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Repetitive model refinement for structural health monitoring using efficient Akaike information criterion

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Abstract. The stiffness of a structure is one of several structural signals that are useful indicators of the amount of damage that has been done to the structure. To accurately estimate the stiffness, an equation of motion containing a stiffness parameter must first be established by expansion as a linear series model, a Taylor series model, or a power series model. The model is then used in multivariate autoregressive modeling to estimate the structural stiffness and compare it to the theoretical value. Stiffness assessment for modeling purposes typically involves the use of one of three statistical model refinement approaches, one of which is the efficient Akaike information criterion (AIC) proposed in this paper. If a newly added component of the model results in a decrease in the AIC value, compared to the value obtained with the previously added component(s), it is statistically justifiable to retain this new component; otherwise, it should be removed. This model refinement process is repeated until all of the components of the model are shown to be statistically justifiable. In this study, this model refinement approach was compared with the two other commonly used refinement approaches: principal component analysis (PCA) and principal component regression (PCR) combined with the AIC. The results indicate that the proposed AIC approach produces more accurate structural stiffness estimates than the other two approaches.

Keywords: Akaike information criterion; repetitive model refinement; multivariate autoregressive; stiffness estimation; structural health monitoring

1. Introduction

As it is difficult to judge whether a structure is safe by visual observation alone, structural health monitoring (SHM) is essential to structural safety and maintenance. The subject of SHM has recently received a growing amount of interest from researchers in diverse fields of engineering (Masri *et al*. 2000). For example, Ho *et al*. (2012) presented a solar-powered, multi-scale, vibration-impedance sensor node on an Imote2 platform for hybrid structural health monitoring (SHM) of a cable-stayed bridge. Chang *et al*. (2003) reviewed a number of research projects focused on improving damage detection methods, including the use of novel signal processing techniques, new types of sensors, and control theory. Sumitro and Wang (2005) qualitatively compared sensor technologies used in SHM systems and introduced some new sensory technologies, such as GPS-based MMS (movement monitoring systems), PDMD (peak displacement memory devices), and FOS (fiber optic sensors). Weng *et al*. (2008) presented two

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modal identification methods that extract dynamic characteristics from output-only data sets collected using a wireless structural monitoring network. Nagayama *et al*. (2007) investigated the effects of time synchronization accuracy and communication reliability in SHM applications and examined coordinated computing for the management of large amounts of SHM data. Lu *et al*. (2008) used a wireless sensor system to detect structural damage with autoregressive (AR) and autoregressive-with-exogenous-inputs (ARX) modeling methods. Jo *et al*. (2013) used a high-sensitivity wireless strain sensor for structural health monitoring. Taylor *et al*. (2010) used a multi-scale wireless sensor node for health monitoring of civil infrastructure and mechanical systems. Kim *et al*. (2013) developed remote structural health monitoring systems for next-generation supervisory control and data acquisition (SCADA). Bennett *et al*. (2010) illustrated wireless sensor networks for underground railway applications. Ahin *et al*. (2011) and Lin (2011) have applied analysis of processed signals to determining dynamic characteristics for the purpose of assessing the condition of civil engineering structures. The results of these tests are expected to help engineers better understand and repair these types of structures in the future (Ahin *et al*. 2011).The formulation of selection criteria is of great importance to the selection process and the accuracy of the results obtained from signal processing because an appropriate selection model depends largely on prudent identification of selection criteria that reflect client and project objectives (Xia *et al*. 2011).

A structure can be damaged by earthquake events in ways that are difficult to detect by visual observations alone, meaning that the effectiveness of visual inspections is limited. Hence, SHM technology is increasingly shaping the development of structural systems. The methods used include emerging sensor-based SHM technologies that offer the potential for cost-effective maintenance of aging civil infrastructure systems (Lin *et al*. 2012, Lin 2010). Signal processing plays an important role in SHM. A signal is usually a piece of information which provides signatures with significant implications. Successful and early detection of a signal's changes are very important as some changes can be detrimental (Chan *et al*. 2012). The stiffness of a structure is one of several structural signals that are useful indicators of the amount of damage that has been done to the structures (Lautour and Omenzetter 2010).

Time series analysis, which was developed to study long sequences of regularly sampled data, is increasingly being used in SHM (Lautour and Omenzetter 2010). Multivariate auto regression (MAR) is a time series analysis procedure that is often used to characterize dynamic systems because of its simplicity (Rogers *et al*. 2010).Methods for health monitoring and damage detection using linear models developed from monitoring data are applicable to a wide range of civil structures, but the linearity of such models limits their usefulness (Lin and Chen 2009). Using higher-order sensitivity terms obtained from Taylor's series or power series expansion, different combinations of damage and nonlinearities in the damage can be modeled (Wong and Barhorst 2006). In addition, although kinematic differential equations are linear, they have time-varying coefficients that make them amenable to solution by substitution with a power series model (Milenkovic 2011).

The Akaike information criterion (AIC) is one of the indexes used to evaluate such models (Milenkovic 2011). The AIC is designed to estimate the predictive accuracy of competing hypotheses and can be applied to comparing the expected performance of competing models in predicting new data (Posada and Buckley 2004). The AIC has often been used to assess the generalization performance of linear models (Figueiredo *et al*. 2011).Principal component analysis (PCA) is perhaps the oldest and best known of the techniques of multivariate analysis (Jolliffe 1986). PCA is, in principle, a data reduction technique (Statheropoulos *et al*. 1998) that finds a set of mathematical spectra (or factors) that contains the maximum number of variations common to all spectra in a data set. This defines a new space within which each spectrum in the original group of data can be modeled using a linear combination of the factors identified (Park *et al*. 2001). In addition, the use of principal component regression (PCR) as a multivariate calibration tool raises the question of what subset of factors, i.e., what principal components (PCs) yield the best calibration model (Depczynski *et al*. 2000).

In this paper, the equations of motion for signal processing are presented, a stiffness parameter is identified from the equations of motion, and the transformation of the equations of motion into three models—a linear series, a Taylor series, and a power series model—is described. The MAR modeling method implemented in each of the three models to estimate structural stiffness is then introduced. The criteria for refinement of the statistical models, including the efficient AIC proposed in this paper, the PCA, and the integration of PCR with AIC, are introduced. Finally, this paper presents the result of a case study conducted to assess the accuracy of structural stiffness estimates obtained using the three criteria by comparing experimental results with theoretical values obtained using the three criteria for the three models.

