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Automated identification of the modal parameters of a cable-stayed bridge: Influence of the wind conditions

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Abstract. This paper was written in the context of a benchmark study promoted by The Hong Kong Polytechnic University using data samples collected in an instrumented cable-stayed bridge. The main goal of the benchmark test was to study the identification of the bridge modes of vibration under different wind conditions. In this contribution, the tools developed at ViBest/FEUP for automated data processing of setups collected by dynamic monitoring systems are presented and applied to the data made available in the context of the benchmark study. The applied tools are based on parametric output only modal identification methods combined with clustering algorithms. The obtained results demonstrate that the proposed algorithms succeeded to automatically identify the modes with relevant contribution for the bridge response under different wind conditions.

Keywords: automated operational modal analysis; cable-stayed bridge; benchmark study

1. Introduction

Nowadays, an increasing interest in permanent monitoring of the structural behaviour of crucial civil infrastructures, such as bridges, has been observed. This is due to the need of controlling a huge number of structures that are reaching their critical age and also to the necessity of validating the performance of new structures with high levels of complexity. Moreover, recent technological advances have contributed to make the installation and operation of permanent monitoring systems more practical and economical and permit a very efficient transmission and processing of the recorded data.

Despite the already considerable number of installed monitoring systems, a large research effort is still needed to improve the present processing capabilities in order to conveniently explore the collected data.

In the particular case of the vibration based structural health monitoring systems, it is very important to accurately estimate the modal parameters (natural frequencies, modal damping ratios and mode shapes) of the instrumented structures from their response to ambient excitation. Therefore, the development of processing tools to automatically identify the modal parameters from datasets continuously collected by dynamic monitoring systems and their validation through

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benchmark studies is very pertinent.

The present research was developed in the context of a benchmark study, that follows other similar benchmark studies (Ni *et al* 2012), and aims the test of algorithms for automated identification of the modal parameters using acceleration data samples collected by 24 accelerometers installed in the deck of the Ting Kau Bridge. One particular aspect of the present analysis is the study of the performance of the identification algorithms in different wind conditions: low wind speeds, intermediate wind speeds and during the occurrence of typhoons. The benchmark test data comprehends ten data samples of training data with known mean wind speeds, six being associated with low wind speeds, four recorded during the occurrence of typhoons, and six data samples of blind data with unknown mean wind speeds, two being associated with low wind speeds, two associated with intermediate wind speeds and two recorded during the occurrence of typhoons. An important goal of the benchmark is also the characterization of the wind characteristics associated with the six data samples of blind data.

2. Tools developed at ViBest/FEUP for automated modal analysis

The application of parametric methods for the identification of modal parameters based on measured structural responses to ambient excitation, such as the SSI-COV or the p-LSCF, implies the selection of the model order. Since it is difficult, or impossible, to forecast the order of the model that will provide the best fit to the experimental data, several model orders have to be tried. The results can then be summarized in stabilization diagrams that enhance the estimates that are consistent for several model orders. These are the ones that are associated with physical modes. The automation of the identification requires the development of algorithms to automate the analysis of these diagrams. The methodology described in the present paper to achieve that goal takes profit from a hierarchical clustering algorithm.

Hierarchical clustering algorithms are based on the construction of a hierarchy of a treelike structure. At the beginning, each object is considered a cluster. In subsequent steps, the two closest clusters (or individuals) are combined into a new aggregate cluster, thus reducing the number of clusters by one in each step. Eventually, all individuals are grouped into one large cluster (Fig. 1). The implementation of the hierarchical algorithms is composed of the following main steps: calculation of the similarity between every pair of objects in the dataset, linking of the objects in a hierarchical tree and, finally, definition of a rule to cut the hierarchical tree at a certain level, assigning all the objects of each branch to a single cluster (Hair *et al.* 1998).

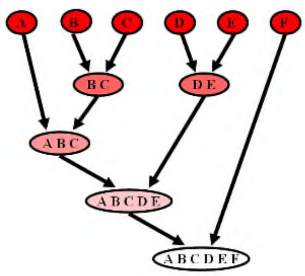


Fig. 1 Hierarchical clustering

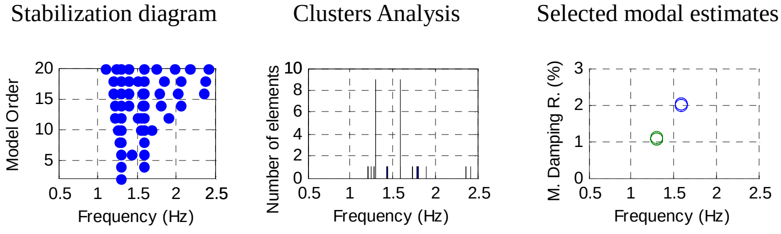


Fig. 2 Steps of the proposed methodology for automated OMA

In the context of the analysis of the data present in stabilization diagrams, in a first step, the similarity between all the pairs of estimated modes is calculated. The proposed methodology relies on a similarity measure that is based on the estimates of the natural frequencies and mode shapes, parameters that can really distinguish two modes. In this way, the distance between two mode estimates (i and j) is calculated with the following formula

$$d_{i-j} = \left| \frac{f_i - f_j}{\frac{1}{2}\left(f_i + f_j\right)} \right| + \left(1 - MAC_{i,j}\right) \tag{1}$$

where f_i is the natural frequency of the mode estimate *i* and $MAC_{i,j}$ is the Modal Assurance Criterion (Allemang and Brown 1982) between the mode shapes of the estimates *i* and *j*. If the distance between the mode estimates is short, that means both estimates present similar natural frequencies and mode shapes. Therefore, they are probably representing the same physical mode, and so they should be included in the same cluster. The distance between two clusters is equal to the smallest distance between objects inside the two clusters (calculated with Eq. (1)).

