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Vibration control of hysteretic base-isolated structures: an LMI approach

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Abstract. Seismic isolation systems are essentially designed to preserve structural safety, prevent occupants injury and properties damage. An active saturated LMI-based control design is proposed to attenuate seismic disturbances in base-isolated structures under saturation actuators. Using a mathematical model of an eight-storied building structure, an active control algorithm is designed. Performance evaluation of the controller is carried out in a simplified model version of a benchmark building system, which is recognized as a state-of-the-art model for numerical experiments of structures under seismic perturbations. Experimental results show that the proposed algorithm is robust with respect to model and seismic perturbations. Finally, the performance indices show that the proposed controller behaves satisfactorily and with a reasonable control effort.

Keywords: vibration control; base-isolated structures; active control; linear matrix inequalities; saturation

1. Introduction

The combination of passive base isolators and feedback controllers (applying forces to the base) has been proposed in recent years with the objective of maintaining the seismic response of structures within safety, service and comfort limits. Some groups have proposed active feedback systems, for instance Barbat *et al*. (2010), Pozo *et al*. (2009) and Pozo *et al*. (2006). More recently, semiactive controllers have been proposed in the same setting with the hope of gaining advantage from their implementation using, for instance, magnetorheological (MR) dampers (Bahar *et al*. 2010, Chang *et al*. 2013, Ghaffarzadeh 2013, Luo *et al*. 2001, Ramallo *et al*. 2002, Rodríguez *et al*. 2012, Yang and Agrawal 2002).

The basic idea of base isolation is to make the structure behave like a rigid body through a certain degree of decoupling from the ground motion. In this way it is possible to absorb part of the energy induced by the earthquake and to diminish the fundamental frequency of structural

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vibration to a value lower than the dominant energy frequencies of earthquake ground options. As a consequence, both the relative displacements of the structure with respect to the base (damage source) and the absolute accelerations it undergoes (which endanger human comfort and the safety of installations) can be simultaneously reduced. The idea of adding a feedback control is based on the premise that a control action is to be applied at the base with force magnitudes that are not excessive due to the high flexibility of the isolators. The main benefit of the inclusion of the control is that the assistance of such a force can help prevent large displacements of the base isolator, which could endanger the integrity of the scheme, but it may also introduce an additional effect reducing the interstory drifts, which are already small due to the effect of the isolator. This may be useful, particularly for structures containing sensitive equipment or important resources, such as hospitals, public services, computer facilities, etc.

In this paper, an active saturated Linear Matrix Inequality (LMI)-based controller for seismic attenuation is developed and applied to a hysteretic base-isolated eight-storied building, similar to existing building in Los Angeles (California). In recent years, LMI techniques have become quite popular in control design (Pujol and Acho 2010, Tang and Yu 2011). The main reason for this popularity has been the discovery of interior point methods for convex programming that allows the numerical solution of LMIs in polynomial time. It has been acknowledged that many control problems can be formulated in terms of LMIs (Apkarian *et al*. 2001, Boyd *et al*. 1994, Ji *et al*. 2009, Oliveria and Peres 2006). Moreover, saturation can produce limit cycles even in linear stable systems (Vincent and Grantham 1997). These limit cycles can induce internal perturbation that can raise instability in structural control systems under seismic perturbation. Because seismic perturbations are unknown, but bounded, there always exists the risk that the actuators reach their maximal available force producing saturation (Pnevmatikos and Gantes 2011). So, the controller has to be well designed to display good performance under seismic perturbation and under saturation effect. Control of systems with saturation actuators have been extensively studied (Hu and Lin 2001, Lan and Wang 2010) and all of this knowledge can also be applied to the field of structural control. Furthermore, the maximal available energy of the actuators can be *a priori* used for control design, which is intuitively correct in civil engineering. The LMI controller design proposed in this work is based on the results obtained in Nguyen and Jabbari (1999), giving an innovative control algorithm for seismic disturbance attenuation in structures employing saturating actuators. The design is based on a simplified model version of the benchmark building system (Narasimhan *et al*. 2006), which is recognized as a state-of-the-art model for numerical experiments of structures under seismic perturbations. Besides, our controller is robust against model and saturation effect, as can be appreciated in the numerical experiments. Performance of the proposed controller, for seismic attenuation, are evaluated by numerical simulations using seven different earthquakes and eight evaluation criteria, such as the peak base shear, the peak base displacement or the peak absolute floor acceleration.