2. Establishment of a power series model

Modal parameters can be used for a variety of purposes, such as active control, and help us to understand the dynamic behavior of a structure (Meo *et al*. 2006). To analyze the measured signal of a test structure and identify the target parameter observed, the equations of motion for the signal processing must first be established. In general, for a multi-degree-of-freedom (MDOF) system subjected only to force excitation, the equations of motion can be written as follows (Pei *et al*. 2004, Lin 2010, Lin 2011)

$$\mathrm{M}\ddot{x}(t) + r\big(x(t), \dot{x}(t)\big) = f(t) \tag{1}$$

where M is the mass matrix; the vectors $\ddot{x}(t)$ and $r$arethe acceleration and restoring force, respectively; $x(t)$ and$\dot{x}(t)$ are the displacement and velocity, respectively; and $f(t)$ is the excitation force, which can be expressed as follows (Lin *et al*. 2001)

$$f(t) = -\mathrm{M}\{\mathrm{I}\}\ddot{x}_G(t) \tag{2}$$

where$\{\mathrm{I}\}$ is the identity column matrix and $\ddot{x}_G(t)$is the ground (base)acceleration induced by the seismic event.

The variation in the restoring force with time, $\dot{r}$, i.e., Δr/Δt, is by convention usually used in the modeling of the restoring force (Lin and Chen 2009). The Bouc-Wen hysteresis model, which is one of the most widely accepted smoothly varying differential models in the engineering field (Li and Meng 2007), is used to represent $\dot{r}$.A generalized Bouc-Wen model can be expressed as follows (Lin and Chen 2009)

$$\begin{aligned}\dot{r}_\varphi &= c_\varphi\big(\ddot{x}_\varphi - \ddot{x}_{\varphi-1}\big) + k_\varphi\big(\dot{x}_\varphi - \dot{x}_{\varphi-1}\big) + f_\varphi[3\big(x_\varphi - x_{\varphi-1}\big)^2\big(\dot{x}_\varphi - \dot{x}_{\varphi-1}\big)] \\ &+ g_\varphi\big|\dot{x}_\varphi - \dot{x}_{\varphi-1}\big|\big|r_\varphi\big|^{power-1} r_\varphi + e_\varphi(\dot{x}_\varphi - \dot{x}_{\varphi-1})\big|r_\varphi\big|^{power} (\varphi = 1,2, \dots s)\end{aligned} \tag{3}$$

where the parameter *power* is given ‘a priori,’ $\ddot{x}_{\varphi-1}$ = $\dot{x}_{\varphi-1}$ = $x_{\varphi-1}$ = 0 in the case of φ = 1, and *s* is the number of building stories. Through simple rearrangement, the formula above can be expressed using four parametersA_1,A_2,A_3, and A_4, as follows

$$\dot{r}(t) = \sum_{i=1}^{Q} \left(A_1 x(t) + A_2 \dot{x}(t) + A_3 \ddot{x}(t) + A_4 r(t) \right)^i \tag{4}$$

where Q is the maximum order of the moment required, which is usually specified by the user (Lin and Chen 2009, Lin 2013). In Eqs. (3) and (4), the parameter A_2refers to the target parameter observed, i.e., the structural stiffness. The third order of moment is usually selected in the series model; a higher-order model would probably yield only a marginal improvement (Huang *et al*. 1998).

The developed model refinement approaches are implemented for the linear series, Taylor series, and power series models of multivariable polynomial expansions, leading to more accurate identification and a more controllable design for system vibration control (Lin and Chen 2009). Substituting Q = 3 into Eq. (4) yields the power series. The Taylor series can be obtained by removing the cross terms from the power series. The linear series can be obtained by retaining only the first-order terms from the Taylor series, as shown in Eq. (5). Removing a few terms from the power series does not influence the meaning of the curve fitting but does influence the accuracy of the prediction of $\dot{r}(t)$ and the accuracy of the structural stiffness estimate obtained.

$$\dot{r}(t) = \sum_{k=1}^{K} \left\{ \underbrace{ \underbrace{ \underbrace{\left[A_1 x(t_{kj}) + A_2 \dot{x}(t_{kj}) + A_3 \ddot{x}(t_{kj}) + A_4 r(t_{kj}) \right]}_{\text{Linear}} + \begin{bmatrix} A_5 x(t_{kj})^2 + A_6 \dot{x}(t_{kj})^2 + A_7 \ddot{x}(t_{kj})^2 + A_8 r(t_{kj})^2 + \\ A_9 x(t_{kj})^3 + A_{10} \dot{x}(t_{kj})^3 + A_{11} \ddot{x}(t_{kj})^3 + A_{12} r(t_{kj})^3 \end{bmatrix} }_{\text{Taylor}} + \begin{bmatrix} A_{13} x(t_{kj}) \dot{x}(t_{kj}) + A_{14} x(t_{kj}) \ddot{x}(t_{kj}) + A_{15} x(t_{kj}) r(t_{kj}) + A_{16} \dot{x}(t_{kj}) \ddot{x}(t_{kj}) + \\ A_{17} \dot{x}(t_{kj}) r(t_{kj}) + A_{18} \ddot{x}(t_{kj}) r(t_{kj}) + A_{19} x(t_{kj})^2 \dot{x}(t_{kj}) + A_{20} x(t_{kj})^2 \ddot{x}(t_{kj}) + \\ A_{21} x(t_{kj})^2 r(t_{kj}) + A_{22} \dot{x}(t_{kj})^2 x(t_{kj}) + A_{23} \dot{x}(t_{kj})^2 \ddot{x}(t_{kj}) + A_{24} \dot{x}(t_{kj})^2 r(t_{kj}) + \\ + A_{25} \ddot{x}(t_{kj})^2 x(t_{kj}) + A_{26} \ddot{x}(t_{kj})^2 \dot{x}(t_{kj}) + A_{27} \ddot{x}(t_{kj})^2 r(t_{kj}) + A_{28} r(t_{kj})^2 x(t_{kj}) + \\ A_{29} r(t_{kj})^2 \dot{x}(t_{kj}) + A_{30} r(t_{kj})^2 \ddot{x}(t_{kj}) + A_{31} x(t_{kj}) \dot{x}(t_{kj}) \ddot{x}(t_{kj}) + \\ A_{32} x(t_{kj}) \dot{x}(t_{kj}) r(t_{kj}) + A_{33} x(t_{kj}) \ddot{x}(t_{kj}) r(t_{kj}) + A_{34} \dot{x}(t_{kj}) \ddot{x}(t_{kj}) r(t_{kj}) \end{bmatrix} }_{\text{Power}} \right\} + E_j \tag{5}$$

3. Multivariate autoregressive method

To estimate the structural stiffness, a suitable computation algorithm for a series model is required. In this study, the MAR modeling method was implemented in the linear series, Taylor series, and power series models. Although a scalar AR model is often used, the MAR process is more appropriate when correlated multi-channel signals are processed simultaneously (Jamoos *et al*. 2010). Using MAR, measures can be derived from the model parameters that represent the relationships within the system in the time and frequency domains (Rogers *et al*. 2010).

