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# Optimal reduction from an initial sensor deployment along the deck of a cable-stayed bridge

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**Abstract.** The ambient vibration measurement is an output-data-only dynamic testing where natural excitations are represented, for instance, by winds and typhoons. The modal identification involving output-only measurements requires the use of specific modal identification techniques. This paper presents the application of a reliable method (the Stochastic Subspace Identification – SSI) implemented in a general purpose software. As a criterion toward the robustness of identified modes, a bio-inspired optimization algorithm, with a highly nonlinear objective function, is introduced in order to find the optimal deployment of a reduced number of sensors across a large civil engineering structure for the validation of its modal identification. The Ting Kau Bridge (TKB), one of the longest cable-stayed bridges situated in Hong Kong, is chosen as a case study. The results show that the proposed method catches eigenvalues and eigenvectors even for a reduced number of sensors, without any significant loss of accuracy.

**Keywords:** ambient vibration; cable-stayed bridge; modal frequencies; mode shapes; parameter identification; sensor deployment

## 1. Introduction

Structural health monitoring (SHM) is an active area of research in civil engineering. Several system identification techniques have been developed over the past few decades, and their application is growing with the availability of instrumentation on civil infrastructures (Vicario *et al*. 2015). Typically, the goal is to estimate parameters of a mathematical model of the structure under study. Parameters identification through dynamic measurements is a discipline originally developed in mechanical and aerospace engineering (Ewins 1986, Ljung 1987, Juang 1994), but in the context of civil engineering, the structures (such as bridges and buildings) behave with their own features.

The parameters to be estimated by dynamic measurements are mainly of a modal nature, such as frequencies, damping ratios, and mode shapes. They will serve as a basis for the input to the finite element modal updating, in detecting and locating damage, as well as in assessing structural safety under special scenarios, as for instance large earthquakes and wind loads.

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There are three main classes of structural dynamic testing, (1) forced vibration testing, (2) free vibration testing, and (3) ambient vibration testing. The first relies on artificial items, such as drop weights, vibrodynes, and shakers, to excite the structure. For large infrastructures these devices are either unavailable or too expensive. In the second class, a free vibration condition is induced, by assessing adequate initial conditions. The main drawback using both these techniques is that the traffic along the infrastructure has to be stopped for a rather long period. The third class of methods does not require the interruption of the service, because it uses the disturbance, either wind or traffic, as an excitation (Ren *et al.* 2004b).

The output-only modal identification methods can be classified into two main groups, namely, frequency domain methods and time domain methods (Peeters and De Roeck 2001). Typically, modal parameters identification is carried out from both input and output measurements through the frequency response functions (FRFs) in the frequency domain, and impulse response functions (IRFs) in the time domain. In civil engineering, sensors are usually installed at different locations, and record the dynamic response (i.e., the output) of the structure. But, to determine the input or the excitation level on the real structure in service conditions is a difficult task. A further benefit from the output-only data is the saving in equipment, since no tools are needed to excite the structure. The ambient vibration measurement is an output data-only dynamic testing where natural excitations are induced by winds and typhoons.

Ambient vibration testing was adopted to study many large-scale bridges. Amongst others, one mentions: the Golden Gate Bridge (Abdel-Ghaffer and Scanlan 1985); the Faith Sultan Mehmet Suspension Bridge (Brownjohn *et al.* 1992); the Tsing Ma Suspension Bridge (Xu *et al.* 1997); the Vasco da Gama Cable-Stayed Bridge (Cunha *et al.* 2001); the Kap Shui Mun Cable-Stayed Bridge (Chang *et al.* 2001); the Roebling Suspension Bridge (Ren *et al*. 2001); the steel girder arch bridge (Ren *et al.* 2004); the Hakucho Suspension Bridge (Siringoringo and Fujino 2008); the Humber Bridge (Brownjohn *et al.* 2010); the Tamar Suspension Bridge (Koo *et al*. 2013), and the Vincent Thomas Suspension Bridge (Karmakar *et al.* 2015).

When adopting the results derived from ambient vibration tests, only response data are caught, because the loading condition on the structure remains unknown. The modal analysis, that involves output-only measurements, requires the use of particular modal identification techniques, which deal with the small magnitude of ambient vibration contaminated by noise without the knowledge of input forces.

The benchmark launched in (Ni *et al.* 2015) aims to study the mechanism behind the output-only modal identification, deficiency in modal identifiability, and criteria to evaluate robustness of the identified modes, but also to apply various methods of output-only modal identification.

Herein, the stochastic subspace identification-data driven method (SSI-data) as implemented in MACEC (Reynders *et al.* 2014) is used to identify modal parameters. The results are then combined with a metaheuristic bio-inspired tool, namely the Firefly Algorithm (Yang 2013), and an optimal reduced sensor deployment is pursued. The aim is to identify the same number of modal parameters (thus showing the robustness of the achieved results), but with a more sustainable economic effort. A strongly nonlinear objective function that takes into account not only the eigenvalues, but also the eigenvectors is introduced, and this is why the solution of the optimization problem is searched via a metaheuristic algorithm.

## 2. Governing relations

### *2.1 Stochastic Subspace Identification (SSI)*

Ambient excitation testing requires elaborations by a modal parameter identification method able to deal with ambient vibration measurement. Among them, the authors selected the stochastic subspace identification (SSI) method, well-implemented in the computational tool utilized: MACEC (Reynders *et al.* 2014) working within the software environment MATLAB® (Matlab 2015).

The reader is referred to (Van Overschee and De Moor 1996, Peeters and De Roeck 1999, Chen and Huang 2012, Reynders *et al.* 2014) among others.

A structural dynamic model is described by a set of linear second-order differential equations with constant coefficient, by introducing the classical equation of motion with a time-dependent vector of input forces.

Such equation can be rewritten as a first-order system of differential equations, as a state-space representation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t)$ is the state vector, $A$ is the state matrix and $B$ is matrix of the excitation coefficient.