This paper is structured as follows. The hysteretic base-isolated structure to be controlled is described in Section 2. The saturated LMI-based controller is developed in Section 3. Numerical simulations to analyze the performance of the proposed controller are presented in Section 4. Final comments are given in Section 5.

2. System description

Consider a nonlinear base-isolated building structure as shown in Fig. 1. For control design, a dynamic model composed of two coupled subsystems, namely, the main structure or superstructure (S_r) and the base isolation (S_c) is employed

$$S_r : \mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{J}\ddot{x}_g + \bar{\mathbf{C}}\dot{\tilde{\mathbf{r}}} + \bar{\mathbf{K}}\tilde{\mathbf{r}}, \tag{1}$$

$$S_c : m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0 x_0 = c_1\dot{r}_1 + k_1 r_1 - \Phi(x_0,t) - m_0\ddot{x}_g + u, \tag{2}$$

where $\ddot{x}_g$ is the absolute ground acceleration, $\mathbf{x} = \left[x_1, x_2, \ldots, x_8\right]^T \in \mathbb{R}^8$ represents the horizontal displacements of each floor with respect to the ground. The mass, damping and stiffness of the i-th storey are denoted by m_i, c_i and k_i, respectively, $\tilde{\mathbf{r}} = \left[r_0, \mathbf{r}^T\right]^T \in \mathbb{R}^9$ and $\mathbf{r} = \left[r_1, r_2, \ldots, r_8\right]^T \in \mathbb{R}^8$, represents the horizontal displacements of the i-th floor relative to the (i-1)–th storey. The base isolation is described as a single degree of freedom with horizontal displacement x_0. It is assumed to exhibit a linear behavior characterized by mass, damping and stiffness m_0, c_0 and k_0, respectively, plus a nonlinear behavior represented by a hysteretic restoring force $\Phi(x_0,t)$. The matrices $\mathbf{M}, \mathbf{C}$, $\mathbf{K}$, $\mathbf{J}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}$ of the structure have the following form

$$\begin{aligned}
\mathbf{M} &= \mathrm{diag}(m_1, m_2, \ldots, m_8) \in \mathbf{R}^{8\times 8},\\
\mathbf{C} &= \mathrm{diag}(c_1, c_2, \ldots, c_8) \in \mathbf{R}^{8\times 8},\\
\mathbf{K} &= \mathrm{diag}(k_1, k_2, \ldots, k_8) \in \mathbf{R}^{8\times 8},\\
\mathbf{J} &= [1, \ldots, 1] \in \mathbf{R}^{8},
\end{aligned}$$

$$\bar{\mathbf{C}} = \left(\bar{c}_{ij}\right) \in \mathbf{R}^{8\times 9}, \quad \bar{c}_{ij} = \begin{cases} c_i, & i \le j \\ c_{i+1}, & j - i = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\bar{\mathbf{K}} = \left(\bar{k}_{ij}\right) \in \mathbf{R}^{8\times 9}, \quad \bar{k}_{ij} = \begin{cases} k_i, & i \le j \\ k_{i+1}, & j - i = 2 \\ 0, & \text{otherwise} \end{cases}$$

The restoring force Φ can be represented by the Bouc-Wen model (Ikhouane *et al.* 2005, Ikhouane and Rodellar 2007) in the following form

$$\Phi(x_0,t) = \alpha K x_0(t) + (1-\alpha) D K z(t) \tag{3}$$

$$\dot{z} = D^{-1}(\bar{A}\dot{x}_0 - \beta|\dot{x}_0||z|^{n-1} z - \lambda \dot{x}_0 |z|^n) \tag{4}$$

where $\Phi(x_0,t)$ can be considered as the superposition of an elastic component $\alpha K x_0$ and a hysteretic component $(1-\alpha)Dkz(t)$, in which the yield constant displacement is $D>0$ and $\alpha \in (0,1)$ is the post- to pre-yielding stiffness ratio; $n \geq 1$ is a scalar that governs the smoothness of the transition from elastic to plastic response and $K>0$. The Bouc-Wen parameters α, k, D, $\bar{A}$, β, n and λ are taken from Narasimhan *et al*. (2006) for simulations and are also defined in Table 3.