Consider the following k-dimensional stationary time series

$$\mathrm{X}(t) = \left(X_1(t), X_2(t) \dots \dots, X_k(t) \right)^T \tag{6}$$

where$(\quad)^T$ denotes transpose (Wada *et al*. 1996). The MAR model can be expressed as follows

$$\mathrm{Y}(t_j) = \sum_{k=1}^{K} A_{jk} X_k(t_j - M_k) + E_j \; j = 1 \dots \dots \dots f \tag{7}$$

where f is the total length of the time over which signals were measured divided by the time interval between signals. In this study, the time interval between signals was 0.005 s. In Eq. (7), M_k is the delay and is set to 0, implying that there is no time delay between the variables, and E_j is white noise with a zero mean vector (Wada *et al*.1996).

4. Criteria for statistical model refinement

The criteria for statistical model refinement examined in this study were an efficient AIC, PCA, and PCR combined with AIC. These three model refinement approaches were applied to the previously described linear series, Taylor series, and power series models, respectively, to assess their effects on the accuracy of stiffness estimation.

4.1 Efficient Akaike information criterion

The algorithm yielding the smallest AIC is considered to be the most desirable (Harada *et al*. 2010). The AIC is defined as follows

$$\mathrm{AIC} = \log(\mathrm{V}) + \frac{2d}{N} \tag{8}$$

where$d$ is the number of estimated parameters, N is the number of values in the estimation data set, and V is the loss function (Ljung 1999).In the AIC, as more parameters are added to the model, the first term in Eq. (8) becomes smaller, indicating an improvement infit, whereas the second component, or penalty term, becomes larger. Indeed, when the sample is large, the number of adjustable parameters makes a negligible difference, and more complex models are favored (Posada and Buckley 2004).

The loss function V is defined by the following equation

$$\mathrm{V} = \det\left\{\frac{1}{N}\sum_{1}^{N} \varepsilon(t,\theta_N)[\varepsilon(t,\theta_N)]^T\right\} \tag{9}$$

where θ_N represents the estimated parameters (Ljung 1999). The quality of fit to the data is usually measured using a loss function. If the class is not too large, good regression functions can be obtained by minimizing the loss (Hutter and Tran 2010). Assuming that the amount of information is fixed, the value of N is thus a constant. The value of d varies with the number of model parameters. In Eq. (8), the term $2d/N$ is proportional to d, as shown in Fig. 1. In general, the value of log (V) decreases nonlinearly as the value of d increases, as shown in Fig. 2.

If the value of d increases by one and the increase represents the real contributing component of the model, the value of $2d/N$ increases very little and the value of log (V) decreases significantly, resulting in a decrease in the AIC value given by Eq. (8). For these reasons, the value of AIC can be used as an index in model refinement. For instance, if the added component of the power series model in Eq. (5) causes a decrease in the AIC value, indicating that the added component represents the real contributing component of the model, the component should be retained. Conversely, if the added component does not reduce the AIC value, it is statistically justifiable to remove the component.

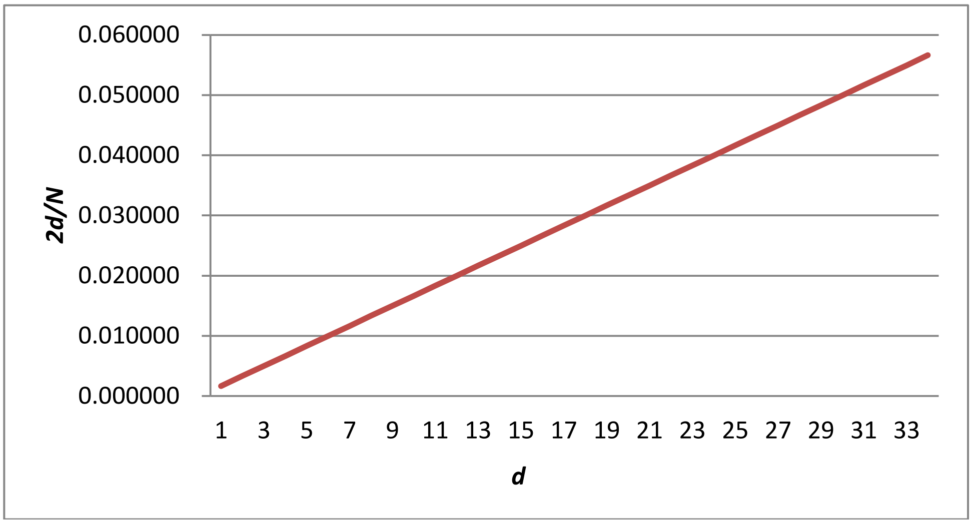

Fig. 1 In the equationAIC $= \log(\mathrm{V}) + \frac{2d}{N}$, the term $2d/N$ is proportional to the number of estimated model parameters d, N is the number of values in the estimation data set, and V is the loss function. Values from the case study described in section 5 are shown

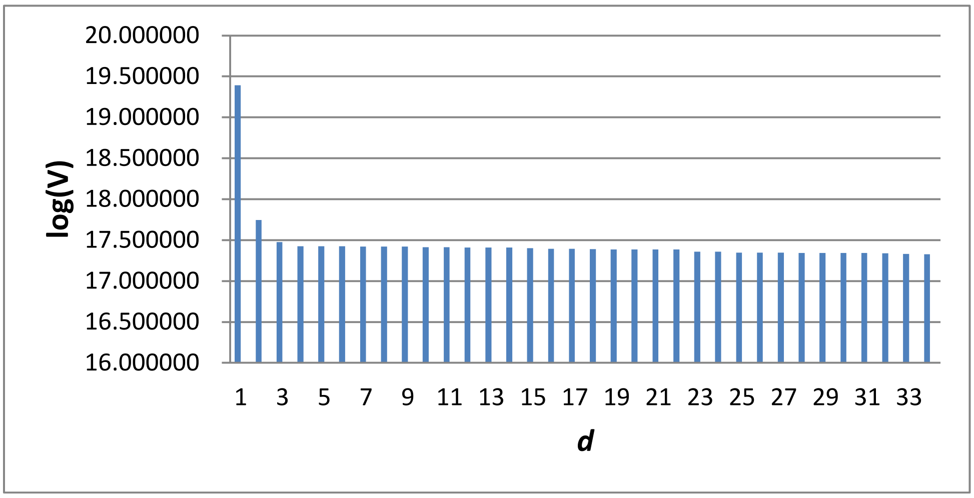

Fig. 2 The value of log (V) in the equation AIC $= \log(\mathrm{V}) + \frac{2d}{N}$ decreases nonlinearly as the value of d increases. Values from the case study described in section 5 are shown

4.2 Principal component analysis

The objective of the multivariate statistical method known as PCA is to reduce the number of predictive variables in a model (Sousa *et al*. 2007, Boukhatem *et al*. 2012). Given a set of p-dimensional vectors x_i $(i = 1, \ldots, n)$ drawn from a statistical distribution with mean $\mathbf{x}$ and covariance matrix R, decomposition of the covariance matrix by singular value decomposition leads to the following (Lautour and Omenzetter 2010)

$$\mathrm{R} = \mathrm{V\Lambda V}^T \tag{10}$$

where $\Lambda = \mathrm{diag}\left({\sigma_1}^2, \ldots, {\sigma_p}^2\right)$ is a diagonal matrix containing the eigenvalues of R ranked in descending order ${\sigma_1}^2 > \cdots > {\sigma_p}^2$ and $\mathbf{V}$ is a matrix containing the corresponding eigenvectors or PCs (Lautour and Omenzetter 2010). For each PC, there is a coefficient, also known as a loading (Jolliffe 1986). The higher the loading of a variable is, the more that variable contributes to the variation accounted for by the particular PC (Abdul-Wahab *et al*. 2005).