Moreover, the output vector, $y(t)$, can be a part of, or a linear combination of system states, as

$$y(t) = Cx(t) + Du(t) \tag{2}$$

where $C$ and $D$ are the real output influence coefficient matrix and the out control influence coefficient matrix, respectively. Eqs. (1) and (2) provide a continuous-time state-space model of the dynamic system. The sample time and noise are always influencing the measurements. Hence, after the sampling such a model modifies into

$$\begin{aligned} x_{k+1} &= \hat{A}x_k + \hat{B}u_k \\ y_k &= Cx_k + Du_k \end{aligned} \tag{3}$$

where $x_k$ is the discrete time state vector, $\hat{A}$ is the discrete state matrix, and $\hat{B}$ is the discrete input matrix. Then, Eq. (3) represents a discrete-time state-space model of a dynamic system.

A further issue is the process noise due to the disturbance and the modeling inaccuracies. If the stochastic components (i.e., noise) are included, Eq. (3) can be extended to consider also a process noise $w_k$ and a measurement noise $v_k$ drawn as a continuous-time stochastic state-space model

$$\begin{aligned} x_{k+1} &= \hat{A}x_k + \hat{B}u_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned} \tag{4}$$

Since it is difficult to find the individual process and measurement noise in an accurate way, some assumptions have to be made. Thus, the process noise $w_k$ and the measurement noise $v_k$ are assumed to be of zero-mean, white and with covariance matrices.

Dealing with practical civil engineering problems, only the responses of the structure are measured when adopting output-only techniques, while the input sequence $u_k$ remains unmeasured. When ambient vibration tests are performed, it is impossible to distinguish the input term $u_k$ from the noise terms $w_k$ and $v_k$. The result is a purely stochastic system

$$\begin{aligned} x_{k+1} &= \hat{A}x_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{5}$$

The input is now implicitly modeled by the noise terms (the second terms in the right hand side of the above equation). By the way, any assumption related to the white noise has to be explicit. The consequence reveals that the such assumption is violated.

Eq. (5) is the basis for the time-domain system identification through ambient vibration measurements. The subspace method is able to identify the state space matrices based on the measurements and using the QR-factorization, singular value decomposition (SVD) and least square, as numerical techniques. Thus, the QR-factorization results in a significant data reduction, while the SVD rejects the noise. Once the mathematical description of the structure, i.e., the state space model, is defined, the modal parameters are determined: natural frequencies, damping ratios and mode shapes.

### *2.2 The MACEC software*

Modal analysis of a structure develops along three principal steps that are the data collection, the system identification and the determination of modal features, such as eigenvalues, damping ratios, mode shapes and so forth. MACEC, a toolbox of MATLAB® (Matlab 2015), is a powerful tool developed by the Catholic University of Leuven in Belgium (Reynders *et al.* 2014) that manages with every step in the modal analysis procedure, and saves for the data collection.

Such tool is herein applied in order to verify the modal parameter under the first set of blind-data provided for this benchmark study.

### *2.3 A bio-inspired approach for structural optimization*

The Firefly Algorithm (FA) is a bio-inspired swarm intelligence method that was developed studying the social behavior of fireflies (Yang 2010). This is a gradient-free algorithm and that means there is no use of derivatives. This feature makes this method very useful in solving optimization problems with strong non-linearity. The greatest advantage of such tools is that they do not trap in any local minimum or maximum (as occurring for same basic genetic algorithms (Casciati 2008 and Casciati 2014)), thus reaching the best value of the objective function. This novel method was firstly introduced for continuous optimization (Yang 2013), and later extended to discrete problems, such as structural control (among the others (Casciati and Elia 2015a, Talatahari *et al.* 2014, Zhou *et al.* 2015)).

In general, FA combines three main strategies: attractiveness, brightness, and distance between each firefly. Indeed, it can be idealized with three main assumptions: first, each firefly is unisex, thus any firefly is attracted by the others nevertheless the sex; second, the objective function determines the brightness of a firefly; and third, the attractiveness is determined by the brightness, i.e., when the maximum (or minimum) of the fitness (objective) function is achieved the brighter firefly attracts the less brighter ones.

## 3. A policy for sensor reduction

### *3.1 Problem statement*

In the present paper, the actual sensor deployment at the bridge deck has been taken into account and the possibility of reducing the number of sensors maintaining enough modal information is pursued. Indeed, one of the goals of the benchmark consists of identifying the second mode when normal excitation is occurring. At this stage, this mode is found only during typhoon conditions.

As from Fig. 1, where the system architecture of the proposed method is sketched, one needs to specify:

a) the input of the optimization tool;
b) the objective function.

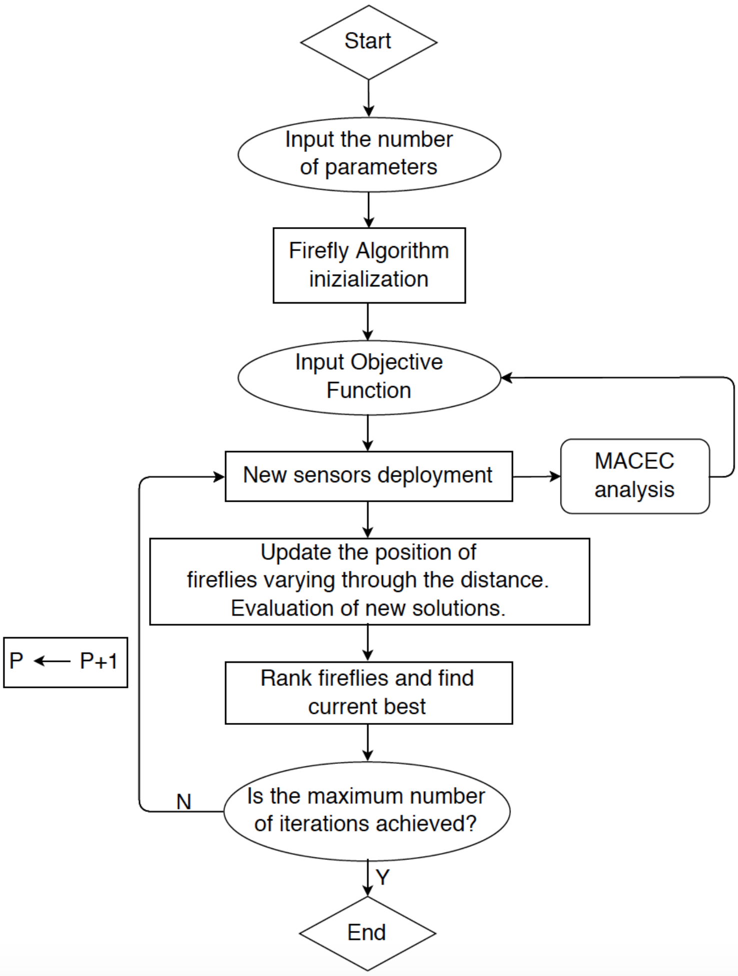


Fig. 1 System architecture

### 3.2 Symbolism

Dealing with SHM, means assessing the performance of a structure, knowing its response under dynamic excitation. Well-known modal analysis tools, employed for linear systems, are applied for obtaining the modal features, i.e., mode shapes and frequencies.

