Smart Structures and Systems, *Vol. 15, No. 1 (2015) 209-225*
DOI: http://dx.doi.org/10.12989/sss.2015.15.1.209

Practical issues in signal processing for structural flexibility identification

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(Received January 10, 2014, Revised May 17, 2014, Accepted May 28, 2014)

Abstract. Compared to ambient vibration testing, impact testing has the merit to extract not only structural modal parameters but also structural flexibility. Therefore, structural deflections under any static load can be predicted from the identified results of the impact test data. In this article, a signal processing procedure for structural flexibility identification is first presented. Especially, practical issues in applying the proposed procedure for structural flexibility identification are investigated, which include sensitivity analyses of three pre-defined parameters required in the data pre-processing stage to investigate how they affect the accuracy of the identified structural flexibility. Finally, multiple-reference impact test data of a three-span reinforced concrete T-beam bridge are simulated by the FE analysis, and they are used as a benchmark structure to investigate the practical issues in the proposed signal processing procedure for structural flexibility identification.

Keywords: impact test; signal processing; flexibility identification; parameter sensitivity

1. Introduction

Various kinds of sensors have been developed and applied for structural health monitoring (SHM). The most frequently used sensors for SHM probably are strain gauges and accelerometers. The strain gauge is widely adopted because it provides strain data with very clear physical meanings which are easy for bridge engineers and owners to use for safety evaluation of key structural elements. The dynamic strain measurement is suitable for long-term SHM, and it can also be used for structural modal analyses (Adewuyi and Wu 2011). Even though the strain gauge has irreplaceable use value in SHM, it has the drawback that it is sensitive to local structural damage thus it is not suitable for global structural parameter identification. In contrast, the acceleration-based SHM are powerful to identify structural global parameters. It has been progressively becoming a mainstream way for health monitoring of civil infrastructures after over thirty years' development (Yu *et al*. 2010, Chen *et al*. 2011, Lei *et al*. 2012a, Lei *et al*. 2012b, ASCE 2011).

The ambient vibration testing is one of the most widely adopted ways for structural field test,

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

especially for large-scale structures like high-rise buildings and long-span bridges, because they are not easy to excite by human-made forces while the ambient vibration test takes the natural forces like traffics and winds as excitations. Structural dynamic responses (e.g., accelerations) measured during the ambient vibration test are the basis of the well developed vibration-based SHM mythologies. Modal analysis has been developed for a long history since the Fast Fourier Transform (FFT) technology developed to transform the accelerations in the time domain to the frequency domain. A number of modal analysis methods have been developed in the literature, like peak picking, ERA (Eigensystem Realization Algorithm), CMIF (Complex Mode Indication Function), PolyMAX, and PPT (Poly Reference Time-domain) methods, to identify structural modal parameters including frequencies, damping ratio and mode shapes from measured accelerations (Catbas *et al*. 2004, Peeters 2004). Almost all large-scale structural health monitoring systems employs the ambient vibration test and modal analysis as the main tools, supplemented by strain gauges and other kinds of sensors. For instance, the ASCE structural identification committee recently published a state-of-the-art structural health monitoring report (ASCE 2011). It collected 5 cases studies for building health monitoring and 10 case studies for bridge health monitoring, almost all of which performed ambient vibration tests and modal analyses (Grimmelsman *et al*. 2007, Carden and Brownjohn 2008, Zhang *et al*. 2009).

However, there is still a wide gap between modal analysis results and structural safety evaluation in engineering practices. Structure owners generally want to know whether their structures are damaged and where are the damages, but modal analysis results are difficult to directly provide such kind of information. This gap significantly prohibits the application of the vibration-based SHM technologies in engineering practices, and it has become the bottleneck problem in the SHM technology development. Even though a few methods using measured accelerations for structural damage detection have been developed, e.g., using genetic algorithms to search optimum structural parameters by minimizing the errors between the simulated and measured accelerations, there is still a far way to apply them in engineering practices. The most practical way using ambient vibration test for structural health monitoring by far probably is the six-step procedure proposed by ASCE structural identification committee (ASCE 2011). It includes field inspection, prior FE modeling, vibration test, signal processing, FE modal calibration, and FE prediction for structural safety evaluation. This six step procedure tried to connect the modal analysis results with structural safety evaluation through the FE calibration, and it has been successfully applied for health monitoring of several long-span bridges, like the Henry Hadson bridge and the Throgs Neck bridge (Grimmelsman *et al*. 2007, Zhang *et al*. 2009). This six-step procedure greatly improve the progress of the structural health monitoring technology, but it drawback is also clear. For instance, due to various kinds of uncertainty exist in experiment, data processing and FE modeling, there may be more than one calibrated FE models perfectly matching the experiment results (Friswell and Mottershead 1995).

Unlike the ambient vibration test only providing structural modal parameters, the impact test is potential to produce not only modal parameters but also structural flexibility. By measuring both structural responses and impacting forces, structural flexibility identification becomes possible, from which structural deflections under any static load can be predicted. It is known that structural deflection measurement from the truck static load test have been defined to be a standard structural safety evaluation index in many bridge design or maintenance codes (AASHTO 2007). The impact test is able to predict structural deflections under any static load, thus it is potential to replace the expensive truck static test and become an effective way for practical bridge safety evaluation. Due to this merit, the impact test has absorbed researchers' interests for bridge health monitoring in recent years. Dr. Aktan and his group members have successfully performed the impact test on a few bridges (Catbas *et al*. 2005). The results show that the deflections predicted from the identified flexibility matrix agreed well with those from the corresponding truck static tests. However, the current technology needs expert's experience to carefully identify structural flexibility from the impact test data due to various kinds of uncertainty existing in experiment and signal processing stages. How to quickly and stably identify structural flexibility and from impact test data is still a challenging problem. Efforts from the following two areas are potential to improve the reliability for structural flexibility identification and deflection prediction: (a) improving the impacting devices; Hammer impacting adopted in traditional impact test generally produces weak impacting forces, and the excited responses are greatly affected by traffic noise. Developing advanced impacting devices to provide stronger impacting force with a wide frequency range will be much helpful to improve the data quality from the point of view of experiment. (b). developing robust signal processing methods; The measured data are unavoidably polluted by noise and other kinds of uncertainty, thus robust signal processing methods are necessary for accurate modal parameter identification especially for flexibility identification because it is much more sensitive to uncertainty. A few researchers are working on the first aspect to develop advanced impacting devices. For instance, Moon and his colleagues are developing a drop hammer device for bridge impact testing which is potential to provide a much stronger impact force for bridge excitations (Zhou *et al*. 2011). In this article, the focus is put on the second aspect: how to accurately and stably identify structural flexibility from impact test data by developing an advanced signal processing method and investigating practical issues in applying the proposed signal processing method. For this purpose, a signal processing procedure identifying structural flexibility from impact test data, which includes a suit of data pre-processing strategies and a Sub-PolyMax data post-processing method, is first proposed. Then, Sensitivity studies of three pre-defined parameters required in the signal processing procedure will be performed to investigate how they affect the identification result through a 3-span concrete reinforced bridge example.

