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Damage detection of multi-storeyed shear structure using sparse and noisy modal data

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Abstract. In the present paper, a method for identifying damage in a multi storeyed shear building structure is presented using minimum number of modal parameters of the structure. A damage at any level of the structure may lead to a major failure if the damage is not attended at appropriate time. Hence an early detection of damage is essential. The proposed identification methodology requires experimentally determined sparse modal data of any particular mode as input to detect the location and extent of damage in the structure. Here, the first natural frequency and corresponding partial mode shape values are used as input to the model and results are compared by changing the sensor placement locations at different floors to conclude the best location of sensors for accurate damage identification. Initially experimental data are simulated numerically by solving eigen value problem of the damaged structure with inclusion of random noise on the vibration characteristics. Reliability of the procedure has been demonstrated through a few examples of multi storeyed shear structure with different damage scenarios and various noise levels. Validation of the methodology has also been done using dynamic data obtained through experiment conducted on a laboratory scale steel structure.

Keywords: shear structure; damage; eigen value; eigen vector; modal data; noise

1. Introduction

During the last three decades, vibration based methods have been developed and applied to detect structural damage in the civil, mechanical and aerospace engineering disciplines (Cawley and Adams 1979, Stubbs and Osegueda 1990, Ricles and Kosmatka 1992). These methods are based on the fact that the vibration characteristics of structures (namely frequencies, mode shapes, and modal damping) are functions of the structural parameters such as mass, stiffness and damping. Structural damage usually causes a decrease in structural stiffness, which produces change in the vibration characteristics of the structure.

Chatterjee *et al.* (1994) have studied the vibration characteristics of a bridge structure when subjected to live loads. Li *et al.* (1999) have used flexibility approach for damage identification of cantilever type shear structures. Some related research work on vibration based damage identification are Santos *et al.* (2000), Dulieu-Barton *et al.* (2003), Darpe *et al.* (2004) and Gupta (2006). Bandyopadhyay and Bhattacharyya (2007) presented a statistical system identification

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technique for health monitoring of beam and truss structures from limited noisy data. For dealing with noise in experimental data Panigrahi *et al.* (2009) developed a technique by optimising the residual force vector using genetic algorithm. The authors validated the methodology on a simulated uniform strength beam with different damage scenarios. Yang and Liu (2009) have presented a methodology by the eigen parameter decomposition of structural flexibility change to identify damage. A number of researchers have reviewed the works done on vibration based damage identification methods (Doebling *et al.* (1998), Carden and Fanning (2004), Humar *et al.* (2006), Fan and Qiao (2011) to name a few).

Damage identification techniques are important for ensuring structural integrity and for health monitoring of structures. In case of an actual structure, mass and stiffness are distributed throughout the structure. Many a times, the structure can still be idealised as a lumped system. Usually, such a lumped mass idealisation leads to satisfactory result. Moreover, the mathematical model of the system becomes simpler to adopt. Thus, for damage identification, a structure may be represented as a lumped mass system for ease in computation. The damage is identified from reduction in the physical properties of a structure between two time-separated inferences.

Although a number of works have been carried out in this particular area, still efforts are going on to refine and develop the analytical models for more accurate results. Udwadia (1994) have developed a methodology for optimum sensor locations for parameter identification in dynamic system. Loh and Ton (1995) studied a system identification approach to detect changes in structural dynamic characteristics on the basis of measurements. The authors used the recursive instrumental variable method and extended Kalman filter algorithm for the development of damage identification methodology. A memory-matrix based identification methodology for structural and mechanical systems has been studied by Udwadia and Proskurowski (1998). Yuan *et al.* (1998) have developed a method that estimates mass and stiffness matrices of shear structure from first two orders of structural mode measurement. Many references related to this topic for system identification of buildings may be found in a review paper by Datta *et al.* (1998). Morita *et al.* (2001) presented a damage detection technique of a five-storey steel frame with simulated damages. Chakraverty (2005) proposed computationally efficient procedures to refine the methods of Yuan *et al.* (1998) to identify the structural parameters from modal test data. The refinement has been obtained by using Holzer criteria (Harker 1983) along with other numerical techniques (Bhat and Chakraverty 2007). Medhi *et al.* (2008) used the system identification technique for health monitoring of shear structure. Casciati (2008) optimized an objective function considering stiffness values as optimisation variables. The computed stiffness values are compared with the designed one to identify the damage. The authors have considered a beam problem for identification of damage. The proposed strategy was adequate to follow the progressive growth of the damage, but it was not successful when the damage was already spread. Meruane and Heylen (2011) implemented a hybrid real-coded genetic algorithm with damage penalization to locate and quantify structural damage. The method is tested with different levels of incompleteness in the measured degrees of freedom. Chaekuk *et al.* (2011) introduced a new damage evaluation method that identifies the structural damage in a shear building based on a genetic algorithm using the structural flexibility matrix with dynamic analyses. The proposed method enables the deduction of the extent and location of structural damage, even when there is insufficient data on the dynamic characteristics and insufficient accurate measurements of the structural stiffness and mass. Panigrahi *et al.* (2013) established a methodology to identify damage in a shear structure using Genetic Algorithm with sparse modal information. The authors validated the methodology for identification of damage on a simulated multi-storeyed shear structure with different damage scenarios with incomplete modal information as input. Kourehli *et al.* (2013) presented a novel approach to structural damage detection and estimation using incomplete modal data and incomplete static response of a damaged structure. The proposed method used available modal data or static displacement to formulate objective function which is further optimized by the simulated annealing algorithm to find out damage location and severity of damage in structural elements. The authors successfully applied the methodology for damage identification in a simply supported beam and a three-storey plane frame with and without noise in modal data and containing several damages.