Table 1 Model coefficients of the base-isolated structure

	mass (kg)	stiffness (N/m)	damping (Ns/m)
base	3565.7	919422	101439
1st floor	2580	12913000	11363
2nd floor	2247	10431000	10213
3rd floor	2057	7928600	8904
4th floor	2051	5743900	7578
5th floor	2051	3292800	5738
6th floor	2051	1674400	4092
7th floor	2051	496420	2228
8th floor	2051	49620	704

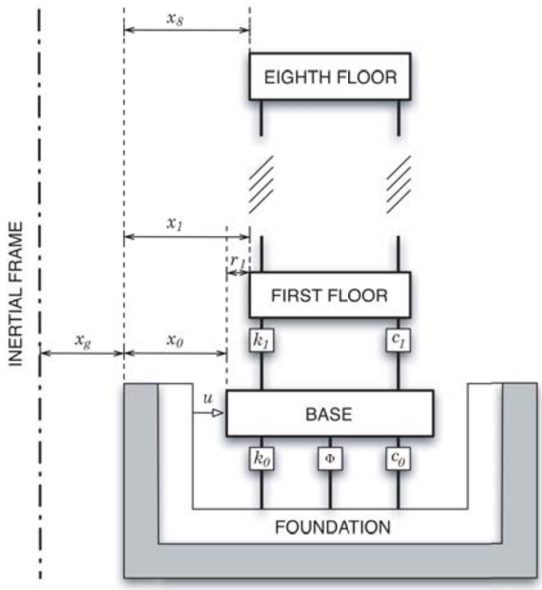

Fig. 1 Base-isolated structure with active control

Finally, u is the control force supplied by an appropriate actuator.

It is well accepted that the movement of the superstructure S_r is very close to the one of a rigid body due to the base isolation (Skinner *et al*. 1992). Then it is reasonable to assume that the interstory motion of the building will be much smaller than the relative motion of the base (Luo *et al*. 2001). Table 1 presents the model coefficients of the base-isolated structure. Consequently, the following simplified equation of motion of the base can be used in the subsequent controller design

$$\tilde{S}_c : m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0x_0 = -\Phi(x_0,t) - m_0\ddot{x}_g + u. \tag{5}$$

The feasibility of this simplification is justified in a more detailed way in Luo *et al*. (2001) and Pozo *et al*. (2008).

Eq. (5) together with (3) can be expressed in matrix form as

$$\begin{aligned}\begin{bmatrix} \dot{x}_0 \\ \ddot{x}_0 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\dfrac{k_0+\alpha K}{m_0} & -\dfrac{c_0}{m_0} \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ -\dfrac{(1-\alpha)DK}{m_0} & -1 \end{bmatrix} \begin{bmatrix} z \\ \ddot{x}_g \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_0} \end{bmatrix} u \\ &= Ax + B_1w + B_2u,\end{aligned} \tag{6}$$

where $x = \begin{bmatrix} x_0 & \dot{x}_0 \end{bmatrix}^T$ is the position and velocity ground displacement. The hysteretic component z and the absolute acceleration $\ddot{x}_g$ are the components of the vector w. The dynamic of the hysteretic variable $z(t)$, defined in Eq. (4), is not considered in Eq. (6). The hysteretic component $z(t)$ is included in the perturbation vector w, and then a linear expression of the structural system is obtained and used to prove stability (Sadek *et al*. 2003). Therefore, the nonlinear model (2)-(4) is considered for simulations.

3. Control design

3.1 Controller synthesis

The key of this Section is to obtain a controller through the solution of a linear matrix inequality (LMI) optimization problem.

To design a controller to achieve an optimal performance objective and to guarantee the inputs to remain less than or equal to the saturation limits, we recover the matrix expression of the Eq. of motion (5) in Eq. (6) and we define our control objective

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u,\\ z_\infty &= C_\infty x,\end{aligned} \tag{7}$$

where $x = \begin{bmatrix} x_0 & \dot{x}_0 \end{bmatrix}^T \in \mathbb{R}^2$ is the state space vector, composed by the base displacement and velocity; $w \in \mathbb{R}^2$ is the disturbance input; $u \in \mathbf{R}$ is the control input; and $z_\infty \in \mathbf{R}$ is the output to be controlled. $A, B_1 \in \mathcal{M}_{2\times 2}(\mathbb{R})$, and $B_2 \in \mathcal{M}_{2\times 1}(\mathbb{R})$, are constant matrices as defined in Eq. (6). The control objective is to find a state feedback saturated controller that guarantees the $\mathcal{L}_2$ gain of γ_∞ from w to z_∞. Since $w = \begin{bmatrix} z & \ddot{x}_g \end{bmatrix}^T$, the control objective is equivalent to the seismic attenuation ($\ddot{x}_g$) and the force mitigation of the nonlinear term of the base isolation restoring force (which depends on $z(t)$). In this paper, the H_∞-performance controlled output $z_\infty \in \mathbf{R}$ is defined by the weighted matrix $C_\infty = \begin{bmatrix} 0.0001 & 1 \end{bmatrix}$. This value of C_∞ is used to give more emphasis to the base velocity rather than to base displacement, because we remark that the internal variable $z(t)$, in Eq. (4), is a function of the velocity.