2. A signal processing procedure for structural flexibility identification

2.1 The signal processing framework

For a typical bridge impact test, N_0 sensors are mounted to key nodes of the monitored bridge. N_i nodes are selected as reference nodes. The impacting device (for instance, the sledge hammer) will hit the reference node one by one, until all referenced nodes are hit. During each hitting, the impact force and structure responses at N_0 nodes will be measured. Based on the number of the reference node selected, the impact test is called the single-reference (N_i =1) or the multiple-reference ($N_i > 1$) impact test. Generally the multi-reference impact test produces better identification results than the single-reference test, because the structural modal responses especially those in high modes are prone to be well excited by multiple-point hammer hitting. In general the impacting device repeatedly hits a reference node a few times, then the averaging technique is used to average these repeated measurements in the data processing stage for random uncertainty reduction.

After multi-reference impact test data are measured, a procedure including a suit of data pre-processing strategies and a data post-processing method as shown in Fig. 1 is proposed to identify structural modal parameters and flexibility matrix. The data pre-processing strategies reduce various kind of uncertainty involved in the test data, while the Sub-PolyMax method is employed as the data post-processing method to identify structural characteristics. Both the data pre-processing and post-processing techniques are presented below.

2.2 Data pre-processing strategies for uncertainty reduction

Due to traffic noise, environment affect, sensor sensitivity and other reasons, impact test data are unavoidably involves various kind of uncertainty. Special attentions are required to reduce the noise level and improve the data quality in the data pre-processing stage for subsequent structural identification. A suit of data pre-processing techniques are first developed to clean the data (Fig. 1). Force and acceleration records are first inspected to identify any malfunctioning sensors or spikes. If an acceleration time series contained repetitive pronounced errors such as bias or large spikes, or the impact force has clear rebound forces or double clicks, the channel is tagged and disregarded from further processing. Subsequently, the sampling rate is scaled up/down under the tradeoff between the computation time and frequency resolution. The affects of the sampling rate on the structural flexibility identification will be investigated later. Thirdly, a digital Butterworth band pass filter with certain cut-off frequencies is designed to remove the low and high frequency components. Following that, the exponential window is used to artificially force the response data to decay to zero at the end of measurement thus minimizing leakage. The mathematical form of the exponential window is defined as

$$\omega(t) = e^{-\beta t} \quad \text{where} \quad \beta = \frac{1}{\tau} \tag{1}$$

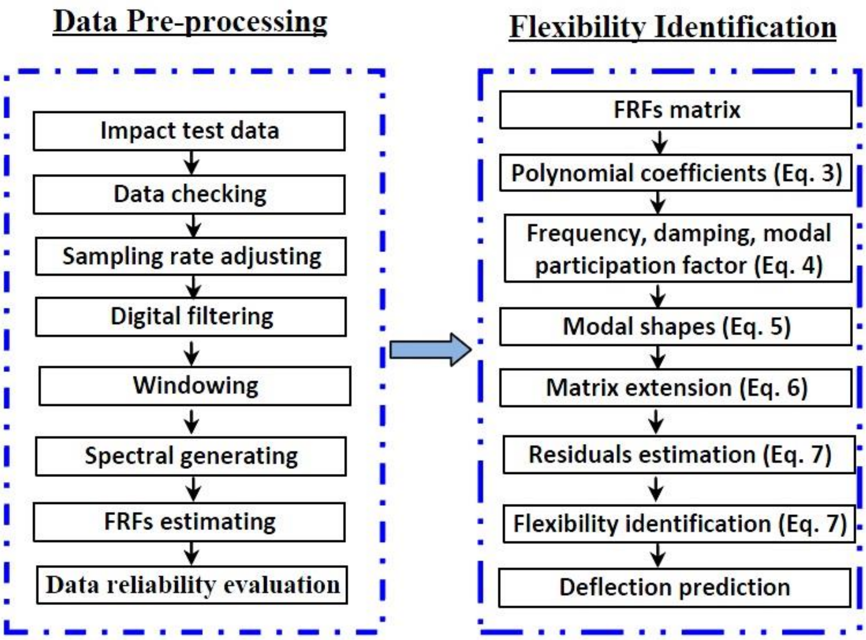


Fig. 1 The framework of the proposed signal processing method

where τ is a parameter determining how fast the response data decay to zero at the end of measurement. This parameter affects the accuracy of the identified flexibility, and it will be investigated in next section. While the exponential window is applied to the structural response, the rectangular window is added to the impact force record to set all the force time series to be zero except the short time window when the impact devices hit the structure. After the windowing, the impact force and accelerations are used to generate spectra. The equations for spectra generation are not provided here for brevity, but it worth to note that the data averaging technique can be used here to reduce random uncertainty. As presented earlier, the impacting device may repeatedly hit the same reference node a few times in the experiment stage. These repeated forces and accelerations are averaged in this step to produce averaged spectra. The average technique is a simple but very effective way to improve the data quality. From the generated spectra, frequency response functions (FRFs) are estimated by using the H1, H2 or Hv method. As the last step of the data pre-processing, a data reliability evaluation procedure is necessary to guarantee the cleaned data have adequate quality. The FRF reciprocity and the coherence function plots are useful to evaluate the quality of the generated FRFs, which are the basis for subsequent data post-processing for structural modal parameters and flexibility identification. In this proposed pre-processing stage, several pre-defined parameters are required and their variations affect the flexibility identification results. They are (a) the parameter, τ, in the exponential window technology, which affects how fast the accelerations decay to zero at the end of the measurement; (b) the fft point number, N_{fft}, which determine the frequency resolution ($\Delta f = Fs/N_{fft}$) of the estimated FRFs, where $Fs = 1/dt$ is sampling frequency and dt is the time interval; (c) decimation number, which adjusts the sampling rate of both the recorded impacting forces and accelerations thus affect the frequency resolution.

