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Effective vibration control of multimodal structures with low power requirement

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Abstract. In this paper, we investigate the vibration control of multimodal structures and present an efficient control law that requires less energy supply than active strategies. This strategy is called modal global semi-active control and is designed to work as effectively as the active control and consume less power which represents its major limitation. The proposed law is based on an energetic management of the optimal law such that the controller follows this latter only if there is sufficient energy which will be extracted directly from the system vibrations itself. The control algorithm is presented and validated for a cantilever beam structure subjected to external perturbations. Comparisons between the proposed law performances and those obtained by independent modal space control (IMSC) and semi-active control schemes are offered.

Keywords: multimodal structures; control; energy; IMSC; accumulator; actuator; semi-active

1. Introduction

Traditionally, there were two categories of vibration control based on the power flow in dynamic subsystems namely passive and active. Active strategies have good performances but necessitate an external power supply to apply control in opposite of the passive ones. A third intermediate strategy has been introduced since the early nineties (Jolly and Margolis 1997) where the system is not passive, yet, on average, more energy flows into it than out of it. It is called a semi-active system (or sometimes semi-passive) (Takakbatake and Ikarashi 2013, Zhou *et al*. 2012). Among the semi-active systems, we find the regenerative systems which offer the possibility of being self-sustainable which would reduce the dependence on an external energy source as for active systems. Jolly and Margolis (1997) examined two practical implementations of a regenerative subsystem which are base-excited suspensions and periodically excited compound mounts. They showed that the proposed suspensions were able to exhibit positive average energy absorption regardless of the nature of excitations in opposite of compound mounts which depend on the input spectrum nature.

In fact, the system vibrations energy will be extracted, converted into a useful form and reused to power a control law instead of being lost. The primary focus has been on regenerative dampers

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in automobile suspension systems. Gupta *et al*. (2006) presented two different regenerative electro- magnetic shock absorbers (linear and rotatory) that use the dissipated energy resulting from the roughness of the roads to enhance the damping. Snamina *et al*. (2009) investigated the feasibility of a regenerative vibration control system constituted of a DC motor used as the energy regenerative damper that converts the vibration energy into electric energy stored in a capacitor or battery. They showed that the energy stored in the capacitor can be used to supply the transducer in skyhook circuit in the case of a truck or cabin suspension. Huang *et al*. (2011) compared two different control modes (Consumptive Full Active (CFA) and Regenerative Semi Active (RSA) modes) for an electromagnetic suspension actuator. They used a main/inner-loop structure for the active control, and the energy flow states of the actuator are analyzed by simplifying the inner-loop control system. They demonstrated that the CFA mode can improve vehicle ride comfort by more than 30 percent, despite battery energy consumption, and in RSA mode, the ride comfort can be improved by up to 10 percent with the battery charged by regenerated energy. Recent works introduced a regenerative control approach called" global semi-active control" (Ichchou *et al*. 2011). Based on the reuse of the energy coming from the system vibrations, the energy management consists in switching between two control schemes that are the optimal active and the semi-active ones. It seems to effectively enhance the vibrations control performances. They offered the required algorithms to calculate the energy management term which will decide the phase switching. Numerical results for discrete systems with a very limited number of degrees of freedom like a quarter vehicle suspension or a deck system were given. They demonstrated that vibration attenuation capacities of the proposed strategy approach those of the pure active one and exceed those of the semi-active one. The stored energy seems to increase which lets hope to significant reduction in the energy consumption for real systems.

In this paper, the vibration control is specifically applied to continuous flexible structures. Indeed, emerging from their potential modes interferences, the control solution might be unstable. It is here necessary to design a multi-mode controller that can effectively suppress vibrations at and near specific natural frequencies of interest, but does not introduce unwanted vibrations at other natural frequencies (i.e., spillover) (Inman 2001). Another specific issue to be addressed is a consequence of these system's high order. The higher the order is, the larger is the amount of real time calculations is. This drawback can limit the controller performances or even prevent it from working properly.

Practical requirement for the controller of this kind of structures is employing a minimum number of sensor-actuator pairs using a simple design structure. The principal methods that can be found in the literature for controlling multi-mode vibrations include: positive position feedback control (PPF) (Wang 2003), independent modal space control (IMSC) (Meirovitch 1990, Nyawako *et al*. 2011) and modified independent modal space control (MIMSC) (Fang *et al*. 2003). In IMSC method, the control law is designed in the modal space for each mode independently which yield to uncoupled motion equation in modal coordinate system. Thus, IMSC scheme requires an appreciable less amount of calculation quantity than the coupled control and also, if the number of controlled modes and actuators is the same, controllability is always satisfied and the control spillover will be minimized which actually presents an important disadvantage restricting its application field. So, MIMSC scheme was developed which is a time-sharing technique possible to be applied when the number of actuators is inferior to the controlled modes. The main advantage of MISMC method is the reducibility of the number of actuators, yet it requires a high computation load imposed by the need to calculate and compare the energies in all modes of interest at every time interval.