In the present paper, a method for locating and quantifying damage in a multi-storeyed shear building structure is presented using minimum number of modal characteristics of the structure. This paper consists of three parts. In the first part, a procedure has been developed to estimate the stiffness distribution matrices of a multi-storeyed shear building structure by using only the full modal data i.e., frequency and mode shape of any mode of vibration and the design values of the mass matrix. In the second part, a technique for identifying the full modal information from partial modal data has been discussed. In the last part, the stated methods are combined and applied for identification of damage at different damage scenarios with known frequency and partial mode shape values of any particular mode of vibration. The analysis is also carried out for situations where measurement noise in the vibration characteristics is present. First the input modal data is generated from the vibration analysis superimposed with different noise levels. Results are shown in tabular and graphical forms to show the efficacy of the method for damage identification. A detail study has been carried out for identifying the number of sensors and best location of sensors for identification of damage with maximum accuracy. The proposed model has also been validated with the available results in literature. Next, dynamic tests have been conducted on a laboratory scale for a five storey steel structure with and without damages and the developed methodology is validated.

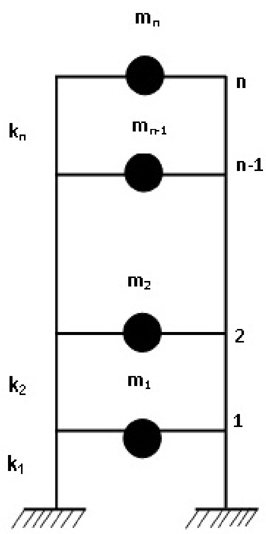

Fig. 1 Multi-storey structure with *n* levels

2. Determination of structural parameters

2.1 Determination of stiffness values

An idealised n-storeyed shear structure is considered as shown in Fig. 1. For a shear structure of n levels, with k_1, k_2..... k_n and m_1, m_2......m_n as the corresponding stiffness and mass values at different levels, the equation of motion of free vibration may be written as

$$M\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \cdot \\ \cdot \\ \ddot{y}_n \end{Bmatrix} + K\begin{Bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} \tag{1}$$

In Eq. (1), M and K are the global mass and stiffness matrices, which may respectively be written as

$$M = \begin{bmatrix} m_1 & 0 & ... & ... & 0 \\ 0 & m_2 & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ ... & ... & 0 & m_{n-1} & 0 \\ 0 & ... & ... & 0 & m_n \end{bmatrix}, \quad K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & ... & 0 \\ -k_2 & k_2+k_3 & -k_3 & ... & 0 \\ ... & ... & & & ... \\ 0 & ... & -k_{n-1} & k_{n-1}+k_n & -k_n \\ 0 & & ... & -k_n & k_n \end{bmatrix}$$

and

$$\{\ddot{y}_1, \ \ddot{y}_2, \ \cdots \ \cdots \ \ddot{y}_n\}^T \ \textit{and} \ \{y_1, \ y_2, \ \cdots \ \cdots \ y_n\}^T$$

are the vectors of acceleration and displacements respectively.

For simple harmonic motion

$$\{y_1, \ y_2, \ \cdots \ \cdots \ , y_n\}^T = \{\phi^{(1)}, \ \phi^{(2)}, \ \cdots \ \cdots \ , \phi^{(n)}\}^T e^{j\omega t}$$

may be assumed, where ω is a natural frequency of the dynamic system. Substituting the same in Eq. (1), we may obtain

$$[K - \lambda_j M]\begin{Bmatrix} \phi_j^{(1)} \\ \phi_j^{(2)} \\ \vdots \\ \phi_j^{(n)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{2}$$

where $\lambda_j = \omega_j^2$ is the j^{th} eigen value and $\phi_j^{(r)}$, $r=1, n$ is the j^{th} mode shape or eigen vector.