2.3 The sub-polyMax method for flexibility identification

A number of modal analysis methods have been developed to identify structural modal parameters from impact test data, e.g., the peak picking method, the CMIF method, and the PolyMax method. However, there are few methods developed for structural flexibility identification. In the authors' known, only the CMIF method is available in the literature for flexibility identification (Catbas *et al*. 2004). In this section, a Sub-PolyMAX method is employed, which not only extends the PolyMax method from modal identification to flexibility identification, but also improve the efficiency and accuracy of the PolyMax method. The basic idea of the Sub-PolyMax method is dividing the whole frequency range to several narrow frequency bands, and performing parameter estimation in each frequency band, finally assembling identification results from all frequency bands for structural flexibility identification. Compared to the PolyMax method identifying structural parameters in the whole frequency range, the Sub-PolyMax method has the merits that computation time is much reduced and the accuracy of the identified results is greatly improved.

Instead of investigating the FRF data in the wide frequency range in the traditional PolyMax method, only a narrow frequency band, $\omega_{ka} \sim \omega_{kb}$, involving only the rth structural mode, ω_r, is studied in the Sub-PolyMax method. The partial fraction of the FRF for the rth mode, $H_p^s(\omega_k)$, is written as

$$H_p^s(\omega_k) = \frac{\sum_{i=1}^{i=n^s} B_{pi}^s z_k^i}{\sum_{i=1}^{i=n^s} A_i^s z_k^i} \tag{2}$$

where the superscript, ^{s}, denotes that the investigation is performed in a subspace of the whole frequency range; k is the kth frequencies lines from ka to kb; n^s is model order; A_i^s and B_{pi}^s are unknown polynomial coefficients, respectively. By writing the FRF partial fraction in the rth mode as a numerator polynomial divided by a denominator polynomial, modal parameters of the rth mode can be estimated by solving the roots of the denominator polynomial. Due to the frequency band considered is narrow and only the rth mode is involved, the required model order is low.

From Eq. (2), the Jacobean LS implementation can be written to solve the polynomial coefficients

$$J^s\theta^s = 0 \tag{3}$$

where, $\theta^s = \begin{bmatrix} \beta_1^s \\ \vdots \\ \beta_{N_o}^s \\ \alpha^s \end{bmatrix}$, $\beta_p = \begin{bmatrix} B_{p0}^s \\ \vdots \\ B_{pn^s}^s \end{bmatrix}, \alpha = \begin{bmatrix} A_0^s \\ \vdots \\ A_{n^s}^s \end{bmatrix}$, $J^s = \begin{bmatrix} Z_1^s & 0 & \cdots & 0 & \aleph_1^s \\ 0 & Z_1^s & \cdots & 0 & \aleph_2^s \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & Z_{N_o}^s & \aleph_{N_o}^s \end{bmatrix}$,

$$Z_p = \begin{bmatrix} 1 & z_{ka} & \cdots & z_{ka}^{n^s} \\ 1 & z_{ka+1} & \cdots & z_{ka+1}^{n^s} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{kb} & \cdots & z_{kb}^{n^s} \end{bmatrix}, \quad \aleph_p^s = \begin{bmatrix} H_p^s(\omega_{ka}) & z_{ka}H_p^s(\omega_{ka}) & \cdots & z_{ka}^{n^s}H_p^s(\omega_{ka}) \\ H_p^s(\omega_{ka+1}) & z_{ka}H_p^s(\omega_{ka+1}) & \cdots & z_{ka}^{n^s}H_p^s(\omega_{ka+1}) \\ \vdots & \vdots & \ddots & \vdots \\ H_p^s(\omega_{kb}) & z_{kb}H_p^s(\omega_{kb}) & \cdots & z_{kb}^{n^s}H_p^s(\omega_{kb}) \end{bmatrix}.$$

It is seen that only parts of FRF data within the frequency band $\omega_{ka} \sim \omega_{kb}$ are used in Eq. (3) to identify modal parameters in a specific structural mode, thus it significantly reduces the Jacobean matrix size by reducing the number of frequency lines. Moreover, the required model order n^s is very low due to only one mode is included in the investigated narrow frequency band, thus it further reduces the size of the Jacobean matrix. In addition, Eq. (2) estimates each mode separately by using FRF values in narrow frequency bands, the identified modal parameters will not be influenced by FRF data outside the investigated the frequency band thus they will be more accurate. Even Eq. (3) was derived on the assumption that only one mode is involved in the investigated narrow frequency band, it is easy to be extended to the case when a narrow frequency band involves more modes.

After denominator polynomial coefficients, α^s, are solved from Eq. (3), structural modal parameters are extracted from the roots of the polynomial. The eigen value Λ and the eigen vector Λ of the matrix α^s can be produced from the eigen analysis, which has the following relationship with structural modal parameters

$$\Lambda_r = e^{-j\gamma_r\Delta t} \tag{4}$$

where $\gamma_r, \gamma_r^* = -\xi_r\omega_r \pm j\sqrt{1-\xi_r^2\omega_r}$, r is the number of structure mode. From the calculated eigen-values, structural frequency and damping ratios are identified as $\omega_r = \mathrm{imag}(\frac{log(\Lambda_r)}{-\Delta t})$, and $\xi_r = \frac{real(\frac{log(\Lambda_r)}{-\Delta t})}{imag(\frac{log(\Lambda_r)}{-\Delta t})}$. Modal participation factor, $L_r \in \Re^{N_i \times 1}$, is identified from eigen-vector, V. It should be noted that the length of L_r is the number of input N_i, not the number of output N_o, thus it needs to be extended to be a matrix with the length of N_o which will be presented later.