As mentioned above, the idea of the control strategy is to use the vibrational energy of the system in order to supply the controller in the case of multimodal structures which is called" Modal global semi-active vibration control". The vibrational energy is extracted via piezoelectric transducers, converted into electric energy and stored in accumulators. If the available energy amount is sufficient, then optimal control is addressed. Otherwise, vibrations are simply damped (semi- actively) through extracting their energy and storing it until the next cycle. A similar work has been recently offered by Makihara *et al*. (2012) where they developed a self-powered semi-active strategy for multimodal vibrations suppression. Yet, they managed to design a digital fully autonomous controller that requires no external energy even for the microprocessor, sensors, switches in opposite to our study, where the re-injection of the stored energy was not undertaken. In fact, they assumed that the piezoelectric transducer works as semi- active vibration suppressor actuator and as power supply to drive the microprocessor, while, in our study, piezoelectric transducers are only used as energy scavengers and switching is based on the energy amount test. They also performed a comparison between their proposed digital and the conventional analog self powered systems (Niederberger *et al*. 2006) and demonstrated that dig- ital systems have the advantage of being programmable and able to implement any sophisticated control scheme and advanced filtering algorithm, thus suitable for complex multi-input/multi- output systems (Shimose *et al*. 2012). In addition, digital self-powered systems have excellent performances and excellent stability.

Inspired from the controlling strategy developed for the continuous modal structures, the paper firstly presents in section 2, the modeling of the system to be studied. The deduction of the relative reduced-order model to be used is part of this first stage. In section 3, the definition of the global semi-active control strategy is presented including the required mathematical tools, constraints to respect, active calculation to derive the optimal law to track and energetic comparisons as well as the development of the modal control algorithm. Finally, in section 4, numerical results discussing the performances of the proposed law are compared to the fully-active and the semi- active ones. The system energy transfer and distribution is also investigated emphasizing the storing phases of the law.

2. Modal space representation

Flexible structures are inherently distributed parameter structures with infinite degrees of freedom which request a high computational effort if using the full system model. That is why, modal reduction methods are preferred. Yet, a feedback controller based on a finite reduced modal model may destabilize the residual modes. This part of unmodeled dynamics may lead to spillover problems on real applications (Meirovitch and Baruk 1983). Since an excited structure has preferable modes of vibration which depend on the spectral content of the excitation, the lower order modes are assumed to be the most significant to the system global response. This way, the full order model can be reduced to those modes with a faithful restitution of the dynamic behavior.

In the following example, a cantilever beam subjected to external forces is considered for vibration control study. The full model equations of motion are derived according to the following Equation with $x(t)$ being the generalized nodal displacement vector.

$$M.\ddot{x}(t)+C.\dot{x}(t)+K.x(t)=Lu(t) \tag{1}$$

where: *M*, *C* and *K* are the structural mass, damping and stiffness matrices of the beam respectively, $\dot{x}(t)$ and $\ddot{x}(t)$ are the velocity and acceleration vectors respectively, *L* is the location matrix of the control force $u(t)$. The eigenvalue problem is then solved by using the modal transformation matrix Ψ and the modal reduced displacement vector $q = \begin{bmatrix} q_1 & q_2 & \dots & q_i \end{bmatrix}^t$, $(i = 1 \quad \dots \quad n)$ such that $x = \Psi.q$. The equations of motion relatives to the reduced *n* eigenmodes, are now uncoupled and can be written as

$$\ddot{q} + diag(2\xi_i w_i)\dot{q} + diag((w_i^2)q = f(t) \tag{2}$$

where w_i and ξ_i are the natural eigenfrequency and the damping ratio of the i[th] mode respectively, and $diag(2\xi_i w_i) = (\Psi^T M \Psi)^{-1i}\Psi^T C\Psi$, $diag(w_i^2) = (\Psi^T M \Psi)^{-1}\Psi^T K\Psi$ and $f(t) = (\Psi^T M \Psi)^{-1}\Psi^T Lu(t)$. The modal control force $f(t) = \begin{bmatrix} f_1 & f_2 & \dots & f_i \end{bmatrix}^t$ is related to the physical control vector $u(t)$. Consequently, each control force f_i corresponding to mode *i* depends on all the modal co- ordinates which leads to the problem of recoupling our decoupled equations.

Methods avoiding this recoupling issue are presented in section 2.1.

The state space approach is the basis of the current control theories and is strongly recommended in the design and analysis of control systems with a great amount of inputs and outputs (Williams *et al.* 2007). Let $X(t)$ be the state vector such that $X(t) = \begin{bmatrix} q(t) & \dot{q}(t) \end{bmatrix}^t$, Eq. (1) can be written in the form of a linear, first-order state space differential equation

$$\dot{X}(t) = AX(t) + Bf(t) \tag{3}$$

with

$$A = \begin{bmatrix} 0 & I \\ -diag(w_i^2) & -diag(2\xi_i w_i) \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

where *A* is the modal state matrix, *B* is the location matrix of the modal control forces, and *I* is the *n*-rank identity matrix.