These rules permit the construction of the hierarchical tree. The next step is the selection of the tree cut level. The criterion used to prone the branches of the hierarchical tree consists of imposing a maximum limit for the distance between any point and its closest point of the same cluster.

After this step, several groups of modal estimates are obtained. It is now important to distinguish the groups that contain estimates associated with physical modes from the ones that contain numerical or spurious estimates. This separation is usually easy, because the estimates associated with physical modes are very consistent for models with different orders. Therefore, the groups that contain estimates of physical modes present a much higher number of members than the groups that contain spurious estimates, which present a higher scatter between models of different orders. Normally, the number of physical modes (n_m) expected in the frequency range of analysis can be anticipated (for instance by a simple preliminary frequency domain analysis), thus the n_m groups with more elements are selected. Fig. 2 illustrates the application of the algorithm to simulated data of a structure with two degrees of freedom (Magalhães *et al*. 2010).

Since modal damping estimates were not yet taken into account, in the modal estimates stored in the groups associated with physical modes, some extreme values of modal damping ratios are expected. These are removed by a classical outliers analysis (Johnson and Wichern 1992) performed within all selected clusters. However, taking into account the relatively small number of observations inside each cluster, more sophisticated outlier analysis can be envisaged (Yuen and

Mu 2012).

The final outputs of the proposed methodology are the average values of the modal parameters (natural frequency, modal damping ratio and mode shape) corresponding to the estimates that belong to the same selected cluster. Further details of the algorithm can be found in (Magalhães *et al*. 2009).

In the context of a dynamic continuous monitoring program, the modal parameters identified in each setup have to be compared with the ones identified in other setups. Therefore, a procedure to link the modal parameters identified from each dataset that are associated with the same physical mode is needed. This can be achieved by comparing each new group of estimates with a set of reference values. In this way, in the adopted methodology, the new estimate for each reference mode is chosen, using the MAC ratio as selection criterion, from a group composed by all the selected clusters mean estimates that have a natural frequency that does not differ more than a predefined percentage from the reference value. The estimate is only accepted if the MAC ratio is higher than a predefined threshold.

At the end of this section, it is relevant to refer that the previously described algorithms were integrated in a software for on-line continuous dynamic monitoring called DynaMo. This software comprehends also tools to manage all the data files created at the monitored structure, statistical tools to remove the environmental and operation effects on the automatically identified modal parameters, the construction of control charts for the detection of abnormal structural behaviors and graphical tools to display the continuously generated results (Magalhães *et al*. 2012).

3. Ting Kau Bridge and its monitoring system

The Ting Kau Bridge in Hong Kong is one of the few multi-span cable-stayed bridges and one of the longest cable-stayed bridges in the world. The bridge comprehends three slender single-leg towers (Ting Kau Tower, Central Tower and Tsing Yi tower) with heights of 170 m, 194 m and 158 m, two main spans of 448 m and 475 m (see Fig. 3) and two side spans of 127 m each.

The bridge deck is separated into two carriageways, each one being supported by two longitudinal steel girders placed along the carriages edges and linked with steel cross girders and a concrete slab on top. The two carriages present a gap of 5.2 m and are connected by cross girders at 13.5 m intervals. The deck is transversely supported at all the towers, in the northern end pier (left side of Fig. 3) and in the southern Tsing Yi abutment (right side of Fig. 3).

The deck is supported by 384 stay cables organized in four planes. Additionally, the three towers are stabilized by cables in the transverse direction and the central tower is also stabilized in the longitudinal direction by two sets of four longitudinal cables that connected the top of the tower with the deck in the vicinity of the side towers (Bergermann and Schlaich 1996).

After the completion of the bridge construction in 1999, a sophisticated long-term monitoring system, called Wind and Structural Health Monitoring System (WASHMS), was devised by the Highways Department of the Hong Kong SAR Government to monitor the structural health and performance of the bridge (Ko and Ni 2005). This monitoring system includes 24 uni-axial accelerometers, 20 bi-axial accelerometers, 1 tri-axial accelerometer, 7 anemometers (ultrasonic-type and propeller-type anemometers), 2 displacement transducers, 83 temperature sensors, 88 strain gauges and a weigh-in-motion sensing system.