The control synthesis is based on the results presented by Nguyen and Jabbari (1999), where the level of performance γ_∞ is given. In this reference, a high-gain controller design to improve disturbance attenuation for systems with input saturation is presented. It shows that if some set of LMI's are feasible, then there exists a saturate state feedback controller that guarantees the $\mathcal{L}_2$ gain of fixed γ_∞ from w to z_∞. In the present paper, a modification of this result is considered: the level performance γ_∞ is taken as a variable, solving an optimization problem over γ_∞ and obtaining a result less conservative. This is the main difference with the work by Nguyen and Jabbari (1999), where γ_∞ is fixed. Consider as a design parameters $\bar{\alpha} > 0$ and $\epsilon > 0$ and define the gain $\gamma^\star = \frac{u_{\lim}}{w_{\max}} > 0$, where $u_{\lim}$ is the maximum value of the saturated control, $w_{\max} = \sqrt{z_{\max}^2 + m_0^2 \ddot{x}_G}$ is the maximum magnitude of the perturbation and $\ddot{x}_G = \max_t \{\ddot{x}_g(t)\}$.

Table 2 Parameters and control law

control parameters	$u_{\lim} = 10m_0$	$\delta = 5 \cdot 10^6$	$w_{\max} = \sqrt{2^2 + (8.58m_0)^2}$
control law	$u = -\mathrm{sat}(0.05980x_0 + 0.14892\dot{x}_0)$		
performance level	$\gamma_\infty = 15.8676$		

Table 3 Parameters of the hysteresis model in Eqs. (3) and (4)

$\alpha = 0.5$	$\bar{A} = 1$	$K = 61224.49$ (N/m)	$\beta = 0.5$
$D = 0.0245$ (m)	$\lambda = 0.5$	$n = 2$	

Proposition 1. Consider the system in Eq. (7). Given $\gamma^{\star} = \dfrac{u_{\lim}}{w_{\max}} > 0$, if there exists a positive definite constant matrix $Q > 0$, scalars $\gamma_{\infty} > 0$ and $\rho > 0$, for some given $\bar{\alpha} > 0$ such that the following LMI's are feasible

$$\begin{bmatrix} AQ + QA^T - \rho B_2 B_2^T & B_1 & QC_{\infty}^T \\ B_1^T & -\gamma_{\infty} & 0 \\ C_{\infty} Q & 0 & -I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} AQ + QA^T + \bar{\alpha} Q - \rho B_2 B_2^T & B_1 \\ B_1^T & -\bar{\alpha} \end{bmatrix} \le 0 \tag{9}$$

$$\begin{bmatrix} 4Q & \rho B_2 \\ \rho B_2^T & \gamma_{\star}^2 \end{bmatrix} > 0 \tag{10}$$

Then, the high-gain nonlinear state feedback controller

$$u = -\mathrm{sat}\left(\frac{\delta \rho}{2} B_2^T Q^{-1} x \right), \tag{11}$$

where

$$\mathrm{sat}(x) = \begin{cases} u_{\lim}, & x > u_{\lim} \\ x, & |x| \le u_{\lim} \\ -u_{\lim}, & x < -u_{\lim} \end{cases}, \tag{12}$$

guarantees quadratic stability with $\mathcal{L}_2$-gain level of $\sqrt{\gamma_{\infty}}$. The term δ in Eq. (11) is any constant larger than one and $u_{\lim}$ is the desired saturation limit.

Proof. The proof is based on results over LMI theory presented by Boyd *et al*. (1994), where it is shown that Eqs. (8) and (9) is a necessary and sufficient condition for the existence of sub-optimal H_{∞} state feedback controller, defined here by the specific structure $u = -\dfrac{\rho}{2} B_2^T Q^{-1} x$ ensuring internal stability. Inequality (10) is a necessary and sufficient condition for $\| u \|_{\infty} \le u_{\lim}$. Then, by Nguyen and Jabbari (1999), Eq. (11) defines a high-gain nonlinear state feedback controller, and the proposition is proved.