The above equation may be expanded and written as,

$$(k_1+k_2)\phi_j^{(1)} - k_2\phi_j^{(2)} - \lambda_j m_1\phi_j^{(1)} = 0$$
$$-k_2\phi_j^{(1)} + (k_2+k_3)\phi_j^{(2)} - k_3\phi_j^{(3)} - \lambda_j m_2\phi_j^{(2)} = 0$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$
$$-k_{n-1}\phi_j^{(n-2)} + (k_{n-1}+k_n)\phi_j^{(n-1)} - k_n\phi_j^{(n)} - \lambda_j m_{n-1}\phi_j^{(n-1)} = 0.$$
$$-k_n\phi_j^{(n-1)} + k_n\phi_j^{n} - \lambda_j m_n\phi_j^{n} = 0. \tag{3}$$

Yuan *et al.* (1998) and Chakraverty (2004) have determined the mass and stiffness values of all the levels by assuming that two frequency parameters and corresponding mode shapes are known. Moreover, the mass of the n^{th} level was assumed to be unity. In the present paper it is assumed that the mass parameters are known and these remain at their original values. A procedure is proposed to obtain the stiffness values of all levels by assuming that frequency and mode shape values of a particular mode of vibration are known.

Eq. (3) may now be rewritten and presented in matrix form as

$$\begin{bmatrix} \phi_j^{(1)} & \phi_j^{(1)}-\phi_j^{(2)} & 0 & \ldots\ldots & & 0 \\ 0 & \phi_j^{(2)}-\phi_j^{(1)} & \phi_j^{(2)}-\phi_j^{(3)} & \ldots\ldots. & & 0 \\ . & . & . & \ldots\ldots.. & \ldots & \\ . & . & . & \ldots\ldots & \ldots & \\ 0 & 0 & \ldots.. & \phi_j^{(n-1)}-\phi_j^{(n-2)} & & \phi_j^{(n-1)}-\phi_j^{(n)} \\ 0 & 0 & \ldots\ldots. & 0 & & \phi_j^{(n)}-\phi_j^{(n-1)} \end{bmatrix} \times \{k_1, k_2, \ldots\ldots\ldots k_{n-1}, k_n\}^T$$

$$= \left\{m_1\lambda_j\phi_j^{(1)}, m_2\lambda_j\phi_j^{(2)},\ \ldots\ldots\ldots.m_{(n-1)}\lambda_j\phi_j^{(n-1)}, m_{(n)}\lambda_j\phi_j^{(n)}\right\}^T \tag{4}$$

Eq. (4) is then written in compact form as

$$[C]_{n\times n}\{c\}_{n\times 1} = \{D\}_{n\times 1} \tag{5}$$

where C is the square matrix and $\{c\}$ and $\{D\}$are the column vectors, representing the stiffness and the modified mass matrices.

To avoid ill conditioning in computing $\{c\}$ of Eq. (5) directly, the following steps are considered

$$C^T C\{c\} = C^T\{D\} \tag{6}$$

First, the Eq. (5) is rewritten by multiplying $[C]^T$ on both sides as
Hence

$$\{c\} = \left[C^T C\right]^{-1}[C]^T\{D\}$$

Now the vector $\{c\}$ of unknown stiffness parameters may be obtained as

$$\{c\} = \left[C^T C\right]^{-1}[C]^T \begin{Bmatrix} m_1\lambda_j\phi_j^{(1)} \\ m_2\lambda_j\phi_j^{(2)} \\ \vdots \\ m_{(n-1)}\lambda_j\phi_j^{(n-1)} \\ m_{(n)}\lambda_j\phi_j^{(n)} \end{Bmatrix} \tag{7}$$

where

$$\{c\} = \{k_1, \quad k_2, \quad \cdots \; k_{n-1}, \; k_n\}^T{}_{n\times 1}$$

Using Eq. (7), stiffness parameter for all the levels of the structure may be obtained in terms of the mass parameters, one frequency parameter λ_j and corresponding mode shape $\phi_j^{(r)}$.

2.2 *Computation of full modal information from partial modal data*

In this section, a method is described to obtain the complete mode shape at a particular frequency from partial modal information (Panigrahi *et al.* 2013).

Eq. (2) may alternatively be expressed as

$$[A]_{n\times n}\{\phi_j\}_{n\times 1} = \{0\} \tag{8}$$

Let total number of levels where modal measurements are made be Q at the j^{th} frequency. The displacement at other levels may be calculated analytically from the existing modal data by rearranging Eq. (8). The levels whose modal component is known may be represented by P_q where q varies from 1 to Q.

Now Eq. (8) may be represented as

$$[A]_{nXn}\{\phi_j\}_{n\times 1} - \sum_{q=1}^{Q} \{A_{P_q}\}\phi_{jP_q} = -\sum_{q=1}^{Q} \{A_{P_q}\}\phi_{jP_q} \tag{9}$$

where A_{Pq} represents the P_q^{th} column of A matrix, ϕ_{jPq} represents the P_q^{th} element of modal matrix ϕ_j.

