Smart Structures and Systems, *Vol. 13, No. 3 (2014) 343-352*
DOI: http://dx.doi.org/10.12989/sss.2014.13.3.343

Sensor placement driven by a model order reduction (MOR) reasoning

Fabio Casciati* and Lucia Faravelli[a]

Department DICAR, University of Pavia, Via Ferrata 1, 27100 Pavia, Italy

(Received September 15, 2013, Revised January 6, 2014, Accepted January 10, 2014)

Abstract. Given a body undergoing a stress-strain status as consequence of external excitations, sensors can be deployed on the accessible lateral surface of the body. The sensor readings are regarded as input of a numerical model of reduced order (i.e., the number of sensors is lower than the number of the state variables the full model would require). The goal is to locate the sensors in such a way to minimize the deviations from the response of the true (full) model. One adopts either accelerometers as sensors or devices reading relative displacements. Two applications are studied: a plane frame is first investigated; the focus is eventually on a 3D body.

Keywords: model order reduction (MOR); numerical model; sensor placement; state variables; truncation

1. Introduction

For long time numerical optimization tools were all proposed in the class of the gradient-based methods (Borzì and Schulz 2011, among others). Genetic algorithms opened the way to the so called "heuristic" methods (Yang 2008) whose potential allowed several authors to reformulate standard optimization problems involving strong nonlinearities, integer variables and logic options in the definition of the objective function. Among these problems the placement of sensors (in structural monitoring (Papadimitriou 2004)) and devices (in structural passive control) is ongoing a significant re-flourishing.

This paper is focused on the sensor placement and adopts a heuristic optimization tool, but the reasoning toward the formulation of the problem differs from the standard one in two aspects:

1) a preliminary effort is addressed to the identification of the suitable number of observed variables to be considered; given a global numerical model of a structural system, find the order of the reduced order model (Schilders *et al*. 2008, Calberg *et al*. 2011): it gives the number of variables to be observed;

2) the global numerical model is reduced by a balanced truncation (Casciati and Faravelli 2014) rather than by paying attention only to the first harmonics in the structural response; the reduced order model is written in terms of state variables which can be derived either from the true response or from the response detected by the sensors at given positions. The sensor placement

*Corresponding author, Professor, E-mail: fabio@dipmec.unipv.it

[a] Professor, E-mail: lucia@dipmec.unipv.it

http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

which minimizes the discrepancies in the state variable reconstruction is regarded as the most suitable.

On the back of the proposed approach there are implicit assumptions which are made explicit below:

a) the mechanical system is linear or is linearized in a narrow range of behaviour;

b) the focus is on assigned load conditions in terms of position and direction, being the amplitude (of the oscillations in time) free.

The paper is organized by relying on the literature for the three required ingredients: namely, optimization tools, sensor placement and model order reduction, so that in section 2 the proposal for the twoclass of sensors is simply formulated. Section 3 shows the application to a plane frame, while section 4 is devoted to a 3D case.

2. Proposed formulation of the problem

Following (Casciati and Faravelli 2014), for a dynamic system discretized by finite elements, the state space formulation sees $2N$ first order ordinary differential equations, where $2N$ is the order of the model, which is two times the number N of the system degrees of freedom

$$\dot{\boldsymbol{z}}\,(t) = \boldsymbol{A}\,\boldsymbol{z}(t) + \boldsymbol{B}\,\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{z}$ is the state variable vector of size $2N$, the superimposed dot denotes time derivative, $\boldsymbol{u}$ is the vector of the external excitations, of size p, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are time invariant matrices of sizes $2N$ by $2N$ and $2N$ by p, respectively. The state variables are not supposed to have any physical meaning. But they are linked to any set of observables variables $\boldsymbol{y}(t)$ (denoted as “observed variables”) by a second set of equations, this time of the algebraic type

$$\boldsymbol{y}(\,t) = \boldsymbol{C}\,\boldsymbol{z}(t) + \boldsymbol{D}\,\boldsymbol{u}(t) \tag{2}$$

