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A novel method to specify pattern recognition of actuators for stress reduction based on Particle swarm optimization method

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Abstract. This paper is focused on stiffness ratio effect and a new method to specify the best pattern of piezoelectric patches placement around a hole in a plate under tension to reduce the stress concentration factor. To investigate the stiffness ratio effect, some different values greater and less than unity are considered. Then a python code is developed by using particle swarm optimization algorithm to specify the best locations of piezoelectric actuators around the hole for each stiffness ratio. The results show that, there is a line called "reference line" for each plate with a hole under tension, which can guide the location of actuator patches in plate to have the maximum stress concentration reduction. The reference line also specifies that actuators should be located horizontally or vertically. This reference line is located at an angle of about 65 degrees from the stress line in plate. Finally two experimental tests for two different locations of the patches with various voltages are carried out for validation of the results.

Keywords: reference line; pattern recognition; piezoelectric actuator; stress concentration; plate under tension

1. Introduction

The best positioning of piezoelectric patches as sensors and actuators on smart structures has been a point of interest in recent years. So, many researchers used optimization algorithms to optimize and find the best position of piezoelectric patches in real smart structures application. Some reviews on optimization of smart structures and optimal placement of the piezoelectric sensors and actuators are available in previous publications (Amezquita-Sanchez *et al*. 2012, Frecker 2003, Gupta *et al*. 2010). Adali *et al.* (2006) worked on control the deflection of L-shaped frames under various loads with minimizing the deflection of the frame tip by considering the actuator locations. To control the vibration of a flexible fin as a smart structure, Mehrabian and Yousefi-Koma (2011) investigated the optimal placement of sensors and actuators on a fin. Also Rader et al. (2007) worked on vibration control of fins by optimizing the configuration of piezoelectric actuators using genetic algorithm. To control the shape of a structure, Mukherjee and Joshi (2002) developed an iterative technique for determination of the optimal location of the piezoelectric patches. Also, Chee *et al*. (2002) and Lin and Nien (2007) worked on shape control

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of a plate with piezoelectric actuators using finite element method. They modeled the actuators as additional plies completely embedded into the host plate and tried to find the best position of actuators on the host plate. Kang *et al*. (2008, 2012) developed topology optimization for shape control of a plate with piezoelectric actuators. They used the error between the desired and actuated shape as the fitness function and tried to find the best position and induced voltage. Silva *et al*. (2004) investigated on the shape control of plate bounded with piezoelectric actuators by using genetic algorithm. They studied numerically and experimentally the best value for induced voltages to piezoelectric actuators so that a predefine plate shape is achieved. To control the shape of various smart composite plates with piezoelectric actuators, a new evolutionary algorithm was developed by Nguyen *et al*. (2004, 2007a, 2007b). They used error function as fitness function and employed the linear least square method to optimize the shape of the active piezoelectric actuators. The optimum voltage for static shape control of smart structures with nonlinear piezoelectric actuators is studied by Sun *et al.* (2004, 2005). They worked on shape control of plate and presented some methods to achieve a pattern for piezoelectric actuators. Using finite element method, Luo and Tong (2006) worked on high precision control for a twisting and bending composite plate. They used linear least square method for electric potential to achieve the desired shapes for a plate. Nakasone and Silva (2010) worked on topology optimization of piezoelectric laminated sensors and actuators. Using finite element method, Xu and Koko (2004) investigated on control the smart structures by piezoelectric patches. Liew *et al*. (2004) worked on dynamic analysis of laminated composite plates with piezoelectric patches using first-order shear deformation plate theory. Using simulated annealing method, Correia *et al*. (2003) worked on a composite laminated plate and studied the optimization of the location of piezoelectric patches to increase the buckling loads. Using shape memory alloys and piezoelectric patches as a device for applying induced strains, Sensharma *et al*. (1993, 1996) investigated on reduction the stress concentration factor around a hole in a plate to control the stress field. Stress concentration reduction around a hole in a plate under tension with piezoelectric patches was carried out by Shah *et al*. (1994). They demonstrated that the piezoelectric layers can embedded on tension or compression area to reduce the stress concentration factor. The optimum pattern for patches placement around the hole in a plate under tension was presented by Jafari Fesharaki and Golabi (2014). They used particle swarm optimization algorithm to show the best location for piezoelectric actuators around the hole.

In this paper, the determination of the best pattern for locating the piezoelectric patches to reduce the stress concentration factor in a plate with a hole under tension is investigated. Then a novel method to specify the pattern recognition for placement of the actuators around the hole is presented using particle swarm optimization (PSO) algorithm. The method proposed in this paper can be used for locating the piezoelectric patches around the hole in their best placements for real applications. The presented results are validated with some experimental tests.

2. Mathematical formulation

The first order shear deformation theory is used to model the mechanical behavior of a thin plate under tension. Based on this theory, the displacements components *u, v* and *w* at any point of the elements of a plate are considered as follows (Reddy 1997)

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\theta_x(x,y)\\
v(x,y,z) &= v_0(x,y) + z\theta_y(x,y)\\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{1}
$$

Where, u_0 , v_0 and w_0 are the components of mid-plane displacements and θ_x and θ_y are the rotations of the normal to the mid-plane about the x and y direction respectively.

The coordinates and displacements inside the element using isoparametric relations, are considered as

$$
u = \sum_{i=1}^{n} N_i \delta_i, \qquad x = \sum_{i=1}^{n} N_i x_i, \qquad y = \sum_{i=1}^{n} N_i y_i \tag{2}
$$

Where, *"n"* is the number of nodes and N_i is the element shape functions.

The piezoelectric constitutive equations with the electric and elastic fields coupling can be assumed as (Ghandhi 1992)

$$
\sigma = Q\varepsilon - dE, \qquad D = e^T \varepsilon + pE \tag{3}
$$

Where $\sigma, \varepsilon, D, Q, d$ and p are the stress, strain, electric displacement vector, elastic constant matrix, the matrix of piezoelectric stress coefficient and dielectric matrix respectively. The electric field vector *"E"* is defined as (Ghandhi 1992)

$$
E = -\nabla \Phi \tag{4}
$$

Where, Φ is the applied electric voltage across the thickness of the piezoelectric actuators.