Proposition 1 shows that if there exists a constant matrix $Q > 0$ and a nonnegative scalar $\rho > 0$, it is possible to optimize the $\mathcal{L}_2$-gain γ_{∞} solving a set of LMI's. This is the main difference with Nguyen and Jabbari (1999), where the gain level γ_{∞} is given.

3.2 LMI-based control algorithm

In this section, the steps to solve the disturbance attenuation control problem are discussed. The final goal is to find a feasible solution for the gain matrices and to get a bound for the performance criterion.

- Step 1. Verify that (A, B_2) is controllable.
- Step 2. Define the saturation limit $u_{\lim}$ and the largest disturbance amplitude $w_{\max}$. Then, define $\gamma^{\star} = \frac{u_{\lim}}{w_{\max}}$. Define matrix C_{∞}, that is, which state variable has to be controlled.
- Step 3. Fix $\bar{\alpha} > 0$. The choice of this value depends on each problem. We have considered $\bar{\alpha} = 0.01$. Then, for this value:
- Step 4. Solve the LMI system in Eqs. (8)-(10) via LMI optimization on γ_{∞}, with LMI variables $Q > 0$, $\rho > 0$ and $\gamma_{\infty} > 0$.
- Step 5. Is this LMI feasible?
- Step 5.1. No? Then go to Step 3 and change the value of $\bar{\alpha}$. By Step 1 the problem is feasible, but the choice of $\bar{\alpha}$ is crucial.
- Step 5.1. Yes? Then check if the performance level γ_0 can be improved and go to Step 3 and increase $\bar{\alpha}$. If the new γ_{new} is worse, keep $\gamma_{\infty} = \gamma_0$. Otherwise, $\gamma_{\infty} = \gamma_{\text{new}}$.
- Step 6. With γ_{∞} fixed, using Proposition 1, the high-gain nonlinear state feedback controller is defined in Eq. (11).
- Step 7. A simulation is made sweeping through $\delta > 1$.

In Step 5.2, γ_0 represents the $\mathcal{L}_2$ level used to observe the disturbance attenuation performance in the interactive algorithm process. Because in Proposition 1, δ can be any constant greater than one, Step 7 is incorporated to look for, by numerical simulations, a sub-optimal value of δ.

4. Numerical results

The model in Eqs. (1) and (2) is used to design an appropriate control law. The applicability and efficiency of the proposed controller is then shown using a more realistic and complex model through the benchmark building presented by Narasimhan *et al*. (2006). In fact, the design is based on a simplified model version of the benchmark building system (Narasimhan *et al*. 2006), which is recognized as a state-of-the-art model for numerical experiments of structures under seismic perturbations. Table 2 presents the design parameters used to obtain the control law $u(t)$ in (11). The value $u_{\lim}$ is determined based on the specifications of the structure under study. Parameter $\delta > 1$ has to be greater enough to compensate the value $\rho B_2^T Q^{-1}$ in Eq. (11). The maximum values $z_{\max}$ and $w_{\max}$ are defined from the benchmark detailed description. From the procedure in Section 3.2, we derive the robust control $u(t)$ in Eq. (11) and the optimal performance level γ_{∞}, both defined in Table 2.