For dealing with the inverse problem, a numerical approach, which consists of performing a finite element analysis where some variables have to satisfy the requirement of minimizing the discrepancies with the measured response, is proposed. For instance, an objective function, which is minimized when the difference of the measured and generated modal parameters, is formulated (Casciati and Elia 2015b).

Traditional modal analyses can be applied to linear system to achieve their own modal features (i.e., frequencies and mode shapes). Such parameters are denoted with the subscript '*F*' because they are referred to the scenario where the complete set of devices is installed.

Namely, $\bar{\boldsymbol{\omega}}_F$ states the $N \times 1$ vector of natural frequencies, while $\bar{\boldsymbol{\Phi}}_F$ the $N \times N$ matrix of corresponding modal shapes.

Whereas, the scenario where a lower number of devices is considered shall be stated with the subscript '*NF*', where both eigenvalues and eigenvectors are stored in a $N \times 1$ vector, $\bar{\boldsymbol{\omega}}_{NF}$, and in a $N \times N$ matrix, $\bar{\boldsymbol{\Phi}}_{NF}(\boldsymbol{x})$, respectively, where $\boldsymbol{x}$ denotes the vector of the positions of the grid node ordered as in Fig. 3.

### 3.3 Formulation of the objective function

The objective function formulated can be easily written in its scalar form as the difference between the full and not full scenarios as

$$F(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{N} \frac{1}{i} \left( \frac{\bar{\omega}_{i,F} - \bar{\omega}_{i,NF}}{\bar{\omega}_{i,F}} \right)^2} + w \max_{1 \le j \le N} \left[ \frac{\sum_{i=1}^{N} \left( \bar{\Phi}_{ij,F} - \bar{\Phi}_{ij,NF}(\boldsymbol{x}) \right)^2}{\sum_{i=1}^{N} \bar{\Phi}_{ij,F}^2} \right] \tag{6}$$

where the *i*-th element is given by $1/i$ so that the lower frequencies are prioritized over the higher one that are disturbed by measurement noise. This term can be adopted equal 1 when adopting only few frequencies. In this paper, only the first eight eigenvalues will be considered. Alternative expressions for Eq. (6) incorporate the spectra instead of the modes and this allows one to account for the actual damping. A comparison between the two different formulations is still in progress.

The second term in Eq. (6) plays and important role because, after a weighted calibration, it enhances the convergence of the method. This is confirmed by the numerical results presented in the following sections. A weight factor, *w*, is introduced and calibrated resulting into an optimal value of 1. It ensures good results in terms of convergence of the method.

It is worth noting that the norm of a matrix is not uniquely defined when considering the eigenvectors. Furthermore, according to (Nair *et al.* 2006), the norm of a $N \times N$ matrix $\mathbf{G}$ is computed as

$$
\begin{aligned}
& l_2\text{norm}\left(\mathbf{G}\right) = \sqrt{\left(\text{greatest eigenvalue of } \mathbf{G}^{\mathrm{T}}\mathbf{G}\right)} \\
& \text{or} \\
& l_2\text{norm}\left(\mathbf{G}\right) = \max_{1\le j\le N} \sum_{i=1,\ldots,N} \left(G_{ij}\right)
\end{aligned}
\tag{7}
$$

Thus, when the $l_2$norm is applied to the $j$-th column of matrix **G**, it becomes the traditional norm of a vector. In a mathematical way, the reader finds $l_2\text{norm}\left(\mathbf{G}_j\right) \equiv \left\|\mathbf{G}_j\right\| = \sqrt{\sum_{i=1,\ldots,N}\left(G_{ij}^2\right)}$ with $j = 1,\ldots,N$. Hence, the second term in the right hand side of Eq. (6) is also expressed in matrix form as

$$
w \max_{1\le j\le N} \left[ \frac{\left\| \mathbf{\Phi}_{j,F} - \mathbf{\Phi}_{j,NF}\left(\mathbf{x}\right)\right\|^2}{\left\|\mathbf{\Phi}_{j,F}\right\|^2} \right] \tag{8}
$$

Finally, the optimization problem is posed as follow

$$
\begin{aligned}
& \text{minimize } F\left(\boldsymbol{x}\right) \\
& \text{under the constraint: } \boldsymbol{x}_{Lb} \le \boldsymbol{x} \le \boldsymbol{x}_{Ub}
\end{aligned}
\tag{9}
$$

where $\boldsymbol{x}_{Lb}$ and $\boldsymbol{x}_{Ub}$ are the $ns \times 1$ vectors of the lower and the upper bounds of each variable in the design parameter space, respectively.

Since the problem presents highly nonlinearity, the minimization of the objective function in Eq. (6) is reached by the adoption of a metaheuristic tool, namely the Firefly Algorithm (FA).

## 4. Ting Kau Bridge

### *4.1 Actual deployment of accelerometers*

The Ting Kau Bridge (TKB) is a three-tower cable-stayed bridge situated in Hong Kong that spans from the Tsing Yi Island to the Tuen Mun Road (Bergermann *et al.* 1996). The two main central spans are 448 m and 475 m long, respectively and there are two side-spans of length 127 m. The bridge deck is divided into two carriageways 18.8 m width. Along the deck, there are three slender single-leg towers 170 m, 198 m, and 158 m high, respectively. Two steel girders along the edges of the deck with steel crossgirders every 4.5 m, and a concrete slab on the top form each carriageway. Furthermore, there is a 5.2 m gap between the two parallel carriageways: they are linked each to the other every 13.5 m by connecting crossgirders. Finally, 384 stay cables in four cable planes support the deck.

A unique feature of the bridge consists in the arrangement of the three single-leg towers, strengthened by longitudinal and transverse cables, with a stabilizing function. There are 8 longitudinal stabilizing cables used to diagonally connect the top of the central tower to the Ting Kau and Tsing Yi Towers (the length reaches 465 m), whereas 64 cables are utilized to strengthen the three towers in the transverse (lateral) direction (Ni *et al.* 2015).