The Hamilton's principle is considered to derive the equation of motion for a plate containing the piezoelectric actuators. For electromechanical coupled systems, the principle is

$$
\int_{a_1}^{a_2} \delta (T - U + W_{ext}) da = 0 \tag{5}
$$

Where a_1 and a_2 are arbitrary constants. *"U", "T"* and "W_{ext} "are the potential energy, kinetic energy and the work done by external forces respectively and are defined as (Reddy 1997)

$$
\begin{aligned}
T &= \int_V \frac{1}{2} \rho \{\dot{k}\}^T \{\dot{k}\} dV\\
U &= \int_V \frac{1}{2} \left[\{\varepsilon\}^T \{\sigma\} - \{E\}^T \{D\} \right] dV\\
W_{ext} &= \sum_{i=1}^{n_f} \{k\}^T \{F_c\}
\end{aligned}
\tag{6}
$$

Where $\dot{k}, \rho, F_c$ and n_f are the vector of velocity, mass density, external force vector, number of applied forces respectively and *"V"* is the volume of the structure.

3. Particle Swarm Optimization (PSO) algorithm

Particle swarm optimization (PSO) is an evolutionary computation method based on the behavior of a colony that proposed by Kennedy and Eberhart (1995). In this algorithm, each particle has three characteristic: particle position, particle velocity and fitness function for each particle.

The initial position of each particle is located randomly and the initial velocity of each particle has zero value and during the implementation of the PSO optimization algorithm, each particle wanders in the design space and remembers the best previous position. All particles communicate their information to the other particles and affect the position and velocity of them. The main steps for implementation of the PSO algorithm are considered as (Rao 2009)

1. Assume the number of particle "N".

2. Consider the initial position for each particle "X" in the upper and lower range randomly as "$X_1, X_2, ..., X_n$". Hereafter, the location of *"j"*th particle and its velocity in iteration "i" are specified as $X_j^{(i)}$ and $V_j^{(i)}$, respectively. Thus the initial position of the particles is specified as $X_1^{(0)}, X_2^{(0)}, ..., X_N^{(0)}$.

3. Evaluate the values of objective function corresponding to the particles as $f(X_1^{(0)}), f(X_2^{(0)}), ..., f(X_N^{(0)})$.

4. set the iteration number as $i=1$ and find the velocity of all particles. The initial velocity of all particles is assumed to be zero.

5. Find the historical best value of particle $X_j^{(i)}$ as $P_{Best,j}$ in the i th iteration, with the highest value of the fitness function $f(X_j^{(i)})$, encountered by particle j in all the previous iterations.

6. Find the historical best value of $X_j^{(i)}$ as G_{Best}, in the i th iteration with the highest value of the objective function $f(X_j^{(i)})$, encountered in all the previous iterations by any of the N particles.

7. Find the velocity of particle j in the i th iteration as follows

$$V_j^{(i)} = V_j^{(i-1)} + c_1 r_1 \left[P_{Best,j} - X_j^{(i-1)}\right] + c_2 r_2 \left[G_{Best} - X_j^{(i-1)}\right];\ j = 1,2,...,N \tag{7}$$

Where, c_1 and c_2 are the individual and group learning rates respectively and are usually assumed to be 2. r_1 and r_2 are selected randomly in the range 0 and 1.

8. Find the position of the j th particle in i th iteration as

$$X_j^{(i)} = X_j^{(i-1)} + V_j^{(i)}\ ;\ j = 1,2,...,N \tag{8}$$

9. Find the objective function values due to each particle as $f(X_1^{(i)}), f(X_2^{(i)}), ..., f(X_N^{(i)})$

10. The method is assumed to converge if the positions of all particles converge to the same set of values. If the convergence criterion is not satisfied, the iteration number set as $i = i+1$ and step 5 is repeated by updating and computing the new values of $P_{Best,j}$ and G_{Best}.

4. Problem definition

A thin rectangular plate with 200 mm length, 100 mm width which has a central hole with 20mm diameter subjected to $1MPa$ tension is considered.

Since the maximum stress concentration factor occurs at top/bottom of the hole, it is clear that locating piezoelectric patches near the hole reduces the maximum stress concentration more effectively. Therefore, a 5×5 mm grid mesh near the hole is assumed for locating the piezoelectric patches. The geometrical definition of the problem and the grid mesh for locating the patches on the host plate are shown in Fig. 1. In order to consider a relationship between the applied electric field E_0 to the patches and applied mechanical load to the host plate, the electric field was assumed such that it would induce a stress $S = 1MPa$ on an infinite piezoelectric plate under plane stress condition (Shah *et al.* 1993). The voltage can induce positive or negative strains on the host plate.

To investigate the best pattern for locating piezoelectric patches around the hole in a plate under tension, some stiffness ratios (Rs) are considered. The stiffness ratio is described for piezoelectric and plate, to show that the stress is divided between the plate and the patches simultaneously. However, the considered Poisson's ratio is constant for plate and piezoelectric patches.

The ratio of piezoelectric patches' stiffness to the host plate's stiffness is nominated "stiffness ratio" or "Rs" in this paper and is assumed as

$$R_s = \frac{\left(E/t \right)_{Piezo}}{\left(E/t \right)_{Plate}} \tag{9}$$

Four stiffness ratios are considered greater and four stiffness ratios are considered smaller than unity. The values of stiffness ratio are assumed as

