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# Recovering structural displacements and velocities from acceleration measurements

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**Abstract.** In this research, an internal model based method is proposed to estimate the structural displacements and velocities under ambient excitation using only acceleration measurements. The structural response is assumed to be within the linear range. The excitation is assumed to be with zero mean and relatively broad bandwidth such that at least one of the fundamental modes of the structure is excited and dominates in the response. Using the structural modal parameters and partial knowledge of the bandwidth of the excitation, the internal models of the structure and the excitation can be respectively established, which can be used to form an autonomous state-space representation of the system. It is shown that structural displacements, velocities, and accelerations are the states of such a system, and it is fully observable when the measured output contains structural accelerations only. Reliable estimates of structural displacements and velocities are obtained using the standard Kalman filtering technique. The effectiveness and robustness of the proposed method has been demonstrated and evaluated via numerical simulations on an eight-story lumped mass model and experimental data of a three-story frame excited by the ground accelerations of actual earthquake records.

**Keywords:** structural health monitoring; modal decomposition; observer design; internal model

## 1. Introduction

Deflection is a global characteristic for any structure in both the construction and service periods. It is essential for estimating the performance of the structure under dynamic loading. There are many different types of sensors that can be used to measure the dynamic or static deflection of a structure. These include models such as the interferometer (Lloret and Rostogi 2003), the dail guage (Kim and Cho 2004), the laser doppler vibrometer (Nassif *et al*. 2005), the linear variable differential transducer (LVDT) (Park *et al*. 2007), and the global positioning system (GPS) (Roberts *et al*. 2012). These sensors have been confirmed through laboratory testing to accurately measure the displacement of a structure. However, for field testing, these sensors may not be feasible for use due to the limitations and challenges in their instrumentation. Direct measurement of displacement requires a reference datum, which is, in many cases, only temporarily available through scaffold (Yoneyama *et al*. 2007, Gindy *et al*.

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2008) or suspended cable. These datums are time consuming, increase the cost of the project, and are ineffective due to the difficulty in maintaining a stationary reference over time. These difficulties make using a reference datum unsatisfactory for long term applications. As a result, indirect displacement measurement methods have been developed. A common method used in practice is to measure accelerations directly and apply integration to calculate the velocities and displacements. However, it can be theoretically shown that small random errors in the measured signals, also referred to as noise, are bound to grow through successive integrations, which leads to significant distortions in the estimated profiles (Astrom 2006). Gindy *et al.* (2008) introduced a post processing method in which a state space analytical model is constructed via a singular value decomposition based algorithm to generate an approximation of the noise-free acceleration signal. By applying appropriate correction techniques, this noise-free approach can be used off-line to derive a displacement profile. Smyth and Wu (Smyth and Wu 2007) proposed a multi-rate Kalman Filtering technique for the data fusion of displacement and acceleration response measurements. Using displacement measurements along with measured accelerations, this technique has been proven to filter and smooth noise contaminated measurements into accurate estimates. Based on a similar concept, Ma and Xu developed a fusion method to accurately estimate the displacement and velocity from the noise tainted acceleration and displacement measurements (Ma and Xu 2007). More recently, the FIR-based method was proposed (Lee *et al.* 2010, Hong *et al.* 2013) and implemented on a wireless platform (Park *et al.* 2013) for reconstruction of structural displacements from acceleration measurements.

Velocity is another global characteristic of a structure, but it is rarely measured in the field due to the difficulty in directly obtaining it. However, structural velocity is important because it is the one variable which can be used to characterize the motion of a structure. In the state space of a structural system, velocity and displacement are independent states of the system. Knowing both means complete knowledge of the dynamics of the structure, which is beneficial in various applications, such as characterization of structural motion, identification of structural parameters, monitoring in-service structural performance, and so on. To the best knowledge of the authors, there have not been reliable methods reported in the literature for velocity measurement or velocity profile reconstruction for ambient structural vibrations. Laser doppler vibrometers are designed for measuring velocities, however, deployment of them in the field is impractical due to limitations such as their prohibitively high cost, delicate nature, and installation difficulties, among others.

Aside from the time-domain signals, dynamics of a structure can be analyzed in the modal domain. Such modal representation describes the structural motion from a different perspective, usually in a more concise and abstract way with most of the information preserved. Methods that extract modal information, i.e., frequencies, mode shapes, and damping ratios, from time-domain signals have gained increasing popularity in both theory development and practical applications (Ewins 2000). Particularly of interest for vibrations of civil infrastructures are methods for identifying these modal parameters with output-only system techniques under ambient vibrations (Brincker *et al*. 2001, Nagarajaiah and Basu 2009, Au and L. 2011). For example, the modal parameters of a high-rise slender structure, more than 400 meters high in its large scale model, were identified using a subspace system identification based algorithm (Liu and Loh 2011). Additionally, six modes and their parameters were identified for a footbridge and a 300 meter tall building in Hong Kong (Au and L. 2011).

In this study, a method based on an internal model is developed to estimate a structure’s displacements and velocities from measured accelerations. Estimates are obtained via reconstruction of the structural modal displacements and modal velocities, which allows for subsequent recovery of the time-domain information using modal superimposition. Based on the modal parameters of the structure, the internal models for the structural modes and excitation are established, which are used to construct an autonomous system for every participating mode that closely mimics the modal dynamics of the structure. The Kalman filtering technique is applied to the individual autonomous systems to estimate the modal responses of every mode. The proposed method is essentially an observer-based approach, and thus is suitable for real-time applications. The measured accelerations can be used as an online performance index to monitor the accuracy of the estimated structural displacements and velocities. Illustrative examples of an eight-story lumped mass shear beam numerical model and a three-story frame experimental model were chosen to demonstrate the implementation of the proposed method and verify its feasibility and robustness through error analysis. The results of these simulations are compared to the known measured values to validate the accuracy and precision of this method.