In practice, only loadings with absolute values greater than 50% are selected for PC interpretation (Lautour and Omenzetter 2010). In this study, we used the loading as the standard for screening, although some literature mentions using the total variance, typically 60–90%, as the screening criterion (Statheropoulos *et al*. 1998, Sousa *et al*. 2007). McAdams *et al.* (2000) stressed that a variable's significance may depend on its associations. Although an effect may be statistically significant, if the sample size is large, there is no reason for it to be retained if it makes only a minuscule difference in the predictions.

4.3 Principal component regression mixed with Akaike information criterion

The third model refinement approach adopts as the criterion PCR combined with AIC. This approach uses an adaptation of PCR that appears to have certain advantages over stepwise regression (McAdams *et al.* 2000). If a regression equation is obtained for PCs rather than predictor variables, the contribution of each transformed variable (i.e., each PC) to the equation can be more easily interpreted than the contributions of the original variables (Jolliffe 1986). A PCR model should be built by applying a selection process to the scores to determine which factors should be used to build a model for each constituent (Park *et al.*2001). The PCR model for time-domain data can be expressed as follows

$$\mathrm{Y} = \alpha\mathrm{Z} + \mathrm{E} \tag{11}$$

Where Y is the matrix of original outputs, E is the appropriate error term, α is the regression coefficient matrix, and Z is the score matrix, which is given by the following equation (Jolliffe 1986)

$$\mathrm{Z} = \mathrm{XA} \tag{12}$$

where X is a matrix of the predictor variable and $\mathbf{A}$ is a matrix of eigenvector (loading) of $X^T\mathrm{X}$.

In this study, we attempted to convert the variables to PCs in a regression setting, with the converted variables being independent and orthogonal to each other. We then introduced the previously described efficient AIC to refine the PCs, and we converted the PCs back to the MAR to assess the stiffness of a structure.

5. Case study

A case study was conducted to assess the stiffness of a structure by comparing experimental results with theoretical values derived on the basis of the three criteria described above for the three models. Estimation of structural parameters such as the stiffness was conducted using the Nonlinear Stress Analysis Technique (NSAT) computer program, which was designed for structural analysis (Tsai 1996). Using NSAT, a static analysis was performed to evaluate the stiffness of each floor of a building. A unit lateral force applied at the top floor of the structure was considered the input, and the relative displacements between each floor were calculated automatically. The stiffness estimates for the first, second, and third floors were 1,652,892.562 N/m, 1,524,390.244 N/m, and 1,582,278.481 N/m, respectively (Lin and Chen 2009).

A flow chart of the experimental design for the case study is shown in Fig. 3. The structural responses to the El Centro and TCU-084 seismic forces were represented by a power series model up to the third order of moments and by a Taylor series model and a linear series model, as shown in Eq. (5). The three models were then represented in the MAR modeling to estimate the stiffnesses of the building floors and compare them to the values obtained from NSAT. Stiffness values obtained from the models refined using the three statistical model refinement approaches (efficient AIC, PCA, and PCR combined with AIC) were also estimated.

5.1 Example of AIC

Using the power series model as an example, the measured data for the first floor of the test structure subjected to the El Centro earthquake were entered into Eq. (5), with 34 components, and then expressed using the MAR method to estimate structural parameter values. The 34 components were added one by one. When each additional component was added, the AIC value was recomputed, until all 34 components were included in the MAR method, resulting in the34 AIC values as listed in Table 1.

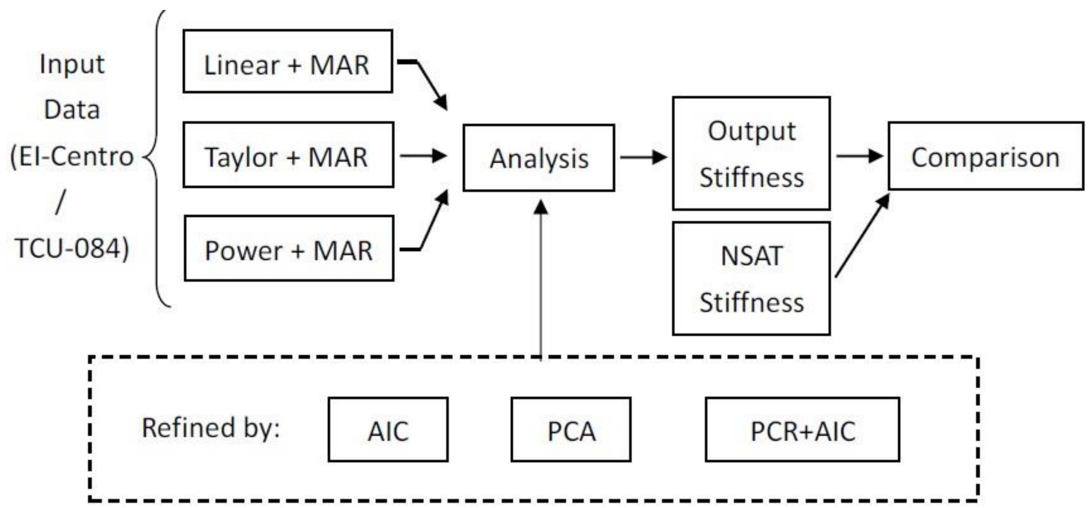

Fig. 3 Flow chart of the experimental design for the case study

Table 1 The AIC value was recomputed each time a component was added, until all34 components were included in the MAR method. The results obtained using the power series model for the first floor of the test structure subjected to the El Centro earthquake are shown