Eq. (9) may be rewritten as

$$[A']_{(n-Q)\times(n-Q)}\{\phi_j'\}_{(n-Q)\times 1} = Z \tag{10}$$

where A' is the reduced A matrix obtained by eliminating P_q^{th} row and P_q^{th} column. Similarly ϕ_j'

is the reduced column matrix from ϕ_j without P_q^{th} element. We define Z as

$$Z = -\sum_{q=1}^{Q} \left\{A'_{P_q}\right\} \phi_{jP_q}$$

Here A'_{Pq} is the P_q^{th} column matrix of A without the P_q^{th} elements in it and ϕ_j' represents the unknown modal matrix. This may be computed from Eq. (9) as

$$\left\{\phi_j'\right\} = (A'^T A')^{-1} (A'^T Z) \tag{11}$$

Assuming that only one sensor is placed at l^{th} level and so, 1^{st} to $(l\text{-}1)^{th}$ and $(l+1)^{th}$ to n^{th} level modal components are unknown. Using the known data for l^{th} level i.e. $\phi_j^{(l)}$ (of the j^{th} mode), Eq. (10) may be expressed as

$$\begin{bmatrix} P_1 & -k_2 & \cdots & 0 & 0 & & 0 & \cdots & 0 & & \\ -k_2 & P_2 & & & & & & \cdots & \vdots & & \\ 0 & & & -k_{l-1} & P_{l-1} & 0 & 0 & \cdots & 0 & & \\ 0 & & & \cdots & 0 & P_{l+1} & -k_{l+2} & \cdots & 0 & & \\ 0 & & & \cdots & & -k_{l+2} & P_{l+2} & \cdots & \vdots & & \\ 0 & & & \cdots & 0 & \cdots & & \cdots & \vdots & & \\ 0 & & \cdots & \cdots & & \cdots & 0 & \cdots & \vdots & & \\ 0 & & & \cdots & & \cdots & & \cdots & \vdots & & \\ 0 & & & \cdots & 0 & \cdots & 0 & \cdots & \vdots & -k_n & k_n - \lambda_j m_n \end{bmatrix} \times \begin{Bmatrix} \phi_j^{(1)} \\ \vdots \\ \phi_j^{(l-1)} \\ \phi_j^{(l+1)} \\ \vdots \\ \vdots \\ \vdots \\ \phi_i^{(n)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ k_l \phi_j^{(l)} \\ k_{l+1} \phi_j^{(l)} \\ \vdots \\ \vdots \\ 0 \end{Bmatrix} \tag{12}$$

where $P_t = k_t + k_{(t+1)} - \lambda_j m_t$ for $t = 1, 2, \ldots l \ldots n$.

The unknown modal matrix may be obtained by using Eq. (11).

The mode shape is obtained only as a ratio of the modal components. Hence, when two measurements are made at r and s levels, one of the values, say $\phi_j^{(r)}$ may be taken as unity and $\phi_j^{(s)}$ may be taken as the ratio of the two measurements. Other modal values are then obtained such that these are scaled with respect to $\phi_j^{(r)}$.

3. Damage assessment

It is presumed that the developed damage in a structural system causes change in the stiffness without any major change in the mass at that level. If the amount of reduction in stiffness and the level at which this reduction takes place can be identified, then extent of damage and location may be identified. In general, the design mass and stiffness matrices of a structure during its construction are known a priori. During the inspection for damage identification of the structure, if one frequency and corresponding partial mode shape values are measured experimentally, the stiffness parameters of the damaged structure may be computed using Eqs. (11) and (7). By comparing each stiffness parameter of damaged state with that of design stiffness parameter, extent of damage and the location of damage may be identified.

Table 1 The design values of stiffness and mass taken for a ten-storeyed structure

Stiffness values in kN/m	k_1	k_2	k_3	k_4	k_5	k_6	K_7	k_8	k_9	k_{10}
	23.5	27	27	27	27	27	27	27	27	27
Mass values in kg.	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}
	12	12	12	12	12	12	12	12	12	10

Table 2 Mass participation factors for various modes of vibration

Mode No.	1st	2nd	3rd	4th	5th
Mass participation factor in %	79.38	8.89	3.25	1.68	1.03

For the illustration purpose, a ten-storeyed shear building structure with design structural parameters as given in Table 1 is considered as an example.

Two numbers of damaged scenarios i.e., examples (a) and (b) are presented here to show the reliability of the methodology. In practical cases, experimentally obtained modal information is used for damage identification. However, in the absence of experimental results, the input modal data generated from the vibration analysis of the known damaged shear structure is superimposed with noise. Hence, for simulating an experimental measurement, here the natural frequencies and the mode shape are perturbed randomly by measurement noise of two different levels:

- **Noise level I**

A random noise of 0.5% on frequency value and a random noise of 1% on each modal component of mode shape are imposed individually.

- **Noise level II**

A random noise of 1% on frequency value and a random noise of 3% on each modal component of mode shape are imposed individually.

The natural frequency and corresponding mode shape (partial/ full) values for the first mode are considered as known. Also the first mode is contributing 79.38% of the total mass for the undamaged structure in mass participation as shown in Table 2.