During the bridge construction, as well as after its completion in 1999 (Wong 2004, Ko *et al.*

2005), more than 230 sensors were installed on the TKB, within a long-term SHM system conceived by the Hong Kong SAR Government Highways Department. Accelerometers, anemometers, strain gauges, temperature sensors, GPS, and weigh-in-motion sensors are deployed across the bridge as in Wong and Ni *et al.* (Wong 2007, Ni *et al.* 2011). 24 uniaxial, 20 biaxial, 1 triaxial accelerometers are permanently installed at the deck of the four spans, on the longitudinal stabilizing cables, on the top the three towers, and at the base of the central tower. They form a total of 67 accelerometers channels and they monitor the dynamic response of the bridge itself. In the paper, only the data collected by the accelerometers placed at the deck are considered. Fig. 2 shows the placement of the accelerometers and the anemometers on the bridge deck, within the general layout of the bridge.

In each of the sections from *A* to *P* in Fig. 2, two accelerometers are installed on the east and west side of the longitudinal steel girders, respectively. They measure the vertical acceleration, while another accelerometer is installed on the central crossgirder and it measures the transverse acceleration. The sampling frequency is 25.6 Hz. Furthermore, 7 anemometers are installed at the top of each of the three towers and two on two main spans. Ultrasonic anemometers are installed on the east and west side of the deck and the sampling frequency of each anemometer is 2.56 Hz. Fig. 3 shows the actual deployment of accelerometers at the bridge deck.

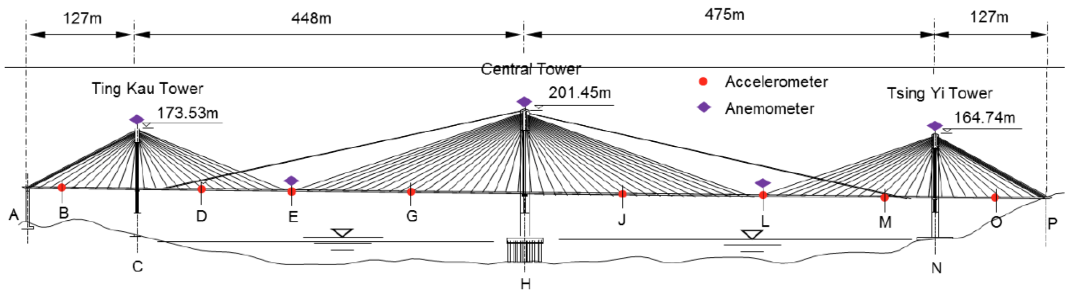


Fig. 2 Deployment of accelerometers and anemometers at bridge deck

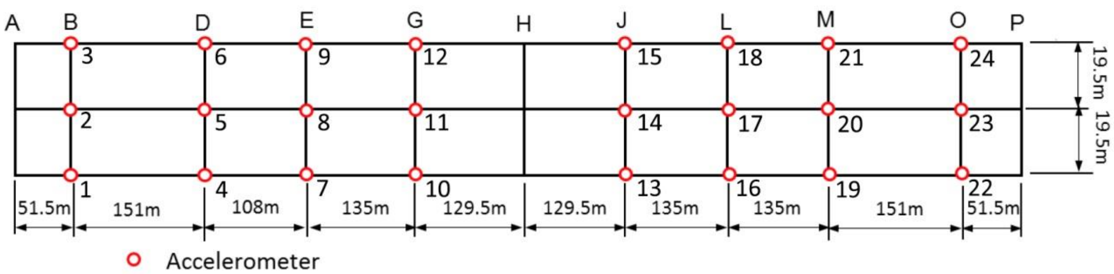


Fig. 3 Actual deployment of accelerometers on the bridge deck

### *4.2 Existing recorded data*

The benchmark on the data collected in 10 years of monitoring on the Ting Kau Bridge was lauched in (Ni *et al.* 2015) and its companion document posted in the web (Dept. of Civil Eng. 2015). Initially this latter document was listing the identified frequencies in Table 1, from where some frequencies were removed as spurious in the currently appended document. They are marked by a star in the table. The frequencies were identified having available 6 sets of data under weak wind conditions, 5 sets of data under typhoon conditions and other sets of monitoring acceleration data (also called “blind dataset”) coming without any specification on the excitation conditions. These data were collected in different period and under different wind speeds duration. The first set of these blind data was utilized in this purpose in order to explain the properties of output-only methods.

#### *4.2.1 The identified modal frequencies*

The data driven stochastic subspace identification (data-driven SSI) technique is regarded as the most powerful class of the known identification techniques for natural input modal analysis in the time domain (Brincker *et al.* 2006). Such technique has been applied to identify the modal frequencies and modal shapes of the Ting Kau Bridge and it works directly with the recorded time domain signals. It is able to identify the space models from the output data only by the application of robust numerical techniques among which the QR factorization, the singular value decomposition and least squares, once the formulation of the state space model is achieved used with the 24 accelerometers in Fig. 2, the identified frequencies are listed in Table 1.

Next section provides more details on the technique in view of the further bits of information makes available.

Table 1 First eight eigenvalues and damping ratios from the first blind-set data for 24 sensors (records of duration 1 hour without a non-particular condition of external excitation) extracted by MACEC and compared with Ni *et al*. (2015)

| Eigenvalue | Frequency [Hz] (MACEC) | Frequency [Hz] (Ni *et al.* 2015) | Damping ratio [%] (MACEC) |
|---|---|---|---|
| 1 | 0.160* | - | 1.34 |
| 2 | 0.162 | 0.162 | 2.85 |
| 3 | 0.178* | - | 4.63 |
| 4 | 0.223 | 0.226 | 2.68 |
| 5 | 0.268 | 0.257 | 4.93 |
| 6 | 0.286 | 0.288 | 4.46 |
| 7 | 0.307 | 0.300 | 1.91 |
| 8 | 0.357 | 0.358 | 2.46 |

*4.2.2 From the actual situation to the reduced one*

In Section 2, the actual deployment of the devices was provided. The current deployment permits one to carry out a general analysis, which by the software MACEC. The attention is focused on the first eight eigenvalues as stated in Table 1.