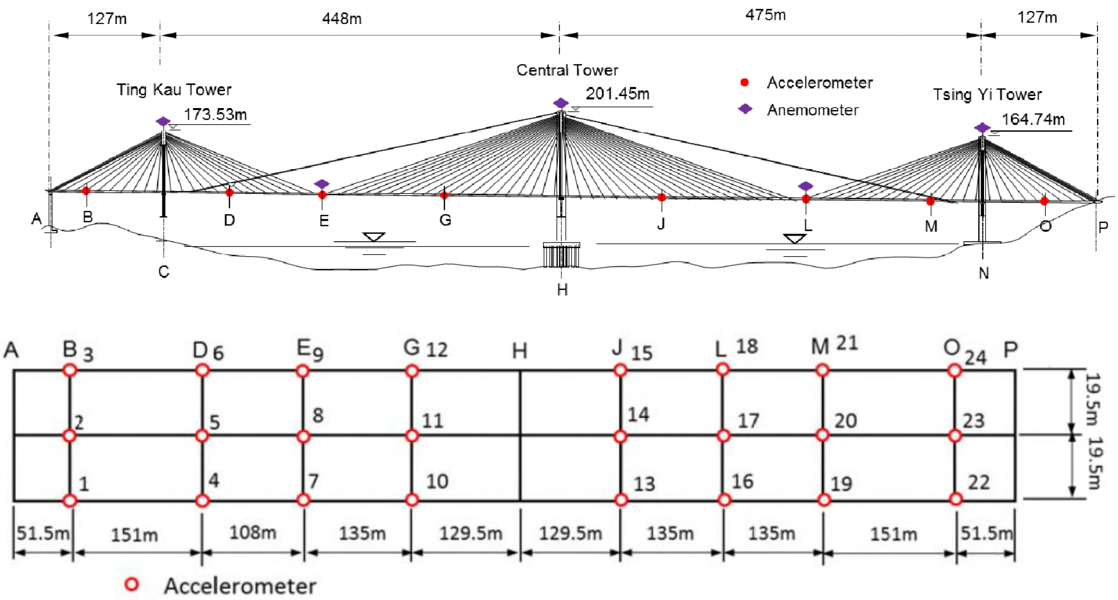


Fig. 3 Deployment of accelerometers and anemometers on Ting Kau Bridge Vertical measurements: 1, 3, 4, 6, 7, 9, 10, 12, 13, 15, 16, 18, 19, 21, 22, 24 Horizontal measurements: 2, 5, 8, 11, 14, 17, 20, 23

In the present study, only the data measured by the accelerometers installed at the deck is considered. The layout of these accelerometers is shown in Fig. 3. The accelerometers (2, 5, 8, 11, 14, 17, 20, and 23) installed in the center of the deck measure transverse accelerations, while the others measure vertical accelerations (Ni *et al*. 2015).

4. Preliminary analysis of the training data

The training data contains the ten data samples characterized in Tables 1 and 2. All of them have a duration of 1 hour and were collected with a sampling frequency of 25.6 Hz. These data samples have been previously analyzed in the paper (Ni *et al.* 2015). Since, as also shown in this paper, the most relevant modes present natural frequencies lower than 0.5 Hz, before being processed the acceleration time series were low-pass filtered with a cutting frequency of 0.512 Hz and down-sampled to a sampling frequency of 1.28 Hz. Since the analysis is focused on a narrower frequency range, this operation makes the data processing for automated identification of the modal parameters simpler and faster.

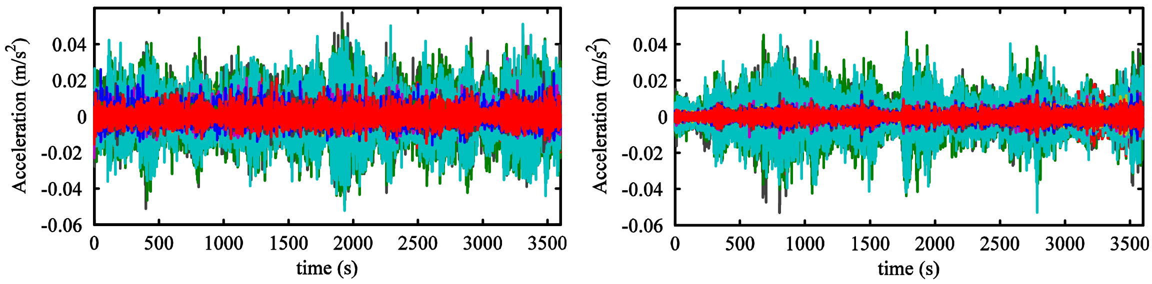
Fig. 4 shows two sets of acceleration time series, one associated with a period of one hour with a 2 m/s mean hourly wind speed and another recorded during the Maggie typhoon, presenting a mean hourly wind speed of about 12 m/s. Despite the very significant difference in terms of mean wind speed, the amplitude of the deck vibrations, considering the contribution of the frequencies between 0 and 0.625 Hz is similar.

