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Nonlinear electromechanical analysis of a functionally graded square plate integrated with smart layers resting on Winkler-Pasternak foundation

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Abstract. This paper presents nonlinear analysis of a functionally graded square plate integrated with two functionally graded piezoelectric layers resting on the Winkler-Pasternak foundation. Geometric nonlinearity was considered in the strain-displacement relation based on the Von-Karman assumption. All the mechanical and electrical properties except Poisson's ratio can vary continuously along the thickness of the plate based on a power function. Electric potential was assumed as a quadratic function along the thickness direction and trigonometric function along the planar coordinate. The effect of non homogeneous index was investigated on the responses of the system. Furthermore, a comprehensive investigation has been performed for studying the effect of two parameters of assumed foundation on the mechanical and electrical components. A comparison between linear and nonlinear responses of the system presents necessity of this study.

Keywords: plate; Winkler-Pasternak foundation; nonlinear responses; functionally graded material; piezoelectric

1. Introduction

The properties of material and selection of the best of them for application in different environments has an important role in design and fabrication of structures. The important problem in selection of materials is existence of opposite conditions that makes hard this process. Combination of opposite conditions has forced engineers and material scientists to propose new materials with variable properties. These materials have been named functionally graded materials (FGM's). The property of these materials can be changed continuously and gradually along the direction of coordinate system. For application of these materials in electromechanical systems as sensor or actuator, structure made of these materials can be integrated with piezoelectric layers. The piezoelectric effect has been presented scientifically by Pierre and Jacques Curie in 1880. Piezoelectric structures are very applicable in the industrial systems as sensor or actuator in various geometries such as plates, cylinders and shells. Derivation of the relation between the applied loads and displacement in a piezoelectric structure such as square plate may be considered as an important subject especially when the plate undergoes large deformation. A foundation has

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important effect on the responses of the electromechanical system and needs more investigations.

Woo and Meguid (2001) investigated the nonlinear analysis of a functionally graded plates and shallow shells. They proposed an analytical solution for the coupled large deflection of the FG plates and shallow shells. Von Karman theory is employed for considering the large transverse deflection. GhannadPour and Alinia (2006) investigated the large deflection analysis of a rectangular FG plate based on the Von Karman theory for simulation of the large deflection. The solution was obtained using minimization of the total potential energy with respect to unknown parameters. Hui-Shen (2007) considered the nonlinear response of a FG plate due to heat conduction. It was assumed that the plate to be shear deformable. Higher order shear deformation theory was employed for analysis of the problem.

Huang *et al*. (2008) presented exact solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations using the three-dimensional theory of elasticity. The effects of stiffness of the foundation, loading and non homogeneous index on mechanical responses of the plates were investigated. Ebrahimi and Rastgo (2008) investigated the free vibration of smart circular thin FG plate using the classical plate theory. The power function is employed for simulation of the material properties distribution along the thickness direction. Plate was composed of a FG layer and two FGP layers at top and bottom of that. The obtained results were verified by those obtained results from three dimensional finite element analyses. Alinia and GhannadPour (2009) investigated the large deflection analysis of a rectangular FG plate with logarithmic distribution of material properties. Sarfaraz Khabbaz *et al*. (2009) investigated the nonlinear analysis of FG plates under pressure based on the higher-order shear deformation theory. The first and higher order shear deformation theories were employed to investigate the large deflection of FG plate. The effect of the thickness and non homogeneous index were investigated on the distribution of the displacements and stresses. Khoshgoftar *et al.* (2009) investigated thermo elastic analysis of a FGP cylinder under pressure. It was assumed that all mechanical and electrical properties except Poisson ratio vary continuously along the thickness direction based on a power function.

Benyoucef *et al*. (2010) presented static analysis of simply supported functionally graded plates subjected to a transverse uniform load resting on an elastic foundation. The material properties of the plate are assumed to be graded in the thickness direction according to a simple power-law distribution in terms of volume fractions of material constituents. The foundation is modeled as a two-parameter Pasternak-type foundation. Ait Atmane *et al*. (2010) studied Free vibration analysis of simply supported functionally graded plates (FGP) resting on a Winkler–Pasternak elastic foundation by a new higher shear deformation theory. The equation of motion for FG rectangular plates resting on elastic foundation was obtained through Hamilton’s principle. Zenkour *et al*. (2011a) studied the bending response of an orthotropic rectangular plate resting on two-parameter elastic foundations. Analytical solutions for deflection and stresses were developed by means of the simple and mixed first-order shear deformation plate theories. The results were compared with those obtained in the literature using three-dimensional elasticity theory or higher-order shear deformation plate theory to check the accuracy of the simple and mixed first-order shear deformation theories.

A functionally graded piezoelectric rotating cylinder as mechanical sensor under pressure and thermal loads was investigated analytically by Rahimi *et al*. (2011) for evaluation of angular velocity of rotary devices. Zenkour (2011b) investigated on the bending response of simply-supported orthotropic plates. The mixed first-order shear deformation plate theory (MFPT) was employed to study the bending responses. The foundation was modeled using Winkler elastic foundation. Zenkour, Allam and Radwan (2013a) presented bending response of an orthotropic rectangular plate resting on two-parameter elastic foundations under thermo-mechanical loadings using a unified shear deformation plate theory. They (Zenkour *et al*. 2013b) also studied bending responses of a functionally graded plate resting on elastic foundations and subjected to a transverse mechanical load. A relationship between the simple and mixed first-order transverse shear deformation theories was presented as the main result of that study. The obtained results using both simple and mixed first-order theories were compared with them. Some nonlinear analysis of functionally graded piezoelectric structures has been presented by the author (Arefi and Rahimi 2011, 2012, Arefi 2013, Arefi and Nahas 2014, Arefi and Khoshgoftar 2014).