After frequencies, damping ratios, and model participation factors, L_r, have been estimated in narrow frequency bands, mode shapes, φ_r, modal scaling factors, residuals, and flexibility matrix can be estimated as presented below. The FRF for each output is written as

$$H_p(\omega_k) = L_r \left[\frac{1}{j\omega_k - \gamma_r} \right] \varphi_{pr} \tag{5}$$

Note that here φ_{pr} includes the pth line of the rth mode shape and its conjugate. Solving φ_{pr} from Eq. (5) is still performed in each narrow frequency band. After φ_{pr} for all outputs and all modes are calculated, they are assembled to the structural mode shape matrix, φ.

It should be known that dimensions of both the modal participation factor, *L*, and the mode shape φ, identified so far are not full matrixes. To overcome this limit, the scaling factor can be used to extend their dimensions to be $N_0 \times N_0$. The scaling factor of the rth mode is calculated by

$$Q_r = L_{ir} / \varphi_{ir} \tag{6}$$

where i denotes the ith input location. By using the identified scaling factor, the modal participate factor is extended to be $L_r^e = Q_r \varphi_r$, where $L_r^e \in R^{N_0 \times 1}$. By using this extended modal participate factor, the dimensions of the estimated mode shape matrix from Eq. (6) is extended to be $N_0 \times N_0$. Through this extension, structural flexibility matrix identified below will also be a full matrix, thus structural deflections when the static load locates at any output node can be predicted.

Since scaling factor has been estimated, it is easy to derive structural flexibility in the following way:

$$f = H(\omega_k = 0) = \sum_{r=1}^{m} \left(\frac{R_r}{-\gamma_r} + \frac{R_r^*}{-\gamma_{ir}^*} \right) \tag{7}$$

where *f* is structural flexibility matrix, and $R_r = L_r \varphi_r^T$ is the residue of the rth mode. After structural flexibility matrix is identified, structural deflections under any static loads can be predicted.

It is seen from the equations presented above that the Sub-PolyMAx method identifies structural parameters in narrow frequency bands, thus the Jacobin matrix dimension is significantly reduced by adopting less frequency lines and a much smaller modal order. This makes computation time be much reduced. Furthermore, structural identification results in a narrow frequency band is not affected by FRF data in other bands, thus it is potential to accurately identify structural parameters in the frequency band with much lower FRF peaks. Especially, the Sub-PolyMax method not only identifies structural modal parameters, but also identifies structural flexibility.

3. Practical issues in structural flexibility identification

3.1 The benchmark structure

A procedure including a suit of data pre-processing strategies and the Sub-PolyMax method has been developed in last section. Due to the complex of structural flexibility identification, it is necessary to use a benchmark structure to investigate how the pre-defined parameters required in the data pre-processing stage affect the identification results and how robust of the proposed

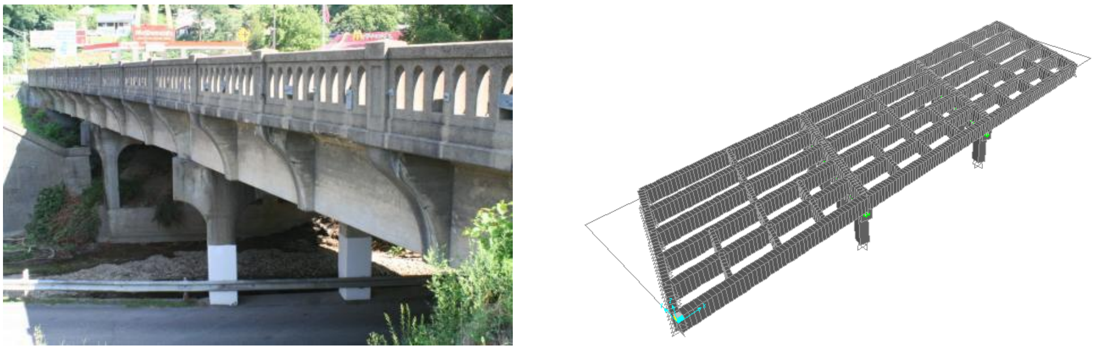
Sub-PolyMax method works for structural flexibility identification. For this purpose, the flexibility of the studied structure should be beforehand known, in order to provide a baseline to compare with the identified results. Due to this reason, the FE model of a real reinforced concrete bridge is developed to serve as the benchmark structure. Impact test data from field test of real bridges are not appropriate for sensitivity analysis performed in this article, because the flexibility of real structures is unknown and there is no way to evaluate the correctness of the identified flexibility.

The prototype of the FE model is a three span reinforced concrete T-beam bridge located in West Virginal. It is a simply supported concrete structure with a skew of approximately 18° (Fig. 2). Each span of this bridge is approximately 14.6 m long; with a width along the skew is 14.6 m as well. The FE model of this bridge is constructed in the SAP2000 software. Because the purpose the FE modeling is for parameter sensitivity investigation and method verification, only the longitudinal and transversal beams and the piers are modeled for simplicity. The bridge deck can be easily modeled by using the shell element but it is excluded in the developed FE model because it make the computation time be much longer in the multi-reference impact test simulation. The beams and piers are modeled by using the frame elements. The beams and the pier caps are connected using rigid links, forcing the components to act compositely. The support conditions at the abutments are pins, while the base of the piers is fixed. In total, the model is comprised of 1408 frame elements, and 336 rigid links.