## 2. Formulation

The dynamic response of a linear structure subjected to an arbitrary excitation can be accurately represented by superposition of a finite number of modal responses (Humar 1990) as

$$\mathbf{u} = \mathbf{\Phi} y \tag{1}$$

where $\mathbf{\Phi}$ denotes an N by m (N $\geq$ m) modal matrix of the structure, in which N is the number of degrees of freedom of the structure and m is the number of modes that contribute significantly to the structural response. Structural response in time and modal domain are denoted using $\mathbf{u} = [\,u_1 \quad u_2 \quad \cdots \quad u_N\,]^T$ and $\mathbf{y} = [\,y_1 \quad y_2 \quad \cdots \quad y_N\,]^T$, respectively. In this study, structural response considered includes displacement, velocity, and acceleration. Clearly, $\mathbf{u}_v = \mathrm{d}\mathbf{u}_d/\mathrm{d}t$ and $\mathbf{y}_v = \mathrm{d}\mathbf{y}_d/\mathrm{d}t$, where subscripts $d$ and $v$ refer to displacement and velocity, respectively. Furthermore, displacement and velocity are defined using the structural base as the reference, thus, in the case where the structural base is fixed during the course of excitation, $\mathbf{u}_a = \mathrm{d}^2\mathbf{u}_d/\mathrm{d}t^2$ and $\mathbf{y}_a = \mathrm{d}^2\mathbf{y}_d/\mathrm{d}t^2$, in which subscript $a$ refers to absolute acceleration, however, in the case of base excitation, such relationship is not valid.

Consider the case of classical damping, the modal responses are governed by the following equation

$$\frac{d^2 y_i}{dt^2} + 2\zeta_i\omega_i\frac{dy_i}{dt} + \omega_i^2 y_i = P_i(t), i = 1,2,\cdots,N \tag{2}$$

where $\xi_i$ and $\omega_i$ are the damping ratio and natural frequency of the $i$th mode, respectively, $y_i$ the $i$th modal displacement, and $P_i(t)$ represents the $i$th modal force, normalized by the $i$th modal mass.

For every mode, define a non-dimensional time variable as $\tau_i = \omega_i t$. Eq. (2) can be written in a non-dimensional form as

$$\ddot{y}_i + 2\xi_i \dot{y}_i + y_i = Q_i(\tau_i) \tag{3}$$

Here $(\dot{\ })$ and $(\ddot{\ })$ denote first and second derivatives of the argument with respect to the non-dimensional time variable, $\tau_i$, and $Q_i(\tau_i) = P_i(\tau_i)/\omega_i^2$.

Eq. (3) can be written in state space as

$$\dot{\mathbf{Y}}_i = \mathbf{A}_i \mathbf{Y}_i + \mathbf{B}_i Q_i(\tau_i) \tag{4}$$

where the state vector of the *i*th mode is defined as $\mathbf{Y}_i = [y_i \quad \dot{y}_i]$, the excitation location matrix of the *i*th mode is $\mathbf{B}_i = [y_i \quad \dot{y}_i]^T$, and the state transition matrix of the *i*th mode is defined as

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ -1 & -2\zeta_i \end{bmatrix} \tag{5}$$

Using the definition of the state vector, the modal acceleration of the *i*th mode, $y_{ai}$ can be written as

$$y_{ai} = \mathbf{C}_i \mathbf{Y}_i + D_i Q_i(\tau_i) \tag{6}$$

where $\mathbf{C}_i = [-1 \quad -2\xi_i]$ and $D_i$ depends on the excitation. For a base-excited structure $D_i$= 0.

### 2.1 Observer design in model domain

In the case where structural accelerations $\mathbf{u}_a$ are measured and the structural modal properties are known, the modal accelerations can be obtained as

$$y_a = \mathbf{\Phi}^{-1} \mathbf{u}_a \tag{7}$$

Note that in the case where N > m, i.e., there are more degrees of freedom than the modes, a pseudo inverse can be used to obtain the response in modal domain. Using the absolute acceleration decomposed in modal domain, the modal displacement and modal velocity for every mode can be estimated with a properly designed observer as

$$\dot{\hat{\mathbf{Y}}}_i = (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i)\hat{\mathbf{Y}}_i + \mathbf{B}_i Q_i(\tau_i) + \mathbf{L}_i y_{ai} \tag{8}$$

Where $\hat{\mathbf{Y}}_i = \begin{bmatrix} \hat{y}_i & \dot{\hat{y}}_i \end{bmatrix}^T$ is the estimated state vector for the *i*th mode, in which $\hat{y}_i$ and $\dot{\hat{y}}_i$ denotes the estimated displacement and velocity for the *i*th mode, respectively, and $\mathbf{L}_i$ is the observer gain, which can be determined using various techniques (Åström and Murray 2008). It is noted that the error of the estimation depends on the accuracy of the information about the modal excitation and modal acceleration measurement. In this study, it is assumed that the modal properties of the structure are identified accurately and there are a sufficient number of structural acceleration measurements, i.e., N $\geq$ m, thus, the modal acceleration can be obtained using (7). In the following section, treatment of modal excitation using the internal model concept is presented.

### *2.2 Observer design using internal model of excitation*

#### *2.2.1 Internal model*

It is known that a mathematical model may be constructed to represent the inherent law of motion or evolution reflected in a deterministic signal. Such a model is also called internal model (Matausek and Stipanovic 1998, Xu and Yang 1999, Xu 2001). For a continuous, deterministic signal x(t), the internal model equation can be written as

$$\Lambda(D)x(t) = 0 \tag{9}$$

where the internal model polynomial is defined as $\Lambda(D) = D^n + \alpha_{n-1}D^{n-1} + \cdots + \alpha_2 D^2 + \alpha_1 D + \alpha_0$, in which $n$ is the order of the model, $D = d/dt$ is a differential operator and $\alpha_0, \alpha_1, \cdots, \alpha_n$ are constant coefficients.