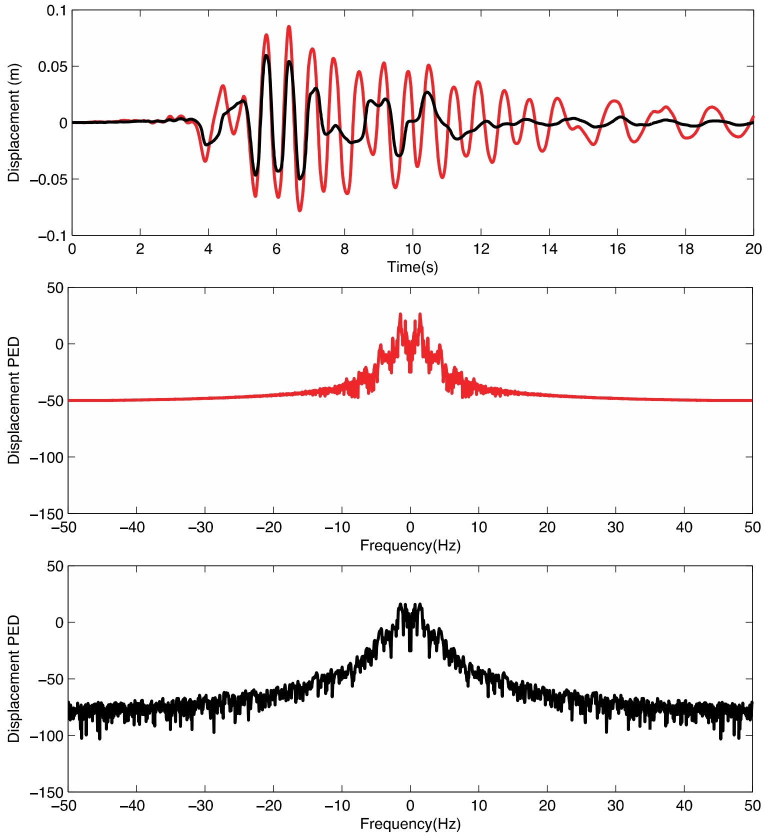

Fig. 2 Time-history response of the isolated building under Newhall excitation (FP- y , FN- x). Displacement of the base, for both the uncontrolled (red) and the controlled (black) situations (up) and its corresponding power spectral density (uncontrolled in the middle and controlled down)

The results of the saturated LMI controller in Table 2 –based on Eq. (1) – are summarized in Table 4, for the fault normal (FN) component and the fault parallel (FP) components. The evaluation is reported in terms of the performance indices described by Narasimhan *et al*. (2006). The controlled structure –whose parameters are described in Tables 1-3– is simulated for seven earthquake ground accelerations as defined by Narasimhan *et al*. (2006) (Newhall, Sylmar, El Centro, Rinaldi, Kobe, Ji-Ji and Erzinkan). All the excitations are used at the full intensity for the evaluation of the performance indices. The performance indices larger than 1 indicate that the response of the controlled structure is bigger than that of the uncontrolled structure. These quantities are highlighted in bold.

In this paper, the controllers are assumed to be fully active. These actuators are used to apply the active control forces to the base of the structure. In this control strategy most of the response quantities are reduced substantially from the uncontrolled cases. The nonlinear model of the main structure (2)-(4) is used to run out the simulation. The Bouc-Wen model (3)-(4) is considered and the parameters of the hysteresis model are defined in Table 3.

4.1 Performance indices

The base and structural shears are reduced between 8 and 55% in a majority of earthquakes (except El Centro and Sylmar). The reduction in base displacement is between 17 and 69% in all cases. Reductions in the inter-storey drifts between 4 and 40% are achieved when compared to the uncontrolled case. The floor accelerations are also reduced by 4-44% in a majority of earthquakes (except Newhall (FP- y), Sylmar (FP- y), El Centro and Ji-ji (FP- x)).

The benefit of the active control strategy is the reduction of base displacements (J_3) and shears (J_1, J_2) of up to 69% without increase in drift (J_4) or accelerations (J_5). The reduction of the peak base displacement J_3 of the base-isolated building is one of the most important criteria during strong earthquakes. Moreover, the index J_6 in the proposed scheme reach to small values, which means that the force generated by all control devices with respect to the base shear of the structure is acceptable.

For the base-isolated buildings, superstructure drifts are reduced significantly compared to the corresponding fixed-buildings because of the isolation from the ground motion. Hence, a controller that reduces or does not increase the peak superstructure drift (J_4), while reducing the base displacement significantly (J_3), is desirable for practical applications (Xu *et al*. 2006). In this respect, the proposed active controller performs well.

4.2 Time-history plots

Figs. 2-4 show the time-history plots of various response quantities for the uncontrolled building, and the building with active controllers using some of the seven earthquakes.