In each analysis, both eigenvalues and eigenvectors are output quantities parameters.

Assume now that one relies on 16 sensors only. For the deployment in Fig. 3, the eigenvalues in Table 1 are obtained by the same algorithm, together with the corresponding eigenvectors.

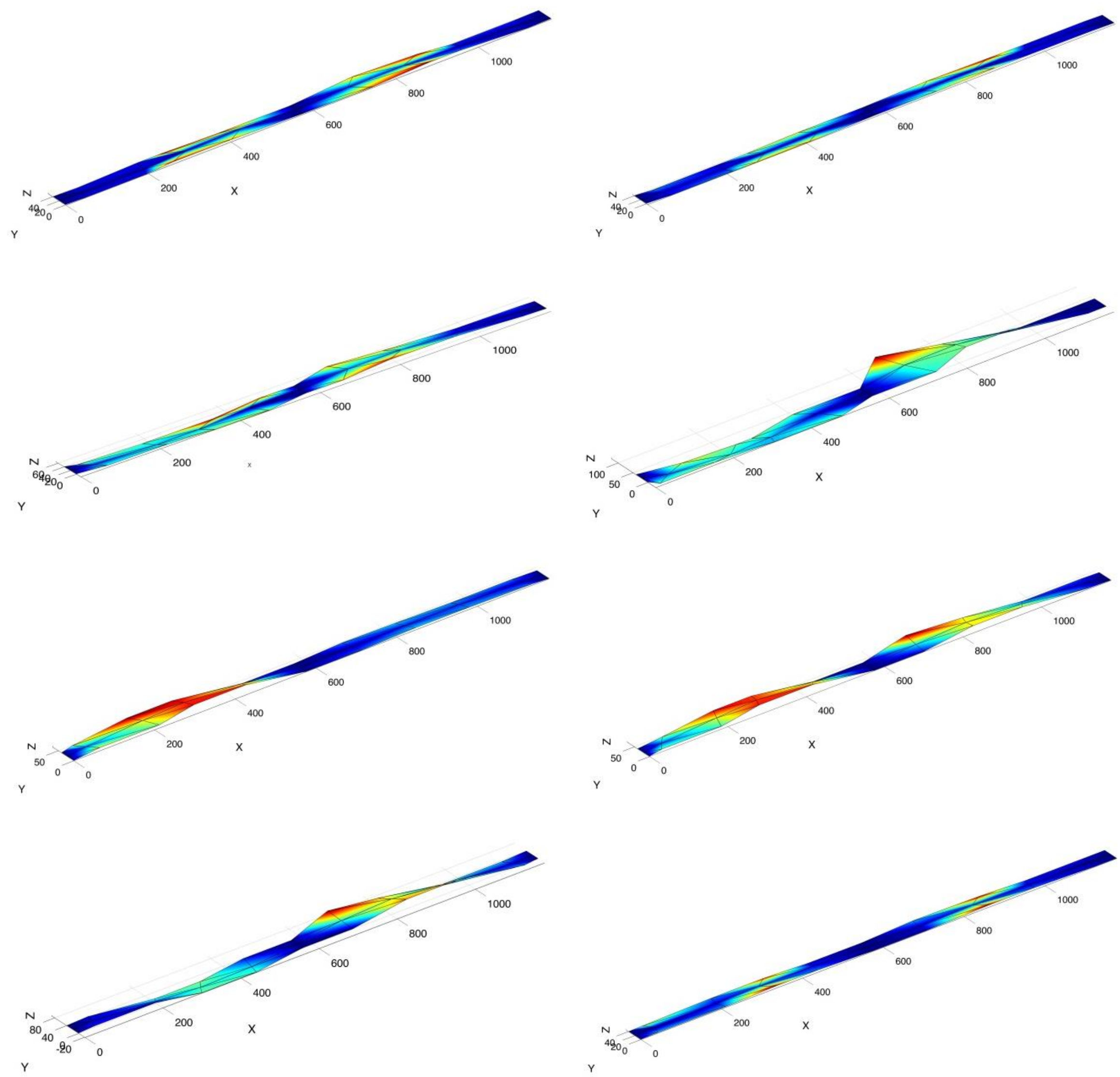

Fig. 4 First 8 eigenvectors in case of 24 sensors at the bridge deck

## 5. Proposed method

### *5.1 Study on the actual situation*

The control parameters of the applied metaheuristic tool are summarized in Table 2. The randomization number, the maximum attractiveness and the absorption coefficient are maintained constant in each the performed analysis, such as in most of the implementations found in literature.

Under these assumptions, the convergence of the solution is achieved with a small computational burden.

### *5.2 Use of MACEC on the whole set of data*

First of all the grid, the number and the position of the degrees of freedom, and the type of finite element used for carrying out the analyses (beam or surface) are set. Fig. 5 shows the bridge deck implemented in MACEC.

Table 2 Control parameters of the Firefly Algorithm

| Control parameters of FA | Adopted value |
|---|---|
| *NP*, size of the initial population of fireflies | 100 |
| $I_{max}$, maximum number of iterations | 40 |
| $\zeta$, randomization number | 0.5 |
| $\beta_{min}$, minimum attractiveness | 0.2 |
| $\gamma$, absorption coefficient | 1.0 |