If the structural excitation is deterministic and the complete information of it can be obtained, it is possible to construct the internal model for the modal excitation for every mode such that

$$\Lambda_{P_i}(D_{\tau_i})Q_i(\tau_i) = 0 \tag{10}$$

where the internal model polynomial for the *i*th mode is $\Lambda_{P_i}(D_{\tau_i}) = D_{\tau_i}^{n_i} + \alpha_{n-1}D_{\tau_i}^{n_i-1} + \cdots + \alpha_2 D_{\tau_i}^2 + \alpha_1 D_{\tau_i} + \alpha_0$, in which $n_i$ denotes the order of the internal model. Here $D_{\tau_i} = d/d\tau_i$ denotes differentiation with respect to the non-dimensional time variable.

Using the internal model (10), the structural modal Eq. (3) can be rewritten in an augmented form as

$$\Lambda_{P_i}(D_{\tau_i})\Lambda_{S_i}(D_{\tau_i})y_i = 0 \tag{11}$$

where $\Lambda_{S_i} = D_{\tau_i}^2 + 2\xi_i D_{\tau_i} + 1$ denotes the internal model polynomial of the *i*th structural mode.

For the case of ambient structural excitations, if the excitations can be treated as the sample functions of a random process, each sample function may be analyzed as a deterministic signal. As long as the frequency-domain properties of the random process do not vary dramatically, it is possible to use one internal model to represent its sample functions, thus, the above analysis is still valid for ambient vibrations.

#### *2.2.2 Observer design*

Using (11), an augmented state-space model of the *i*th mode can be written as

$$\dot{\mathbf{Y}}_{i,avg} = \mathbf{A}_{i,avg}\mathbf{Y}_{i,avg} \tag{12}$$

where the augmented state vector is defined as $\dot{\mathbf{Y}}_{i,avg} = \begin{bmatrix} y_i & \dot{y}_i & \ddot{y}_i & \cdots & y_i^{(n_i-1)} \end{bmatrix}^T$ and the state transition matrix is defined in a canonical form as

$$\mathbf{A}_{i,avg} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & 1 \\ -\beta_0 & -\beta_1 & -\beta_2 & \cdots & -\beta_{n_i+1} \end{bmatrix} \tag{13}$$

where coefficients $\beta_j$, $j = 0, 1, 2 \cdots$ are determined from the internal model polynomials, i.e., $\Lambda_{P_i}(D_{\tau_i})$ and $\Lambda_{S_i}(D_{\tau_i})$. Note that as the dynamics of the excitation are included in the system equation, the augmented system becomes an autonomous one.

The absolute structural acceleration can be written in standard state-space representation as

$$y_{ai} = \mathbf{C}_{i,avg}\mathbf{Y}_{i,avg} \tag{14}$$

where $\mathbf{C}_{i,avg} = \begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$ for the case of fixed base, i.e., the excitation does not include a component related to the motion of the base, whereas in the case of base-excitation, $\mathbf{C}_{i,avg} = \begin{bmatrix} -1 & -2\xi_i & 0 & 0 & \cdots & 0 \end{bmatrix}$. It can be shown that the pair $(\mathbf{A}_{i,avg}, \mathbf{C}_{i,avg})$ is observable, thus the following observer can be constructed to estimate the states of the augmented system.

$$\dot{\hat{\mathbf{Y}}}_{i,avg} = (\mathbf{A}_{i,avg} - \mathbf{L}_{i,avg}\mathbf{C}_{i,avg})\hat{\mathbf{Y}}_{i,avg} + \mathbf{L}_{i,avg}y_{ai} \tag{15}$$

where $\hat{\mathbf{Y}}_{i,avg} = \begin{bmatrix} \hat{y}_i & \dot{\hat{y}}_i & \ddot{\hat{y}}_i & \cdots \end{bmatrix}^T$ is the estimated state vector of the augmented system, in which $\hat{y}_i$ and $\dot{\hat{y}}_i$ are the estimated *i*th modal displacement and velocity, respectively. The observer gain is denoted as $\mathbf{L}_{i,avg}$.

In practice, ambient Excitations applied to a structure are, in general oscillatory, and thus can be expressed using Fourier Series (Tolstov 1976). For an excitation with zero mean, a possible form of the internal model may be chosen as

$$\Lambda_{P_i}(D_{\tau_i}) = (D_{\tau_i}^2 + r_1^2)(D_{\tau_i}^2 + r_2^2)\cdots(D_{\tau_i}^2 + r_p^2) \tag{16}$$

where $r_j = \overline{\omega}_j / \omega_i$, $j = 1, 2, \cdots, p$, in which $\overline{\omega}_1 < \overline{\omega}_2 < \cdots < \overline{\omega}_p$ are the lowest $p$ dominant frequencies of the excitation. Note that among these frequencies, only those that are close to the natural frequencies of the structure contribute significantly to the structural response, whereas those further away, even at higher energy levels, do not have much influence on the structural response. Thus, the natural frequencies of the structure can be used as a guide to the construction of the internal model for the excitation, especially for applications where such modal properties of the excitation are not available as *a priori*. For every mode, the normalized frequencies in the internal model can be selected as

$$r_{ij} = 1 \pm \eta_{ij}, \quad j = 1, 2, \cdots, q \tag{17}$$

where subscript *i* refers to the *i*th mode and the distribution of the normalized frequencies $r_j$ is symmetrical about the normalized structural natural frequency. Symbol $\eta_{ij}$ denotes the spacing between the frequencies, and 2*q* is the number of total frequencies. Note that the half-power-bandwidth of every mode is $2\xi_i$, for structures with low damping levels, small frequency spacing, e.g., on the same order of magnitude as the damping levels, and small number of frequencies can be used to reduce the order of the resulted system.