This paper tries to present nonlinear electromechanical responses of a functionally graded square plate with functionally graded piezoelectric layers resting on the Winkler-Pasternak foundation. The nonlinear analysis was performed using the energy method (Arefi and Rahimi 2010, 2012, 2014, Arefi *et al*. 2012). The effect of non homogeneity and Winkler-Pasternak foundation is considered on the responses of the system.

2. Formulation

Nonlinear electromechanical analysis of a FGM plate embedded with two smart layers at top and bottom is analyses using the classic plate theory (CPT) in this paper. Based on the CPT, the displacement of every layer is defined by two terms including the displacement of mid-plane and rotation about the mid-plane (Ugural 1981, Ebrahimi and Rastgo 2008). Therefore, we will have

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\dfrac{\partial w(x,y)}{\partial x} \\ v(x,y,z) = v_0(x,y) - z\dfrac{\partial w(x,y)}{\partial y} \\ w(x,y,z) = w(x,y) \end{cases} \tag{1}$$

where, u_0, v_0, w are displacement components of the plate mid-plane ($z = 0$) and $\vec{u} = (u, v, w)$ is displacement vector. Considering Eq. (1), the nonlinear strain components are obtained as (Lai *et al*. 1999)

$$\begin{gathered} \{\varepsilon\} = \tfrac{1}{2}\left\{\nabla\vec{u} + \nabla^T\vec{u} + (\nabla^T\vec{u})(\nabla\vec{u})\right\} \rightarrow \\ \varepsilon_{ij} = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right) \end{gathered} \tag{2}$$

where, ε_{ij} are the strain components and ∇ is del operator. The nonlinear components of the strains are obtained using substituting Eq. (1) into Eq. (2) as follows

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$
$$\varepsilon_{yy} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{3}$$
$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)$$

After defining the strain-displacement relation, the constitutive equations for both FGM and FGPM sections can be presented. Two assumed sections are depicted in Fig. 1.

Shown in Fig. 1 is the schematic figure of a functionally graded square plate embedded with functionally graded piezoelectric layers at top and bottom of plate resting on the Winkler-Pasternak foundation. Stress-strain relations for FGM and FGPM sections in general state are (Khoshgoftar *et al*. 2009)

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{ijk}E_k \tag{4}$$

where, σ_{ij} and ε_{kl} are the stress and strain components, E_k is electric field, C_{ijkl} and e_{ijk} are the stiffness and piezoelectric coefficients.

For FGM section $-h_e \le z \le h_e$ where electric potential hasn't effect on stress components, constitutive equations can be expressed as follows

$$\begin{cases} \sigma_{xx} = C^e{}_{xxxx}\varepsilon_{xx} + C^e{}_{xxxy}\varepsilon_{xy} + C^e{}_{xxyy}\varepsilon_{yy} \\ \sigma_{yy} = C^e{}_{yyxx}\varepsilon_{xx} + C^e{}_{yyxy}\varepsilon_{xy} + C^e{}_{yyyy}\varepsilon_{yy} \\ \sigma_{xy} = C^e{}_{xyxx}\varepsilon_{xx} + C^e{}_{xyxy}\varepsilon_{xy} + C^e{}_{xyyy}\varepsilon_{yy} \end{cases} \tag{5}$$

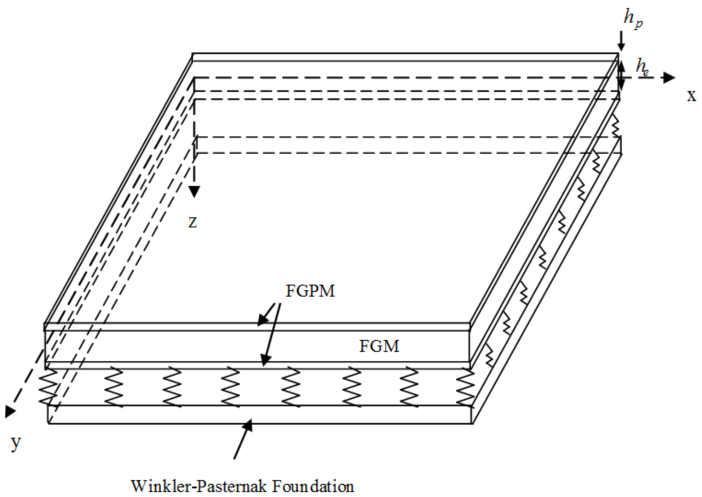

Fig. 1 The schematic figure of a FG square plate with piezoelectric layers resting on Winkler-Pasternak foundation

In Eq. (5), normal stresses are depending on the normal strains and shear stress is a function of shear strain only. Due to these assumptions $C^e{}_{xxxy} = C^e{}_{yyxy}\varepsilon_{xy} = C^e{}_{xyxx} = C^e{}_{xyyy} = 0$. Furthermore, due to small ratio of the plate thickness with respect to the length and width of the plate, the normal stress σ_{zz} and shear stresses σ_{xz} , σ_{yz} is ignorable.