Table 1 Six data samples selected under weak wind conditions

Sample	Time and date	Mean hourly wind speed (m/s)
Sample 1	15:00-16:00, 28 Dec 1999	2.00
Sample 2	15:00-16:00, 18 Fev 1999	3.40
Sample 3	15:00-16:00, 01 Mar 1999	3.34
Sample 4	15:00-16:00, 21 Jun 1999	3.41
Sample 5	15:00-16:00, 24 Jul 1999	6.17
Sample 6	15:00-16:00, 12 Aug 1999	4.20

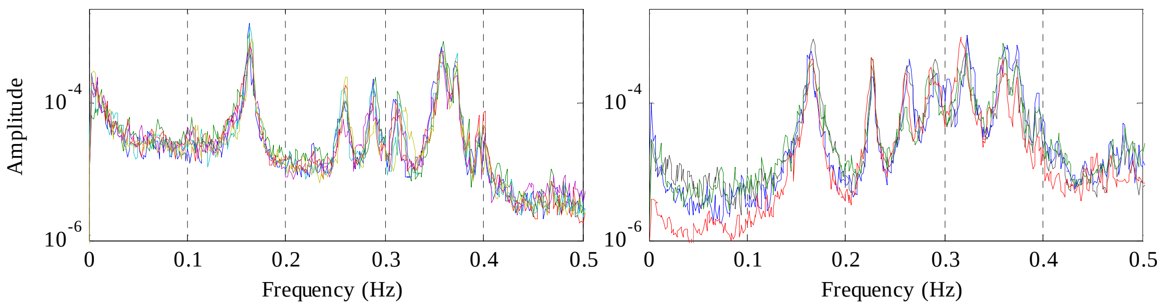
Table 2 Four data samples selected under typhoon conditions

Typhoon	Time and date	Mean hourly wind speed (m/s)
Maggie	03:00-04:00, 07 Jun 1999	12.11
Sam	02:00-03:00, 23 Aug 1999	15.62
York	06:00-07:00, 01 Sep 1999	21.72
York	15:00-16:00, 21 Sep 1999	15.91



(a) Sample 1 (low wind speeds) (b) Maggie Typhoon

Fig. 4 Acceleration time series of the 24 channels (larger values associated with vertical accelerations)



(a) Low wind speed (Sample 1 to Sample 6) (b) High wind speed (Sample 7 to Sample 10)

Fig. 5 First singular values spectra of the data samples

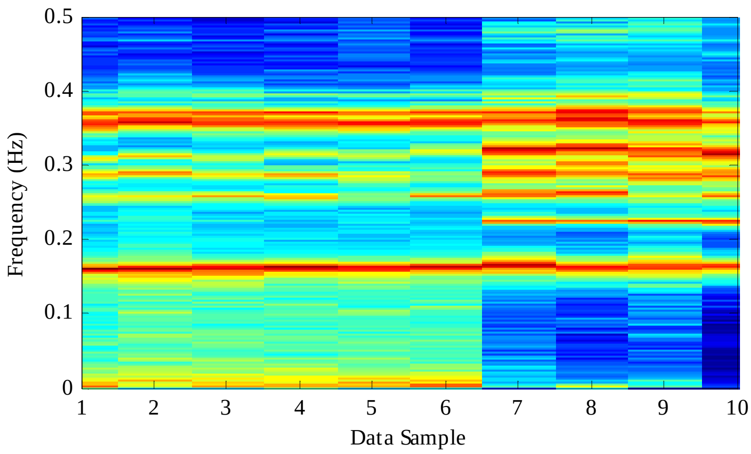


Fig. 6 Color map with the first singular values spectra, 10 training datasets

Fig. 5 shows the first singular values of the spectra matrices associated with the 10 data samples under study (one plot for each setup), organized according to the mean wind speed. In the analysis of both groups of singular value spectra, it is clear the appearance of a new resonant frequency around 0.226 Hz in the data samples associated with higher wind speeds. This can be further confirmed with a top view of these singular values spectra, as presented in Fig. 6, where the index in the horizontal axis represents the data sample number (for the data samples recorded during the occurrence of typhoons it is used the order of appearance in Table 2, from sample 7 to sample 10) and hotter colors are adopted for higher amplitudes.

5. Identification of the modal properties using the training data

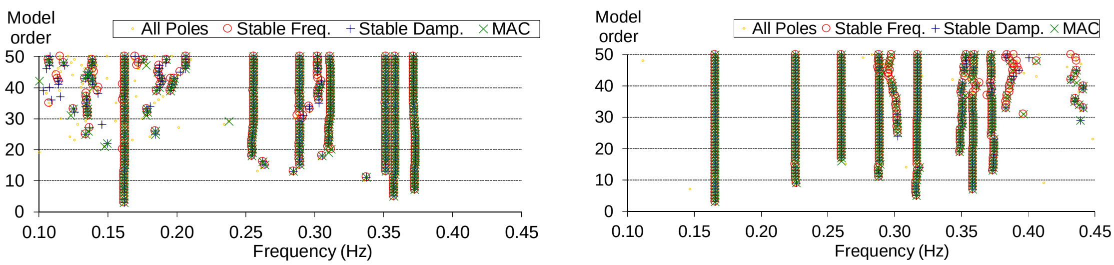
In this section, after the preliminary analysis previously shown, the algorithms presented in section 2 are applied to the data samples of the training data. As algorithm for output only modal identification, it was selected the SSI-COV method (Peeters and Roeck 1999). As usual, the parameters of the identification algorithm were defined after testing different alternatives and taking profit from the experience on the possessing of similar data. The length of the correlation functions was set to 20 points (15.6 seconds), all the measured points were selected as references and model orders between 2 and 50 were adopted.