### *2.3 Estimation error*

Assuming that the internal model of the excitation for the *i*th mode is selected as

$$\Lambda_{P_i}(D_{\tau_i}) = \prod_j \left[D_{\tau_i}^2 + (1-\eta_j)^2\right] \cdot \left[D_{\tau_i}^2 + (1+\eta_j)^2\right] \tag{18}$$

where $\prod$ is the product operator. Using such an internal model and the structural modal equation, the following augmented state-space representation of the system can be obtained.

$$\begin{aligned} \dot{\mathbf{Y}}_{i,avg} &= \mathbf{A}_{i,avg}\mathbf{Y}_{i,avg} + \mathbf{B}_{i,avg}R_{Qi} \\ y_{ai} &= \mathbf{C}_{i,avg}\mathbf{Y}_{i,avg} + v_i \end{aligned} \tag{19}$$

where $R_{Qi}$ denotes the residual modal excitation resulting from the internal model, $v_i$ the measurement noise for the *i*th mode, and $\mathbf{B}_{i,avg} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^T$ denotes the location of the excitation in the augmented system.

The estimation error of an observer designed in the form of Eq. (15) can be written as

$$\dot{\mathbf{e}}_{i,avg} = (\mathbf{A}_{i,avg} - \mathbf{L}_{i,avg}\mathbf{C}_{i,avg})\hat{\mathbf{e}}_{i,avg} + \mathbf{B}_{i,avg}R_{Qi} + \mathbf{L}_{i,avg}v_i \tag{20}$$

where the error is defined as $\mathbf{e}_{i,avg} = \mathbf{Y}_{i,avg} - \hat{\mathbf{Y}}_{i,avg}$. Note that the measurement noise $v_i$ includes the hardware noises introduced directly by the measuring instruments as well as error generated from data processing, i.e., modal expansion. When the mode shapes of dominant modes are identified with satisfactory accuracy, which can be normally achieved by using a sufficient number of well distributed sensors, the effect of such error may be negligible. In the case where the dominant modes of the excitation are well captured in the internal model (18), the residual modal excitation $R_{Qi}$ is insignificant. Eq. (19) takes the same form as the standard Kalman filtering problem, where an optimal observer gain can be obtained (Kalman 1960) when the levels of noises are known. It is noted that when the excitation is with nonzero mean, the residual modal excitation $R_{Qi}$ will include a static component, which will generate a static error in the final estimation. If the static component of the excitation is known, the associated error can be minimized by explicitly incorporating it into the Kalman filter. For the case of unknown nonzero mean, the static error cannot be effectively compensated in the filtering process, and thus the estimated structural response will deviate from the actual values by some unknown static errors. Compensation for such errors in real time requires further in-depth investigation, which is not included in this study.

## 3. Illustrated examples

In this section, results from two illustrative examples are presented to demonstrate the effectiveness of the proposed approach. The examples comprise numerical and experimental studies of building structures subject to seismic excitations. A general sketch of the two models is shown in Fig. 1. In both examples, accelerations of all floor levels are used as the only measurements to reconstruct the displacements and velocities relative to the base. Results obtained using the N-S ground acceleration record of the 1940 El Centro earthquake are presented. Ground acceleration records of other major earthquakes were also applied to the structural models. The results obtained were similar and thus are omitted herein.

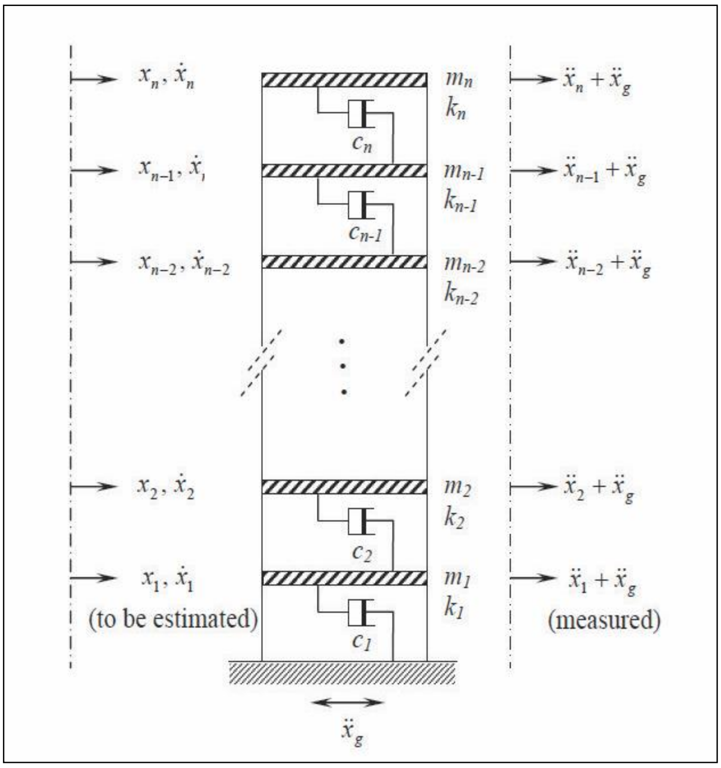



Fig. 1 Sketch of shear beam lumped mass models

### *3.1 Example 1: eight-story lump mass model*

The first structure considered was an eight-story lumped mass shear beam numerical model (Yang 1982, Spencer *et al*. 1994, Ma *et al*. 2008). The mass, stiffness, and damping coefficient for each floor level were assumed to be $m = 3.456\times 10^5$ *kg*, $k = 3.404\times 10^8$ *N/m* and $c = 4.0\times 10^6$ N $\cdot s/m$ respectively. The first three natural frequencies of the model are 1.05, 3.11, and 5.06 Hz, respectively. These frequencies correspond to damping ratios of 3.0%, 8.9%, and 14.5%, respectively. The highest natural frequency of the model is 11.17 Hz. According to the Nyquist-Shannon sampling theorem, the sampling frequency should be at least twice higher than the highest frequency in the signal to avoid loss of useful information. Accelerations of all floors were assumed to be measured at a sampling frequency of 50 Hz in this example. The measurements were also assumed to be contaminated with 20% noise. The same frequency was used to sample the simulated displacements and velocities.