The constitute equations for piezoelectric sections of the plate (FGP) $h_e < |z| \le h_e + h_p$ are expressed as

$$\begin{cases} \sigma_{xx} = C^p{}_{xxxx}\varepsilon_{xx} + C^p{}_{xxxy}\varepsilon_{xy} + C^p{}_{xxyy}\varepsilon_{yy} - e_{xxx}E_x - e_{xxy}E_y - e_{xxz}E_z \\ \sigma_{yy} = C^p{}_{yyxx}\varepsilon_{xx} + C^p{}_{yyxy}\varepsilon_{xy} + C^p{}_{yyyy}\varepsilon_{yy} - e_{yyx}E_x - e_{yyy}E_y - e_{yyz}E_z \\ \sigma_{xy} = C^p{}_{xyxx}\varepsilon_{xx} + C^p{}_{xyxy}\varepsilon_{xy} + C^p{}_{xyyy}\varepsilon_{yy} - e_{xyx}E_x - e_{xyy}E_y - e_{xyz}E_z \end{cases} \tag{6}$$

The assumptions expressed after Eq. (5) must be considered for piezoelectric section $C^p{}_{xxxy} = C^p{}_{yyxy} = C^p{}_{xyxx} = C^p{}_{xyyy} = 0$. Electric field E_k is obtained using gradient of a potential function $\phi(x,y,z)$ with minus sign as follows (Khoshgoftar *et al*. 2009)

$$\phi = \phi(x,y,z) \rightarrow E_k = -\frac{\partial\phi(x,y,z)}{\partial k}, k = x,y,z \tag{7}$$

The electric displacement D_i which must satisfy Maxwell's equation in the electromechanical systems is defined as (Khoshgoftar *et al*. 2009)

$$D_i = e_{ijk}\varepsilon_{jk} + \eta_{ik}E_k \tag{8}$$

where, η_{ik} are the dielectric coefficients.

The electric displacement equations for the piezoelectric sections of the FGP plate $h_e \le z \le h_e + h_p$ are

$$\begin{cases} D_x = e_{xxx}\varepsilon_{xx} + e_{xxy}\varepsilon_{xy} + e_{xyy}\varepsilon_{yy} + \eta_{xx}E_x + \eta_{xy}E_y + \eta_{xz}E_z \\ D_y = e_{yxx}\varepsilon_{xx} + e_{yxy}\varepsilon_{xy} + e_{yyy}\varepsilon_{yy} + \eta_{yx}E_x + \eta_{yy}E_y + \eta_{yz}E_z \\ D_z = e_{zxx}\varepsilon_{xx} + e_{zxy}\varepsilon_{xy} + e_{zyy}\varepsilon_{yy} + \eta_{zx}E_x + \eta_{zy}E_y + \eta_{zz}E_z \end{cases} \tag{9}$$

After definition of necessary mechanical and electrical components, we can employ energy method to evaluate the nonlinear mechanical and electrical responses of the system.

The energy per unit volume of the plate $\bar{u}$ is evaluated by

$$\begin{aligned} &\bar{u} = \tfrac{1}{2}\{\varepsilon^T\sigma - E^T D\} \rightarrow \\ &\bar{u} = \tfrac{1}{2}\{\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + 2\sigma_{xy}\varepsilon_{xy} - D_xE_x - D_yE_y - D_zE_z\} \end{aligned} \tag{10}$$

By introduction of the potential energy, the total energy equation of the plate under uniform or non-uniform pressure is expressed by (Ugural 1981)

$$U = \iint_A \int_{-(h_e+h_p)}^{(h_e+h_p)} \bar{u}(x,y,z)\,dz\,dx\,dy - \iint_A p(x,y)w\,dx\,dy - \frac{1}{2}\iint_A L(x,y)w\,dx\,dy \tag{11}$$

where, $L(x, y)$ is distributed pressure on the plate due to foundation. In the general state, for mentioned foundation we will have

$$L(x,y) = f_f \tag{12}$$

where, f_f is force due to Winkler-Pasternak foundation which in general form have direct and shear effects as follows

$$f_f = -kw + G\nabla^2 w \tag{13}$$

where, in two dimensional coordinate system x, y, $\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$. The energy equations for two different sections of the plate are presented as follows

$$\begin{aligned}
U = &\iint_A \int_{-h_e}^{h_e} \tfrac{1}{2}\{\sigma^e{}_{xx}\varepsilon_{xx} + \sigma^e{}_{yy}\varepsilon_{yy} + 2\sigma^e{}_{xy}\varepsilon_{xy}\}dzdxdy + \\
&2\iint_A \int_{h_e}^{h_e+h_p} \tfrac{1}{2}\{\sigma^p{}_{xx}\varepsilon_{xx} + \sigma^p{}_{yy}\varepsilon_{yy} + 2\sigma^p{}_{xy}\varepsilon_{xy} - D_x E_x - D_y E_y - D_z E_z\}dzdxdy \\
&-\iint_A p(x,y)wdxdy - \frac{1}{2}\iint_A \{-kw + G(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})\}wdxdy
\end{aligned} \tag{14}$$

The nonlinear differential equations of the system can be obtained by taking variations with respect to employed unknown functions. Alternatively, we can propose series solution using trigonometric terms for displacement components and electric potential.