Table 4 Numerical results for the proposed saturated LMI controller. Case A refers to (FP- x, FN- y) and B refers to (FP- y, FN- x)

Earthq.	Case	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
Newhall	A	0.700	,668	0.697	0.791	0.751	0.174	0.533	0.708
	B	0.689	0.717	0.401	0.658	**1.155**	0.499	0.282	0.814
Sylmar	A	0.765	0.751	0.637	0.801	0.797	0.288	0.598	0.816
	B	0.956	**1.013**	0.702	0.892	**1.067**	0.358	0.497	0.906
El Centro	A	0.881	**1.032**	0.307	0.601	**1.362**	0.606	0.230	**1.219**
	B	0.878	0.933	0.349	0.818	**1.279**	0.859	0.274	**1.052**
Rinaldi	A	0.913	0.917	0.833	0.891	0.852	0.157	0.599	0.827
	B	0.714	0.656	0.570	0.744	0.828	0.354	0.413	0.763
Kobe	A	0.493	0.454	0.493	0.718	0.559	0.206	0.458	0.506
	B	0.705	0.664	0.664	0.716	0.726	0.195	0.407	0.529
Ji-Ji	A	**1.003**	0.972	0.776	0.815	**1.231**	0.284	0.534	0.742
	B	0.678	0.667	0.564	0.961	0.958	0.467	0.535	0.815
Erzin.	A	0.827	0.806	0.636	0.883	0.824	0.373	0.559	0.877
	B	0.520	0.485	0.408	0.767	0.749	0.428	0.311	0.493

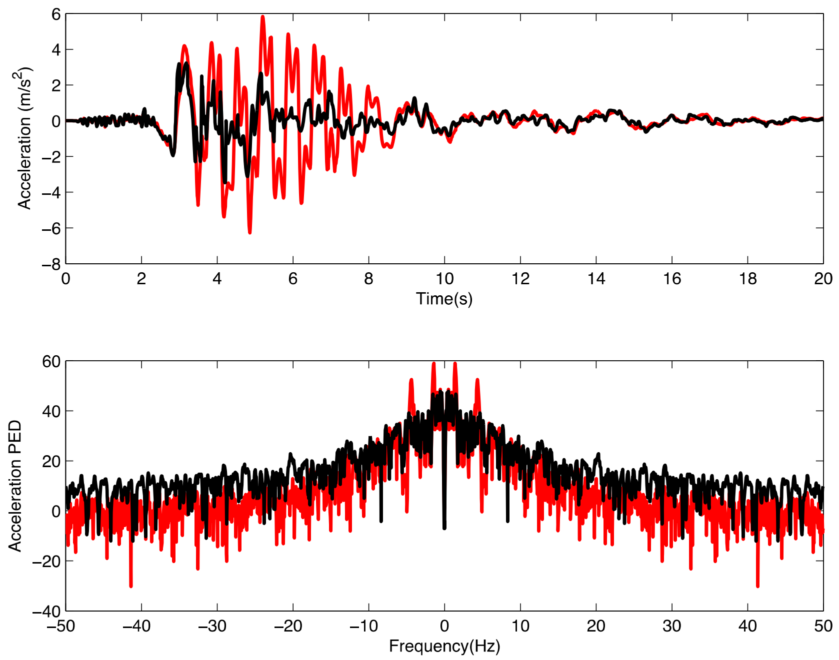

Fig. 3 Time-history response of the isolated building under Erzinkan excitation (FP- y, FN- x). Absolute acceleration of the base, for both the uncontrolled (red) and the controlled (black) situations (up) and its corresponding power spectral density (down)

More precisely, Figs. 2 present the plots for the base displacement under Newhall (FP- y, FN- x) for both the uncontrolled and the controlled situations. The plotted quantities in Fig. 3 are the absolute acceleration of the base, also for both the uncontrolled and the controlled situations. The resulting saturated control force under Newhall is depicted in Fig. 4.

Looking at Fig. 2, it can be seen that the controlled relative displacement of the base is significantly reduced compared to the uncontrolled case. Fig. 3 shows that the reduction in the absolute base acceleration is not as drastic, but it is still significant.

4.3 Power spectral density plots

Fig. 2 also shows the power spectral density (PED) plot of various response quantities for the uncontrolled building, and the building with active controllers using some of the seven earthquakes. The idea of the PED, as a metric, is that if the area under the curve in the controlled case is smaller than the area under the curve in the uncontrolled case, the vibrations are then reduced. We can clearly observe this behaviour in this figure. In Fig. 3 the same kind of power spectral plots can be found but with respect to the base acceleration. In this case, the controlled curves are not placed below the uncontrolled ones, but the closed-loop peaks are significantly reduced, an important issue on structural effort reduction.