An analysis of the spectra of the excitation and the structural responses (Fig. 2) revealed that under this relative broad-bandwidth excitation, only the first structural mode contributed significantly to the structural displacements, while higher modes participated slightly more at lower floors. Such higher modes, however, contributed to the accelerations at much higher levels. Since the accelerations were assumed to be measured in this study, only the dominant mode was sufficient for a good estimation of the displacements and velocities. In this example, the internal model was constructed using only the first structural mode. The corresponding modal parameters, i.e., frequency, mode shape, and modal damping, were identified using the frequency domain decomposition (Brincker *et al*. 2001). As the half-power-bandwidth of the first mode was approximately 0.06 Hz, the frequency spacing of $\eta_{11} = 0.1$ (Eq. (17)) was used. A Kalman filter was designed using the assumed measurement noise level. The variance of the residual modal excitation $R_{Qi}$ was assumed to be 0.01 in the filter design.

The estimated structural displacements and velocities are presented in Figs. 3 and 4. Only the quantities for the top floor are shown for simplicity. A time period of five seconds, in which the largest response occurred, was chosen to provide a clear view of the two lines in the figures. Fig. 3 compares the displacements of the top floor. It is clear the estimates follow the actual measurements very closely. The velocity profile for the top floor is also compared in Fig. 4. Similarly, the estimates closely mirror the actual velocity response. A summary of the maximum errors in the estimated peak values for all floor levels is presented in Table 1. It is seen that the proposed method estimated the peak/trough values accurately for all floor levels. The errors for peak/trough displacements and velocities are, with the only exception of the velocity at the $1^{st}$ level, below 10%. The correlation coefficients (Rahman 1968) were also calculated using the entire time histories of the results. It was found that the coefficients increase monotonically with higher floor levels. The lowest and highest values for displacements were found to be 98.83% and 99.11% for the first and top floor, respectively. The correlation coefficients for velocities were smaller, ranging from 89.03% for the first floor to 91.52% for the top floor. It is interesting to note that the estimates are not sensitive to the accuracy of the identified modal damping ratios. In this example, the first modal damping ratios, ranging from 1% to 8%, were used in the proposed method for the estimates. Compared with the actual damping ratio of 3%, the considered damping ratios covered a range of over 200% difference or identification error. Such significant discrepancies, however, only introduced very slight, unnoticeable changes in the estimates. Such robustness to damping ratios is beneficial in practice as damping ratios are less easily identified with the same level of accuracy as natural frequencies using ambient vibration measurements.

Similar to the traditional observer-based estimation/filtering techniques, the accuracy of the proposed method in real-world applications can only be checked or monitored using system outputs, which are the actual measured accelerations compared with the filtered values in this study. Figs. 5 and 6 respectively show the filtered accelerations of the first and top floors obtained using the first mode and the first three modes. As discussed previously, higher modes contribute to accelerations at significantly higher levels as compared to their share in displacements or velocities, especially at lower levels.