A common selection for $\boldsymbol{y}$ is to contain N (relative to the base) displacements followed by N (relative) velocities. Further observed variables could be the absolute accelerations which come, as a further vector of length N, from the equilibrium equations: be computed as the vector of the (absolute) accelerations, provided no external action enters the equilibrium equation (i.e., p=1 and $u(t)$ represents the base excitation only)

$$\boldsymbol{y}_a(t) = -\left[\left(\frac{\boldsymbol{K}}{\boldsymbol{M}}\right)\ \left(\frac{\boldsymbol{c}}{\boldsymbol{M}}\right)\right]\boldsymbol{y}(t) + \boldsymbol{B}_a\left(\frac{\overline{\boldsymbol{u}}}{\boldsymbol{M}}\right) \tag{3}$$

where $\boldsymbol{M}$, $\boldsymbol{c}$ and $\boldsymbol{K}$ are the matrices of mass, damping and stiffness, respectively. $\overline{\boldsymbol{u}}$ denotes the vector of the external excitation without any ground motion component (which is already accounted in the r.h.s.). Matrix $\boldsymbol{D}$ is assumed to be zero in the following.

2.1 Model order reduction (MOR)

Equations from (1) to (3) can be re-written adding the index “$_R$“ (for reduced) to all the quantities except the observed variables (Casciati and Faravelli 2013) when the reduced order model is pursued by balanced truncation

$$\dot{\boldsymbol{z}}_R(t) = \boldsymbol{A}_R\,\boldsymbol{z}_R(t) + \boldsymbol{B}_R\boldsymbol{u}\,(t) \tag{4}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}_R\, \boldsymbol{z}_R(t) \tag{5}$$

$$\boldsymbol{y}_a(t) = -\left[\left(\frac{\boldsymbol{K}}{\boldsymbol{M}}\right)\ \left(\frac{\boldsymbol{c}}{\boldsymbol{M}}\right)\right]\boldsymbol{y}(t) + \boldsymbol{B}_a\left(\frac{\bar{\boldsymbol{u}}}{\boldsymbol{M}}\right) \tag{6}$$

The number of state variables is now $2n$, with n significantly lower than N.

Alternatively, model order reduction can be achieved by identifying some degrees of freedom as masters, to which other degrees of freedom are linked. This occurs for instance when all the storey nodes of a plane frame are given the same horizontal displacement. In this case the number of actually detectable observed variables is reduced, as well as the dimension of mass, damping and stiffness matrices. Equations from (1) to (3) can be re-written adding the index "$_\rho$", with the number of state variables $2v$, with v significantly lower than N

$$\dot{\boldsymbol{z}}_\rho(t) = \boldsymbol{A}_\rho\, \boldsymbol{z}_\rho(t) + \boldsymbol{B}_\rho \boldsymbol{u}(t) \tag{7}$$

$$\boldsymbol{y}_\rho(t) = \boldsymbol{C}_\rho\, \boldsymbol{z}_\rho(t) \tag{8}$$

$$\boldsymbol{y}_{a\rho}(t) = -\left[\left(\frac{\boldsymbol{K}_\rho}{\boldsymbol{M}_\rho}\right)\ \left(\frac{\boldsymbol{c}_\rho}{\boldsymbol{M}_\rho}\right)\right]\boldsymbol{y}_\rho(t) + \boldsymbol{B}_{a\rho}\left(\frac{\bar{\boldsymbol{u}}}{\boldsymbol{M}_\rho}\right) \tag{9}$$

As a further option, one can apply the balanced truncation scheme to the Eqs. (7)-(9), writing an even more reduced model for the initial dynamic system, with the number of state variables $2v_R$, with v_R significantly lower than N, n and v

$$\dot{\boldsymbol{z}}_{\rho R}(t) = \boldsymbol{A}_{\rho R}\, \boldsymbol{z}_{\rho R}(t) + \boldsymbol{B}_{\rho R} \boldsymbol{u}(t) \tag{10}$$

$$\boldsymbol{y}_\rho(t) = \boldsymbol{C}_{\rho R}\, \boldsymbol{z}_{\rho R}(t) \tag{11}$$

$$\boldsymbol{y}_{a\rho}(t) = -\left[\left(\frac{\boldsymbol{K}_\rho}{\boldsymbol{M}_\rho}\right)\ \left(\frac{\boldsymbol{c}_\rho}{\boldsymbol{M}_\rho}\right)\right]\boldsymbol{y}_\rho(t) + \boldsymbol{B}_{a\rho}\left(\frac{\bar{\boldsymbol{u}}}{\boldsymbol{M}_\rho}\right) \tag{12}$$

where equations from (1) to (3) are re-written adding the indexes "$_{R\rho}$".