The multiple reference impact test data of the Smithers Bridge is simulated by using the developed FE model. The dynamic analysis of the whole FE model is performed, but only the responses of the left span with the instrumentation plan presented below is used for structural identification because these three spans have very weak coupling and each span can be seen as an individual structure. 18 nodes as shown in Fig. 3 are selected as output nodes (N_0=18) whose structural accelerations under each impacting are recorded. Four input nodes (N_i=4) as shown in Fig. 3 are selected as reference nodes. Namely, the impacting force is applied on one of the reference node each time until all reference nodes are impacted. The impacting forces used in the FE simulation are from the recorded hammer hitting forces in a real bridge impact test. To simulate the observation noise and other kinds of uncertainty existing in the experiment stage, the simulated data are polluted by noises in the way as described below. It is known that traffic noise is one of the main uncertainty sources. Therefore, a series of traffic vibration induced accelerations recorded in a bridge ambient vibration test are added into the simulated structural responses as the traffic noise. The magnitude of the added traffic noise is 15%, which means that the standard deviation of the added traffic noise is 15% of that of the simulated acceleration (Zhang *et al*. 2008). Furthermore, to simulate other kinds of uncertainty especially to simulate the noise involved in the impact forces, 3% Gaussian noises are also added to both the structural responses and impacting forces, where the magnitude of the noise is defined in the same way as described above. It is seen that extremely strong noises (15% traffic noise plus 3% Gaussian noise) are added into the simulated data aiming at investigating the sensitivity of the pre-defined parameters of the data pre-processing algorithms and the robustness of the proposed procedure for structural flexibility identification.

3.2 Senstivity analysis

After the multi-reference impact data are simulated, the proposed signal processing procedure as illustrated in Fig. 1 is employed for structural flexibility identification and deflection prediction.



(a) (b)

Fig. 2 The bridge model; (a) The bridge prototype, and (b) The FE model

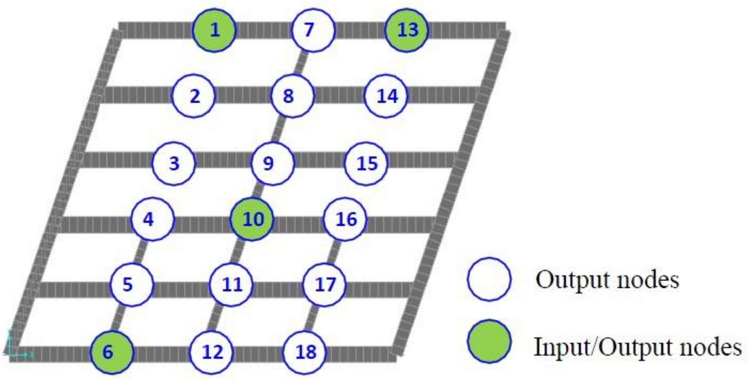


Fig. 3 Impact test sensor layout

In the data pre-processing stage, data checking, filtering, windowing (rectangular and exp windows for forces and accelerations, respectively) technologies are employed to clean the data, and FRF reciprocity and coherence function plot are used to check the quality of the cleaned data. In the data post-processing, the Sub-PolyMax method is executed to identify structural flexibility matrix. As pointed out in Section 2, 3 pre-defined parameters required in the data pre-processing stage affect the flexibility identification results, like the parameter, τ, in the exponential window technology, the FFT point number, N_{fft}, and the sampling rate, Fs = 1/*dt*. To investigate how these parameters affect structural identification results, sensitivity analyses of these three parameters are performed by employing structural identification with the variations of these parameters. The averaged errors between the deflections predicted from the identified flexibility matrix and those from FE simulation are calculated as an index to evaluate the parameter sensitivity.

Fig. 4(a) illustrates how the parameter, τ, affects the predicted deflections. Figs. 4(b) and 4(c) provide other ways to evaluate the parameter sensitivity. In Fig. 4(b), the "FRF" denotes the errors between the FRFs directly estimated from the test data and those reconstructed from the estimated structural modal parameters using Eq. (2). The "FRF reciprocity" denotes the errors between the FRFs when the location of force and response are exchanged. Theoretically the FRF data from a measurement should be identical if we exchange the locations of force and response for a linear and time invariant structure in the single input case, thus reciprocity check the FRFs is a simple but useful index to check the parameter sensitivity. The FRF reciprocity check for H(1,13) and H(13,1) are shown in Fig. 5, which are for the cases using $\tau = 0.1\%$, and $\tau = 1$ in the exponential window, respectively. In Fig. 4(c), the "Frequency" denotes the averaged errors of the identified frequencies in the first 6 modes and those from the FE simulation, and the "MAC" means the averaged MAC errors between the identified modal shapes and those from the FE model. It is seen that Figs. 4(a)-4(c) uses the identified deflection, the reconstructed FRF, the FRF reciprocity, the frequency, and the MAC, respectively, as indexes to evaluate how the parameter, τ, affect structural identification results. It is found that better identification results are produced while employing a smaller τ. The optimum exponential window parameter is selected as $\tau = 0.1\%$ for structural modal parameters and flexibility identification.

The sensitivity of the FFT point number, N_{fft}, is also studied and its results are shown in Fig. 6(a). Generally, the FFT is implemented to transform the time domain test data to the frequency domain by using a standard blocksize of $\bar{N}_{fft} = 2^p$, where p is the first value such that 2^p is larger than the time series length. The normalized FFT points in Fig. 6(a) means how many times the fft point number is the standard blocksize, $\bar{N}_{fft}$. The FFT point number affects frequency resolution. It does affect structural identification results, but a higher frequency resolution for the FRFs doesn't guarantee better deflection prediction results. The optimum value for the FFT point number is selected as the standard FFT blocksize, $\bar{N}_{fft}$, which yielded a frequency resolution of 0.1467 Hz.