2.1 Independent Modal Space Control (IMSC)

Independent modal space control method is used to derive the global control force $f(t)$, since it has the advantage of restating the problem as a set of independent modal equations which will permit decoupling equations and thus simplifying the controller design. For that, the global control force $f(t)$ will be composed of N_C chosen modal feedback forces such that $f(t) = \begin{bmatrix} f_1(t) & f_2(t) & \dots & f_i(t) \end{bmatrix}^t$, $i = 1 \dots N_c$ and the modal feedback control force $f_i(t)$ depends on the corresponding modal coordinates $q_i(t)$ and $\dot{q}_i(t)$ through the following equation

$$f_i(t) = -g_{1_i} q_i - g_{2_i} \dot{q}_i \tag{4}$$

Thus, the global modal control force is written as

$$\begin{aligned} f(t) &= \left[diag(g_1) \quad diag(g_2) \right] X(t) \\ &= gX(t) \end{aligned} \tag{5}$$

The global gain matrix g is calculated through an optimal scheme consisting on the minimization of a quadratic performance index J

$$J = \int_0^{\infty} \left(X^T Q X + f^T R f \right) dt \tag{6}$$

where Q is the positive definite or semi-positive definite weightening matrix and R is the positive factor that weights the importance of minimizing the vibration with respect to the control forces.

Instead of minimizing the global performance index J, we chose to limit the study to N_C modal cost functions J_i such that $J = \left[J_1 \ J_2 \ \cdots \ J_i \right]^t, i = 1 \cdots N_c$ (Zhang *et al.* 2008). These latter modal cost functions depend on the modal control force f_i (to minimize the control input effort), the mode states q_i (through minimizing the potential energy of the structure $w^2 q^2$) and $\dot{q}_i$ (through minimizing the kinetic energy $\dot{q}_i^{\,2}$) and can finally be written as

$$J_i = \int_0^{\infty} \left[\left(w_i^2 q_i^2 + \dot{q}_i^2 \right) + r_i f_i^{\,2} \right] dt, \quad i = 1 \ldots N_c \tag{7}$$

The closed-form solution g_1 and g_2 of the gain matrix which permit minimizing the performance index J_i can be obtained by the formulation given by Meirovitch (1981) such that

$$\begin{aligned} g_{1_i} &= -w_i^2 + w_i \sqrt{w_i^2 + \frac{1}{R}} \\ g_{2_i} &= \sqrt{-2w_i^2 + \frac{1}{R} + 2w_i \sqrt{w_i^2 + \frac{1}{R}}} \end{aligned} \tag{8}$$

By substituting the expression of J_i in Eq. (6), the expression of matrix Q and R can be deduced to $Q = \left[diag(w^2) \quad eye(N_c) \right]$ and $R = \left[diag(r_i) \right]$ respectively.

2.2 Modified Independent Modal Space Control (MIMSC)

Independent modal space control method is useful as it allows system decoupling and efficient control of vibrations. However, it does not take into consideration the previous vibration history of each mode meaning that we are not necessarily controlling the modes that are the most excited by the disturbance forces at each time *t*. In addition, using IMSC method necessitates having as much actuators as the modes to be controlled. To overcome these limitations, ISMC method was improved and the energy of each mode is calculated at a specific interval of time so that the control will be directed to the modes with the highest energy for the period of interest. This method is called Modified Independent Modal Space Control (MIMSC) (Fang *et al.* 2003). Since only the highest energy mode checked at instant t is attenuated, which is different from that corresponding to instant $t^* \neq t$, the number of actuators can also be reduced.

Now the reduced system model is obtained and the global control force $f(t)$ determined, the global semi-active vibration control scheme to use can be completely introduced.

3. Modal global semi-active vibration control law

The control strategy globally manages the system energy which consists of two categories: the energy extracted from vibrations and stored in the accumulators and the dissipated energy used to power the actuators in a way these latter switch between two control types which are the optimal scheme and the semi-active one. Actuators will operate under the purely-active law when the available energy allows them to or under the semi-active one if not which is possible through extracting energy from vibrations and storing it in the accumulators. It is clear then that storage devices are needed in this control scheme (called accumulators) as well as an energy management device responsible of the switching operation between the two different control types. A major advantage can be instantly deduced from the proposed law which is the reduction of the energy consumption required for the control law. We will present in section 4 the performances of the global semi-active scheme which approach very much those of the fully-active ones as well as the energy evolution of the system.