Example (a)

A ten-storeyed shear building structure having some damage is considered. It is assumed that the third and eighth storeys of the structure are damaged partially to an extent of 40% and 20%

respectively. Damage factor is the ratio of stiffness value of damaged structure to that of undamaged one. Hence, the damage factor associated with third and eighth storeys are 0.6 and 0.8 respectively. For other storeys, the prescribed damage factor is 1.0. Dynamic analysis has been carried out for the damaged shear building structure by reducing the stiffness values of third and eighth level by 40% and 20% respectively. The first natural frequency and corresponding modal information of the damaged structure are shown in Table 3.

Stiffness values are computed using the above modal information for the first natural frequency and design mass values (shown in Table 1). Damage factors and the percentage errors are calculated by comparing these values with the design ones.

The analysis is repeated by superimposing noise levels I and II on the simulated modal information. Corresponding results are shown in Table 4. In the absence of noise, damage is correctly identified. With noise level I, the maximum error percentage in computing the damage factors is around 8% and for noise level II this is around 12%.

Next, the cases of known frequency value and corresponding partial mode shape have been utilized for the identification. Here four cases of sensor placements have been considered. The damage factors and the maximum % error have been computed and presented in Tables 5 and 6.

From the above two Tables 5 and 6, a conclusion on sensor location and their effect on damage identification has been done and presented in Table 7.

Table 3 Values of first natural frequency and corresponding mode shape

Frequency in Hz.	$\phi_1^{(1)}$	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$	$\phi_1^{(5)}$	$\phi_1^{(6)}$	$\phi_1^{(8)}$	$\phi_1^{(8)}$	$\phi_1^{(9)}$	$\phi_1^{(10)}$
1.0531	0.1527	0.2826	0.4900	0.6048	0.7079	0.7973	0.8711	0.9422	0.9808	1.0

Table 4 Identified damage factors for problem (a) from first natural frequency and corresponding full mode shape for different noise levels

Story No.	**Theoretical damage factor**	**Without noise**		**Noise level I**		**Noise level II**	
		Identified damage factor	**% of error**	**Identified damage factor**	**% of error**	**Identified damage factor**	**% of error**
1	1.0	1.0	0.0	0.992	0.8	1.0	0.0
2	1.0	1.0	0.0	1.0	0.0	1.0	0.0
3	**0.6**	0.6	0.0	0.570	5.0	0.576	4.0
4	1.0	1.0	0.0	1.0	0.0	0.991	0.9
5	1.0	1.0	0.0	0.931	6.9	1.0	0.0
6	1.0	1.0	0.0	1.0	0.0	0.957	4.3
7	1.0	1.0	0.0	1.0	0.0	1.0	0.0
8	**0.8**	0.8	0.0	0.736	**8.0**	0.703	**12.1**
9	1.0	1.0	0.0	0.979	2.1	0.895	10.5
10	1.0	1.0	0.0	1.0	0.0	1.0	0.0

Table 5 Damage factors obtained using the first frequency value and corresponding partial mode shape (i) at 1st and 2nd level and (ii) at 1st, 3rd and 5th level

Story No.	Theoretical Damage Factor	Sensors placed at 1st and 2nd level (Case-a)			Sensors placed at 1st, 3rd and 5th level (Case-b)		
		without noise	**with noise level I**	**with noise level II**	**without noise**	**with noise level I**	**with noise level II**
1	1.0	1.0	1.0	**0.782**	0.998	0.962	0.982
2	1.0	0.99	0.999	1.0		1.0	0.803
3	**0.6**	0.6	0.654	0.645	0.595	**0.675**	0.569
4	1.0	1.0	0.891	0.927	1.0	0.998	1.0
5	1.0	1.0	0.970	1.0	**0.991**	0.926	0.959
6	1.0	0.997	0.950	0.959	0.998	1.0	0.922
7	1.0	1.0	1.0	0.881	1.0	0.973	**0.820**
8	**0.8**	0.807	**0.688**	0.867	0.795	0.783	0.692
9	1.0	0.994	0.887	0.833	1.0	0.940	1.0
10	1.0	**0.99**	0.898	0.848	0.999	0.968	0.905
Maximum % of error		**1**	**14**	**22**	**1**	**12.5**	**18**

Table 6 Damage factors obtained using the first frequency value and corresponding partial mode shape (i) at 1st and 10th level and (ii) at 1st, 5th and 10th level

Story No.	Theoretical Damage Factor	Sensors placed at 1st and 10th level (Case-c)			Sensors placed at 1st, 5th and 10th level (Case-d)		
		without noise	**with noise level I**	**with noise level II**	**without noise**	**with noise level I**	**with noise level II**
1	1.0	1.0	0.912	0.952	0.998	0.968	0.978
2	1.0	0.997	0.942	1.0	1.0	1.0	0.992
3	**0.6**	**0.597**	0.655	0.676	0.602	0.587	0.655
4	1.0	1.0	1.0	0.889	0.999	0.912	1.0
5	1.0	0.998	0.912	1.0	1.0	0.931	0.952
6	1.0	0.995	0.898	0.879	0.997	0.973	0.986
7	1.0	1.0	1.0	**0.845**	1.0	0.983	0.973
8	**0.8**	0.802	0.732	0.762	0.8	**0.721**	0.692
9	1.0	0.999	1.0	1.0	1.0	0.989	**0.861**
10	1.0	1.0	**0.880**	0.899	**0.995**	1.0	1.0
Maximum % of error		**0.5**	**12**	**15.5**	**0.5**	**10.0**	**14.0**