2.2 Looking for the solution of the inverse problem: the required number of sensors

For sake of exemplification, consider a plane frame of three stories subject to horizontal base excitation. It is easy to obtain (Casciati *et al*. 2012) that $2v_R = 6$. The inverse problem requires that one selects 6 observed values linked to the state variables $\boldsymbol{z}_{\rho R}$ by an invertible matrix, so that the state variables can be obtained from the knowledge of the time histories of those observed values, as obtained, for instance, from sensors. A sensor measures one of the three kinematic quantities displacement, velocity or acceleration. But one has not the possibility to place 6 accelerometers, since the storey degrees of freedom are simply 3. In addition to the three accelerometers one has to place 3 sensors measuring either displacements or velocities. A simplification would be to derive the relative velocity time history from the absolute acceleration record by removing the base contribution (i.e., a further sensor is required) and subsequent integration. Translating words in formulae, the attention is focused on absolute accelerations and relative velocities at given locations, provided $\boldsymbol{B}_a$ is nul

$$\begin{Bmatrix}\breve{\boldsymbol{y}}_{aR}\\ \breve{\boldsymbol{y}}_{vR}\end{Bmatrix} = \begin{bmatrix}\breve{\boldsymbol{C}}_{aR\rho}\\ \breve{\boldsymbol{C}}_{vR\rho}\end{bmatrix}\boldsymbol{z}_{R\rho} = \begin{bmatrix}[\breve{\boldsymbol{C}}_{aR\rho 11}] & [\breve{\boldsymbol{C}}_{aR\rho 12}]\\ [\breve{\boldsymbol{C}}_{vR\rho 21}] & [\breve{\boldsymbol{C}}_{vR\rho 22}]\end{bmatrix}\begin{Bmatrix}\boldsymbol{z}_{R\rho 1}\\ \boldsymbol{z}_{R\rho 2}\end{Bmatrix} \tag{13}$$

where the accent denotes that a special selection of observed variables has been chosen and the vector $\boldsymbol{z}_{R\rho}$ is simply divided in two vectors of the same length. Then the following algebra applies

$$\boldsymbol{z}_{R\rho 1} = \left[\breve{\boldsymbol{C}}_{aR\rho\,11}\right]^{-1}\left\{\breve{\boldsymbol{y}}_{aR} - \left[\breve{\boldsymbol{C}}_{aR\rho\,12}\right]\boldsymbol{z}_{R\rho 2}\right\} \qquad (14)$$
$$\breve{\boldsymbol{y}}_{vR} = \boldsymbol{v}(t) = \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\boldsymbol{z}_{R\rho 1} + \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\boldsymbol{z}_{R\rho 2}$$

where

$$\boldsymbol{v}(t) = \{\boldsymbol{v}(t-\Delta t) + (\breve{\boldsymbol{y}}_{aR} - \boldsymbol{a}_{\mathbf{b}})\Delta t\} = \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\boldsymbol{z}_{R\rho 1} + \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\boldsymbol{z}_{R\rho 2} =$$
$$= \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{aR\rho\,11}\right]^{-1}\breve{\boldsymbol{y}}_{aR} - \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{aR\rho\,11}\right]^{-1}\left[\breve{\boldsymbol{C}}_{aR\rho\,12}\right]\boldsymbol{z}_{R\rho 2} + \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\boldsymbol{z}_{R\rho 2}$$

From the last equation, after re-arranging , one obtains

$$\boldsymbol{z}_{R\rho 2} = \left\{\left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right] - \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{aR\rho\,11}\right]^{-1}\left[\breve{\boldsymbol{C}}_{aR\rho\,12}\right]\right\}^{-1}\left\{\boldsymbol{v}(t-\Delta t) + (\breve{\boldsymbol{y}}_{aR} - \boldsymbol{a}_{\mathbf{b}})\Delta t - \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{aR\rho\,11}\right]^{-1}\breve{\boldsymbol{y}}_{aR}\right\} \qquad (15)$$

Eqs. (14) and (15) express the state variables $\boldsymbol{z}_{R\rho}$, in number of $2\nu_{R\rho}$ as function of the $\nu_{R\rho}$ observed accelerations $\breve{\boldsymbol{y}}_{aR}$. It is worth noticing that $\breve{\boldsymbol{y}}_{aR}$ denotes the absolute acceleration and therefore also the base acceleration $\boldsymbol{a}_{\mathbf{b}}$ must be monitored "by a further sensor".