Fig. 7 shows two examples of the obtained stabilization diagrams. In the analyzed frequency band, for low wind speeds, there are seven clear vertical alignments of stables poles. As expected, after the preliminary analysis in the frequency domain, in the presence of higher wind speeds a new alignment of stables poles appears. With the use of models with higher orders more modes can be identified, but the physical modes with an important contribution for the bridge response to ambient excitation are the ones clearly identified in the presented stabilization diagrams and also associated with the most relevant peaks of the depicted spectra. In the context of a structural health monitoring program, the analysis should be focused in the modes that are well identified, since the modal parameter estimates of these modes are more accurate and therefore more adequate to draw conclusions about the structural performance of the bridge.

In order to avoid a subjective selection of the poles that better represent each physical mode and also to make the processing faster and adequate to be applied in the context of the data processing of a large number of datasets collected by a continuous dynamic monitoring system, the stabilization diagram associated with each data sample were automatically post-processed using the tools described in section 2.

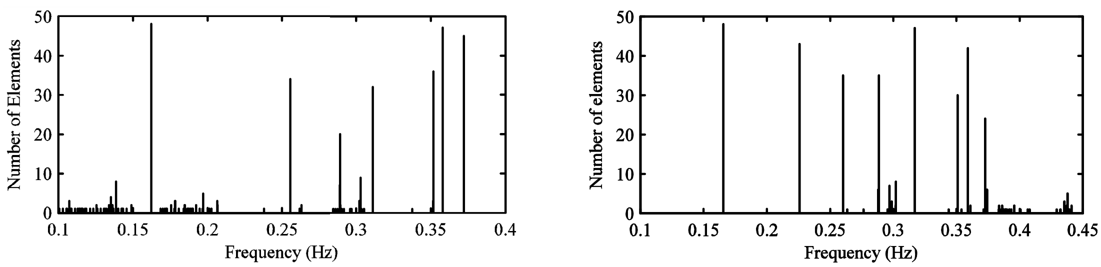
Fig. 8 shows the results of the cluster analysis applied to the previously presented stabilization diagrams, using a maximum distance for the poles inside the same cluster of 0.02 (calculated using Eq. (1)). Each cluster is represented by a vertical line with a height proportional to the number of points contained in the cluster and positioned in the horizontal axis in the mean natural frequency of the poles included in the cluster. It is evident the correspondence between the clusters containing a larger number of elements with the alignments of stable poles that stand out in the stabilization diagrams.

Fig. 9 characterizes the natural frequency and the modal damping ratio of the poles contained in the clusters with a larger number of elements (7 clusters for sample 1 and 8 clusters for sample 10). Since, some modes are identified with almost all model orders, some of these clusters contain more than 40 poles. After the elimination of outliers, quite consistent estimates are obtained for the modes inside the same cluster. In order to increase the chances of identifying in all the setups all the modes with important contributions for the measured responses to ambient excitation, the algorithm for automated identification was configured to select in each setup the 10 clusters with more elements (two or three more clusters than expected modes).



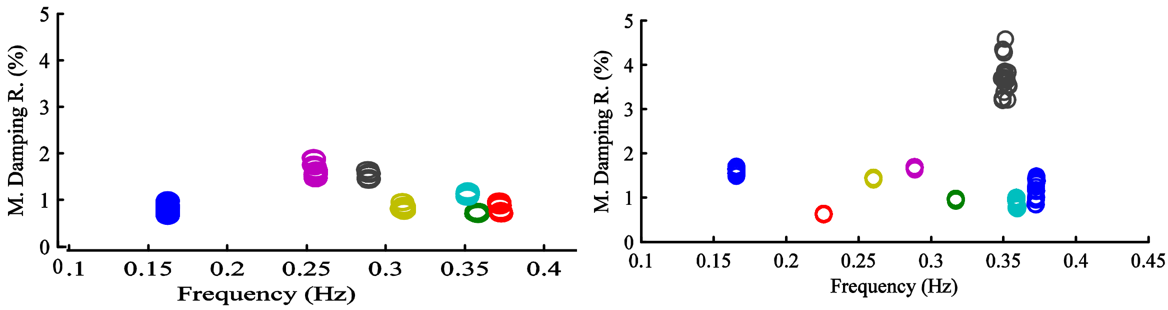
(a) Low wind speed (Sample 1) (b) High wind speed (York Typhoon, sample 10)

Fig. 7 Stabilization diagrams produced by the SSI-COV method



(a) Low wind speed (Sample 1) (b) High wind speed (York Typhoon, sample 10)

Fig. 8 Cluster analysis



(a) Low wind speed (Sample 1) (b) High wind speed (York Typhoon, sample 10)

Fig. 9 Modal parameters of the poles inside the clusters with more than 10 elements

With the purpose of linking the estimates obtained from each data sample, the natural frequencies and mode shapes estimates obtained with data sample 10 were used as references. These are characterized in Fig. 10, the modes shapes being either vertical bending modes or lateral bending modes with torsion of the deck. The presented mode shapes should be analyzed with some care. The modal ordinates are connected with lines, but between modal ordinates a completely different shape can be present. As an example, in the 4^{th} mode the estimated modal ordinates suggest that the modal ordinate in the connection of the deck with the central pier is close to zero, but this is not reflected in the presented mode shape, because experimental results are not available for that point.

The second bending mode, the one that is only excited with high wind speeds, presents a well-defined shape, with consistent values in the upstream and downstream side of the deck, with a torsion movement that is compatible with the lateral bending, but its shape suggests relevant lateral movements in the connection of the deck with the central pier. Taking into account the lateral connection of the deck with the central tower and the tower stiffness at the deck level, this deserves further investigation.