$$R_s = 10, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/10 \tag{10}$$

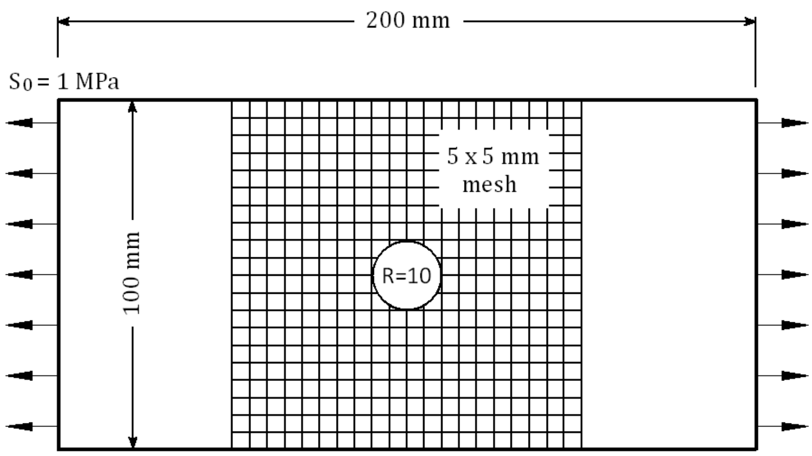

Fig. 1 The geometry of plate

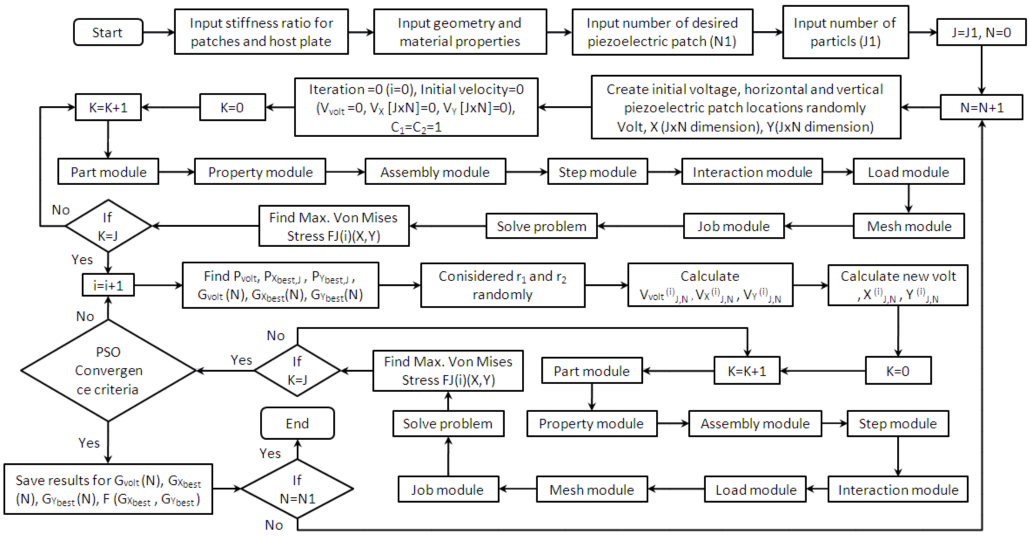

Fig. 2 Flowchart of PSO for consideration problem

To simulate the problem and used particle swarm optimization algorithm, a python code is developed for Abaqus software. In developed code, for each stiffness ratio, the PSO algorithm is used to find the best position, voltage and location of piezoelectric patches around the hole. Hence, the percentage of occupied surfaces by piezoelectric patches is given to the code firstly. Then, according to the mesh area, the particle swarm algorithm finds the best voltage and location of the patches around the hole. The flowcharts of implementation the PSO algorithm and optimization procedure are shown in Fig. 2. To find the value of fitness function for each particle, the finite element analysis is done in each step and the value of maximum von misses stress in host plate and the piezoelectric patches are returned to the PSO algorithm as the value of objective function. When the program converges, the best voltage and locations of the patches in mesh area are specified as output data.

5. Results and discussion

By implementation the python code with each stiffness ratio (Rs), patterns for optimum placement of piezoelectric patches around the hole are achieved. Figs. 3 to 11 show these patterns for various stiffness ratios.

From Figs. 3 to 7, it can be seen that when stiffness ratios is considered $R_s = 1, 2, 3, 4, 10$ the best locations of patches are began from the left/right of the hole. Then the locations are expanding along the length and width of the plate around the hole. But for stiffness ratios near the $R_s = 1$, the growth patches in width direction are more than that in length direction while for stiffness ratio near the $R_s = 10$, the growth of patches in the length direction of plate are more than that in the width direction. Therefore, in real application, the best location of the patches around the hole is different for considering any stiffness ratio of patches and plate.