Table 7 Comparison chart for identification error with sensors placed at different positions (1st mode considered)

Case	**Sensor position**	**Max. % of error in damage identification**	
		with noise level I	with noise level II
Case (a)	Sensors placed at 1^{st} and 2^{nd} level	**14.0**	**22.0**
Case (b)	Sensors placed at 1^{st} , 3^{rd} and 5^{th} level	**12.5**	**18.0**
Case (c)	Sensors placed at 1^{st} and 10^{th} level	**12.0**	**15.5**
Case (d)	Sensors placed at 1^{st}, 5^{th} and 10^{th} level	**10.0**	**14.0**

Table 8 Damage factors obtained using the first frequency value and corresponding partial mode shape at 1^{st}, 5^{th} and 10^{th} level

Story No.	**Theoretical Damage Factor**	**without noise**	**with noise level I**	**with noise level II**
1	1.0	0.988	**0.913**	0.882
2	0.8	**0.789**	0.750	0.903
3	1.0	0.991	0.961	**0.868**
4	1.0	0.985	0.951	0.927
5	0.55	0.546	0.508	0.601
6	1.0	1.0	1.0	0.959
7	1.0	1.0	0.976	1.0
8	0.7	0.71	0.684	0.679
9	1.0	1.0	0.987	0.898
10	1.0	0.986	1.0	1.0
Maximum % of error		**1.5**	**9.0**	**13.0**

From Table 7 it is observed that the accuracy in damage assessment depends upon the level of noise, no of sensors and their placement. The accuracy decreases as the noise level increases as obtained in all the four cases. It may be worth mentioning that the sensor placement location has significant role on accuracy of damage identification. Case (d) gives better accuracy in comparison to case (b) where same number of sensors are placed in both the cases but in case (d) it is distributed in comparison to case (b). Similarly, the case (c) is more accurate in comparison to case (b) even though additional sensor is placed in case (b). From the above cases it is observed that for better accuracy, the sensors are to be placed uniformly throughout the structure when modal information of first mode is known. Similar results were obtained when second frequency and corresponding partial mode shape values are considered but not presented herein. For best results of damage identification, sensors are to be placed in such a way that the curve of the mode considered is almost covered.

Example (b)

In this example, it is assumed that the second, fifth and eighth storeys of the structure are damaged partially to an extent of 20%, 45% and 30% respectively. Two noise levels as taken in the previous example are again considered here. Results are shown in Table 8 for first mode of vibration with sensors placed at first, fifth and tenth levels. It is observed that the error in damage detection is 1.5% for without damage case, 9% with noise level I and 13% with noise level II.

4. Validation of the methodology

4.1 Validation through available results

For validation of the proposed formulation, a comparison has been made with the results of Yang and Liu (2009). Yang and Liu (2009) proposed a new flexibility-based method to provide an insight to the characteristics of structural damage. The method proposed by the authors made use of the eigen parameter decomposition of structural flexibility change and approached the damage identification problem in a decoupled manner. The authors presented the results for a ten-storeyed shear building with an irregular distribution of mass and stiffness as shown in Fig. 2. Damage is simulated as a loss of stiffness of 15% in level 2 and 20% in level 6 considering 5% noise on modal information. The damage was identified using full modal information of four modes.

The same example has been analysed in the present paper and damage factors are obtained using the proposed methodology with known first frequency value and corresponding partial mode shape values at first, fifth and tenth level. Results have been obtained considering two situations. In the first situation, a random noise of 5% on both frequency value and on partial mode shape values is considered. Next, a case where 1% random noise on frequency value and 3% noise on modal information is considered. Results obtained in these two situations are compared with that of Yang and Liu (2009) in Table 9.

Table 9 Comparison of damage factors between Yang and Liu (2009) and present model considering noise on modal information

Story No.	**Actual Damage factor**	**Yang and Liu (2008) (Full modal data with 5% noise of first four modes known)**		**Present Method (frequency values of 1st mode and corresponding partial mode shape at 1st, 5th and 10th level known)**			
				Situation (a) 5% noise on both frequency and mode shape		**Situation (b) 1% noise on frequency and 3% noise on mode shape**	
		Damage factor	**Error %**	**Damage factor**	**Error %**	**Damage factor**	**Error %**
2	0.85	0.7708	9.0	0.697	**18.0**	0.735	**13.5**
6	0.80	0.6954	**13.0**	0.708	11.5	0.752	6.0