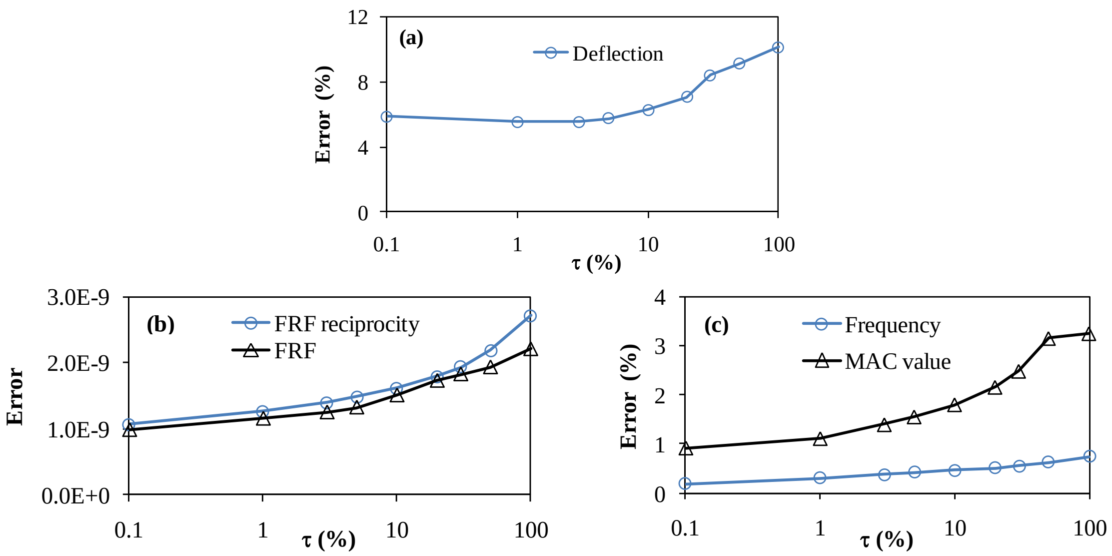


Fig. 4 Sensitivity analysis of the parameter τ

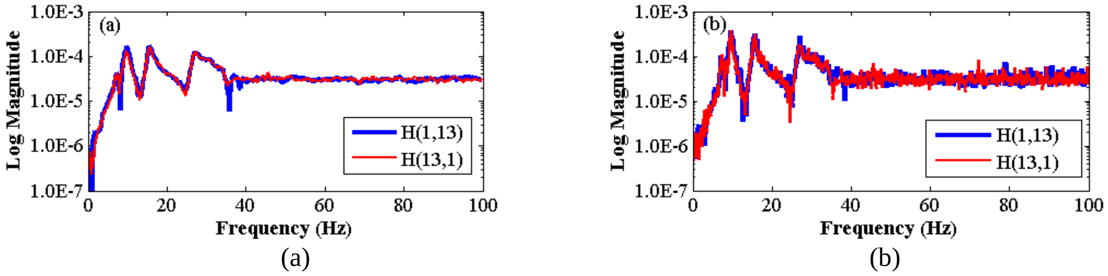


Fig. 5 Reciprocity check with (a) $\tau = 0.1\%$, and (b) $\tau = 1$

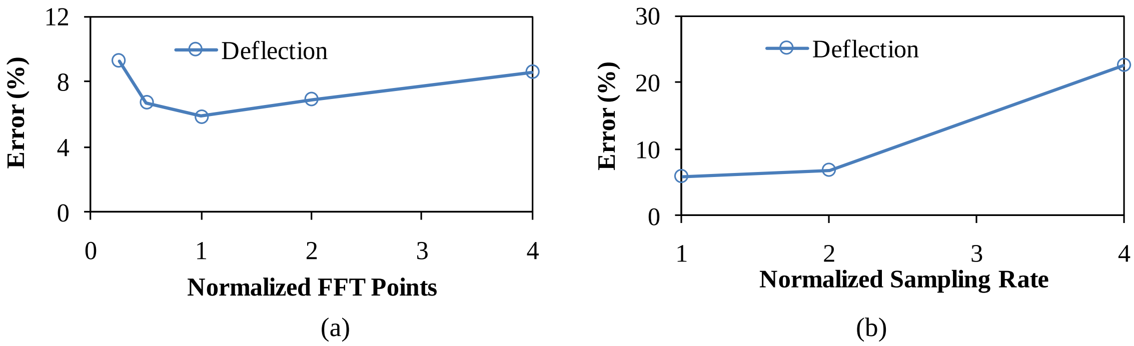


Fig. 6 Sensitivity analyses of (a) FFT point number, and (b) Sampling rate

Sensitivity analysis of the sampling rate is performed as well as shown in Fig. 6(b) to study how the sampling rate affects the deflection prediction results. The normalized sampling rate in Fig. 6(b) means how many times to scale up the sampling rate which is 0.0004165 s in this study corresponding with a sampling frequency of 2400 Hz. In the impact test, a sampling rate needs to be setup before recoding the data. The measurements of both the impact force and the accelerations are recorded with the same sapling rate. It is seen from Fig. 6(b) that the sampling rate greatly influences the deflection identification results. The reason is that a very high sampling frequency is required to recorder the impact force peaks during the impacting device contacts the bridge surface which occurs in a very short time window. It is learning from here that a much higher sampling frequency should be set in impact test for flexibility identification, which generally should be at least 10 times of that in usual ambient vibration tests.

3.3 Flexibility identification results

Practical issues in applying the proposed signal processing method for structural flexibility identification have been investigated by performing sensitivity analyses of several pre-defined parameters required in data processing. This section illustrates the detailed flexibility identification results from the proposed Sub-PolyMax method. As presented earlier, the Sub-PolyMax method performs parameter identification in subspaces of the whole frequency range, to achieve efficiency and accurate flexibility identification. In this example, two frequency segments, 4.25 Hz ~18.92

Hz, and 18.92 Hz ~ 33.6 Hz, are selected from the whole frequency range, each segment with 100 frequency lines. The Sub-PolyMAX method was implemented in each segment independently to identify denominator polynomial coefficients by Eq. (3) and subsequent structural modal parameters by Eq. (4). It should be noted that the model order n^s should be selected in Eq. (2), and it should be larger than the number of structural modes involved in the investigated narrow frequency band. This induces that parts of polynomial roots of $\sum_{i=1}^{i=n^s} A_1^{n^s} z_k^i$ are system roots involving structural modes, while others are spurious roots due to uncertainty. Stabilization diagrams are used to separate system poles from spurious poles. The poles satisfying the criteria $(\zeta_r > 0, \Delta\omega_r < 0.1, \Delta\zeta_r < 0.2, MAC > 0.96)$, where Δ means frequency or damping ratio change with model order increasing, and *MAC* denotes model assurance criterion) are selected as stable system poles. The stabilization diagram is also helpful to select the appropriate mode order. Fig. 7 shows the stabilization diagrams in two selected frequency bands, from which the model order is selected to be $n^s = 20$. From the identified structural modal parameters, FRFs are synthesized as shown in Fig. 8. It is seen that for the studied structure, its FRF curve has much larger peak values at the low frequency range than those in the high frequency range. In the un-weighted PolyMAX method, the least squares errors at all frequency lines are equally weighted, thus structural modal parameters in high modes are very difficult to accurately identify. By dividing the whole frequency range to narrow frequency bands which only involves a few structural modes, only FRF data in a narrow frequency band are used thus the identified structural modal parameters are not affected by FRF values in other frequency bands. As shown in Fig. 8, the synthesized FRF are comparable to the FRF identified directly from input/output test data, even in the high frequency range. This is the merit of the proposed SubPolyMax method.