Component	AIC value	$\dot{x}(t)\ddot{x}(t)$	17.4302	$\dot{x}(t)^2r(t)$	17.3994
$x(t)$	19.3932	$\dot{x}(t)r(t)$	17.4316	$\ddot{x}(t)^2x(t)$	17.3893
$\dot{x}(t)$	17.7480	$\ddot{x}(t)r(t)$	17.4332	$\ddot{x}(t)^2\dot{x}(t)$	17.3904
$\ddot{x}(t)$	17.4790	$x(t)^3$	17.4255	$\ddot{x}(t)^2r(t)$	17.3920
$r(t)$	17.4329	$\dot{x}(t)^3$	17.4193	$r(t)^2x(t)$	17.3914
$x(t)^2$	17.4332	$\ddot{x}(t)^3$	17.4202	$r(t)^2\dot{x}(t)$	17.3913
$\dot{x}(t)^2$	17.4334	$r(t)^3$	17.4192	$r(t)^2\ddot{x}(t)$	17.3922
$\ddot{x}(t)^2$	17.4339	$x(t)^2\dot{x}(t)$	17.4195	$x(t)\dot{x}(t)\ddot{x}(t)$	17.3918
$r(t)^2$	17.4343	$x(t)^2\ddot{x}(t)$	17.4204	$x(t)\dot{x}(t)r(t)$	17.3914
$x(t)\dot{x}(t)$	17.4352	$x(t)^2r(t)$	17.4220	$x(t)\ddot{x}(t)r(t)$	17.3872
$x(t)\ddot{x}(t)$	17.4307	$\dot{x}(t)^2x(t)$	17.4236	$\dot{x}(t)\ddot{x}(t)r(t)$	17.3836
$x(t)r(t)$	17.4297	$\dot{x}(t)^2\ddot{x}(t)$	17.3978		

As Table 1 shows, when the component$x(t)^2$ was added in the MAR method, the AIC value increased from 17.4329 to 17.4332. According the model refinement rule introduced in section 4.1, this component should be removed. Similarly, the components $\dot{x}(t)^2$, $\ddot{x}(t)^2$, $r(t)^2$, $x(t)\dot{x}(t)$, $\dot{x}(t)\ddot{x}(t)$, $\dot{x}(t)r(t)$, $\ddot{x}(t)r(t)$, $\ddot{x}(t)^3$, $x(t)^2\dot{x}(t)$, $x(t)^2\ddot{x}(t)$, $x(t)^2r(t)$, $\dot{x}(t)^2x(t)$, $\dot{x}(t)^2r(t)$, $\ddot{x}(t)^2\dot{x}(t)$, $\ddot{x}(t)^2r(t)$, and$r(t)^2\ddot{x}(t)$ caused the AIC value to increase, so these components should be removed from the MAR. When the model refinement is complete, the remaining components were placed in the MAR one by one to recalculate their AIC values, which are listed in Table 2.

Table 2 listed the results for the second round of the model refinement approach using the AIC. Similarly, it was judged to be statistically sustainable to remove those components that caused the AIC value to rise, including the components $x(t)r(t)$, $r(t)^2\dot{x}(t)$, $x(t)\dot{x}(t)\ddot{x}(t)$, $x(t)\dot{x}(t)r(t)$, $x(t)\ddot{x}(t)r(t)$, and$\dot{x}(t)\ddot{x}(t)r(t)$. The remaining components were then placed in the MAR one by one to recalculate their AIC values. This model refinement approach was repeated until the components' AIC values were obtained in descending order. In other words, the computed AIC value of each component in the refined model was smaller than that of the preceding component, provided that the model was arranged in an orderly fashion. The final results are listed in Table 3.

Table 2 Results of the second round of the model refinement using the AIC, using the remaining components placed into the MAR one by one to recalculate their AIC values

Component	AIC value	$x(t)^3$	17.4301	$x(t)\dot{x}(t)\ddot{x}(t)$	17.4049
$x(t)$	19.3932	$\dot{x}(t)^3$	17.4227	$x(t)\dot{x}(t)r(t)$	17.4064
$\dot{x}(t)$	17.7480	$r(t)^3$	17.4218	$x(t)\ddot{x}(t)r(t)$	17.4080
$\ddot{x}(t)$	17.4790	$\dot{x}(t)^2\ddot{x}(t)$	17.4051	$\dot{x}(t)\ddot{x}(t)r(t)$	17.4095
$r(t)$	17.4329	$\ddot{x}(t)^2x(t)$	17.4034		
$x(t)\ddot{x}(t)$	17.4313	$r(t)^2x(t)$	17.4024		
$x(t)r(t)$	17.4326	$r(t)^2\dot{x}(t)$	17.4041		

Table 3 Final results of the model refinement approach using the AIC, using the power series model for the first floor of the test structure subjected to the El Centro earthquake

Component	AIC value	$r(t)$	17.4329	$r(t)^3$	17.4217
$x(t)$	19.3932	$x(t)\ddot{x}(t)$	17.4313	$\dot{x}(t)^2\ddot{x}(t)$	17.4056
$\dot{x}(t)$	17.7480	$x(t)^3$	17.4306		
$\ddot{x}(t)$	17.4790	$\dot{x}(t)^3$	17.4226		

Through the model refinement using the AIC, the number of components in the power series model was reduced from the original 34 to 9.These final nine components were placed into the MAR to estimate the corresponding value of the component$\dot{x}(t)$, i.e., the structural stiffness, as1,793,000 N/m. Comparison of the estimated stiffness with the reference value obtained using NSAT indicates a relative errorof8.4765%.

5.2 Example of PCA

Using the Taylor series model as an example, the measured data for the first floor of the test structure subjected to the El Centro earthquake was entered into Eq. (5), with 12 components, and then expressed in the MAR method to calculate the loadings. The results are shown in Table 4.

As Table 4 shows, using the PCA refinement approach, the components with eigenvector absolute values greater than 0.5 are$x(t)$, $\dot{x}(t)$,$\ddot{x}(t)$, $r(t)$,$x(t)^2$, $\dot{x}(t)^2$, $\ddot{x}(t)^2$, $r(t)^2$, $x(t)^3$, $\dot{x}(t)^3$, $\ddot{x}(t)^3$, and$r(t)^3$. These components were selected in the refined model and placed into the MAR to obtain an estimateof1, 789,000 N/m for the structural stiffness. Comparison of the estimated structural stiffness with the reference value obtained suing NSAT indicates a relative error of 8.23%.

Table 4 Principal component eigenvector of the Taylor series model for the first floor of the test structure subjected to the El Centro earthquake

Variable	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12
$x(t)$	0.374	-0.151	0.091	0.310	0.078	-0.023	-0.248	**0.773**	-0.206	-0.067	-0.107	0.091
$\dot{x}(t)$	-0.074	-0.452	0.479	-0.236	0.052	0.020	-0.052	-0.160	-0.230	**-0.648**	-0.028	0.024
$\ddot{x}(t)$	-0.387	-0.148	-0.026	0.315	0.078	-0.105	-0.054	0.216	**0.630**	-0.276	0.432	-0.043
$r(t)$	0.407	0.112	0.020	-0.148	-0.018	**-0.521**	0.262	-0.014	-0.172	-0.056	**0.650**	0.055
$x(t)^2$	-0.364	0.264	0.019	-0.388	-0.049	-0.092	0.033	0.295	-0.004	-0.038	-0.055	**0.737**
$\dot{x}(t)^2$	0.010	-0.165	-0.341	-0.305	**0.828**	-0.107	-0.246	-0.032	-0.002	0.063	0.006	-0.019
$\ddot{x}(t)^2$	-0.024	0.367	0.421	0.202	**0.520**	0.305	**0.525**	0.074	-0.033	-0.016	0.026	-0.027
$r(t)^2$	-0.020	0.446	0.466	0.153	0.095	-0.272	**-0.647**	-0.229	0.0266	0.078	0.011	-0.003
$x(t)^3$	0.362	-0.258	-0.013	0.370	0.087	0.160	-0.012	-0.40	0.182	0.065	0.027	**0.659**
$\dot{x}(t)^3$	-0.053	-0.447	0.493	-0.230	-0.004	-0.116	0.089	0.117	0.208	**0.650**	0.030	-0.028
$\ddot{x}(t)^3$	-0.332	-0.151	-0.064	0.430	0.099	**-0.631**	0.263	-0.098	-0.260	0.040	-0.346	0.049
$r(t)^3$	0.401	0.137	0.057	-0.210	0.005	-0.306	0.178	0.029	**0.576**	-0.239	**-0.503**	-0.067