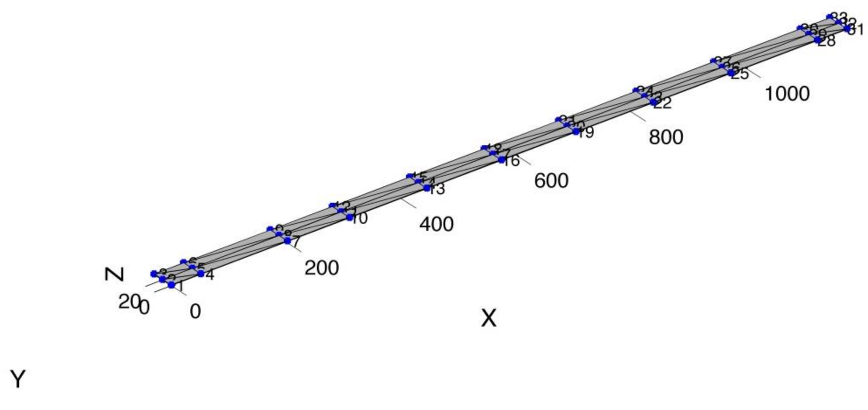


Fig. 5 Bridge deck configuration in MACEC

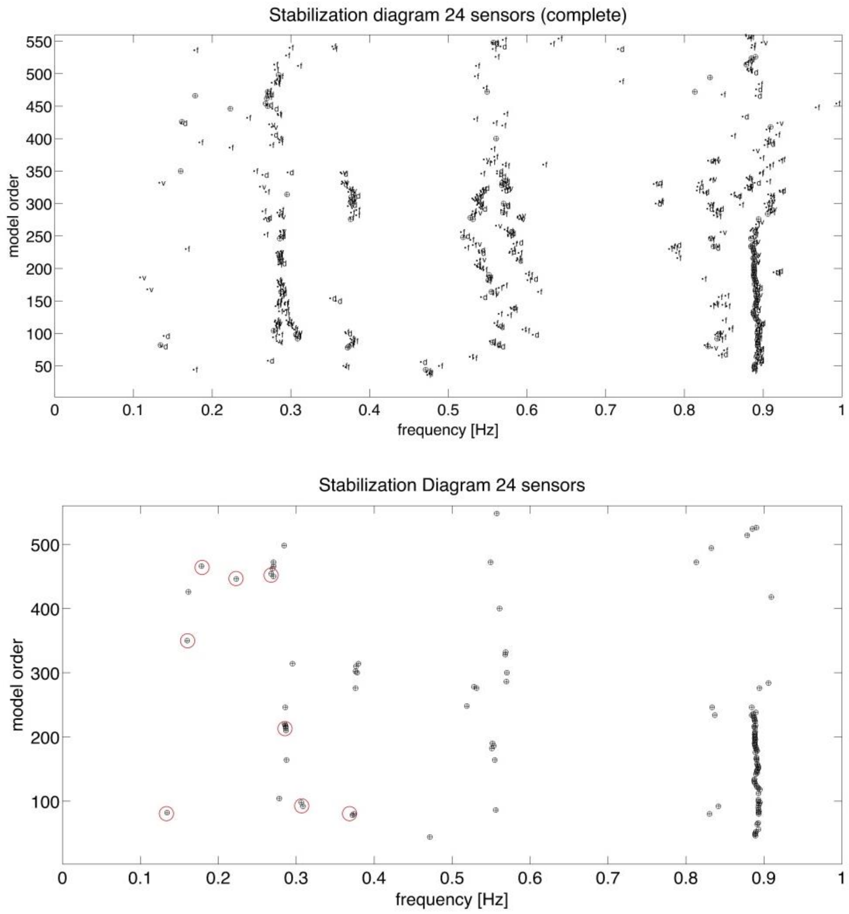


Fig. 6 Stabilization diagrams for 24 sensors

After a phase where the signal is processed, the modal parameters are extracted from the modal analysis toward a stabilization diagram. Fig. 6 shows such diagram, which contains all the modal parameters. Fig. 6 also shows a comparison between the full stabilization diagram and the stabilization diagram where only stable modes are denoted. In the upper figure, different dots represent both stable modes and modes that satisfy all stabilization criteria except for the damping and mode shape differences. While in the lower figure, modes that fulfill all the criteria are represented, and they coincide with the set provided within the benchmark study. Then the selected first eight eigenvalues are extracted, and they are compared with the ones found by exploiting the first blind-dataset provided in this benchmark study.

Before applying this step, robust numerical technique as the QR factorization and the singular value decomposition are applied and the singular values for the maximum system order are obtained. For sake of completeness, each analysis is performed on a Windows 7 notebook, 64-bit, 2.67 GHz Intel® Core™ i7 processor with 4GB ram.

### *5.3 Reducing the number of sensors on the bridge deck*

Once the complete model is defined, i.e., with 24 sensors, the process of reduction that consists in the elimination of some sensors (for instance 8 sensors) starts. These sensors can be considered superfluous, but the new deployment guarantees efficiency in terms of collection of modal parameters.

Furthermore, this purpose wants to minimize the economic effort, but always maintaining the lifeguard and the control of the structure.

The adopted procedure for the sensor deployment is the same to the previous one explained, and the algorithm parameters are kept constant for each analysis.

According to the objective function, the 16 sensors model is compared with the 24 sensors one and several deployments are proposed.

The best solution considers a value of the objective function and a deployment vector as shown in the following statement

$$\begin{aligned} z &= 0.0793 \\ x_{sens} &= [2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 16\ 17\ 18\ 19\ 21\ 22\ 24] \end{aligned} \tag{10}$$

Such the new configuration guarantees, albeit with a minimum error, to note the model parameters and hence to have a lower economic impact. The new stabilization diagram and the new sensor deployment are shown in Figs. 7 and 8.

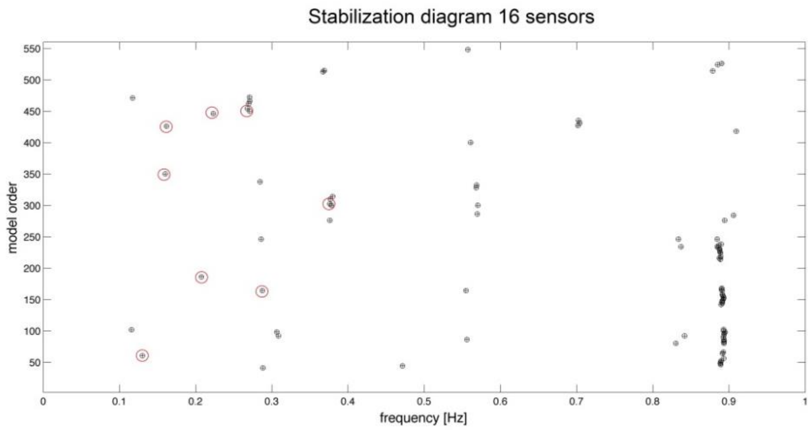


Fig. 7 Stabilization diagram for 16 sensors

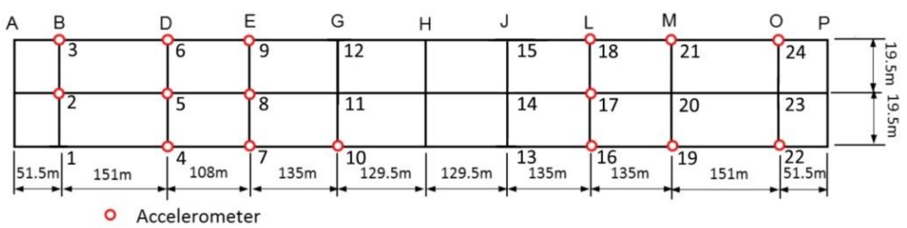


Fig. 8 New proposed deck sensor configuration from MACEC

Table 3 Identified modal damping ratios under unknown excitation for a reduced set of 16 sensors (the optimal one)