3.1 Constraints on accumulators and actuators

In order to be able to apply the global semi-active control, storage (accumulators) and actuation (actuators) devices are needed. Consequently, two constraints that may affect the control performances have to be introduced. The first limitation is related to the stored energy amount which itself depends on the accumulators capacities meaning that, at each time t, the stored energy amount has to be bounded by two extreme values E_{min} and E_{max} such that

$$E_{\min} \leq E(t) \leq E_{\max}, \qquad \forall t \in \left[t_0 \quad t_f \right] \tag{9}$$

The second limitation deals with the control force that the actuators are supposed to deliver. In fact, the control is displayed by piezoelectric actuators (collocated piezoelectric patches bounded on the beam) and the control force $f(t)$ is proportional to the feedback control voltage $V(t)$. In this case, $V(t)$ is the physical control force and matrix L includes the electro-mechanical constants of the piezoelectric patches. The voltage limitation of the of the piezoelectric actuators results in the limitation of the produced control force such that

$$\| f \| \leq f_{\max}, \qquad f \max > 0 \tag{10}$$

Moreover, for stability reasons, actuators saturation must be avoided i.e., they must be able to function regardless of the available stored energy level and also extract energy from the system even if one accumulator is already full. A Boolean function *b(t)* is therefore introduced to define the sequence disconnection between actuators and accumulators, and we have $b(t) = 1$ when actuators and accumulators are connected and optimal control force is delivered and $b(t) = 0$ otherwise i.e., actuators extract energy and store it in the accumulators which make them operating like conventional semi-active actuators. The switching between these two states is decided in function of an energy management term Γ which will be introduced in the following section.

3.2 Algorithm of the control strategy

The minimization of the cost criterion J resulted in the determination of the optimal control force further denoted $f_{gsa}(t)$ (section 2.1) , the knowledge of the initial conditions as well as the definition of the constraints on the accumulators and actuators (section 3.1) permits setting the global minimization problem denoted below as (P)

$$\begin{array}{ll} \dot{X} = AX + Bf & \\ E_{\min} \le E(t) \le E_{\max} & \forall t \in \left[t_0 \quad t_f \right] \\ \| f \| \le f_{\max} & f\max > 0, \quad when \quad b(t) = 0 \\ X(t_0) = X_0, & E(t_0) = E_0 \end{array}$$

In order to be able to apply the global semi-active control to a multimodal structure, the optimization problem will be addressed for each mode apart (i.e., each mode will be considered as a 1 *d.o.f* system). The needed power amount for the piezoelectric actuators to deliver the optimal control force denoted $f_{gsa_i}(t)$ corresponding to each mode i (which is supposed to be supplied from the accumulators) is electrical and can be calculated by Shu *et al*. (2006) through the following expression

$$\begin{aligned} \dot{E}_i &= bwC_{PZT} \frac{\left(V_{gsa_i}\right)^2}{2} \\ &= b\frac{wC_{PZT}}{2\alpha^2}\left(f_{gsa_i}\right)^2 \end{aligned} \tag{11}$$

where w, α and C_{PZT} are the radial frequency of voltage, the piezoelectric constant and capacitance respectively. The equivalent model of the system for mode i is represented in Fig. 1.

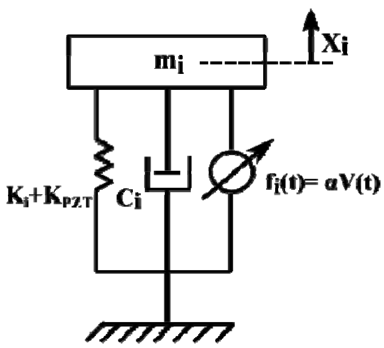


Fig. 1 Equivalent model of the system in neighbour of mode *i*

Consequently, we will have N_c modal global minimization problems to solve through minimizing their corresponding Hamiltonian functions H_i relative to each mode separately, and we get

$$\begin{aligned} H_i = &\left(X_t Q_i X_i + f_i^t r_i f_i\right) + \lambda^t\left(A_i X_i + b_i f_i - \dot{X}_i\right) \\ &+\Gamma_i\left(E_i - b.\frac{wC_{PZT}}{2\alpha^2} f_i^t f_i\right) - \gamma_{1_i}\left(E_i - E_{\min}\right) + \gamma_{2_i}\left(E_i - E_{\max}\right) \\ &+\left(-\beta_{1_i}\left(f_i - f_{\max}\right) + \beta_{2_i}\left(f_i - f_{\max}\right)\right) \end{aligned} \tag{12}$$

with $\lambda_i, \gamma_{1_i}, \gamma_{2_i}, \beta_{1_i}, \beta_{2_i}$ and Γ_i being the set of required Lagrange multipliers. A_i, B_i, Q_i are the reduced state matrix, actuator location matrix and weightening matrix corresponding to mode i respectively.