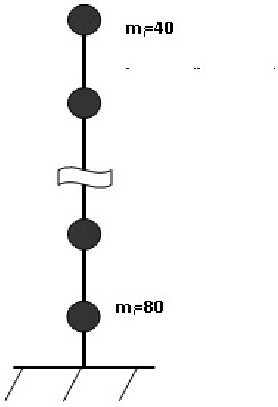

Fig. 2 Ten-story shear structure by Yang and Liu (2009)

Fig. 3 Laboratory Scale 5-Storeyed Steel Structure

Results given by Yang and Liu (2008) show a maximum of 13% error in damage identification. The authors have used four frequency values and corresponding complete mode shape values superimposed with 5% noise. As discussed earlier, practically it is difficult to calculate up to four modes and getting modal information from all the levels. In the present paper only first frequency values have been used and sensors are placed at three levels only and identified damage with 18% of error when modal information is superimposed with 5% noise (Situation (a)). At lower noise level as in situation (b), the % of error in damage identification is 13.5% which is almost same as of Yang and Liu (2009).

4.2 Validation through experimental results

For experimental verification, a test frame has been fabricated which is a five- storey steel structure as shown in Fig. 3. Floor height is 300mm and total height is 1.5 m. Floor plan is 200 mm × 300 mm. The beams, columns and floor plates are connected by bolts and tightened by spring washers and nuts to make the structure more rigid.

The accelerometers used are PCB Piezotronics, model 352C33 which are of high sensitivity (100 mV/g). Accelerometers are put at each floor level. Dynamic analysis has been carried out for both the intact and damaged structure. Here excitation to the structure has been given using a hammer. FFT of the accelerometer output has been done using the software SIGVIEW. The frequency domain graph is shown in Fig. 4. Time domain data has also been collected at ambient conditions. The signature record has been found to be noisy, but lower frequencies are accessible. Those data have not been shown here.

In this experiment, first natural frequency and corresponding mode shape have been considered for identification of damage. Theoretically first natural frequency and corresponding mode shape have been computed of the intact structure. Next, these are compared with the experimental ones as shown in Table 10. Here, the eigen vector is normalised so that modal value of the top floor will be unity. The comparison graph may also be seen in Fig. 5.

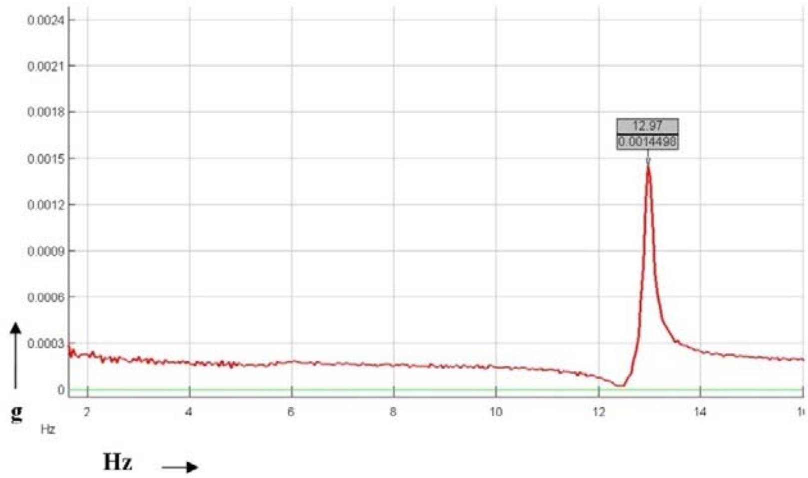

Fig. 4 Response in Frequency domain after FFT

Table 10 Theoretical and Experimental Frequency values and corresponding mode shapes (Intact Structure)

	First Natural Frequency in Hz	Mode Shape Values after normalisation				
		$\phi_1^{(1)}$	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$	$\phi_1^{(5)}$
Expt.	12.97	0.2747	0.5563	0.7651	0.89	1.0
Theo.	13.77	0.2846	0.5462	0.7635	0.9189	1.0

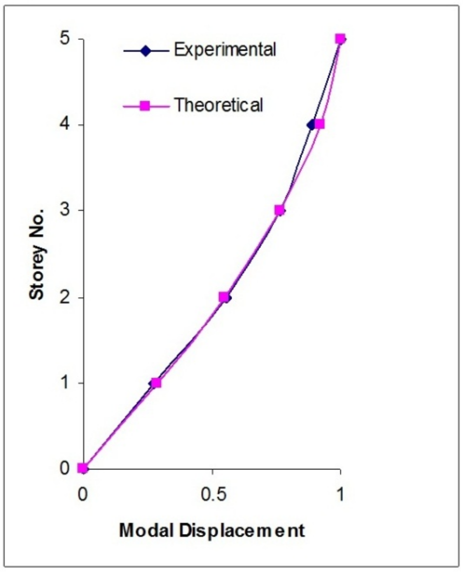

Fig. 5 Comparison of theoretical and experimental mode shapes for intact structure

The discrepancy between finite-element model data and measured data as shown in Table 10 may be due to model structure errors, model order errors, model parameter errors and errors in measurements (Friswell and Mottershead 1995). Here, the theoretically computed frequency value is updated by changing the stiffness values of each floor.