The plate is fixed to four simply support edges and therefore, the displacements at four edges are considered zero. Furthermore the homogeneous boundary conditions are considered for the electric potential (Ebrahimi and Rastgo 2008). Top and bottom of piezoelectric layers are short circuited. The solution procedure may be continued with assumption of four fields for the displacements and electric potential. The sinusoidal function is employed for these assumptions as follows (GhannadPour and Alinia 2006, Alinia and GhannadPour 2009)

$$\begin{aligned}
u_0(x,y) &= \sum_p \sum_q U_{pq} \sin(\frac{2p\pi x}{L})\sin(\frac{(2q-1)\pi y}{L}) \\
v_0(x,y) &= \sum_p \sum_q V_{pq} \sin(\frac{(2p-1)\pi x}{L})\sin(\frac{2q\pi y}{L}) \\
w(x,y) &= \sum_p \sum_q W_{pq} \sin(\frac{(2p-1)\pi x}{L})\sin(\frac{(2q-1)\pi y}{L}) \\
\phi(x,y,z) &= f(z)\sum_p \sum_q \Phi_{pq} \sin(\frac{(2p-1)\pi x}{L})\sin(\frac{(2q-1)\pi y}{L})
\end{aligned} \tag{15}$$

where, U_{pq}, V_{pq}, W_{pq} and Φ_{pq} describes the amplitudes of the displacement components and electric potential, p and q defines the number of the required sentences for definition of the four fields and L is plate length. Short circuit condition of piezoelectric layers can be satisfied if we use

$f(z)$ as follows: (Ebrahimi and Rastgo 2008)

$$\phi(z = h_e) = \phi(z = h_e + h_p) = 0 \rightarrow f(z) = (1 - \{\frac{2z - 2h_e - h_p}{h_p}\}^2) \quad (16)$$

The solution of the system can be obtained by minimizing the energy equation (Eq. (14)) with respect to four amplitudes $U_{pq}, V_{pq}, W_{pq}, \Phi_{pq}$.

$$\mathrm{R_i} = \frac{\partial \mathrm{U}}{\partial q_i} = 0, i = 1,2,3,4 \ , \ q_i = U_{pq}, V_{pq}, W_{pq}, \Phi_{pq} \quad (17)$$

3. Results and discussion

3.1 Material properties

For FGM layer, it is assumed that the bottom of the plate is steel and top of that is ceramic. Therefore the distribution of the material properties for FG layer is (Ebrahimi and Rastgo 2008)

$$E(z) = (E_c - E_m)(\frac{1}{2} + \frac{z}{2h_e})^n + E_m \qquad -h_e \le z \le h_e \quad (18)$$

where, $E(z = -h_e) = E_m$, $E(z = h_e) = E_c$, $2h_e$ is thickness of elastic solid section of the plate and *n* is the non- homogeneous index. The distribution of the mechanical and electrical properties for the two FGP layers can be supposed as a power function along the thickness direction as follows (Khoshgoftar *et al*. 2009)

$$E(z) = E_i (\frac{|z|}{h_e})^n \qquad h_e < |z| \le h_e + h_p \quad (19)$$

where, E_i represents the value of the all mechanical and electrical components at $|z| = h_e$ and h_e is thickness of the piezoelectric section. The numeric values for material properties and geometric parameters are (Ebrahimi and Rastgo 2008, Khoshgoftar *et al*. 2009)

$$E_c = 3.8\times10^{11} Pa, E_m = 2\times10^{11} Pa, E_{h_e} = 7.6\times10^{10} Pa,$$
$$e_{1h_e} = 0.35Cm^{-2}, e_{2h_e} = -0.16Cm^{-2}, \eta_{1h_e} = 9.03\times10^{-11} C^2N^{-1}m^{-2}, \eta_{2h_e} = 5.62\times10^{-11} C^2N^{-1}m^{-2}$$
$$h_e = 10\times10^{-3} m, h_p = 2\times10^{-3} m, L = 0.2m$$

Due to very large values of Winkler-Pasternak parameters (k, G), required dimensionless parameters are introduced as: $w/L, kEI/L^4, GEI/L^2, \Phi/\Phi_O$ where reference electric potential $\Phi_O = E_c/ep$ and $I = \frac{h^3}{12}$ is moment of area for unit width.

3.2 The effect of parameters of Winkler-Pasternak parameters on transverse displacement

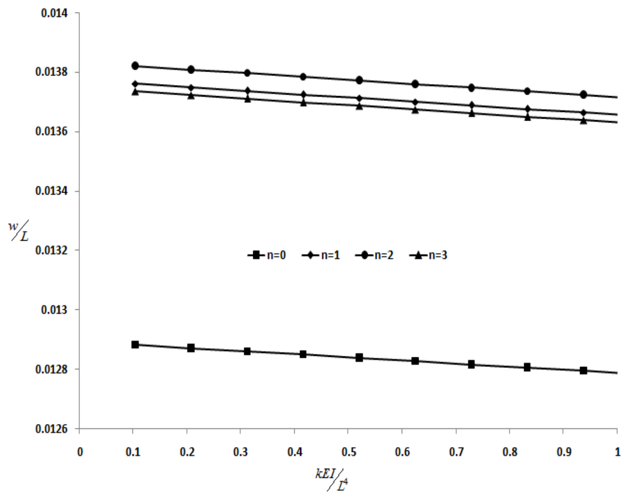

Fig. 2 The distribution of transverse displacement in terms of dimensionless stiffness parameter for different non-homogeneous indexes and $G \neq 0$

The effect of parameters of Winkler-Pasternak parameters (k, G) can be considered on the responses of the system concurrently with considering the effect of non- homogeneous index. Shown in Figs. 2 and 3 are the distribution of maximum transverse displacement in terms of stiffness parameter of foundation (k) for different values of non homogeneous index. This investigation has been performed for two different values of shear parameter of foundation ($G = 0$ and $G \neq 0$).