| Frequency [Hz] | Damping ratio [%] |
|---|---|
| 0.161 | 1.46 |
| 0.167 | 2.94 |
| 0.171 | 4.63 |
| 0.228 | 3.10 |
| 0.267 | 4.87 |
| 0.283 | 5.19 |
| 0.292 | 5.37 |
| 0.382 | 5.21 |

For such configuration, also the value of the damping ratios are shown in Table 3.

Both Tables 1 and 3 frame the first eight frequencies obtained from the deployment of 24 and 16 sensors respectively, under the same excitation (i.e., the first set of the blind-data). Such these values can be comparable and one can observe that also reducing the number of sensors, the second mode is identified, so one of the challenge of this benchmark is correctly pursued.

For the evaluation of the eigen-properties of the system, namely frequencies and damping ratios as shown in Table 3, the approach presented in Reynders and De Roeck (Reynders and De Roeck 2008), has been adopted.

For sake of completeness, a comparison between the two different scenarios is presented in Table 4.

Finally, the path to convergence of the objective function is shown in Fig. 9.

Table 4 Comparison between the frequencies for two different sensors deployment at the bridge deck, where the second set with optimal sensor deployment (with 16 sensors)

| Eigenvalue | Frequency [Hz] with 24 sensors | Frequency [Hz] with 16 sensors |
|---|---|---|
| 1 | 0.160 | 0.161 |
| 2 | 0.162 | 0.167 |
| 3 | 0.178 | 0.171 |
| 4 | 0.223 | 0.228 |
| 5 | 0.268 | 0.267 |
| 6 | 0.286 | 0.283 |
| 7 | 0.307 | 0.292 |
| 8 | 0.357 | 0.382 |

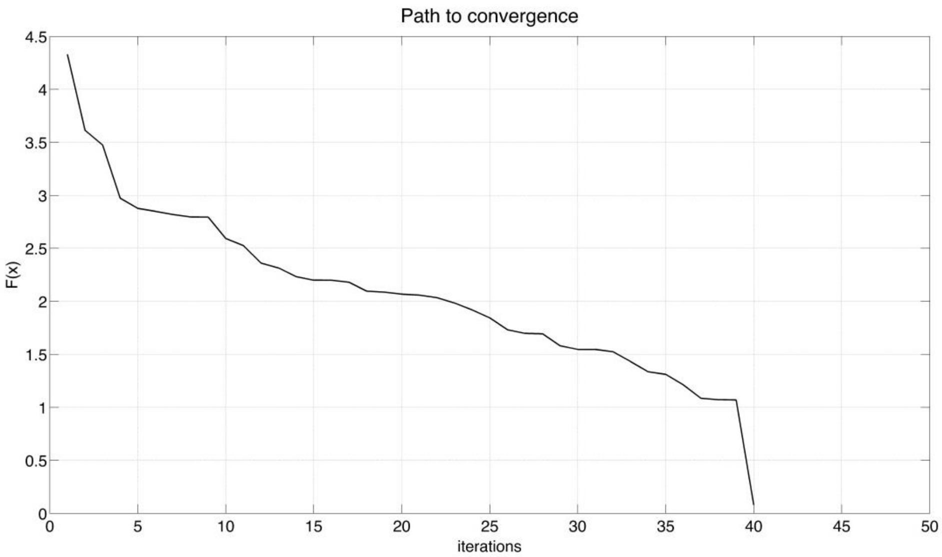


Fig. 9 Path to convergence

## 5. Conclusions

This paper was prepared within the benchmark study on the cable-stayed bridge located in Hong Kong, namely the Ting Kau Bridge.

An application to a modal analysis method with a bio-inspired metaheuristic algorithm is herein implemented. Such application is adopted with a highly nonlinear objective function in order to find an optimal sensor deployment across a large civil engineering structure.

The results have shown that the proposed method identifies the frequencies, even adopting a reduced number of sensors. In terms of computational burden and convergence, the performance of the adopted optimization tool is fully satisfactory.

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## References

Abdel-Ghaffer, A.M. and Scanlan, R.H. (1985), “Ambient vibration studies of the Golden Gate Bridge. I: Suspended structure”, *J. Eng. Mech. - ASCE*, **111**(4), 463-482.

Bergermann, R. and Schlaich, M. (1996), “Ting Kau Bridge, Hong Kong”, *Struct. Eng. Int.*, **6**(3), 152-154.

Brinker, R. and Andersen P. (2006), “Understanding stochastic subspace identification”, *Proceedings of the 24th International Modal Analysis Conference,* St. Louis, Missouri, USA.

Brownjohn, J.M.W., Dumanoglu, A.A. and Severn, R.T. (1992), "Ambient vibration survey of the Faith Sultan Mehmet (Second Bosporus) suspension bridge", *Earthq. Eng. Struct. D.*, **21**, 907-924.

Brownjohn, J.M.W., Magalhaes, F., Caetano, E. and Cunha, A. (2008), "Ambient vibration re-testing and operational modal analysis of the Humber Bridge", *Eng. Struct.*, **32**(8), 2003-2018.

Casciati, F., Elia, L. and Faravelli, L. (2014), "Optimization of Sensors Location for Structural Monitoring", *Proceedings of the OPT-i – International Conference on Engineering and Applied Sciences Optimization*, Kos, Greece.

Casciati, S. (2008), "Stiffness identification and damage localization via differential evolution algorithms", *Struct. Control Health Monit.*, **15**(3), 436-449.

Casciati, S. (2014), "Differential evolution approach to reliability-oriented optimal design", *Probabilist. Eng. Mech.*, **36**, 72-80.