Fig. 11 shows the natural frequencies identified with the ten data samples. It is confirmed that the second mode is only identified in the data samples recorded during the occurrence of typhoons. The 6^{th} mode is only identified in data samples 5, 7, 9 and 10. The remaining setups also present three clusters of stable poles in the frequency range 0.35 Hz – 0.40 Hz, but two of them are associated with mode shapes similar to the one of mode 7 represented in Fig. 10.

The modal damping ratios identified for modes 1 and 2 are characterized in Fig. 12. The first mode presents ratios between 1 and 2%, except in the setup with higher wind speeds where it reached values around 3%. The second mode presents stable values around 0.6%. In order to establish valid correlations between modal damping ratios and wind characteristics a much larger number of data samples is needed. The mode shapes identified in each setup are quite consistent, being the MAC ratios with regard to the reference values generally higher than 0.9 for the modes that are clearly present in all the setups.

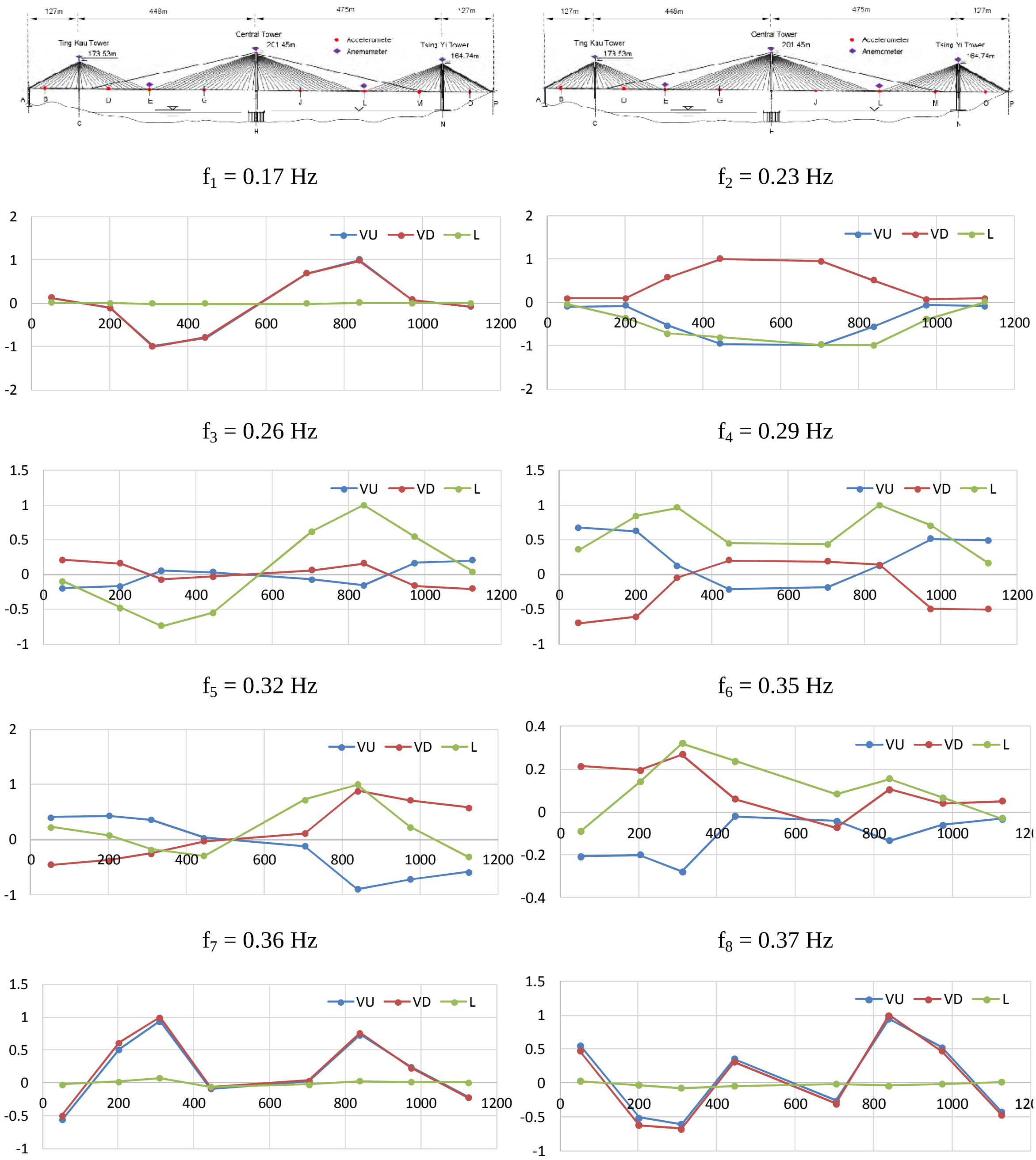


Fig. 10 Mode shapes of the modes identified with Sample 10, recorded during the York Typhoon. VU – vertical upstream, VD – vertical downstream, L – lateral.