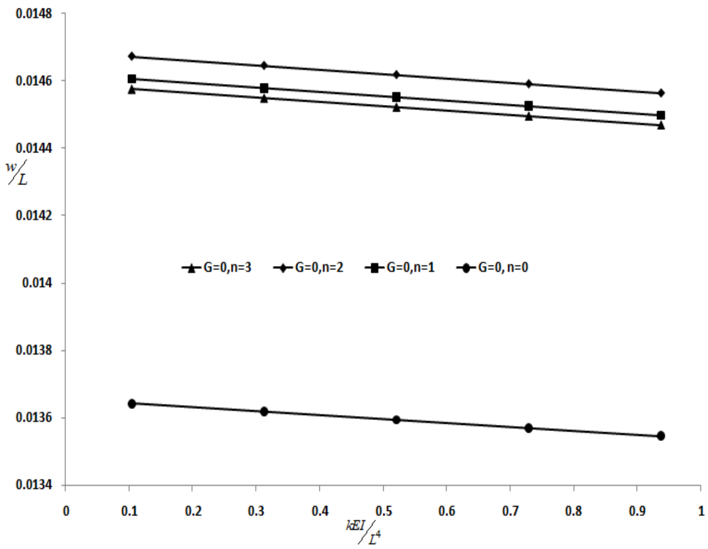

Fig. 3 The distribution of transverse displacement in terms of dimensionless stiffness parameter for different non-homogeneous indexes and $G = 0$

The obtained results in Figs. 2 and 3 indicate that the transverse displacement has maximum value for n=2. In the other word, the foundation has minimum stiffness coefficient for n=2.

The same investigation has been performed to evaluate the effect of shear parameter G on the transverse displacement of the plate. Shown in Figs. 4 and 5 are the distribution of maximum transverse displacement in terms of shear parameter of foundation (G) for different values of non homogeneous index.

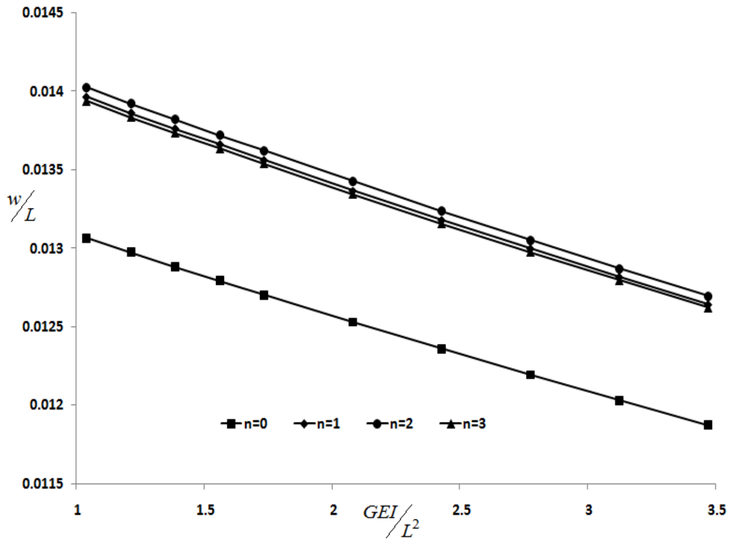

Fig. 4 The distribution of transverse displacement in terms of dimensionless shear parameter for different non-homogeneous indexes and $k \neq 0$

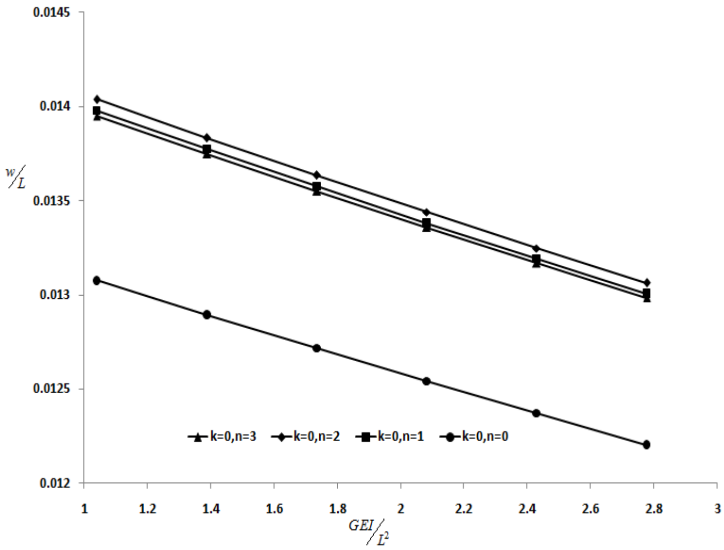

Fig. 5 The distribution of transverse displacement in terms of dimensionless shear parameter for different non-homogeneous indexes and $k = 0$

3.3 The effect of parameters of Winkler-Pasternak foundation on electric potential

Electric potential can be considered as another parameter under effect of Winkler-Pasternak parameters. Shown in Figs. 6 and 7 are distribution of electric potential in terms of stiffness parameter of foundation for different values of non homogeneous index and two zero and nonzero shear parameter of foundation. It is obvious that with increasing the non homogeneous index, electric potential monotonically and considerably increases.