5.3 Example of PCR mixed with AIC

Using the power series model as an example, the measured data for the first floor of the test structure subjected to the El Centro earthquake were entered into Eq. (5), with 34 components, and then expressed in the MAR method to calculate the PC loadings. These loadings were then multiplied by the corresponding components in the power series to form PCs, a converted set of independent and orthogonal 34 components. The 34 converted components (i.e., PCs) were added into the PCR one by one. When each additional PC was added, the AIC value was recomputed, until all34 PCs were included, resulting in the AIC values listed in Table 5.

According to the model refinement approach using PCR combined with AIC, the components $\vec{x}(t)^2$, $\overrightarrow{x(t)\dot{x}(t)}$, $\overrightarrow{x(t)r(t)}$, $\overrightarrow{\dot{x}(t)^2r(t)}$, $\overrightarrow{r(t)^2x(t)}$ and $\overrightarrow{r(t)^2\ddot{x}(t)}$ were removed because each of them results in an AIC value greater than the preceding AIC value. The remaining components were then placed into the PCR to recalculate the AIC values. This component sifting process was repeated until the components could not be refined any further, which suggests that all of the components in the final set are statistically justifiable. The results are shown in Table 6.

Table 6 indicates that the final results of the model refinement approach using PCR combined with AIC results in a set of only 4 PCs and their corresponding AIC values. By converting the PCs back to their original variables and placing the variables in the MAR method, it was possible to estimate the structural stiffness as 1,674,000 N/m. Comparison of the estimated stiffness with the reference value obtained using NSAT indicates a relative errorof1.277%.

Table 5 When each additional PC was added, the AIC value was recomputed, until all34 PCs, converted from the power series, were included in the PCR, for the first floor of the test structure subjected to the El Centro earthquake

Component	AIC value	$\overrightarrow{\dot{x}(t)\ddot{x}(t)}$	3.7655	$\overrightarrow{\dot{x}(t)^2r(t)}$	2.3547
$\vec{x}(t)$	4.5088	$\overrightarrow{\dot{x}(t)r(t)}$	3.7633	$\overrightarrow{\ddot{x}(t)^2x(t)}$	2.354
$\vec{\dot{x}}(t)$	3.8076	$\overrightarrow{\ddot{x}(t)r(t)}$	3.7597	$\overrightarrow{\ddot{x}(t)^2\dot{x}(t)}$	2.3364
$\vec{\ddot{x}}(t)$	3.7865	$\vec{x}(t)^3$	3.7084	$\overrightarrow{\ddot{x}(t)^2r(t)}$	2.32
$\vec{r}(t)$	3.7782	$\vec{\dot{x}}(t)^3$	3.7061	$\overrightarrow{r(t)^2x(t)}$	2.3211
$\vec{x}(t)^2$	3.7797	$\vec{\ddot{x}}(t)^3$	3.6344	$\overrightarrow{r(t)^2\dot{x}(t)}$	2.1608
$\vec{\dot{x}}(t)^2$	3.774	$\vec{r}(t)^3$	2.876	$\overrightarrow{r(t)^2\ddot{x}(t)}$	2.1614
$\vec{\ddot{x}}(t)^2$	3.771	$\overrightarrow{x(t)^2\dot{x}(t)}$	2.5789	$\overrightarrow{x(t)\dot{x}(t)\ddot{x}(t)}$	2.0957
$\vec{r}(t)^2$	3.7709	$\overrightarrow{x(t)^2\ddot{x}(t)}$	2.3651	$\overrightarrow{x(t)\dot{x}(t)r(t)}$	2.0065
$\overrightarrow{x(t)\dot{x}(t)}$	3.772	$\overrightarrow{x(t)^2r(t)}$	2.3613	$\overrightarrow{x(t)\ddot{x}(t)r(t)}$	1.962
$\overrightarrow{x(t)\ddot{x}(t)}$	3.7715	$\overrightarrow{\dot{x}(t)^2x(t)}$	2.3573	$\overrightarrow{\dot{x}(t)\ddot{x}(t)r(t)}$	1.9516
$\overrightarrow{x(t)r(t)}$	3.7729	$\overrightarrow{\dot{x}(t)^2\ddot{x}(t)}$	2.3535		

Table 6 Final results of the model refinement approach using PCR combined with AIC, indicating the PC and the corresponding AIC value, using the power series model for the first floor of the test structure subjected to the El Centro earthquake

Component	$\vec{x}(t)$	$\vec{\dot{x}}(t)$	$\vec{\ddot{x}}(t)$	$\vec{r}(t)$
AIC value	4.5088	3.8076	3.7865	3.7782

5.4 Experimental results

Following the flow chart of the experimental design for the case study shown in Fig. 3, the responses of the three-story structure subjected to the El Centro and TCU-084 earthquakes were represented by linear series, Taylor series, and power series models, each of which was refined and represented in the MAR modeling to estimate the structural stiffness of the building. These estimates were then compared to the values obtained using NSAT. Table 7 lists the average relative errors of the estimates of the three-story structural stiffness, using the three statistical model refinement approaches: the efficient AIC, the PCA, and the PCR combined with AIC.

Table 7 demonstrates that for the power series model, the model refinement approach using the AIC yields the best results, with the smallest average relative error of 2.887%, which is 1.418% lower than that for the original power series model. Comparisons with the results obtained with the other two refinement approaches, the PCA and the PCR combined with AIC, were also conducted, and the smallest average relative errors obtained were 3.266% and 3.518%, respectively, for the refined Taylor series. However, the model refinement approach using the PCA did not reduce the average relative error for the original linear series, the Taylor series, or the power series model, and neither did the approach using the PCR combined with AIC. The remaining components in the refined power series model for the El Centro earthquake obtained using the AIC for the estimation of the 3-story structural stiffness are listed in Table 8.

As Table 8 shows, the refined power series model only contains either 8 or 9 components for the 3-story structure, a drastically reduced model in comparison to the original model with 34 components.