The minimization of H_i with respect to the state coordinates gives the following expression of λ_i

$$\begin{aligned} &\frac{\partial H_i}{\partial X_i} = \left(X_i^t Q_i + \lambda_i^t A_i - \dot{\lambda}_i\right) = 0 \\ &\Rightarrow \dot{\lambda}_i = -Q_i X_i - A_i^t \lambda_i \end{aligned} \tag{13}$$

and the modal control force is obtained by

$$\begin{aligned} \frac{\partial H_i}{\partial f_i} &= \left(f_i^t r_i + \lambda_i^t B_i - \Gamma_i b.\frac{wC_{PZT}}{2\alpha^2} f_i^t\right) \\ &= \left(r_i f_i + B_i^t \lambda_i - \Gamma_i b.\frac{wC_{PZT}}{2\alpha^2} f_i\right) = 0 \\ \Rightarrow f_i &= \frac{-B_i^t \lambda_i}{r_i - \Gamma_i b\dfrac{wC_{PZT}}{2\alpha^2}} \end{aligned} \tag{14}$$

The minimization of H_i with respect to the power amount provides the expression of $\dot{\Gamma}_i$ at instant t

$$\begin{aligned} &\frac{\partial H_i}{\partial E_i} = \left(-\gamma_{1_i} + \gamma_{2_i} + \dot{\Gamma}_i\right) = 0 \\ &\Rightarrow \dot{\Gamma}_i = \gamma_{1_i} - \gamma_{2_i} \end{aligned} \tag{15}$$

The minimization problem relative to each mode i can then be written as

$$\dot{X}_i = A_i X_i + B_i f_i$$
$$\dot{\lambda}_i = -Q_i X_i - A^t{}_i \lambda_i$$
$$f_i = \frac{-b^t{}_i \lambda_i}{r_i - \Gamma_i b \dfrac{wC_{PZT}}{2\alpha^2}}$$
$$\dot{\Gamma}_i = \gamma_{1_i} - \gamma_{2_i}$$

3.2.1 Available energy

In order to calculate the value of the available energy, we need to have a direct relationship between Γ_i and $\dot{E}_i$. So, we substitute the expression of the control force

$$\dot{E}_i = b\frac{wC_{PZT}}{2\alpha^2}.\left(\frac{B_i^t \lambda_i}{r_i - \Gamma_i b \dfrac{wC_{PZT}}{2\alpha^2}}\right)^2 \tag{16}$$

An additional constraint relative to the energy management term Γ_i arises from the previous Eq. (16). In fact, because Γ_i is related to the amount of energy stored in the accumulators $\dot{E}_i$, it must rely in an eligibility interval $\left[\Gamma_{\min} \quad \Gamma_{\max}\right]$ to respect the physical limitations of both actuators and accumulators. When these last conditions are satisfied, we can calculate the value of the energy management term and we get

$$\Gamma_i = \frac{2\alpha^2}{wC_{PZT}}.\left(r_i - \frac{B_i^t \lambda_i}{\sqrt{\dfrac{2\alpha^2 \dot{E}_i}{bwC_{PZT}}}}\right)^2 \tag{17}$$

However, it is necessary to study the state of the system at instant t_n where a control switching is required i.e. the corresponding value of Γ_i which is equal to its previous value at instant t_{n-1} causes a saturation of the accumulator and thus has to be readjusted to a new one denoted $\hat{\Gamma}_i$ satisfying Eq. (16). Let denote $\tilde{\Gamma}_i$ the displayed value of Γ_i at the switching instant from which we can deduce the value of the displayed control force $\tilde{f}_i$ which, itself, is not satisfying the accumulator limitations, such that

$$\tilde{f}_i(t_n) = \frac{-B_i^t \lambda_i(t_n)}{r_i - \tilde{\Gamma}_i(t_n) b \dfrac{wC_{PZT}}{2\alpha^2}} \tag{18}$$

Now, the value of the readjusted energy management term can be calculated from Eq. (17) and we have

$$\tilde{\Gamma}_i(t_n) = \frac{2\alpha^2}{wC_{PZT}} \cdot \left(\frac{B_i^t \lambda_i(t_n)}{r_i - \sqrt{\dfrac{2\alpha^2 \dot{E}_i(t_n)}{bwC_{PZT}}}} \right) \tag{19}$$

By replacing the expression of the displayed control force (Eq. (18)) in (Eq. (19)), we get

$$\hat{\Gamma}_i(t_n) = \frac{1}{\sqrt{\dfrac{2\alpha^2 \dot{E}_i(t_n)}{bwC_{PZT}}}} \left(br_i \tilde{f}_i(t_n) \frac{2\alpha^2}{wC_{PZT}} + \tilde{\Gamma}_i(t_n) \right) + r_i \frac{2\alpha^2}{bwC_{PZT}} \tag{20}$$

The value of $\dot{E}_i(t_n)$ in Eq. (20) enables us to calculate the value of the maximum available instantaneous force further denoted $\hat{f}_i$ through the relationship Eq. (11) and the expression of the readjusted value of Γ_i can be finally obtained by

$$\hat{\Gamma}_i(t_n) = \left(\frac{br_i \tilde{f}_i(t_n) \dfrac{2\alpha^2}{wC_{PZT}} + \tilde{\Gamma}_i(t_n)}{\hat{f}_i(t_n)} \right) + r_i \frac{2\alpha^2}{bwC_{PZT}} \tag{21}$$