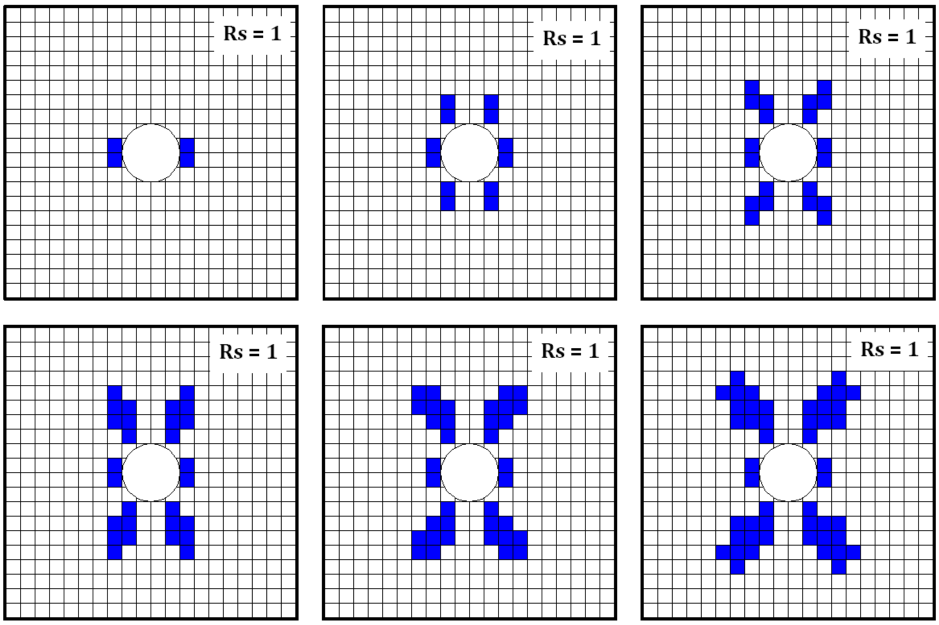

Fig. 3 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=1

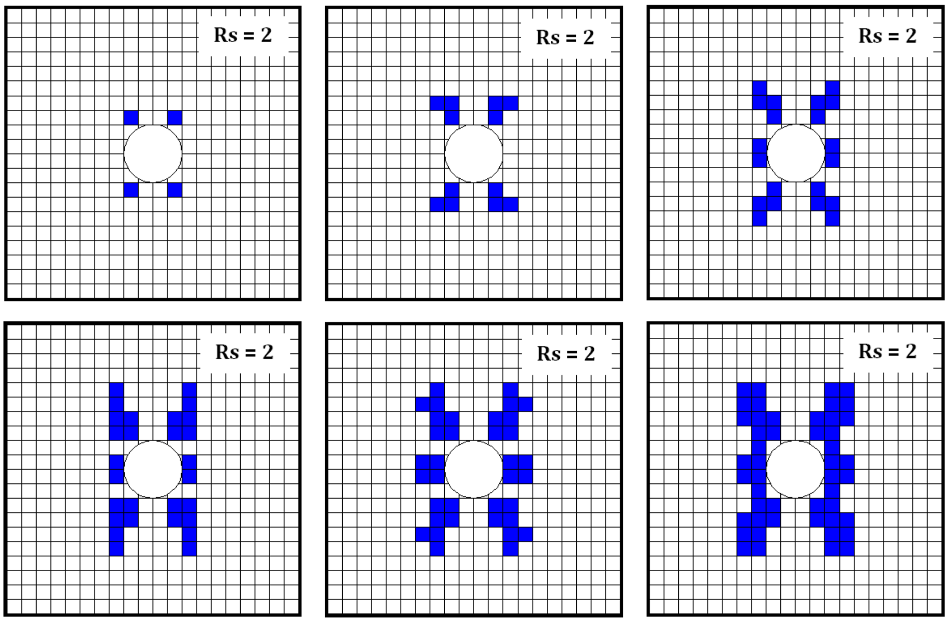

Fig. 4 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=2

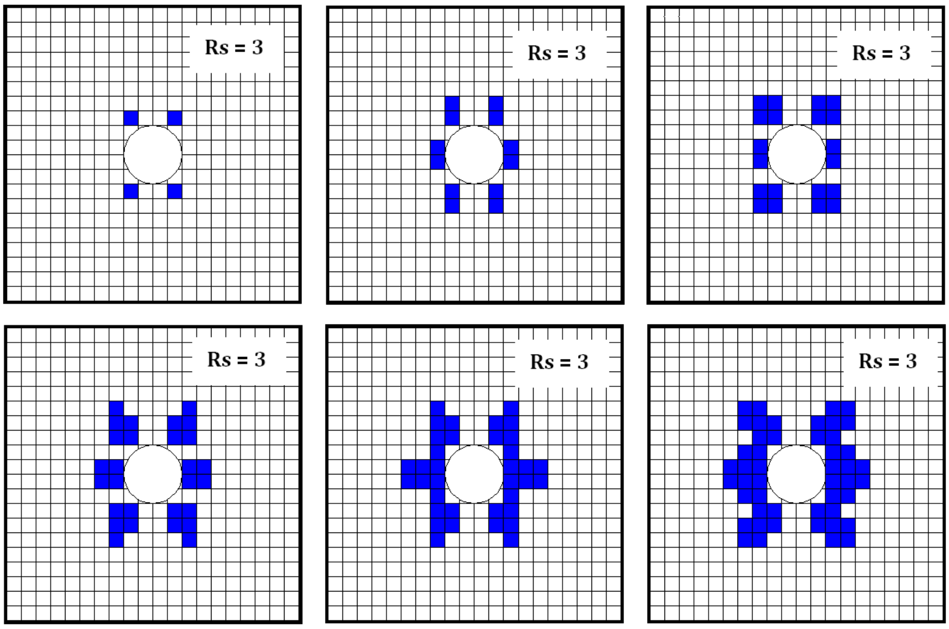

Fig. 5 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=3

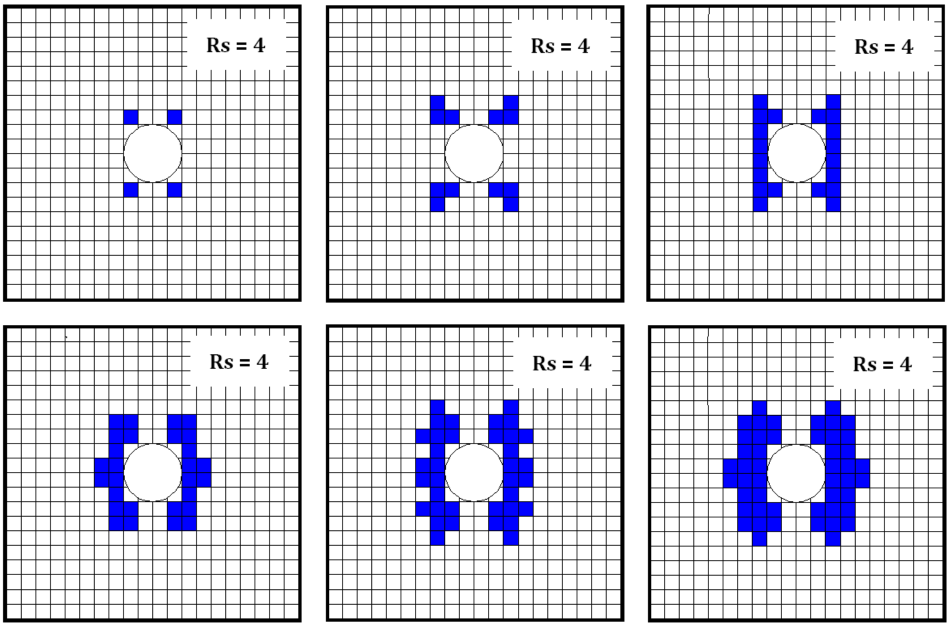

Fig. 6 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=4

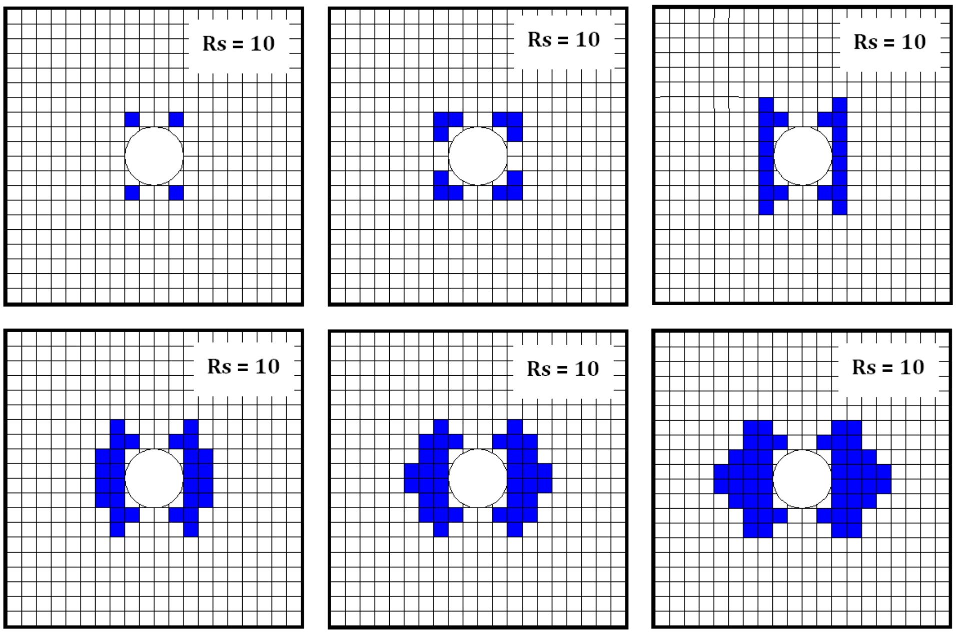

Fig. 7 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=10

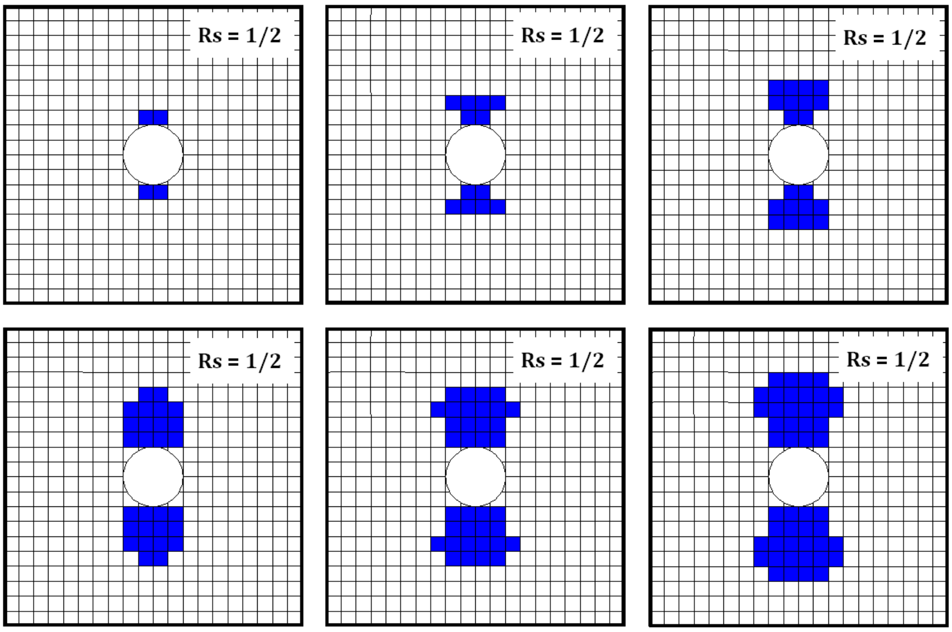

Fig. 8 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=1/2

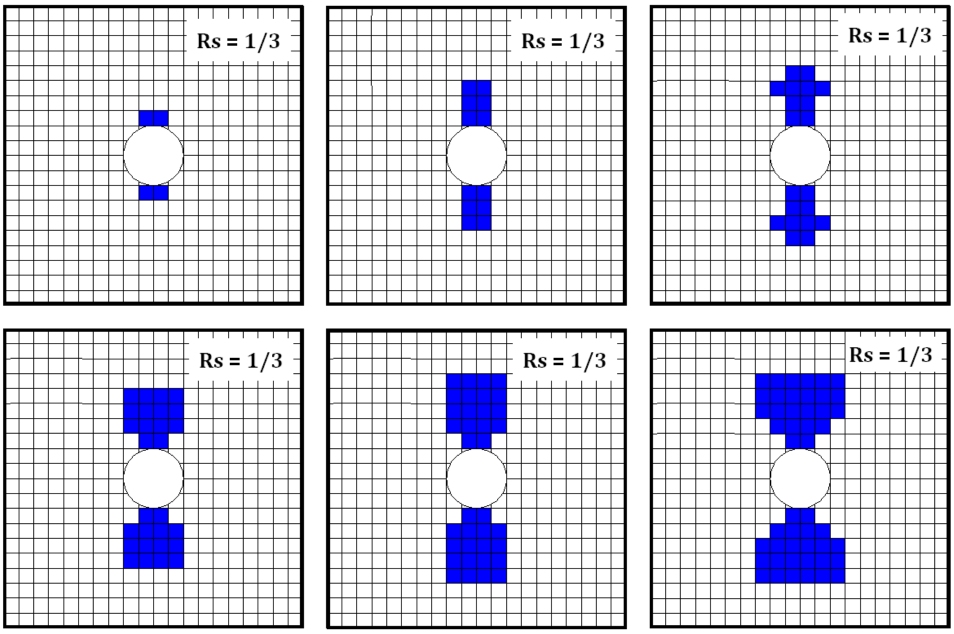

Fig. 9 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=1/3

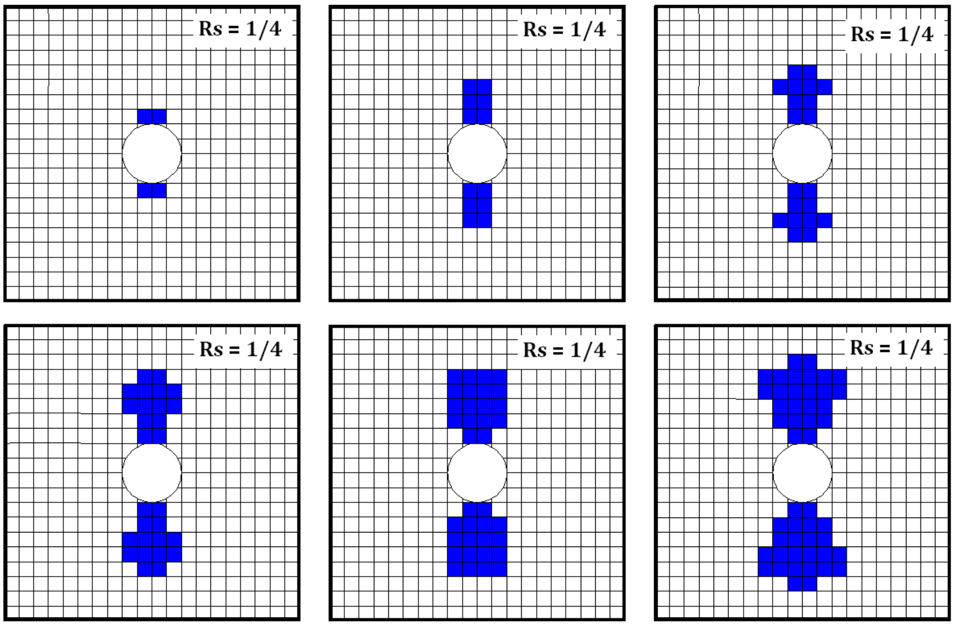

Fig. 10 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=1/4

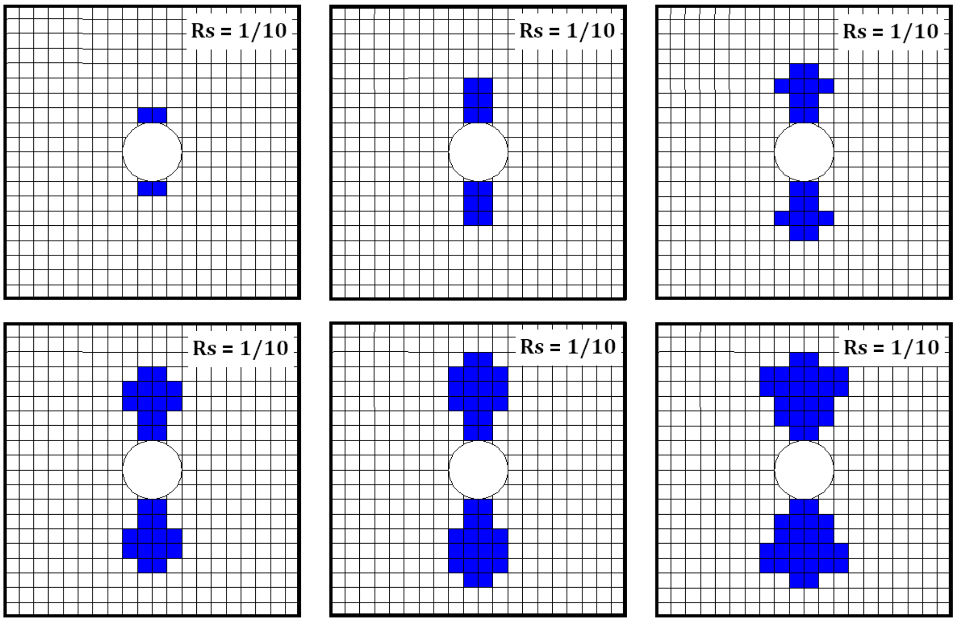

Fig. 11 Optimum pattern of piezoelectric patches around the hole stiffness ratio Rs=1/10

Figs. 8 to 11 show the best pattern and location of the patches around the hole for the stiffness ratios $R_s = 1/2, 1/3, 1/4, 1/10$. It can be observed from these figures that for stiffness ratios greater than unity, the location of the patches are began from top/bottom of the hole and expand vertically along the width of the plate. Furthermore, it can be seen from these figures that the stiffness ratios greater than unity have more effect on best location and pattern of the piezoelectric actuators around the hole.