Table 7 Average relative errors of the estimates of the three-story structural stiffness obtained using the three statistical model refinement approaches (efficient AIC, PCA, and PCR combined with AIC), for two earthquakes and three original models

	El Centro	TCU-084	Total average
Original linear series	3.106%	6.195%	4.651%
Refined linear by AIC	3.106%	6.195%	4.651%
Refined linear by PCA	3.106%	6.195%	4.651%
Refined linear by PCR+AIC	3.127%	6.196%	4.661%
Original Taylor series	4.733%	1.80%	3.266%
Refined Taylor by AIC	4.268%	1.576%	2.922%
Refined Taylor by PCA	4.733%	1.80%	**3.266%**
Refined Taylor by PCR+AIC	4.669%	2.366%	**3.518%**
Original power series	3.886%	4.724%	4.305%
Refined power by AIC	3.740%	2.034%	**2.887%**
Refined power by PCA	3.382%	7.671%	5.526%
Refined power by PCR+AIC	2.562%	7.039%	4.801%

Table 8 Refined power series model for the El Centro earthquake using the AIC for estimation of the 3-story structural stiffness: the symbol " • " denotes a component retained in the refined model

Component	Story 1	Story 2	Story 3
$x(t)$	•	•	•
$\dot{x}(t)$	•	•	•
$\ddot{x}(t)$	•	•	•
$r(t)$	•	•	•
$x(t)^2$			
$\dot{x}(t)^2$			
$\ddot{x}(t)^2$			
$r(t)^2$			
$x(t)\dot{x}(t)$			
$x(t)\ddot{x}(t)$	•		
$x(t)r(t)$			
$\dot{x}(t)\ddot{x}(t)$			•
$\dot{x}(t)r(t)$			
$\ddot{x}(t)r(t)$			
$x(t)^3$	•		
$\dot{x}(t)^3$	•	•	
$\ddot{x}(t)^3$			
$r(t)^3$	•	•	
$x(t)^2\dot{x}(t)$			•
$x(t)^2\ddot{x}(t)$			
$x(t)^2r(t)$			
$\dot{x}(t)^2x(t)$			•
$\dot{x}(t)^2\ddot{x}(t)$	•	•	•
$\dot{x}(t)^2r(t)$			
$\ddot{x}(t)^2x(t)$			
$\ddot{x}(t)^2\dot{x}(t)$			
$\ddot{x}(t)^2r(t)$			
$r(t)^2x(t)$			
$r(t)^2\dot{x}(t)$			
$r(t)^2\ddot{x}(t)$			
$x(t)\dot{x}(t)\ddot{x}(t)$			
$x(t)\dot{x}(t)r(t)$			
$x(t)\ddot{x}(t)r(t)$		•	
$\dot{x}(t)\ddot{x}(t)r(t)$			

6. Conclusions

Structural health monitoring plays an important role in the maintenance and repair of structures. For structural health monitoring to be effective, precise signal processing is necessary and influences the accuracy of the estimation of structural stiffness. In this study, an efficient statistical model refinement approach using the AIC was developed and applied to a3-story test structure constructed in a laboratory for use in structural health monitoring research. If a newly added component of the model decreases the AIC value compared to the AIC value of the previously added component(s), it is statistically justifiable to retain this new component in the model; otherwise, it should be removed. This model refinement process is repeated until all of the components in the model are judged to be statistically sustainable.

This paper describes the establishment of structural models for representing the restoring force, MAR modeling for purposes of structural stiffness estimation, and the establishment and verification of statistical model refinement approaches. For the power series model, the model refinement approach using AIC was found to yield the best results, with the smallest average relative error of 2.887%, which is 1.418% lower than that of the original power series model without model refinement. The smallest average relative errors produced using the other two model refinement approaches, PCA and PCR combined with AIC, were3.266% and 3.518%, respectively, for the refined Taylor series model. However, the PCA model refinement approach did not reduce the average relative error for the original linear series, the Taylor series, or the power series model, and neither did the approach using the PCR combined with the AIC.

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References

Abdul-Wahab,S.A., Bakheit,C.S. and Al-Alawi, S.M. (2005), “Principal component and multiple regression analysis in modelling of ground-level ozone and factors affecting its concentrations”, *Environ. Modell. Softw.*, **20**(10), 1263-1271.

Ahin, A.Ş., Bayraktar, A., Özcan, D.M., Sevim, B., Altunışık, A.C. and Türker, T. (2011), “Dynamic field test, system identification, and modal validation of an RC minaret: preprocessing and post processing the wind-induced ambient vibration data”, *J. Perform. Constr. Fac.*, **25**(4), 336-356.

Bennett, P.J., Soga, K., Wassell, I., Fidler, P., Abe, K., Kobayashi, Y. and Vanicek, M. (2010), “Wireless sensor networks for underground railway applications: case studies in Prague and London”, *Smart Struct. Syst.*, **6**(5-6), 619-639.

Boukhatem, B., Kenai, S., Hamou, A.T., Ziou D.J. and Ghrici, M. (2012), “Predicting concrete properties using neural networks (NN) with principal component analysis (PCA) technique”, *Comput. Concr.*, **10**(6), 557-573.

Chan, C.M., Tang, S.K., Wong, H. and Lee, W.L. (2012), “On weak unsteady signal detection using