In general, to induce and control the damage in a structure either bolts are to be loosened or part of the column to be cut as discussed by Morita *et al*. (2001). However, here the authors have given the damage to a floor by reducing the width of the column at the floor level where damage is to be induced. However the mass removed has not been considered as it is very less compared to the floor mass considered. Many damages situations have been created to validate experimentally the proposed methodology for damage identification and optimization of sensor placements. However, only two cases of damage situations have been presented here. In the first case (a), second and fourth floor were damaged to a percentage of 10 and 30 respectively. Next in case (b), second, third and fifth floors were damaged to a percentage of 15, 7 and 25 respectively. Accelerometers are placed on first, third and fifth floors only.

Full modal information for each case has been computed from experimentally obtained modal data of first, third and fifth floors using Eq. (11).

Next, stiffness values have been computed from Eq. (7). Obtained stiffness values are compared with those of intact structure and damage factors have been identified which are shown in Table 11.

For the damaged situation of case (a), the frequency value has been obtained as 12.48 Hz. whereas the experimentally obtained frequency of intact structure was 12.97 Hz. (Table 10).

Fig. 6 shows a comparison between the computed mode shapes of the intact and damaged structure (case (a)) obtained from the experimental displacements recorded using three sensors placed on first, third and fifth floors only.

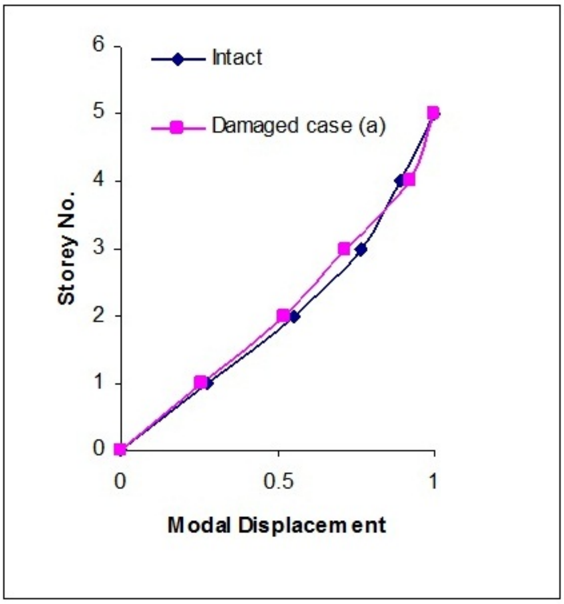

Fig. 6 Comparison of experimental mode shapes for intact and damaged structure case (a)

Table 11 Identified damage factors for case (a) and case (b) from first natural frequency and corresponding partial mode shape at 2nd, 3rd and 5th floor

Case (a)				Case (b)			
Storey No.	Actual damage factor introduced	Identified damage factor from experiment	% of error	Damaged Storey No.	Actual damage factor introduced	Identified damage factor from experiment	% of error
1st	1.0	1.0	0.0	1st	1.0	0.98	2.0
2nd	0.9	0.85	5.6	2nd	0.85	0.82	3.8
3rd	1.0	1.0	0.0	3rd	1.0	0.99	1.0
4th	0.7	0.64	8.5	3rd	0.93	0.87	6.5
5th	1.0	0.98	2.0	5th	0.75	0.69	8.0

From the Table 11, it is observed that damage factors are identified with reasonable accuracy using the proposed methodology. The maximum percentage of error in identification of extent of damage is 8.5% which is permissible. It is also observed that the proposed method is making a false identification of the undamaged floor as damaged floor by assigning some value to the computed damage factor other than unity. However, in these cases also we get the deviation from unity by 2% only which may also be permissible.

5. Conclusions

Methodology presented in this paper is effective in damage identification when the frequency and corresponding partial mode shape values of any particular mode are known. Using this technique it will be easier to identify the damaged storey and the extent of damage of a multi-storey shear structure. This damage identification method is first validated with known first natural frequency and corresponding full modal data. Moreover, it is also validated with known first natural frequency and corresponding partial mode shape data. From the results of damage factors and percentage errors, it is evident that the proposed damage identification method gives good results when the frequency and corresponding mode shape have lesser measurement noise. In the absence of measurement noise, the percentage error in damage detection is practically absent. It may also be pointed out that the inaccuracy in damage identification increases as the noise level increases. Number of sensors and their location have also significant effect on accuracy of the methodology. It is observed that less number of sensors placed at suitable locations gives more accuracy than more number of sensors placed randomly. However, the method is very effective for damage identification where less noisy experimental data of one frequency and corresponding partial modal information is known. The limitation of this methodology is that the modal data of more than one mode cannot be used simultaneously to further increase the accuracy in damage identification, even if they are known to us.

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