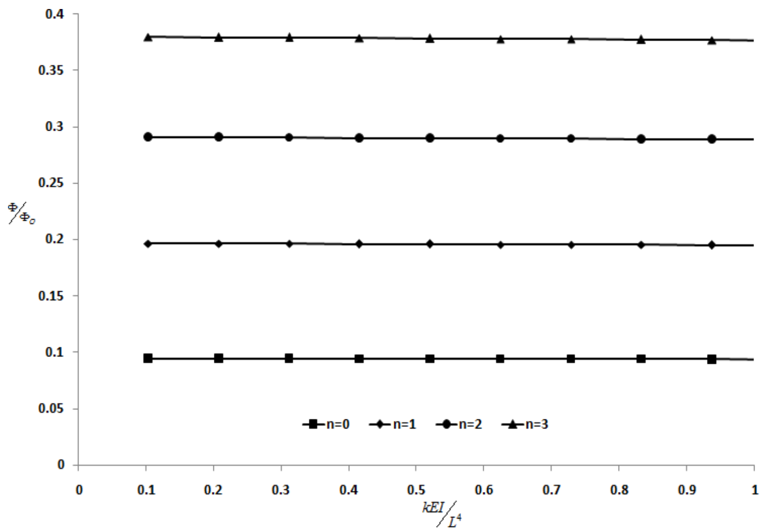

Fig. 6 The distribution of electric potential in terms of dimensionless stiffness parameter for different non-homogeneous indexes and $G \neq 0$

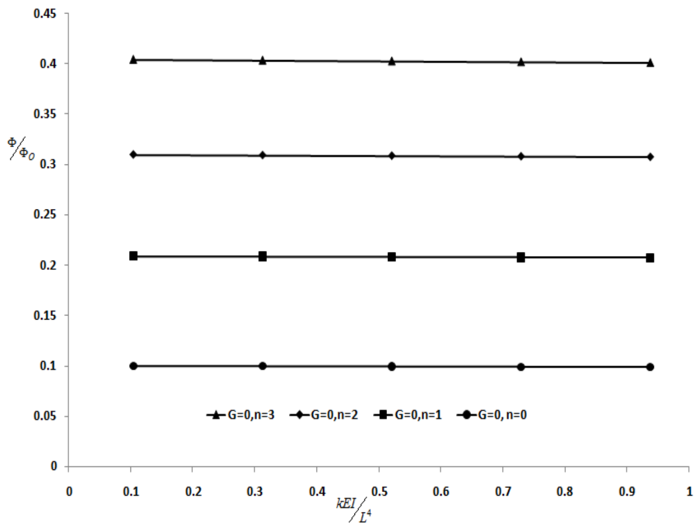

Fig. 7 The distribution of electric potential in terms of dimensionless stiffness parameter for different non-homogeneous indexes and $G = 0$

Shown in Figs. 8 and 9 are distribution of electric potential in terms of shear parameter of foundation for different values of non homogeneous index and two zero and nonzero stiffness parameter of foundation. It can be concluded that with increasing the non homogeneous index, electric potential monotonically and considerably increases.

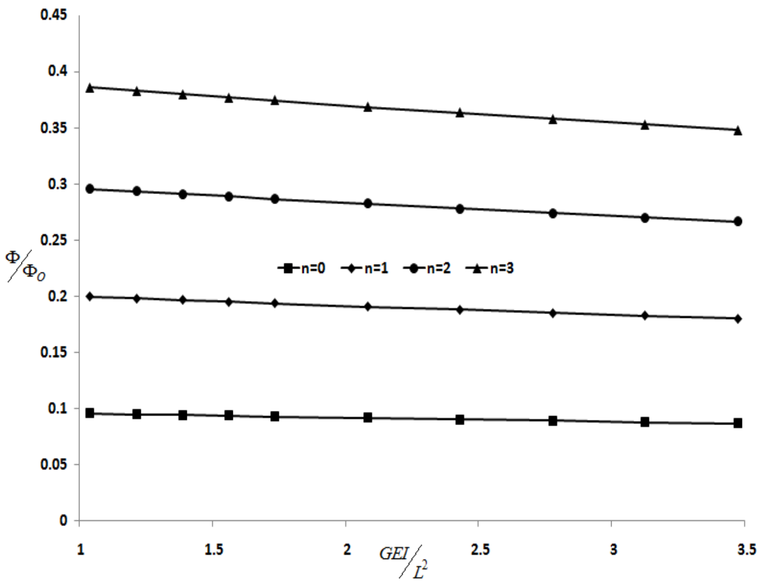

Fig. 8 The distribution of electric potential in terms of dimensionless shear parameter for different non-homogeneous indexes and $k \neq 0$