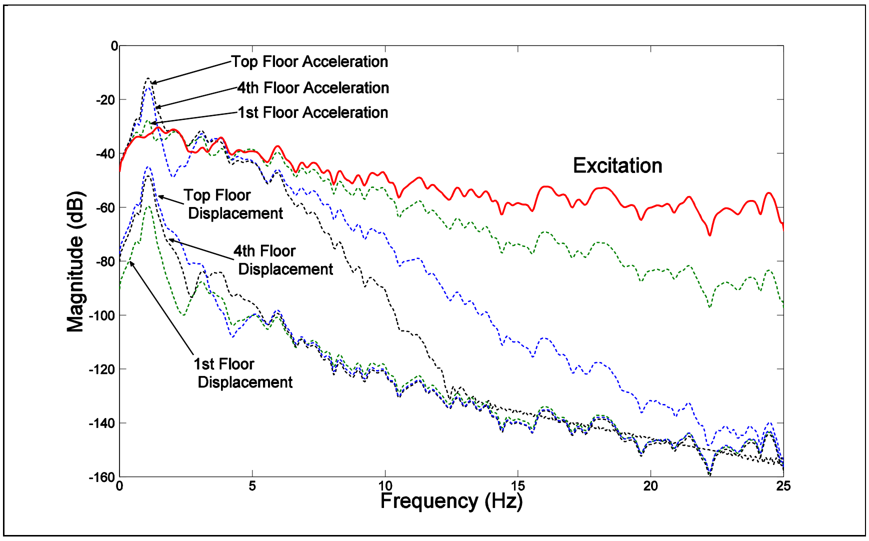



Fig. 2 Spectra of excitation and structural responses

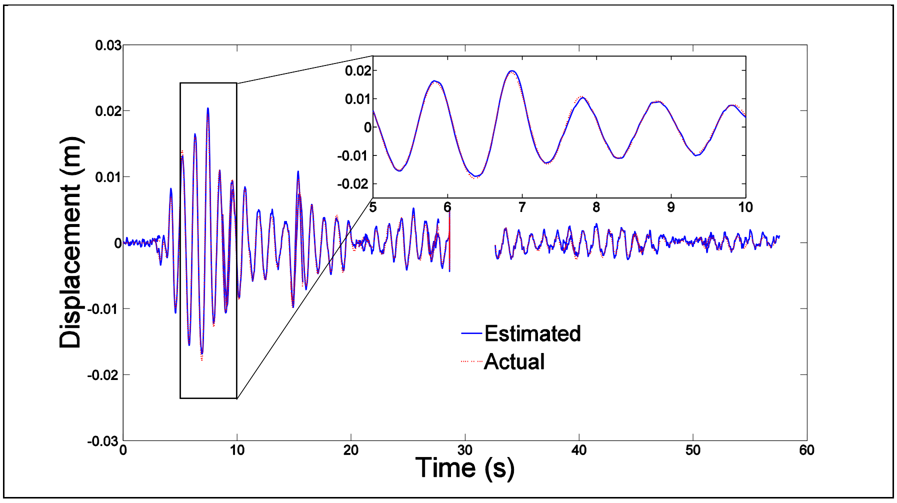



Fig. 3 Estimated displacement of the top floor

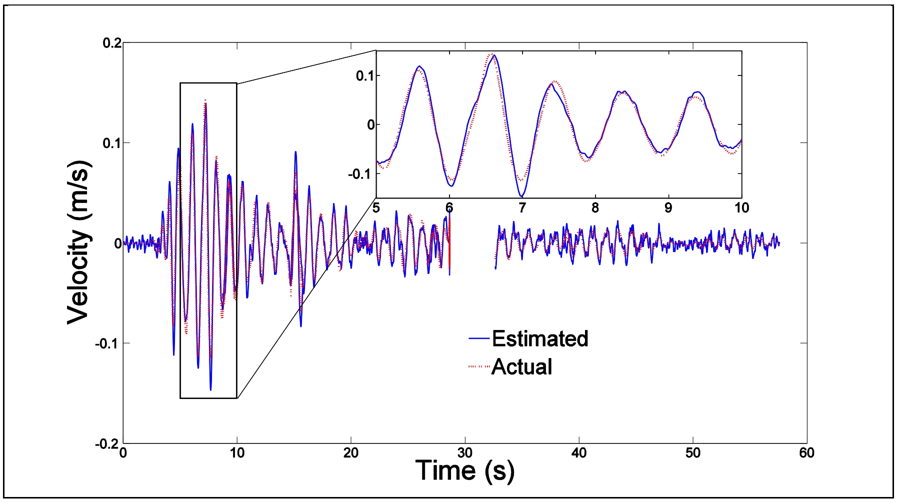



Fig. 4 Estimated velocity of the top floor

Table 1 Maximum percent error in estimated peak/trough values

| Floor Level | Actual Displ.(m) | Est. Displ. (m) | \|Diff.\| (%) | Actual Vel.(m/s) | Est. Vel.(m/s) | \|Diff.\| (%) |
|---|---|---|---|---|---|---|
| 1 | 0.0035 | 0.0037 | 6.73 | 0.0197 | 0.0217 | 10.59 |
| 2 | 0.0069 | 0.0073 | 6.13 | 0.0391 | 0.0427 | 9.34 |
| 3 | 0.0101 | 0.0107 | 5.75 | 0.0572 | 0.0622 | 8.81 |
| 4 | 0.0130 | 0.0137 | 5.48 | 0.0731 | 0.0797 | 9.03 |
| 5 | 0.0154 | 0.0162 | 4.93 | 0.0864 | 0.0944 | 9.28 |
| 6 | 0.0174 | 0.0182 | 4.37 | 0.0971 | 0.1058 | 8.95 |
| 7 | 0.0188 | 0.0195 | 3.83 | 0.1052 | 0.1137 | 8.08 |
| 8 | 0.0195 | 0.0202 | 3.59 | 0.1096 | 0.1177 | 7.39 |

Thus, using only the first mode is sufficient in recovering displacements and velocities despite that the errors in the filtered accelerations with only one mode are noticeably larger at lower floors. Therefore, in real-world applications, relatively larger errors in system output comparison (accelerations) may be allowed at lower floors.

Note that in this example, the highest excited mode was the 3$^{rd}$ mode (5.06 Hz), reasonable accuracy can still be achieved with sampling frequencies lower than 50 Hz. Simulations were conducted with different sampling frequencies. The results showed no noticeable changes in accuracy when the measurements were sampled at 20 Hz and up to 500 Hz. Relatively larger errors occurred when the sampling frequency approached 10 Hz, which was close to the critical frequency as the highest dominant frequency was 5.06 Hz. Much larger errors were observed when further reducing the sampling frequency. As in many civil engineering applications, the structural response is dominated by relatively low frequencies, e.g., up to 20 Hz, a sampling frequency of 50 - 100 Hz should be sufficient.

### *3.2 Example 2: three-story scaled building model*

In addition to the numerical structure, an experimental three-story scaled building model was analyzed. The floors were identically built with a dimension of 0.314 $m \times$ 0.386 $m \times$ 0.401 $m$. The floor masses for the lowest, second and top floors were 6.67 kg, 6.64 kg, and 5.03 kg, respectively. Using structural deflections under static loads, the inter-story stiffness coefficients were determined experimentally to be 31.01 N/mm, 34.07 N/mm, and 31.47 N/mm for the first, second, and top floor, respectively. Using a shear beam lumped mass model, the natural frequencies were theoretically determined to be 5.10 Hz, 14.19 Hz, and 20.41 Hz. The N-S component of the ground accelerations of the 1979 El Centro earthquake was scaled so that the peak value was about 0.8 g and used as the base excitation. The structural accelerations were measured at all three floors. The displacements of the three floors were also measured relative to a fixed reference using LVDTs. Both accelerations and displacements were measured at a sampling frequency of 100 Hz based on the Nyquist-Shannon sampling theorem to avoid loss of useful information.