To show the effect of voltage and stiffness ratio of the patches and the plate on stress reduction around the hole, Figs. 12 and 13 are considered for stiffness ratios grater and less then unity respectively. It can be seen that the voltage can reduce the stress around the hole. However, by adequately increasing the voltage, the location of maximum stress around the hole changed from top of the hole to another point around the hole. Therefore, there is a limitation for increasing the voltages. The new location for maximum stress concentration was about 65 degree from the longitudinal line in the plate.

To compare the capability of various stiffness ratios in stress reduction around the hole, Fig. 14 is presented. It can be seen that the stiffness ratios near and less than unity can reduce the stress more effectively and the stiffness ratio $R_s = 1/2$ can reduce the stress about 57% in the host plate. Moreover, it can be observed that by increasing the area of patches, the stress reduction in plate is increased.

By considering Figs. 15 and 16 for locating the patches around the hole for stiffness ratio greater and less that unity respectively, the main procedure for locating the patches around the hole is specified. It can be seen that the best location and pattern of piezoelectric patches are limited between the lines with about 65 degrees as mentioned above. This line that is called "reference line" by authors can specify the location of the patches around the hole. It can be seen that all of the patches for every stiffness ratios are located between this line and a horizontal/vertical line from the center of hole in the host plate. Fig. 15 shows that the piezoelectric patches with stiffness ratios near the unity are located near the reference line and by increasing the stiffness ratio; the patches are approached the horizontal line. This tendency also can be observed for stiffness ratios less than unity. Moreover, it can be concluded from Fig. 16 that for stiffness ratios near the unity, the patches are tended to the reference line and for stiffness ratios less than unity, the patches are tended to the vertical line. The authors resulted that there are some reference lines for every shape that have stress concentration with different stress flows and by specifying this reference line for each shape, the optimum placement of patches to achieve the maximum reduction in stress concentration factor will be determined.