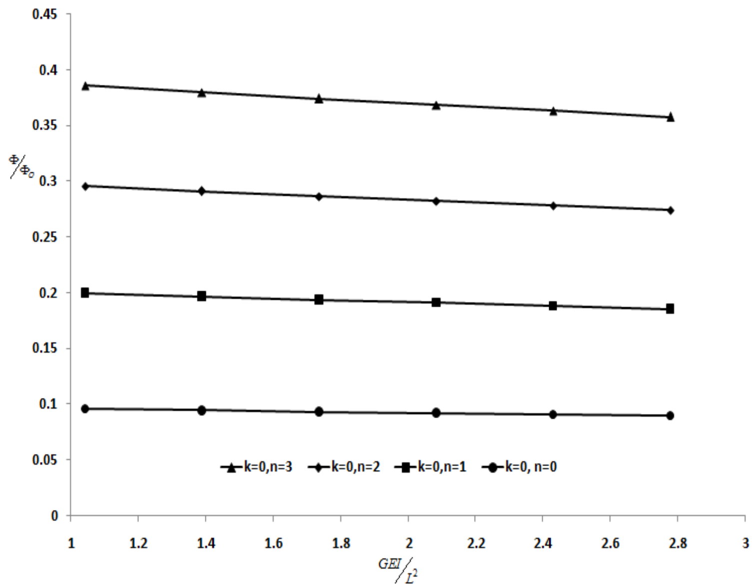

Fig. 9 The distribution of electric potential in terms of dimensionless shear parameter for different non-homogeneous indexes and $k = 0$

3.4 Linear analysis, comparison with nonlinear responses

As mentioned in previous sections, this paper evaluates the nonlinear analysis of a functionally graded piezoelectric plate resting on Winkler Pasternak foundation. The analysis has been performed using a nonlinear analysis using nonlinear strain-displacement equations. In order to study the effect of used nonlinear analysis on the electro mechanical responses of the system, this section evaluates the responses using linear strain-displacement equations. Linear strain-displacement relations are presented in Eq. (20). the comparison between linear and nonlinear responses can justify necessity of employing a nonlinear analysis especially in analysis of a structure with piezoelectric layers as sensor or actuator. This investigation can be performed using comparison between responses of the linear and nonlinear analyses in terms of two parameters of foundation. Shown in Figs. 10 and 11 are linear and nonlinear transverse displacements in terms of stiffness and shear parameters of foundation, respectively.

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_{yy} &= \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \\ \varepsilon_{xy} &= \tfrac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y}\right) \end{aligned} \tag{20}$$

Shown in Figs. 12 and 13 are linear and nonlinear electric potential in terms of stiffness and shear parameters of foundation, respectively.

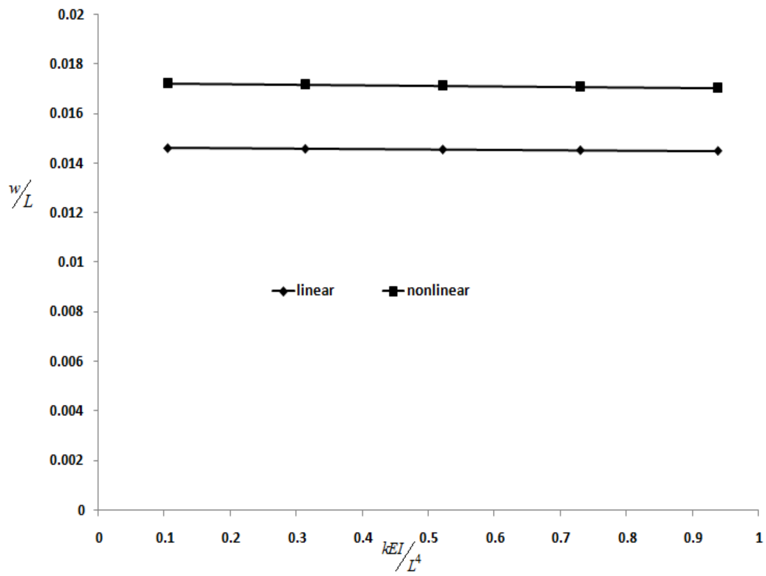

Fig. 10 Comparison between linear and nonlinear transverse displacement in terms of dimensionless stiffness parameter

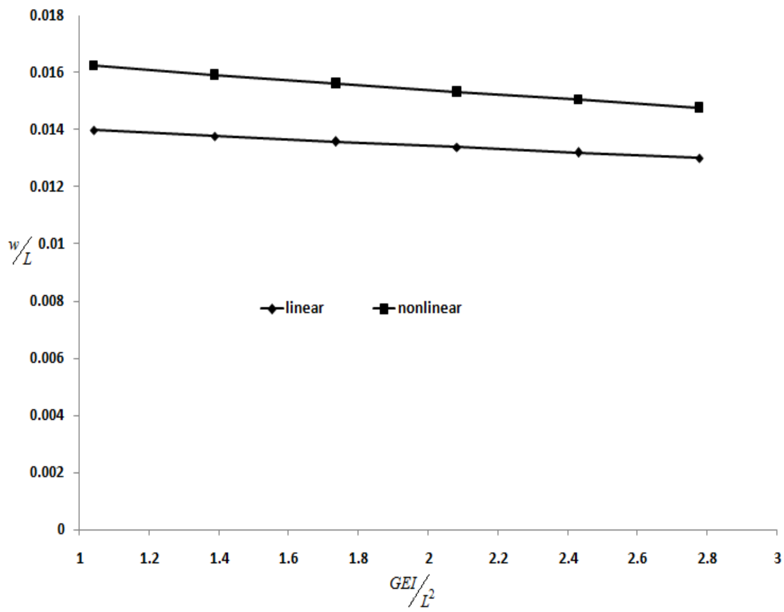

Fig. 11 Comparison between linear and nonlinear transverse displacement in terms of dimensionless shear parameter