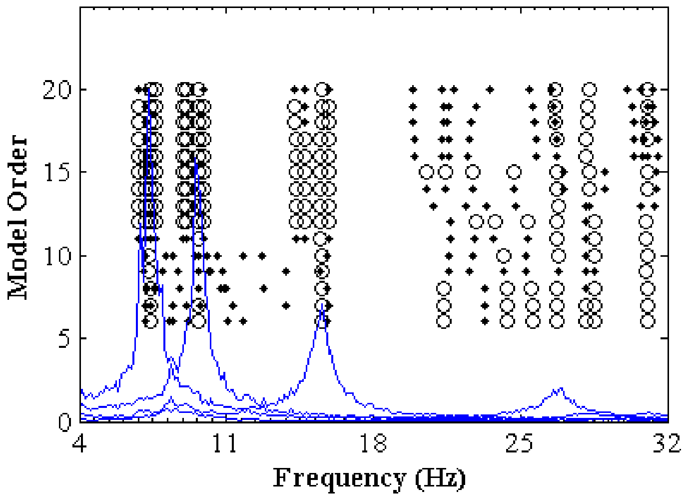


Fig. 7 Stabilization diagram (circles denote stable poles, triangles denotes unstable poles, the blue lines in the figure denotes CMIF plots)

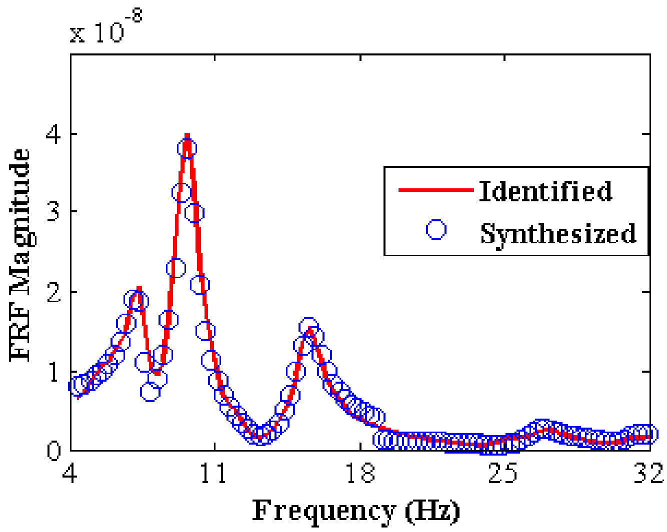


Fig. 8 Synthesized FRF

After structural frequencies, damping ratios and modal participation factors are extracted from the identified polynomial coefficients from Eq. (4), structural modal shapes are estimated by scaling the modal participation factors from Eq. (5). It should be known that the number of reference nodes (N_i=4) is lower than the number of the output nodes (N_0=18), thus the modal shapes identified so far is only a partial part of the full modal shape matrix, thus it needs to extend to a full matrix by using the scaling factor as calculated from Eq. (6). Identified frequencies and mode shapes of the studied structure are plotted in Fig. 9, and the frequencies and mode shapes calculated from FE simulation are also provided in Fig. 9 for comparison. The MAC values calculated from the identified and simulated mode shapes are also plotted in Fig. 10. It is seen that identified structural modal parameters from the proposed signal processing procedure are very accurate, even though the simulated data are polluted by strong noises (15% traffic noise plus 3% Gaussian noise as presented earlier).

After that, residuals and flexibility matrix are estimated. As shown in Eq. (7), the flexibility is calculated by using the identified modal parameters in the first *m* modes. The identified flexibilities using different mode combination are used to predict deflections, and they are compared with the static results from FE simulation. In the static load case, point static loads with the magnitude of 1000 KN are applied on nodes 2, 3, 8, and 9, and the simulated displacenet on nodes 1 to 18 are simulated. Structural deflections under the static load from the static test and those calculated from the identified flexibility matrix were plotted in Fig. 11(a) for comparison. Similarly, Fig 11(b) plots the predicted deflections and the ones from the static test when 1000 KN static loads are on the 7th to 12th nodes. It is seen that structural flexibility identified by using modal parameters in the first 6 modes produces very accurate deflections.