Fig. 7 shows the spectra of the accelerations at all floors. It is clear that three modes dominated in the structural response. Using frequency domain decomposition, the dominant frequencies were identified as 4.79 Hz, 15.19 Hz, and 21.63 Hz. The identified mode shapes are shown in Fig. 8. Using the measured floor masses and stiffness coefficients, the modal assurance criterion (MAC) (Allemang 1980, Allemang and Brown 1982) of the identified mode shapes was calculated to be 0.9958, 0.9756, and 0.9418 for the first, second and third mode, respectively.

The proposed method was applied to the measured accelerations with the same parameters of the Kalman filter used in the numerical example. Once again, the damping ratios played an insignificant role in the estimation, thus, the results obtained using a fixed damping ratio of 0.05 for all three modes are presented. Fig. 9 shows the filtered accelerations of the three floor levels as compared to the actual measured values. It is seen that the filtered accelerations closely follow the actual measurements. The peak values were captured with excellent accuracy. The estimated inter-story drifts for the top two floors are shown in Fig. 10. The estimated displacements are in good agreement with the measured response. The accuracy of the lower floor estimates deviate slightly. The velocity profile of the model is shown in Fig. 11.

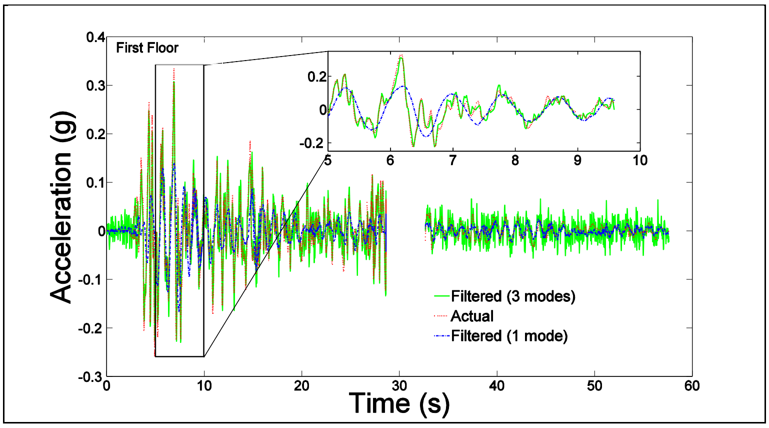



Fig. 5 Filtered acceleration of the first floor

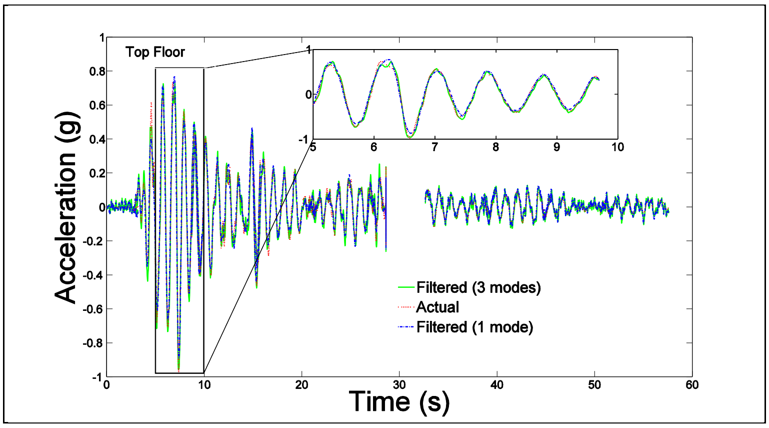



Fig. 6 Filtered acceleration of the top floor

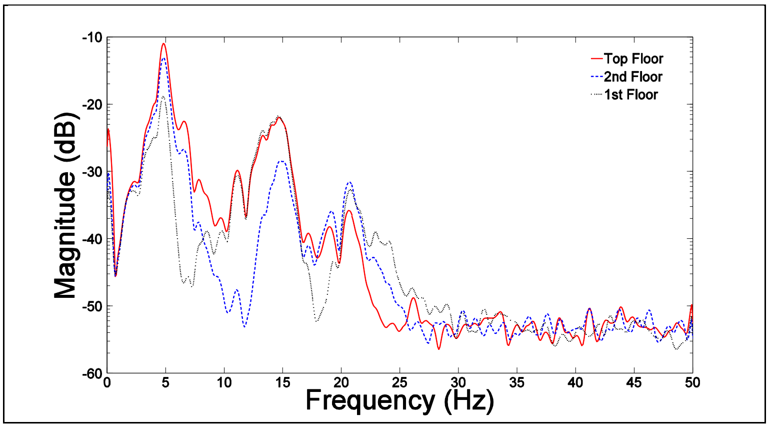



Fig. 7 Frequency content of accelerations at all floors

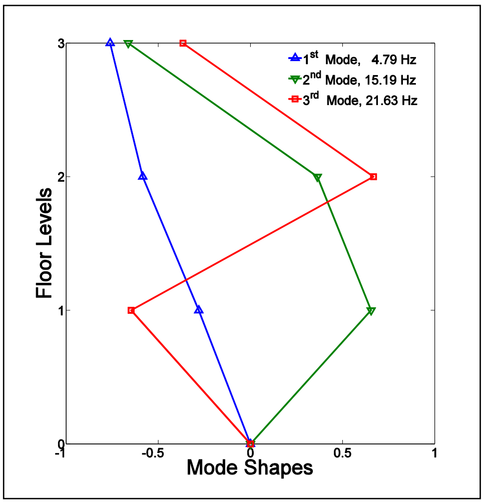
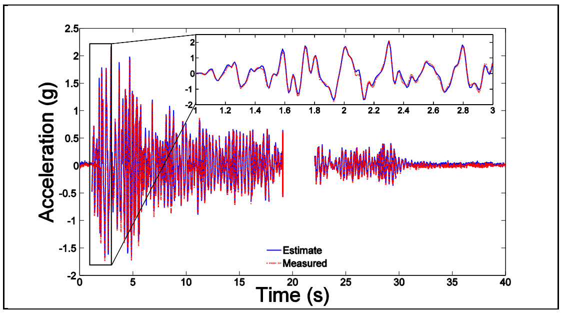
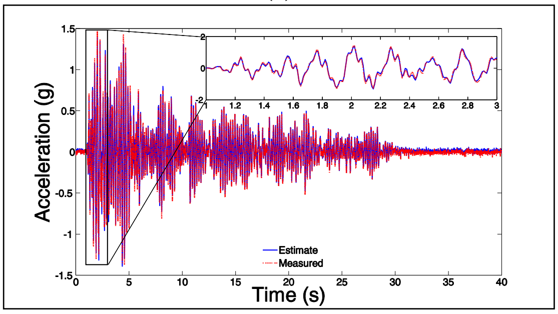