Casciati, S. and Elia, L. (2015), "Innovative, Bio-inspired, Metaheuristic Methods to SHM Diagnostic Goals, *Proceedings of the 7SHMII – 7th International Conference on Structural Health Monitoring of Intelligent Structures*, Turin, Italy.

Casciati, S. and Elia, L. (2015a), "Potential of two metaheuristic optimization tools for damage localization in civil structures", submitted for publication.

Casciati, S. and Elia, L. (2015b), "The potential of the firefly algorithm for damage localization and stiffness identification", *Studies in Computational Intelligence*, **585**, 163-178.

Chang, C.C., Chang, T.Y.P. and Zhang, Q.W. (2001), "Ambient vibration of long-span cable-stayed bridge", *J. Bridge Eng. - ASCE*, **6**(1), 46-53.

Chen, H.P. and Huang, T.L. (2012), "Updating finite element model using dynamic perturbation method and regularization algorithm", *Smart Struct. Syst.*, **10**(4-5), 427-442.

Cunha, A., Caetano, E. and Delgado, R. (2001), "Dynamic tests on large cable-stayed bridge", *J. Bridge Eng. - ASCE*, **6**(1), 54-62.

Dept. of Civil and Environmental Engineering, The Hong Kong Polytechnic University, (2015), http://zn903.com/benchmarkstudy/.

Ewins, D.J. (1986), *Modal Testing: Theory and Practice*, Research Studies Press Ltd., England.

Farrar, C.R., Park, G., Allen, D.W. and Todd, M.D. (2006), "Sensor network paradigms for structural health monitoring", *Struct. Control Health Monit.*, **13**(6), 210-225.

Juang, J.N. (1994), *Applied System Identification*, Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Karmakar, D., Ray-Chaudhuri, S. and Shinozuka, M. (2015), "Finite element model development, validation and probabilistic seismic performance evaluation of Vincent Thomas suspension bridge", *Struct. Infrastruct. E.*, **11**(2), 223-237.

Ko, J.M. and Ni, Y.Q. (2005), "Technology developments in structural health monitoring of large-scale bridges", *Eng. Struct.*, **27**(12), 1715-1725.

Koo, K.Y., Brownjohn, J.M.W., List, D.I. and Cole, R. (2013), "Structural health monitoring of the Tamar suspension bridge", *Struct. Control Health Monit.*, **20**(4), 609-625.

Ljung, L. (1987), *System Identification: Theory for the User*, Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Matlab (2015), *Matlab User Manual*, Mathworks Inc.: Lowell, MA, USA.

Nair, K.K., Kiremidjian, A.S. and Law, K.H. (2006), "Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure", *J. Sound. Vib.*, **291**, 349-368.

Ni, Y.Q., Wang, Y.W. and Xia, Y.X. (2015), "Investigation of mode identifiability of a cable-stayed bridge: comparison from ambient vibration responses and from typhoon-induced dynamic responses", *Smart Struct. Syst.*, **15**(2), 447-468.

Ni, Y.Q., Wong, K.Y. and Xia, Y. (2011), "Health checks through landmark bridges to sky-high structures", *Adv. Struct. Eng.*, **14**(1), 103-119.

Peeters, B. and De Roeck G. (2001), "Stochastic system identification for operational modal analysis: a review", *J. Dynam. Syst. Measurement Control*, **123**(4), 659-667.

Peeters, B. and De Roeck, G. (1999), "Reference-based stochastic subspace identification for output-only modal analysis", *Mech. Syst. Signal Pr.*, **13**(6), 855-878.

Ren, W.X., Harik, I.E., Lenett, M. and Basehearh, T. (2001), “Modal properties of the Roebling Suspension Bridge – FEM modeling and ambient testing”, *Proceedings of the IMAC-XX: A Conf. on Structural Dynamic*, Kissimmee, Florida, February 5-8.

Ren, W.X., Zhao, T. and Harik, I.E. (2004), “Experimental and analytical modal analysis of a steel arch bridge”, *J. Struct. Eng. - ASCE*, **130**(7), 1022-1031.

Ren, W.X. and Zong, Z.H. (2004), “Output-only modal parameter identification of civil engineering structures”, *Struct. Eng. Mech.*, **17**(3), 429-444.

Reynders, E. and De Roeck, G. (2008), “Reference-based combined deterministic-stochastic subspace identification for experimental and operational modal analysis”, *Mech. Syst. Signal Pr.*, **22**(3), 617-637.

Reynders, E., De Roeck, G. and Schevenels, M. (2014), *MACEC 3.3 User’s Manual*, Department of Civil Engineering, Catholic University of Leuven, Belgium.

Siringoringo, D.M. and Fujino, Y. (2008), “System identification of suspension bridge from ambient vibration response”, *Eng. Struct.*, **30**(2), 462-477.

Talatahari, S., Gandomi, A.H. and Yun, G.J. (2014), “Optimum design of tower structures using firefly algorithm”, *Struct. Des. Tall Spec*., **23**(5), 350-361.

Van Overschee, P. and De Moor, B. (1996), *Subspace Identification for Linear Systems: Theory, Implementation and Applications*, Kluwer Academic Publishers, Dordrecht, Netherlands.

Vicario, F., Phan, M.Q., Betti, R. and Longman, R.W. (2015), “Output-only observer/Kalman filter identification ($O^3$KID)”, *Struct. Control Health Monit.*, **22**(5), 847-872.

Wong, K.Y. (2004), “Instrumentation and health monitoring of cable-supported bridges”, *Struct. Control Health Monit.*, **11**(2), 91-124.

Wong, K.Y. (2007), “Design of a structural health monitoring system for long-span bridges”, *Struct. Infrastruct.* E., **3**(2), 169-185.

Xu, Y.L., Ko, J.M. and Zhang, W.S. (1997), “Vibration studies of Tsing Ma Suspension Bridge”, *J. Bridge Eng. - ASCE*, **2**, 149-156.

Yang, X.S. (2010), *Nature-inspired Metaheuristic Algorithms*, 2nd Ed., Luniver Press.

Yang, X.S. (2013), “Multiobjective firefly algorithm for continuous optimization”, *Eng. Comput.*, **29**(2), 175-184.

Zhou, G.D., Yi, T.H., Zhang, H. and Li, H.N. (2015), “Energy-aware wireless sensor placement in structural health monitoring using hybrid discrete firefly algorithm”, *Struct. Control Health Monit.*, **22**(4), 648-666.