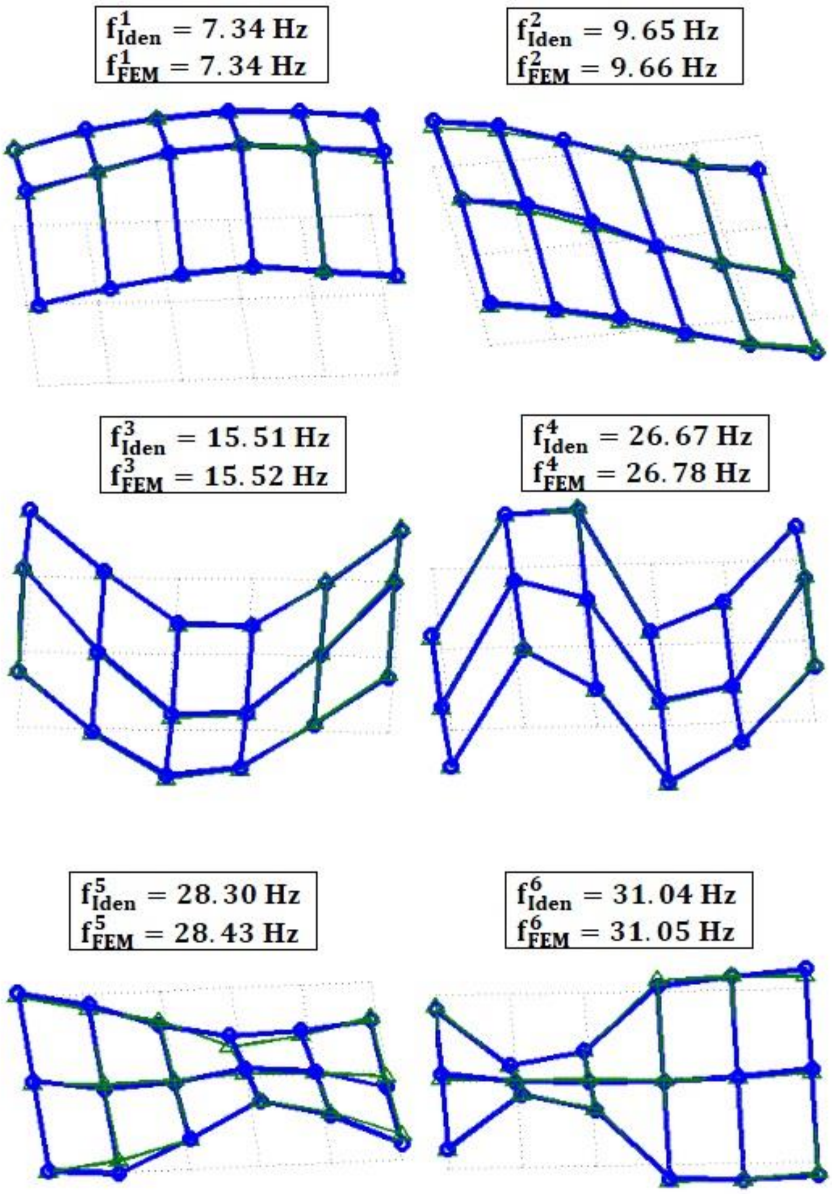


Fig. 9 Identified modal parameters

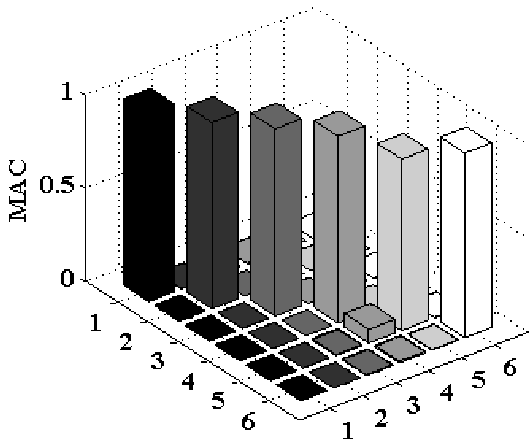


Fig. 10 MAC values between the identified and simulated modal shapes

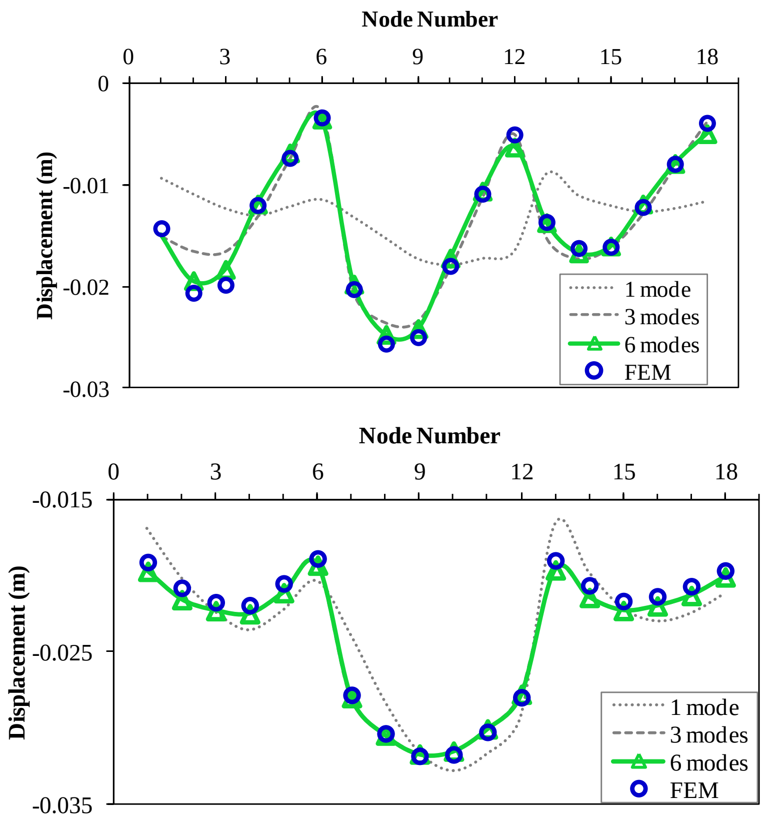


Fig. 11 Deflection prediction from the identified flexibility

4. Conclusions

The following conclusions have been drawn based on the research so far:

(1) A signal processing procedure has been presented for structural flexibility from the multi-reference impact test data. It includes a suit of data pre-processing strategies to improve the data quality, and a Sub-PolyMax method for structural identification in narrow frequency bands.

(2) Practical issues in using the signal processing procedure for structural flexibility identification have been investigated. How the pre-defined parameters affect the identified flexibility results have been revealed.

(3) The investigation throughout this paper takes the FE model of a real three-span reinforced concrete T-beam bridge as the benchmark structure, in which extremely strong noises including 15% traffic noise and 3% Gaussian noise have been added into the simulated impact test data. Impact test data from field test of real bridges are not appropriate for sensitivity analysis performed in this article, because the flexibility of real structures is unknown and it is difficult to evaluate the correctness of the identified flexibility.

Acknowledgements

This work was sponsored by the National Science Foundation of China (51108076) and the Supporting Program of the Twelve Five-year Plan for Science & Technology Research of China (2011BAK02B03). The first author would particularly like to acknowledge the support from Drs. F. Moon and A.E. Aktan of the Drexel University.

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