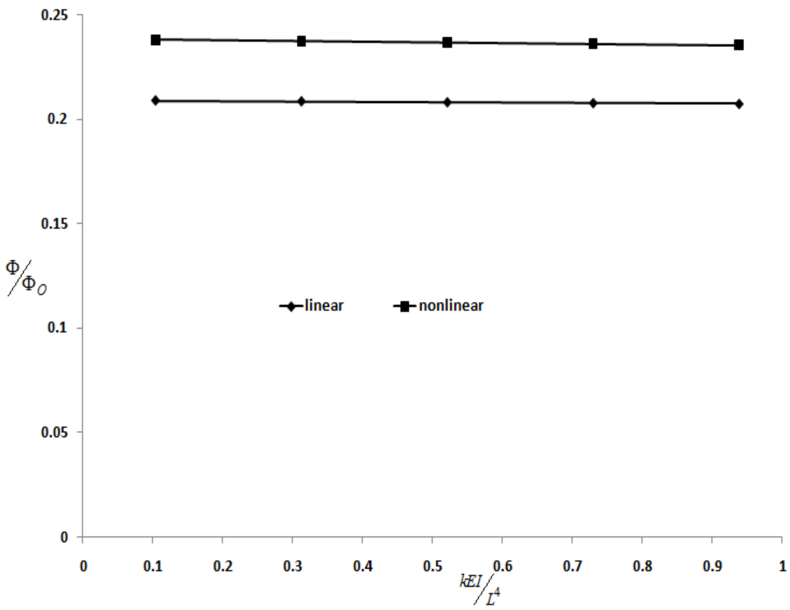

Fig. 12 Comparison between linear and nonlinear electric potential in terms of dimensionless stiffness parameter

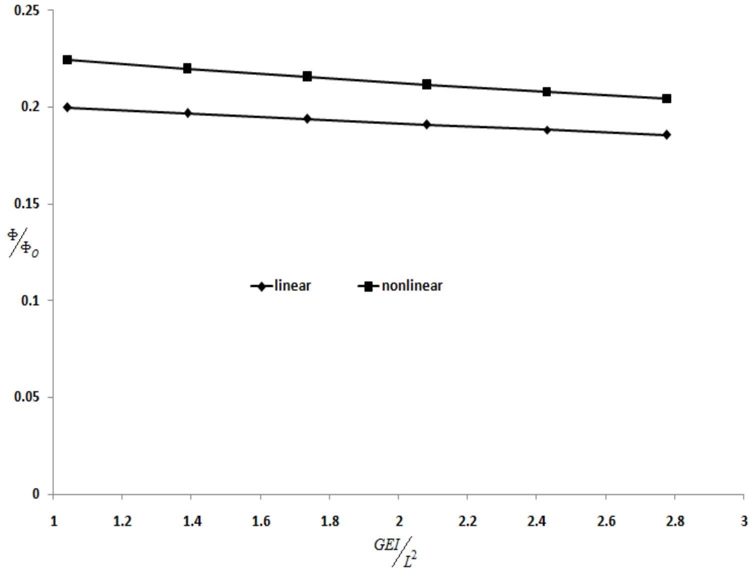

Fig. 13 Comparison between linear and nonlinear electric potential in terms of dimensionless shear parameter

4. Conclusions

Nonlinear electromechanical analysis of a functionally graded plate embedded with functionally graded piezoelectric layers resting on the Winkler-Pasternak foundation has been performed in this paper. The effect of different parameters such as non-homogeneous index and foundation parameters was studied on the mechanical and electrical responses. In order to evaluate the effect of a nonlinear analysis on the responses of the system rather than a linear analysis, the comparisons between linear and nonlinear responses has been performed for mechanical and electrical parameters.

1. It can be concluded form Figs. 2 and 3 that with increasing both parameters of foundation (k, G), transverse displacement decreases. This is due to increasing the stiffness of the foundation.
2. Investigation on the effect of stiffness parameters of foundation indicates that the value of non homogeneous index has important role on the transverse displacement of plate. Transverse displacement for n=2 is maximum value for all values of stiffness parameter.
3. Investigation on the electric potential indicates that varying the stiffness parameter has no considerable effect on electric potential distribution. In contrary, increasing the shear parameter has considerable decreasing manner for electric potential.
4. A linear analysis is performed and the obtained results are compared with those results that are extracted from the nonlinear analysis. This comparison indicates that employing a nonlinear analysis has important effect on improvement of the results rather than a linear analysis.

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Nomenclature

C_{ijkl} stiffness coefficient

$C^{e}{}_{ijkl}, C^{p}{}_{ijkl}$ stiffness coefficient for FG and FGP layer

K, G parameters of foundation

D_i electric displacement

e_{ijk} piezoelectric coefficient

E_k electric field components

$2h_e$ thickness of FG layer

h_p thickness of FGP layer

L length of the square plate

p applied pressure

$E(z)$ distribution of material properties

n non-homogeneous index

x, y, z components of coordinate system

u, v, w displacement components at a general point

u_0, v_0, w displacement components at mid-plane

ε_{ij} strain components

σ_{ij} stress components

$\bar{u}$ energy per unit volume

U total energy of system

η_{ik} dielectric coefficient

$U_{mn}, V_{mn}, W_{mn}, \Phi_{mn}$ amplitude of assumed function

ϕ electric potential

p, q the number of terms for displacement and electric field