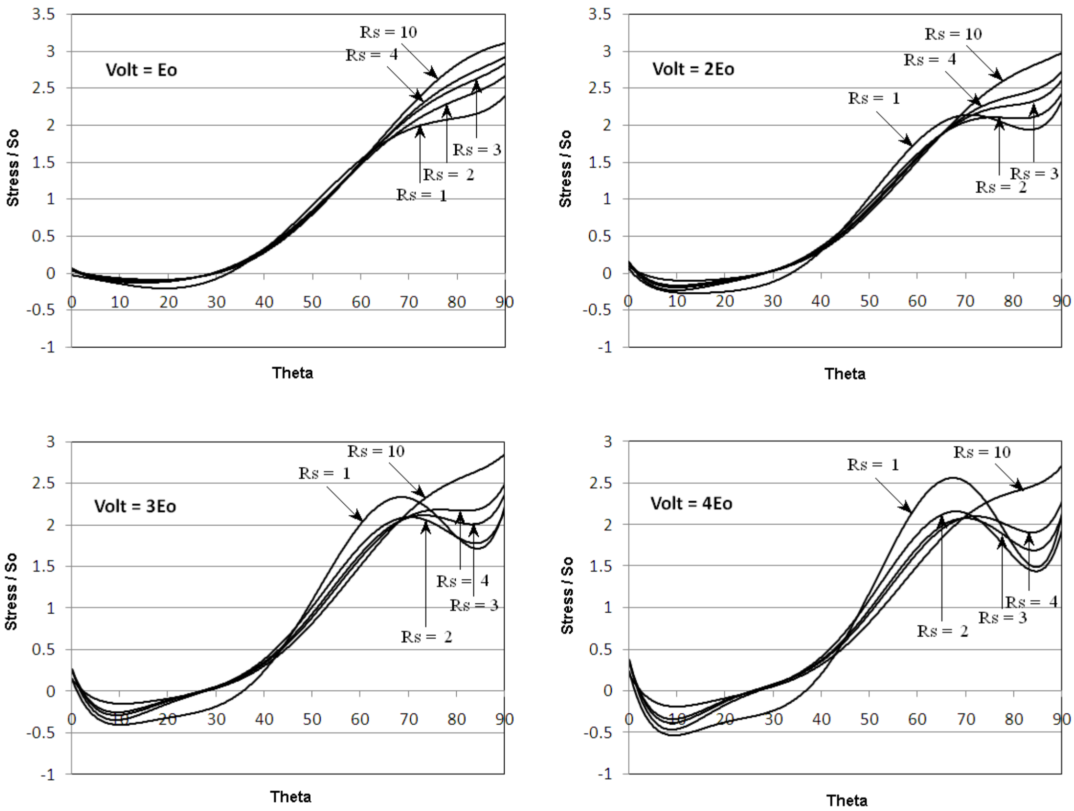

Fig. 12 Effect of stiffness ratio and voltage on stress concentration for Rs>1

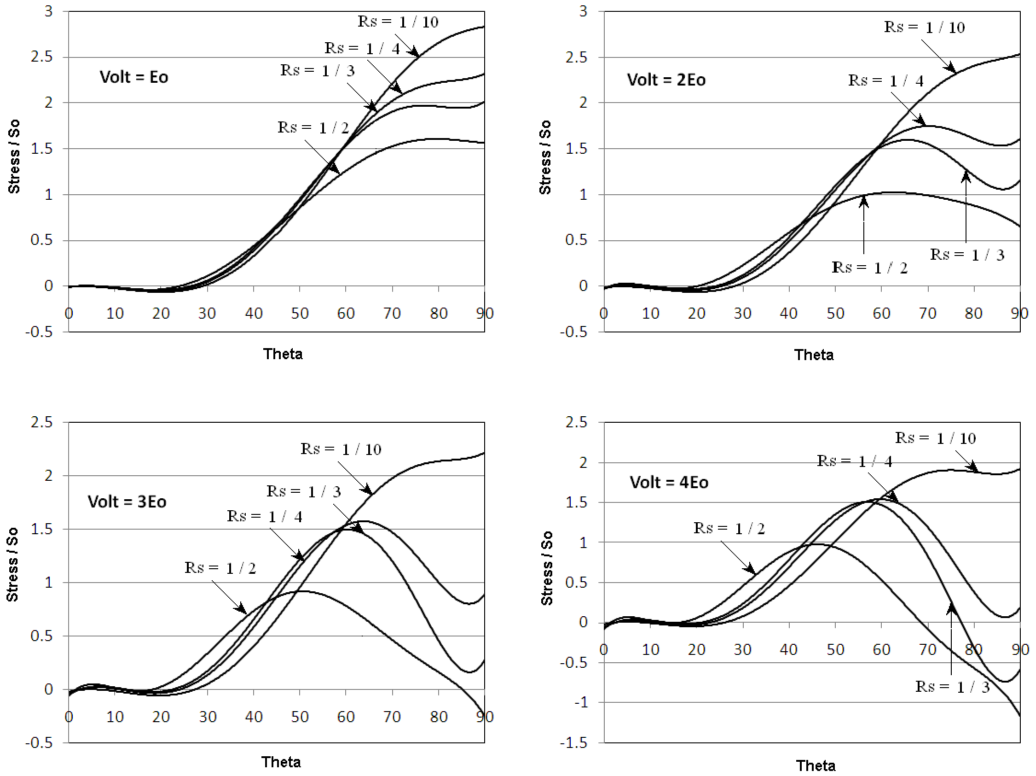

Fig. 13 Effect of stiffness ratio and voltage on stress concentration for Rs<1

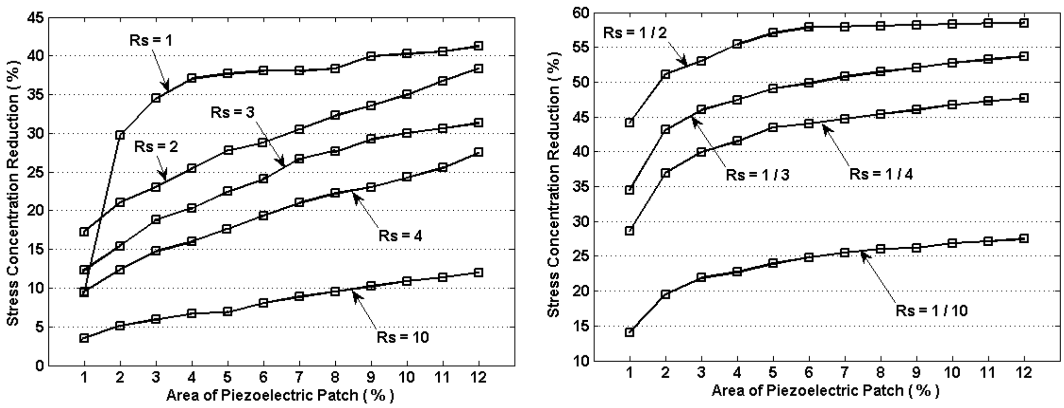

Fig. 14 Effect of stiffness ratio on stress concentration reduction

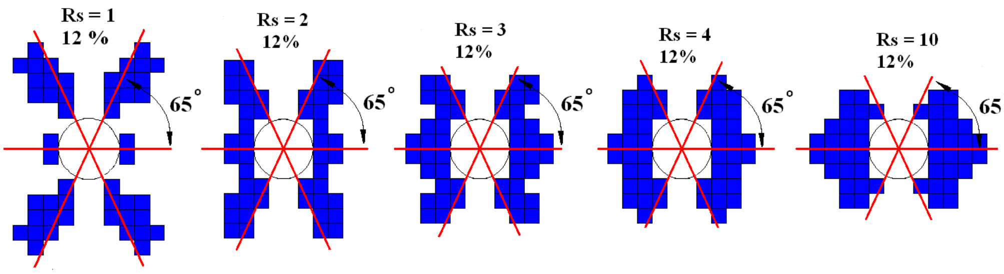

Fig. 15 Best pattern of patches and reference line for Rs>1

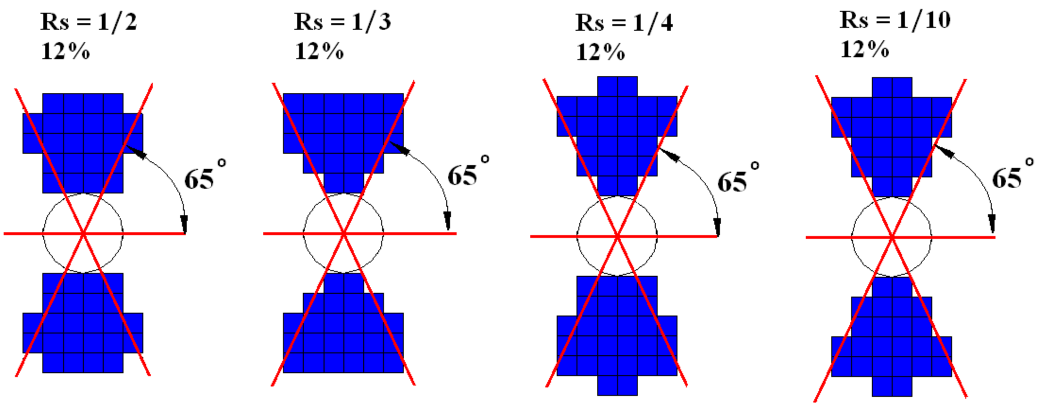

Fig. 16 Best pattern of patches and reference line for Rs<1

6. Results validation

To validate the results two experimental tests are carried out in this paper. The dimensions of the host plate are the same as the values presented in Fig. 1 and made from aluminum subjected to $1\,MPa$ tension. The patches are made from PZT-4. The thickness of the host plate and piezoelectric patches are considered 1 and 0.5 mm respectively. So the stiffness ratio is considered $R_s = 2$.