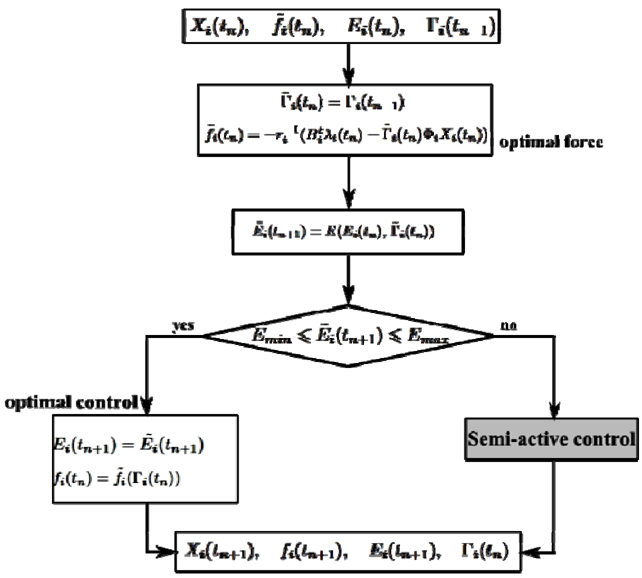


Fig. 2 Algorithm of modal global semi-active control

The global management term Γ at instant t_n, when switching is required, is chosen to equal the modal energy management term $\hat{\Gamma}_i$ corresponding to the highest modal control force $\hat{f}_i$ to be applied i.e., the mode with the highest vibrations level. With this procedure, we are sure that at each time path, we are addressing the mode that influences the most the overall response of the beam and thus obtaining the best vibration attenuation performances with the lowest control effort.

From an industrial implementation consideration, readjusting the value of Γ_i at each time path is not practical and certainly not attractive. So, a suboptimal algorithm is developed in order to avoid this heavy calculation effort and instead, the switching will occur as soon as a saturation in the accumulator is depicted (i.e., limits of $E_i(t)$ are not respected). The algorithm is detailed in Fig. 2.

4. Simulations and discussions

In this section, the capacity of the proposed modal global semi-active control (MGSA) to efficiently attenuate a cantilever beam vibrations (see Fig. 3) is investigated and the performances of this method are compared to the purely active ones obtained with IMSC scheme as well as to the semi-active (SA) scheme. The SA scheme performed herein is driven by shock absorbers producing an actuating force opposite to the vibration's level (in our case the velocity level). The power requirement is expressed by the product of the actuating force and the relative velocities at the actuators' ends. The modal response of the beam is initially reduced to the five first modes which appeared to be the most energetic and here the most interesting to the overall response.

The corresponding natural frequencies are listed in Table 1. The time domain response of the cantilever beam is given (transverse displacement) for two types of disturbance: harmonic and random. The harmonic disturbance is a sinusoidal signal whose frequency is close to the first natural frequency of the system. The frequency bandwidth of the random disturbance ranges from 10 to 600 Hz.

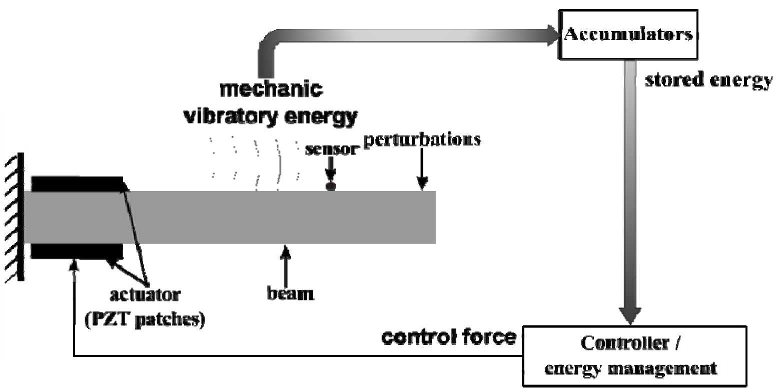


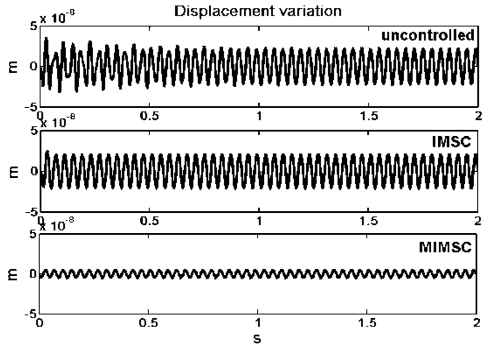
Fig. 3 Global semi-active of a cantilever beam

Table 1 Natural frequencies of the beam

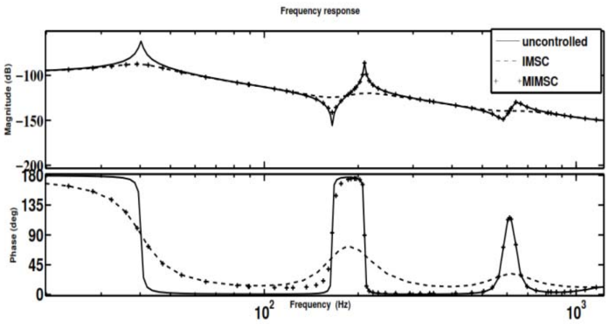
	Mode1	Mode2	Mode3	Mode4	Mode5
Frequency (Hz)	40.26	221.26	641.74	$1.23\ 10^3$	$2.06\ 10^3$