An alternative way consists adopting sensors measuring displacements from where the relative velocity time histories are obtained by numerical derivation. Translating words in formulae, once again, the attention is focused on relative displacements and velocities at given locations, and no assumptions on $\boldsymbol{B}_{\mathrm{a}}$ are required

$$\begin{Bmatrix}\breve{\boldsymbol{y}}_{vR}\\ \breve{\boldsymbol{y}}_{dR}\end{Bmatrix} = \begin{bmatrix}\breve{\boldsymbol{C}}_{vR\rho}\\ \breve{\boldsymbol{C}}_{dR\rho}\end{bmatrix}\boldsymbol{z}_{R\rho} = \begin{bmatrix}\left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right] & \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\\ \left[\breve{\boldsymbol{C}}_{dR\rho\,21}\right] & \left[\breve{\boldsymbol{C}}_{dR\rho\,22}\right]\end{bmatrix}\begin{Bmatrix}\boldsymbol{z}_{R\rho 1}\\ \boldsymbol{z}_{R\rho 2}\end{Bmatrix} \qquad (16)$$

where $\breve{\boldsymbol{y}}_{vR}$ is replaced by

$$\breve{\boldsymbol{y}}_{vR} = \{(\breve{\boldsymbol{y}}_{dR}(t) - \breve{\boldsymbol{y}}_{dR}(t-\Delta t)\Delta t^{-1}\}$$

Therefore

$$\begin{cases}\boldsymbol{z}_{R\rho 2} = \\ \left\{-\left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{dR\rho\,11}\right]^{-1}\left[\breve{\boldsymbol{C}}_{dR\rho\,12}\right] + \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\right\}^{-1}\left\{(\breve{\boldsymbol{y}}_{dR}(t) - \breve{\boldsymbol{y}}_{dR}(t-\Delta t)\Delta t^{-1} - \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]\left[\breve{\boldsymbol{C}}_{dR\rho\,11}\right]^{-1}\breve{\boldsymbol{y}}_{dR}\right\} \\ \boldsymbol{z}_{R\rho 1} = \left[\breve{\boldsymbol{C}}_{vR\rho\,11}\right]^{-1}\left\{\breve{\boldsymbol{y}}_{dR} - \left[\breve{\boldsymbol{C}}_{vR\rho\,12}\right]\boldsymbol{z}_{R\rho 2}\right\}\end{cases} \qquad (17)$$

Eq. (17) expresses the state variables $\boldsymbol{z}_{R\rho}$, in number of $2\nu_{R\rho}$ as function of the $\nu_{R\rho}$ observed relative displacements $\breve{\boldsymbol{y}}_{dR}$.

It would be only tedious for the reader to retype Eqs. (14) and (15) or Eq. (16) for the cases in which the model is governed by state variables $\boldsymbol{z}_R$, in number of *n*, or $\boldsymbol{z}_\rho$ in number of ν. The general conclusion is that the number of sensors required is equal to one half of the number of the state variables.

2.3 Looking for a sensors placement criterion

When the structural monitoring is on, the sensors collect, at given positions, the time histories $\widehat{\breve{\boldsymbol{y}}}$ and $\widehat{\breve{\boldsymbol{y}}}_a$, which coincides with those in Eqs. (2) and (3), provided the full model is correct and the sensors do not generate noise. From the knowledge of these time histories one can obtain estimates of the time histories of the state variables, i.e., in the two cases of relying on accelerometers or relative displacement sensors, respectively:

$$\begin{cases} \hat{\breve{\boldsymbol{z}}}_{R\rho 2} = \\ \left\{-[\breve{\boldsymbol{C}}_{vR\rho\,11}][\breve{\boldsymbol{C}}_{aR\rho\,11}]^{-1}[\breve{\boldsymbol{C}}_{aR\rho\,12}] + [\breve{\boldsymbol{C}}_{vR\rho\,12}]\right\}^{-1} \left\{\boldsymbol{v}(t-\Delta t) + (\widehat{\breve{\boldsymbol{y}}}_{aR} - \boldsymbol{a}_{\mathbf{b}})\Delta t - [\breve{\boldsymbol{C}}_{vR\rho\,11}][\breve{\boldsymbol{C}}_{adR\rho\,11}]^{-1}\widehat{\breve{\boldsymbol{y}}}_{aR}\right\} \\ \hat{\breve{\boldsymbol{z}}}_{R\rho 1} = [\breve{\boldsymbol{C}}_{aR\rho\,11}]^{-1}\{\widehat{\breve{\boldsymbol{y}}}_{aR} - [\breve{\boldsymbol{C}}_{aR\rho\,12}]\hat{\breve{\boldsymbol{z}}}_{R\rho 2}\} \end{cases} \quad (18)$$