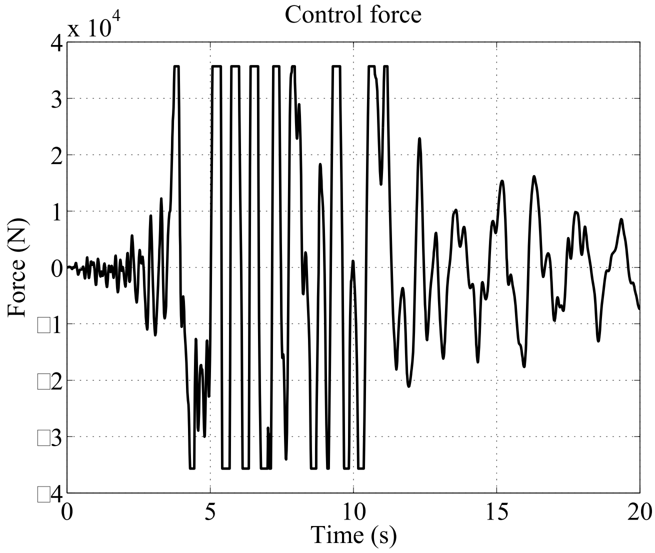

Fig. 4 Saturated LMI-based control force under Newhall excitation (FP- x, FN- y)

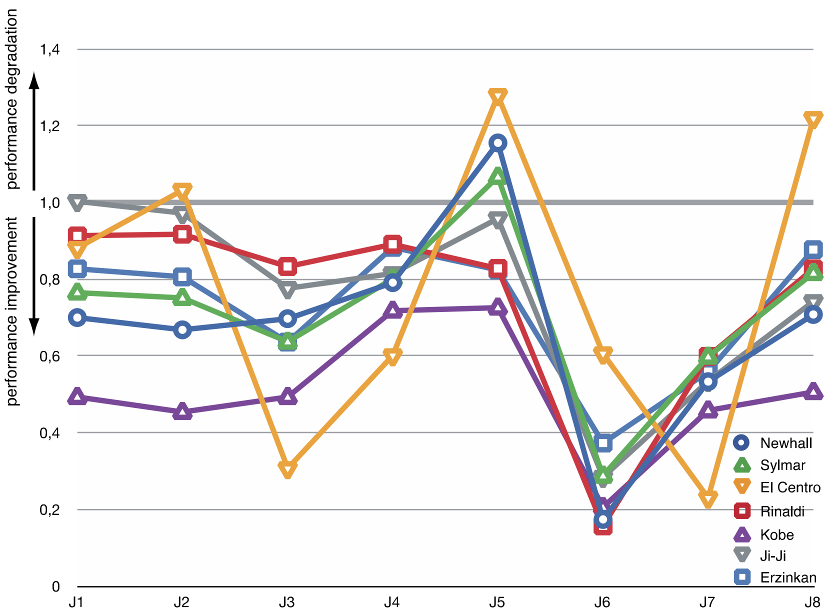

Fig. 5 Comparison of performance indices for various ground motions (Newhall, Sylmar, El Centro, Rinaldi, Kobe, Ji-Ji and Erzinkan) (FP- x, FN- y)

4.4 Comparison

Fig. 5 shows that, as compared to the uncontrolled system –and for the seven ground motions–, the proposed active control provides significant performance improvement in terms of reducing both peak and normed responses. Only in three cases (Newhall, Sylmar and El Centro) there is performance degradation with respect to the peak absolute floor acceleration J_5). Nevertheless, in these three cases, the peak base displacement is reduced substantially. The results of the performance index J_5 can then be improved by tuning the parameters $u_{\lim}$ and δ and obtaining a less drastic reduction for the index J_3.

Remark 1. In the results presented in this Section, we have considered *the same* expression for the control law, no matter what earthquake we are considering for simulation. This results can be improved if the parameters $u_{\lim}$ and δ in Table 2 are chosen for each earthquake in an independent way, according to the characteristics of the different seismic zones.

5. Conclusions

From a structural point of view, the objective of an active control component, as part of a hybrid seismic control system for buildings (and other structures), is to keep the base displacement relative to the ground, the interstory drift and the absolute base acceleration within a reasonable range (which can be affected by the design of the base isolator). In this work, we have proposed and applied a saturated LMI-based controller for seismic attenuation of base-isolated structures. The simulation results illustrate that the base and structural shears, the base displacement, the interstory displacements and the floor accelerations have been significantly reduced by using the proposed saturated controllers as compared with the purely passive isolation scheme. One of the key points of the proposed control scheme is the simplicity of the control law.

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