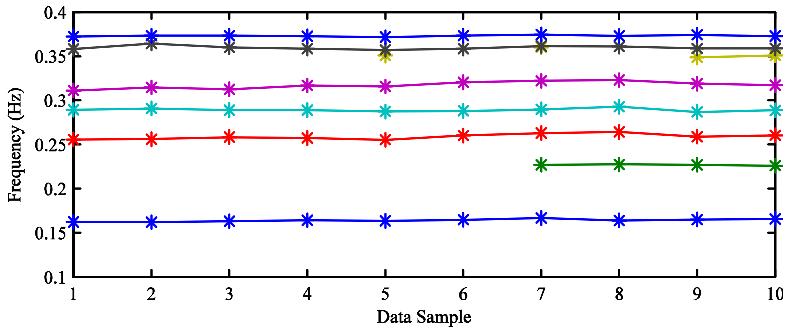


Fig. 11 Natural frequencies automatically identified

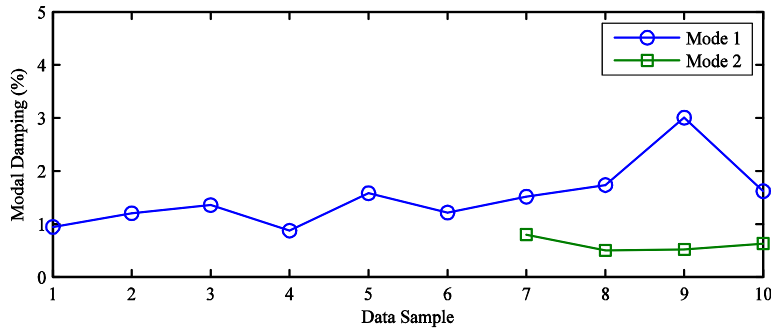


Fig. 12 Modal damping ratios of modes 1 and 2 automatically identified

6. Identification of the modal properties using the benchmark test blind data

This section comprehends the analysis of other six data samples with the duration of one hour, associated with unknown wind characteristics. One of the goals of the benchmark study it the characterization of the wind characteristics associated with these datasets.

In a first instance, a color map with the singular value spectra was calculated. The analysis of the color map presented in Fig. 13 shows three groups of data with distinct behavior: the first two data samples present singular values spectra with a very well defined peak around the frequency 0.23 Hz, the spectra associated with the last two data samples do not present any peak in the frequency interval 0.2 – 0.25 Hz, and finally, the data samples 3 and 4 exhibit spectra with a diffuse peak in the referred frequency interval.

From the comparison of this color map with the one presented in Fig. 6, it is already possible to infer that the last two data samples were recorded with low wind speeds (the second mode is not present), the first two were probably collected during the occurrence of typhoons and the data samples three and four were probably collected under intermediate conditions, meaning a mean wind speed between 6 and 12 m/s.

After this preliminary analysis, these six data samples were automatically processed using the same routines and input parameters that were adopted with the training data.

Fig. 14 shows the identified natural frequencies. The identification of the second mode only on the first four setups corroborates the analysis performed in the preliminary processing. The 6th

vibration mode is only identified in data sample 6. In the training data this mode was also only identified in some setups. Surprisingly, mode 3 is not identified in data sample 3. This mode was identified in all the setups of the training data.

The modal damping ratios of modes 1 and 2 are presented in Fig. 15. With regard to mode 1, the majority of the values are between 1 and 2%. Still, the setups that were associated with higher wind speed present the lowest values, which seems to indicate it is not possible to establish a positive correlation between the damping of this mode and the wind speed. In the data samples 1 and 2, mode 2 presented values consistent with the ones observed in training data recorded during the occurrence of typhoons, which supports the hypothesis that the these two data samples were recorded in similar conditions. The training data did not include data samples with intermediate wind speeds (between 6 and 12 m/s). So, the damping ratios associated with data samples 3 and 4 cannot be validated. Nevertheless, it should be referred that damping estimates present a considerable scatter and therefore definitive conclusions are only possible with the analysis of a very large number of setups.

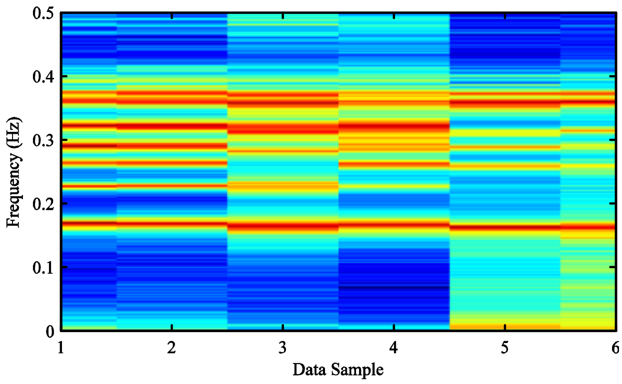


Fig. 13 Color map with the first singular value spectra, 6 blind datasets

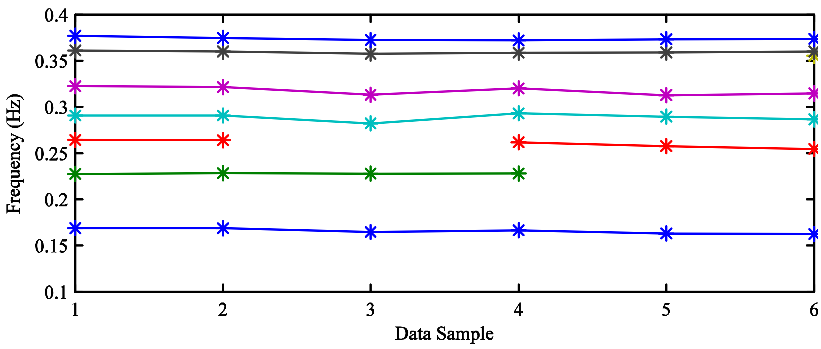


Fig. 14 Natural frequencies automatically identified, benchmark test blind datasets