Table 2 RMS values of the cantilever beam transverse displacement

RMS values	Uncontrolled	IMSC	MIMSC
Beam Transverse displacement (m)	0.1638 10-7	0.1520 10-7	$0.0348\ 10^{-7}$



(a) Transverse displacement variation



(b) Frequency response

Fig. 4 IMSC / MIMSC response comparisons

4.1 IMSC and MIMSC comparisons

The control here addresses the first three modes of the beam for the IMSC scheme and the MIMSC one. The comparison between these schemes is based on the RMS values (Root Mean Square) of the displacement and the velocity time responses. In addition, the frequency responses are also analyzed. We notice that MIMSC provides better performances in the time domain (see Fig. 4(a)) since for the specified chosen time interval, the mode with the highest energy (which is governing the overall response of the beam) is controlled, so the vibration's attenuation is enhanced in comparison to IMSC scheme (see Table 2). However, for frequency responses, MIMSC (++ marked curve) scheme shows identical performances to the IMSC scheme (dashed curve) for the first mode (see Fig. 4(b)).

This can be explained by the fact that, at each time, the first mode is the dominant one. Therefore, it catches all the action of the controller. MIMSC scheme appears to be less efficient for the mode 2 and 3, yet approaching the performances of the IMSC one.

For these reasons, the IMSC scheme will be the only scheme to draw the comparison with the proposed modal global semi-active control (MGSA).

4.2 IMSC, MGSA and SA comparisons

In this work, the efficiency of the modal global semi-active law (MGSA) is intended to be assessed. That is why, its performances are to be compared along with those of the semi-active scheme (SA) to the IMSC active ones. One of the main aspects of the proposed control strategy (MGSA) is to use harvested energy from the structure itself (Fig. 5) in order to supply the actuators (Makihara 2012). The energy-harvesting circuit consists on an AC-DC rectifier (diode bridge) followed by a filtering capacitance C_e used to smooth the DC voltage (Shu *et al*. 2006).

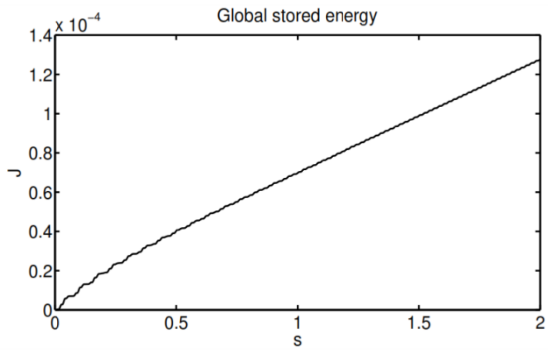


Fig. 5 Evolution of the stocked energy

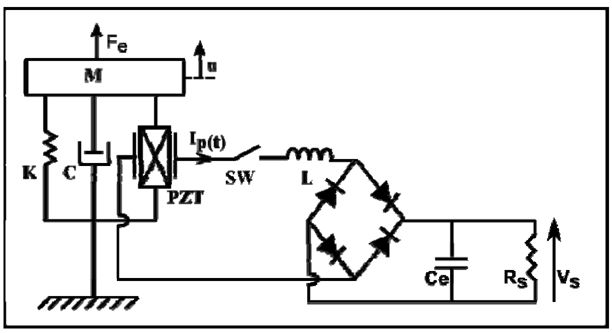


Fig. 6 AC/DC harvesting circuit

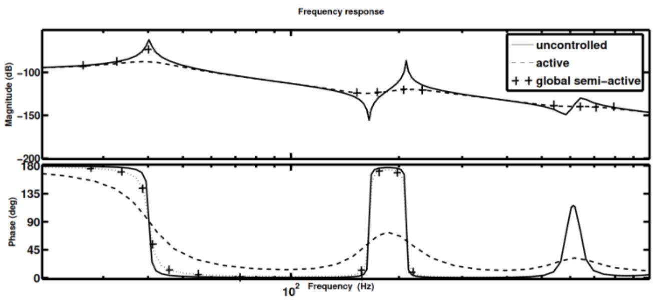
The harvested power is calculated through a resistive load R mounted in parallel to C_e, as shown in Fig. 6, and is expressed as follows

$$P_i(t) = \frac{V_{C_i}^2(t)}{R} = \dot{E}_i(t) \tag{22}$$

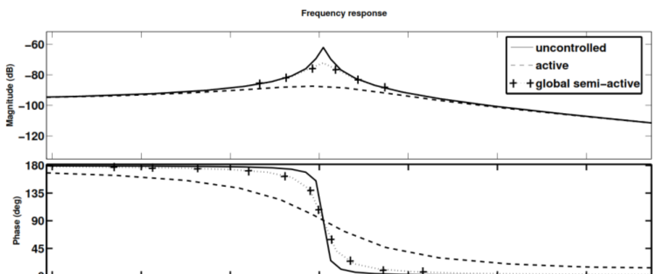
where $V_{C_i}(t)$ is the rectified voltage across the resistive load R and $I_i(t)$ is the current flowing into the circuit for the corresponding mode vibration i.