The material properties for piezoelectric patches and plate are presented in Table 1

For the first test, 4% and for second test 14% of the meshed area from Fig. 1 is occupied with piezoelectric patches. The piezoelectric patches are glued with UHO plus graphite powder to the host plate. To determine the stress concentration in host plate, two strain gauges are located at top/bottom of the hole on the host plate. Fig. 17 shows the schematic view and test setup for experimental tests. In the experimental tests, by induction of electric voltage to the piezoelectric patches, the strain is induced to the host plate from the patches and the stress flow in host plate is changed from the initial condition. Two strain gauges read the new strains from top/bottom of the hole with data logger. The average values of these two strain gauges are considered as the strain at top/bottom of the hole to decrease the error of measuring strain because of inaccurate placement of strain gauges. The experimental tests are simulated by developed python code as explained in previous sections. Table 2 and 3 show the results of the first and second experimental tests and the results of analysis from developed python code respectively.

Table 1 Material property

Plate: Aluminum: E=70 GPa, ν = 0.3								
PZT-4: ν = 0.3								
Elastic Constants (GPa)								
q_{11}	q_{22}	q_{33}	q_{12}	q_{13}	q_{23}	q_{44}	q_{55}	q_{66}
139	139	115	74	74	74	25.6	25.6	25.6
Piezoelectric Constants (C/m^2)								
d_{31}	d_{32}	d_{33}	d_{24}	d_{15}				
-5.2	-5.2	15.08	12.71	12.71				
Dielectric Constants (C^2/Nm^2)								
k_{11}	k_{22}	k_{33}						
6.75×10^{-9}	6.75×10^{-9}	5.87×10^{-9}						

Table 2 Comparison the results of analysis and experimental test for 4% patches

Voltage	Strain from Test	Strain from analysis	Error
0	39.5×10^{6}	41.4×10^{6}	4.59%
16.75 (E_0)	34.84×10^{6}	37.0×10^{6}	5.84%
33.5 ($2E_0$)	30.18×10^{6}	32.6×10^{6}	7.42%
50.25 ($3E_0$)	25.52×10^{6}	28.2×10^{6}	9.50%
67 ($4E_0$)	20.86×10^{6}	23.8×10^{6}	12.35%

Table 3 Comparison the results of analysis and experimental test for 14% patches

Voltage	Strain from Test	Strain from analysis	Error
0	31.21×10^{6}	33.3×10^{6}	5.07%
16.75 (E_0)	25.62×10^{6}	28.14×10^{6}	6.86%
33.5 ($2E_0$)	20.01×10^{6}	22.67×10^{6}	8.16%
50.25 ($3E_0$)	14.41×10^{6}	16.98×10^{6}	9.11%
67 ($4E_0$)	8.81×10^{6}	11.66×10^{6}	11.97%

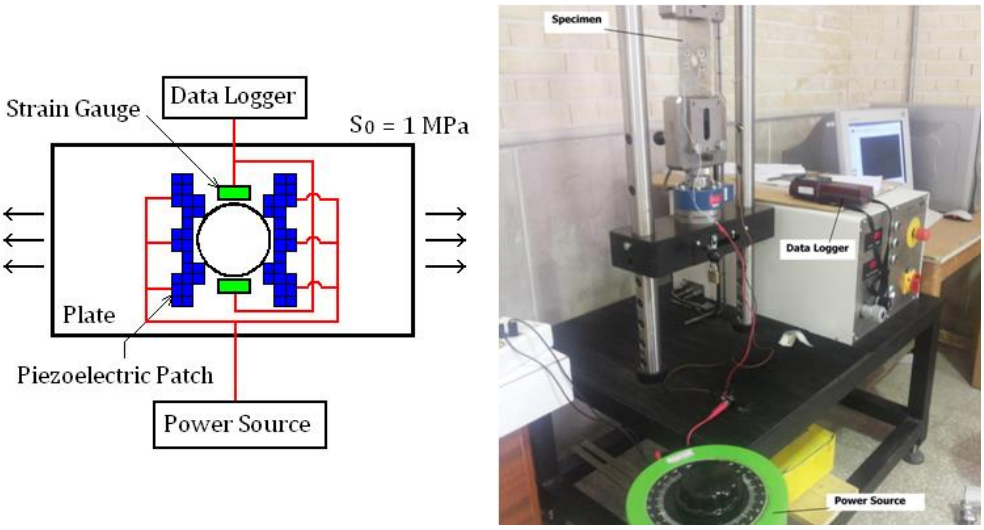

Fig. 17 Schematic of experimental test setup

By considering the errors in two experimental tests from Tables 2 and 3, it can be seen that there is good agreement between the results of analysis and experimental tests. There are about 5% error between tests and analysis for zero voltage at the beginning of tests. But the results show that for each voltage, the strains from analysis have values greater than experimental test. One reason is the placement of strain gauges that are not located on the edge of the hole exactly. The second reason of errors is the adhesive effect for bounding the piezoelectric patches to the host plate.

7. Conclusions

This paper is focused on stiffness ratio effect on best pattern of piezoelectric patches placement around a hole in a plate under tension to reduce the stress concentration factor. For this purpose various stiffness ratios are considered. Then a python code is developed by using particle swarm optimization algorithm, to find the optimum pattern for locating the patches around the hole. In particle swarm optimization algorithm, the objective function is von misses stress in plate and the patches and the location of patches are the position of particles. To implementing the PSO algorithm 20 particles are considered and the ratios 2 are intended for individual and social learning. The results for each stiffness ratios show the adequacy of particle swarm optimization method in engineering problems. After implementation the PSO algorithm, by comparing all optimum patterns a new line that called “reference line” is purposed to locate the piezoelectric patches. By using this reference line, the location of the patches around the hole, with any stiffness ratio effect, can be specified approximately. To confirm the analysis and presented results, two experimental tests for location of the patches with various voltages are performed.

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