondimensional velocity and nonlocal parameter for different nondimensional FG indexes

Table 4 shows the effect of K_s on the fundamental and second frequencies of FG nanobeams for different values of L/h. It is shown that increasing the L/h decreases the effect of K_s and increasing the nonlocal parameter increases the effect of K_s on fundamental and second frequencies.

Table 4 Fundamental and second nondimensional frequency variation of cantilever nanobeam with respect to shear deformation factor with nondimensional nonlocal parameter for different nondimensional angular velocity, n=1, Φ=0

		Fundamental frequency						Second frequency					
		K_S=1/12	1/6	2/6	4/6	5/6	6/6	K_S=1/12	1/6	2/6	4/6	5/6	6/6
L/h=5	bμ=0	2.063713	2.25273	2.367054	2.430434	2.443665	2.452593	7.699379	9.575017	11.37308	12.79475	13.14831	13.40022
	bμ=0.2	2.0585	2.265436	2.392698	2.46394	2.478874	2.488964	5.996634	7.539379	9.016156	10.17667	10.46396	10.66833
	bμ=0.4	2.052093	2.317991	2.490722	2.590669	2.611925	2.626346	3.973277	4.970463	5.892976	6.597408	6.769458	6.891369
L/h=10	bμ=0	2.378773	2.443852	2.478267	2.495975	2.499558	2.501954	11.62919	13.1711	14.22132	14.84849	14.98379	15.07597
	bμ=0.2	2.405639	2.47899	2.518003	2.538138	2.542217	2.544946	9.179817	10.44362	11.30478	11.81957	11.93072	12.00648
	bμ=0.4	2.507766	2.611606	2.667976	2.697387	2.703372	2.707382	5.983967	6.750467	7.263353	7.56696	7.632256	7.67671
L/h=20	bμ=0	2.378773	2.443852	2.478267	2.495975	2.499558	2.501954	11.62919	13.1711	14.22132	14.84849	14.98379	15.07597
	bμ=0.2	2.405639	2.47899	2.518003	2.538138	2.542217	2.544946	9.179817	10.44362	11.30478	11.81957	11.93072	12.00648
	bμ=0.4	2.507766	2.611606	2.667976	2.697387	2.703372	2.707382	5.983967	6.750467	7.263353	7.56696	7.632256	7.67671
L/h=40	bμ=0	2.509745	2.514314	2.516607	2.517755	2.517985	2.518138	15.38637	15.57648	15.67411	15.72359	15.73354	15.74018
	bμ=0.2	2.553822	2.559035	2.561653	2.562964	2.563226	2.563402	12.25987	12.41765	12.49876	12.53988	12.54816	12.55368
	bμ=0.4	2.720437	2.72815	2.732028	2.733973	2.734362	2.734622	7.824804	7.917091	7.964469	7.988478	7.993305	7.996529
L/h=100	bμ=0	2.517706	2.518442	2.51881	2.518995	2.519031	2.519056	15.72146	15.75337	15.7694	15.77743	15.77904	15.78011
	bμ=0.2	2.562908	2.563749	2.564169	2.564379	2.564421	2.564449	12.53809	12.56464	12.57797	12.58466	12.58599	12.58688
	bμ=0.4	2.73389	2.735137	2.735761	2.736073	2.736135	2.736177	7.987431	8.002923	8.010703	8.014601	8.015381	8.015901

5. Conclusions

In this article, bending mechanical vibrational behavior of the rotating FG nanobeams based on Timoshenko beam theory and Eringen's nonlocal elasticity theory constitutive equations is investigated. The Hamilton's principle was used to obtain the governing equations and the related boundary conditions. Then generalized differential quadrature (GDQ) method was applied to discretize the governing differential equations corresponding to clamped–free boundary conditions. In this article, the influence of the several parameters such as angular displacement, FG gradient index and nonlocal parameter on the free vibration of rotating nanobeam was examined.

The following main results could be highlighted from this paper:

1. It should be noted that except fundamental frequency of cantilever FG nanobeam, nondimensional frequencies decrease with nonlocal parameter.

2. The non-dimensional frequency is decreased, while the nonlocal parameter increases. The reason is that the presence of the nonlocal parameter tends to decrease the stiffness of the nanobeam and finally decreases the values of non-dimensional frequency. But, in the first mode of the rotating cantilever nanobeam, the nonlocal parameter behavior is different and, with the increase in nonlocal parameter, non-dimensional fundamental frequency increases.
3. It is illustrated that the FG nanobeam model yields smaller natural frequency than the classical (local) beam model. Therefore, by increasing FG index, all frequencies decrease.

We found that, the nonlocal parameter, rotary effect and the FG gradient index have significant roles and in any studies on nanobeams, they should be examined. As well, because of the lack of modality in cantilever rotating nanobeam, fundamental frequency behaves differently in some cases. Presented herein will be helpful for understanding the vibration features of nanobeams and can be useful for engineers who are designing nanoelectromechanical, nanosensors, nanoactuators and in which nanobeams act as basic elements.

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