From this observation, one can expect this technique to reduce the structure's vibrations by taking some of its energy at least. Time responses (through RMS values) and frequency responses are used to draw these comparisons. Fig. 7(a) indicates the frequency response comparisons when using the different control strategies. It is noted that MGSA scheme is able to attenuate the vibrations of the structure with performances approaching those of the IMSC scheme. The precise analysis of the FRF around the first eigenfrequency (Fig. 7(b)) actually confirms this remark where MGSA control (++ dotted curves) is located between the uncontrolled (solid curve) and the IMSC (dashed curve) responses.

This observation is even confirmed by the time response results of the beam for harmonic and random excitations (Fig. 8). The RMS values for the three types of control (IMSC, MGSA and SA) and the different types of excitations (harmonic and random) are summarized in Table 3. The performances of the modal global semi-active strategy rank it between the active and the semi-active schemes.

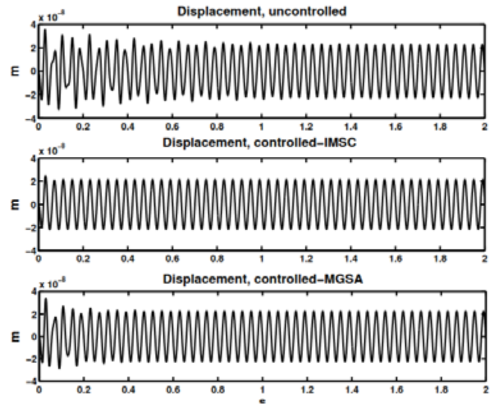


(a) Frequency response: IMSC (dashed) /MGSA (++ dotted) comparisons

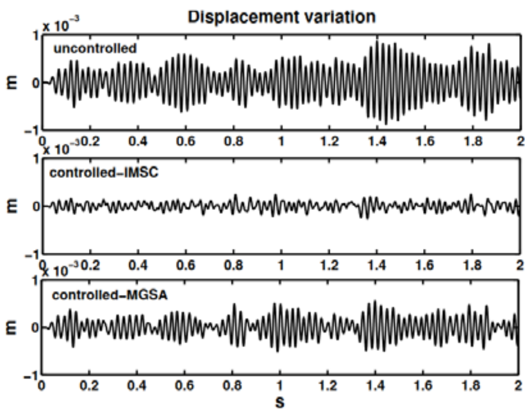


(b) Zoom of the first mode frequency response

Fig. 7 Frequency response of the beam



(a) Harmonic excitations



(b) Random excitations

Fig. 8 IMSC/ MGSA time response comparisons of transverse displacement

Table 3 RMS values of the cantilever beam transverse displacement (IMSC / MGSA / SA)

	displacement (m)			
	uncontrolled	IMSC	MGSA	SA
Sinusoidal	$0.1638\ 10^{-7}$	$0.1520\ 10^{-7}$	$0.1612\ 10^{-7}$	$0.1632\ 10^{-7}$
Random	$0.3277\ 10^{-3}$	$0.0820\ 10^{-3}$	$0.1991\ 10^{-3}$	$0.1996\ 10^{-3}$

5. Conclusions

Among the vibration control techniques, the active scheme is the most efficient in terms of vibration reduction performances. However, it is limited by the high energy needs it requires. A global semi-active control strategy was presented by Ichchou *et al.* (2011) with the aim to reduce the energetic requirements while maintaining good vibrations attenuation performances.

In this paper, the global semi-active control scheme was developed for flexible structures" modal global semi-active control (MGSA)" and the corresponding algorithm was presented. The MGSA strategy is based on a switching between a semi-active law and an optimal one according to the level of the available energy in the accumulators. This energy is extracted from the system vibrations then stored in the accumulators.

The main advantage of the proposed control scheme is its low power requirement since it re-uses the energy of vibrations to supply the actuators. If the stored energy is sufficient to follow the optimal scheme, this last is applied. Otherwise, the controller switches to the semi-active one (dissipative one). The control law was addressed to a cantilever beam and the results showed its good performances approaching those of the optimal law and exceeding those of the semi- active one. Moreover, a reduction in the energy consumption is noticed. It presents an attractive achievement in comparison with the pure active strategy.

Future implementations or real full-size flexible structures are considered, using piezoelectric transducers mounted on the vibrating structure to extract its energy and convert it into a useful form (electric energy).

The aim is to increase the stored energy amount in the accumulators. Thus, the control performances of the law will be enhanced and the energy needs further reduced.

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