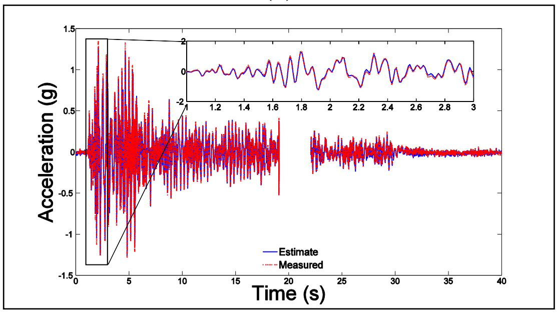

Fig. 8 Identified mode shapes




(a)




(b)

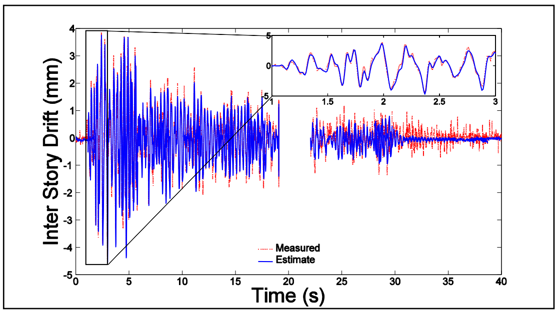



(c)

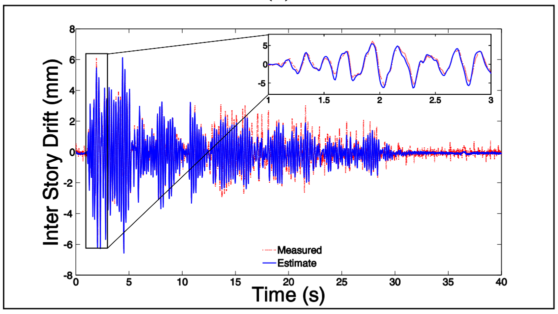
Fig. 9 Estimated accelerations compared with measured. (a) Top floor accelerations, (b) Second floor accelerations and (c) First floor accelerations

## 4. Conclusions

A method has been proposed for structural displacement and velocity estimation using only measured accelerations. When the structural modes are adequately excited, an internal model of the excitation can be established based only on limited modal information of the structure. Using such an internal model, an autonomous dynamic system can be obtained, the states of which can be accurately estimated using standard filtering/observer technique. A numerical example of an eight story, lump mass shear beam model of a building structure subject to seismic excitation has been presented to demonstrate the effectiveness of the proposed method. It has been shown that under a real seismic excitation, the structural displacement and velocity profiles can be reconstructed accurately with only the information of the first structural mode. The accuracy of the estimation can be monitored using the acceleration measurements compared with the filtered ones from the algorithm. As accelerations, particularly at lower floors, contain significant amount of energy at higher modes, larger errors may exist in the filtered accelerations at lower levels if such higher modes are not included in the algorithm. While these errors do not indicate poor estimation accuracy, one may use a few more modes in real-world applications to ensure a relatively uniform performance in acceleration filtering. In such a case, the estimation accuracy can be reliably deducted from the errors in the filtered accelerations. The proposed method has also been validated on a set of experimental data collected from a scaled building model, excited by a scaled ground acceleration record from a real earthquake. There is good agreement between the estimated response and the actual measurements. It is noted that the proposed method does not require precise identification of all the modal parameters. Only partial knowledge of the structural modal parameters, such as that of the dominating modal frequencies and mode shapes, is needed. Discrepancies in the dominant frequencies of the structure are compensated by the spectrum of the internal model, while errors in structural modal damping and modes are treated as system noise and rejected largely in filtering, i.e., the Kalman filter in this study. As a result, the structural properties of the structure are no longer needed to get an estimate of its behavior under ambient excitations. This allows for a more practical approach while monitoring a structure in the field. By simply placing accelerometers, one can determine in real time whether the structure is exceeding its critical displacement and velocity limits.




(a)




(b)

Fig. 10 Estimated inter story drift compared with measured. (a) Top floor inter story drifts and (b) Second floor inter story difts

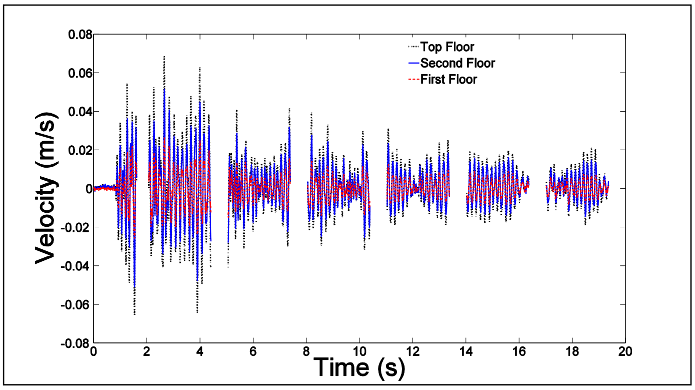



Fig. 11 Estimated structural velocities

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