statistical tests", *Appl. Acoust*., **73**(2), 164-172.
Chang, P.C., Flatau, A. and Liu, S.C. (2003), "Review paper: health monitoring of civil infrastructure", *Struct. Health Monit.*, **2**(3), 257-267.
Depczynski, U., Frost, V.J. and Molt, K. (2000), "Genetic algorithms applied to the selection of factors in principal component regression", *Anal. Chimica Acta*, **420**(2), 217-227.
Figueiredo, E., Figueiras, J., Park, G., Farrar, C.R. and Worden, K.(2011), "Influence of the autoregressive model order on damage detection", *Comput. Aided Civil Infrastruct. Eng*., **26**(3), 225-238.
Harada, T., Ariyoshi, N., Shimura, H., Sato, Y., Yokoyama, I., Takahashi, K., Yamagata, S.I., Imamaki, M., Kobayashi, Y., Ishii, I., Miyazaki, M. and Kitada, M. (2010), "Application of Akaike information criterion to evaluate warfarin dosing algorithm", *Thrombosis Res.*, **126**, 183-190.
Ho, D.D., Lee, P.Y., Nguyen, K.D., Hong, D.S., Lee, S.Y., Kim, J.T. and Shinozuka, M. (2012). "Solar-powered multi-scale sensor node on Imote2 platform for hybrid SHM in cable-stayed bridge", *Smart Struct. Syst.*, **9**(2), 145-164.
Huang, N.E., Shen, Z., Long, S.R. Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.C., Tung, C.C. and Liu, H.H.(1998), "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis", *Procedures of the Roy. Soc. London*, **454**(1971), 903-995.
Hutter, M. and Tran, M.N. (2010), "Model selection with the loss rank principle", *Comput. Statist. Data An*., **54**(5), 1288-1306.
Jamoos, A., Grivel, E., Shakarneh, N. and Abdel-Nour, H. (2010), "Dual optimal filters for parameter estimation of a multivariate autoregressive process from noisy observations", *Signal Process*., **5**(5), 471-479.
Jolliffe, I.T. (1986), *Principal Component Analysis*, second edition, Springer, New York, NY, USA.
Jo, H., Park, J.W., Spencer, B.F., Jr. and Jung, H.J. (2013), "Development of high-sensitivity wireless strain sensor for structural health monitoring", *Smart Struct. Syst.*, **11**(5), 477-496.
Kim, S., Torbol, M. and Chou, P.H. (2013), "Remote structural health monitoring systems for next generation SCADA", *Smart Struct. Syst.*, **11**(5), 511-531.
Lautour, O.R. and Omenzetter, P. (2010), "Damage classification and estimation in experimental structures using time series analysis and pattern recognition", *Mech. Syst. Signal Pr*., **24**(5), 1556-1569.
Li, H.G. and Meng, G. (2007), "Nonlinear dynamics of a SDOF oscillator with Bouc–Wen hysteresis", *Solitons Fract.*, **34**(2), 337-343.
Lin, J.W., Betti, R., Smyth, A.W. and Longman, R.W. (2001), "On-line identification of non-linear hysteretic structural systems using a variable trace approach", *Earthq. Eng. Struct. D*., **30**(9), 1279-1303.
Lin, J.W. and Chen, H.J. (2009), "Repetitive identification of structural systems using a nonlinear model parameter refinement approach", *Shock Vib*., **16**, 229-240.
Lin, J.W., Chen, C.W. and Hsu, T.C. (2013), "A novel regression prediction model for structural engineering applications", *Struct. Eng. Mech.*, **45**(5), 693-702.
Lin, J.W. (2011), "A hybrid algorithm based on EEMD and EMD for multi-mode signal processing", *Struct. Eng. Mech.*, **39**(6), 813-831.
Lin, J.W., Chen, C.W. and Chung, S.H. (2012), "Modeling and assessment of bridge structure for seismic hazard prevention", *Natural Hazards*, **61**(3), 1115-1126.
Lin, J.W. (2010), "Mode-by-mode evaluation of structural systems using a bandpass-HHT filtering approach", *Struct. Eng. Mech*., **36**(6), 697-714.
Ljung, L. (1999), *System Identification: Theory for the User*, Prentice-Hal PTR, New Jersey.
Lu, K.C., Loh, C.H., Yang, Y.S., Lynch, J.P. and Law, K.H. (2008), "Real-time structural damage detection using wireless sensing and monitoring system", *Smart Struct. Syst.*, **4**(6), 759-77.
Masri, S.F., Smyth, A.W., Chassiakos, A.G., Caughey, T.K. and Hunter, N.F. (2000), "Application of neural networks for detection of changes in nonlinear systems", *J. Eng. Mech. - ASCE*, **126**(7), 666-676.
McAdams, H.T., Crawford, R.W. and Hadder, G.R. (2000), *A vector approach to regression analysis and its application to heavy-duty diesel emissions, Society of Automotive Enginee*rs, Inc. 2000-01-1961.
Meo, M., Zumpano, G., Meng, X., Cosser, E., Roberts, G. and Dodson, A. (2006), "Measurements of dynamic properties of a medium span suspension bridge by using the wavelet transforms", *Mech. Syst.*

Signal Pr., **20**(5), 1112-1133.
Milenkovic, P. (2011), "Solution of the forward dynamics of a single-loop linkage using power series", *J. Dynam. Syst. Measurement Control*, **133**(6), 061002 (9 pages).
Nagayama, T., Sim, S.H., Miyamori, Y. and Spencer, B.F. (2007), "Issues in structural health monitoring employing smart sensors", *Smart Struct. Syst.*, **3**(3), 299-320.
Park, B., Chen, Y.R., Hruschka, W.R., Shackelford, S.D. and Koohmaraie, M. (2001), "Principal component regression of near–infrared reflectance spectra for beef tenderness prediction", *T. Am. Soc. Agricultural Engineers,* **44**, 609-615.
Pei, J.S., Smyth, A.W. and Kosmatopoulos, E.B. (2004), "Analysis and modification of Volterra/Wiener neural networks for the adaptive identification of non-linear hysteretic dynamic systems", *J. Sound Vib*., **275**(3-5), 693-718.
Posada, D. and Buckley, T.R. (2004), "Model selection and model averaging in phylogenetics: advantages of Akaike information criterion and Bayesian approaches over likelihood ratio tests", *Systematic Biologists*, **53**(5), 793-808.
Rogers, B.P., Katwal, S.B., Morgan, V.L., Asplund, C.L. and Gore, J.C. (2010), "Functional MRI and multivariate autoregressive models", *Mag. Reson. Imaging*, **28**(8), 1058-1065.
Sousa, S.I.V., Martins, F.G., Alvim-Ferraz, M.C.M. and Pereira, M.C. (2007), "Multiple linear regression and artificial neural networks based on principal components to predict ozone concentrations", *Environ. Modell. Softw*., **22**(1), 97-103.
Statheropoulos, M., Vassiliadis, N. and Pappa, A. (1998), "Principal component and canonical correlation analysis for examining air pollution and metrological data", *Atmos. Environ*., **32**(6), 1087-1095.
Sumitro, S. and Wang, M.L. (2005), "Sustainable structural health monitoring system", *Struct. Health Monit.*, **12**(3-4), 445-467.
Taylor, S.G., Farinholt, K.M., Park, G., Todd, M.D. and Farrar, C.R. (2010), "Multi-scale wireless sensor node for health monitoring of civil infrastructure and mechanical systems", *Smart Struct. Syst.*, **6**(5-6), 661-673.
Tsai, C.S. (1996), *Nonlinear Stress Analysis Technique-NSAT*, Feng Chia University, Taiwan, R.O.C.
Wada, M., Ogawa, T., Sonoda, H. and Sato, K. (1996), "Development of relative power contribution ratio of the EEG in normal children: a multivariate autoregressive modeling approach", *Electroencephalography Clinical Neurophysiology*, **98**(1), 69-75.
Weng, J.H., Loh, C.H., Lynch, J.P., Lu, K.C., Lin, P.Y. and Wang, Y. (2008), "Output-only modal identification of a cable-stayed bridge using wireless monitoring systems", *Eng. Struct*., **30**(7), 1820-1830.
Wong, S.C. and Barhorst, A.A. (2006), "Polynomial interpolated Taylor series method for parameter identification of nonlinear dynamic system", *J. Comput. Nonlinear Dynam*., **1,**248-256.
Xia, B., Chan, A.P.C. and Yeung, J.F.Y. (2011), "Developing a fuzzy multicriteria decision-making model for selecting design-build operational variations", *J. Constr. Eng. M. - ASCE*, **137**(12), 1176-1184.

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