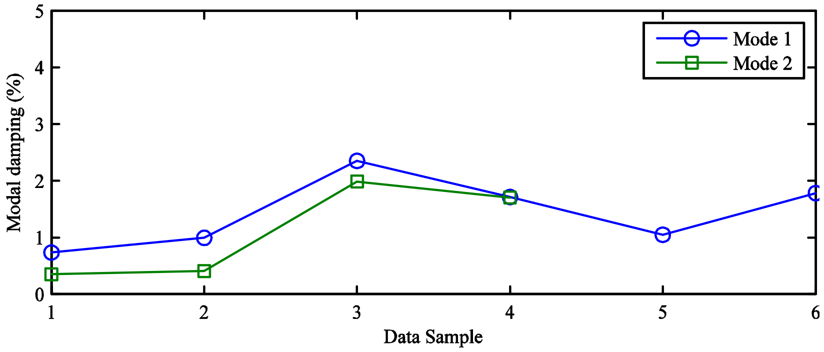


Fig. 15 Modal damping ratios of modes 1 and 2 automatically identified, benchmark test blind datasets

7. Conclusions

This work was developed in the context of a benchmark study promoted by the Hong Kong Polytechnic University whose main goal was to study the identification of the bridge modes of vibration under different wind conditions. In this contribution, the tools developed at ViBest/FEUP for automated data processing of setups collected by dynamic monitoring systems were applied to the available data samples. The obtained results demonstrate that the proposed algorithms succeeded to automatically identify the modes with relevant contribution for the bridge response under different wind conditions. It is relevant to stress that, since the applied tools were designed to be applied in the context of dynamic monitoring systems, only the estimates with reliable results were kept, which means that some modes were not characterized in some setups. Especially, the second vibration mode is just automatically identified in the setups with relevant wind excitation. It is possible to identify this mode with manual analysis, but the obtained modal parameters are not reliable.

After the processing of the training data, it was possible to draw conclusions about the wind speeds presumably associated with the blind data samples.

At the end, it should be referred that, with the proposed processing tools, it is possible to very efficiently process a large number of datasets. This would permit to draw stronger conclusions about the effect of the wind characteristics on the bridge modal properties, since that with a large number of results the uncertainty always associated with the identification techniques could be minimized.

References

Allemang, R.J. and Brown, D.L. (1982), "A Correlation coefficient for modal vector analysis", *Proceedings of* the *IMAC 1, International Modal Analysis Conference*, Orlando, Florida, USA.

Bergermann, R. and Schlaich, M. (1996), "Ting Kau Bridge, Hong Kong", *Struct. Eng. Int.*, **6**(3), 152-154.

Hair, J., Anderson, R., Tatham, R. and Black, W. (1998), *Multivariate Data Analysis*, Prentice Hall.

Johnson, R.A. and Wichern, D.W. (1992), *Applied Multivariate Statistical Analysis*, Prentice Hall.

Ko, J.M. and Ni, Y.Q. (2005), "Technology developments in structural health monitoring of large-scale bridges", *Eng. Struct.*., **27**(12), 1715-1725.

Magalhães, F., Cunha, Á. and Caetano, E. (2009) “Online automatic identification of the modal parameters of a long span arch bridge”, *Mech. Syst. Signal Pr.*, **23**(2), 316-329.
Magalhães, F. (2010), *Operational Modal Analysis for Testing and Monitoring of Bridges and Special Structures*, Ph.D. Dissertation, Faculty of Engineering, University of Porto, Porto, Portugal.
Magalhães, F., Amador, S., Cunha, A. and Caetano, E. (2012), “Dynamo - software for vibration based structural health monitoring”, *Proceedings of the 6th Int. Conf. on Bridge Maintenance, Safety and Management, IABMAS 2012*, Stresa, Lake Maggiore, Italy.
Ni, Y.Q., Xia, Y., Lin, W., Chen, W.H. and Ko, J.M. (2012), “SHM benchmark for high-rise structures: a reduced-order finite element model and field measurement data”, *Smart Struct. Syst.*, **10**(4), 411-426.
Ni, Y.Q., Wang, Y.W. and Xia, Y.X. (2015), “Investigation of mode identifiability of a cable-stayed bridge: comparison from ambient vibration responses and from typhoon-induced dynamic responses”, *Smart Struct. Syst.*, **15**(2), 447-468.
Peeters, B. and De Roeck, G. (1999), “Reference-based stochastic subspace identification for output-only modal analysis”, *Mech. Syst. Signal Pr.*, **13**(6), 855-878.
Yuen, K.V. and Mu, H.Q. (2012), “A novel probabilistic method for robust parametric identification and outlier detection”, *Probabilist. Eng. Mech.*, **30**, 48-59