$$\begin{cases} \hat{\breve{\boldsymbol{z}}}_{R\rho 2} = \\ \left\{-[\breve{\boldsymbol{C}}_{vR\rho\,11}][\breve{\boldsymbol{C}}_{dR\rho\,11}]^{-1}[\breve{\boldsymbol{C}}_{dR\rho\,12}] + [\breve{\boldsymbol{C}}_{vR\rho\,12}]\right\}^{-1} \left\{(\widehat{\breve{\boldsymbol{y}}}_{dR}(t) - \widehat{\breve{\boldsymbol{y}}}_{dR}(t-\Delta t)\Delta t^{-1} - [\breve{\boldsymbol{C}}_{vR\rho\,11}][\breve{\boldsymbol{C}}_{dR\rho\,11}]^{-1}\widehat{\breve{\boldsymbol{y}}}_{dR}\right\} \\ \hat{\breve{\boldsymbol{z}}}_{R\rho 1} = [\breve{\boldsymbol{C}}_{vR\rho\,11}]^{-1}\{\widehat{\breve{\boldsymbol{y}}}_{dR} - [\breve{\boldsymbol{C}}_{vR\rho\,12}]\hat{\breve{\boldsymbol{z}}}_{R\rho 2}\} \end{cases} \quad (19)$$

In the previous equations, the accent " $\breve{}$ " reminds that the result is achieved for a given sensor placement, while the accent " $\widehat{}$ " reminds that the values are read by sensors or are elaborated by such readings. For a given excitation time history, one obtains the time histories $\hat{\breve{\boldsymbol{z}}}$ from the sensors, i.e., by integrating the ordinary differential Eq. (1) in the absence of noise and the time histories $\boldsymbol{z}_{R\rho}$ by integrating the ordinary differential Eq. (10).

The most suitable placement of the sensors is pursued as the one which minimize the discrepancies between computed and estimated state variables, or in mathematical form, the one which minimize the objective function

$$\breve{\delta} = \sqrt{\frac{\left\|\sum_{\mathrm{i}=1}^{T}\left(\mathbf{z}_{\mathrm{i}}* - \hat{\breve{\mathbf{z}}}_{\mathrm{i}}*\right)^2\right\|}{N}} \quad (20)$$

where the "*" denotes that each quantity is made dimensionless by dividing its value by the maximum value in the actual time history made of T steps. One is led therefore to minimize Eq. (20) over all the sets (as the accent " $\breve{}$ " says) of potential allocations of the accelerometer sensors, which are in number one half of the state variable number required by the balanced reduced order theory, plus the ground acceleration sensor.

Equivalently, one can repeat the process by using displacement sensors which are in number one half of the state variable number required by the balanced reduced order theory. The minimization of Eq. (20), due to the strong nonlinearity of the problem, requires the adoption of a suitable numerical tool, as the genetic algorithm adopted in (Casciati 2008) or any suitable tool among those introduced in (Yang 2008) or more generally in the broad literature covering the optimization tools.

Before the theoretical developments above illustrated are applied to numerical examples, a remark applies. For any given structural problem, once the model order is established, one could decide to introduce such a number of sensors. If they were reading the response of the reduced order model, in a linear context, the state variable time histories would be exactly estimated. But the sensors read the actual response of the structural system (even forgetting intrinsic sensor noise) and this will be shown to affect the results in a way which depends on the nature of the sensors, i.e., accelerometers or relative displacement transducers.

3. A 2D numerical example

The benchmark building analyzed in this paper was first introduced in (Othori *et al*. 2004). It is made by a 20 stories steel frame (as sketched in Figs. 1 and 2), with an inter-storey height of 3.96 m (except for the first storey which is 5.49 m high) for a total height of 80.77 m above the ground level, and with a rectangular plan of sizes 30.48 m by 36.58 m, namely, 5 bays of 6.10 m each along the N-S direction and 6 bays along the E-W direction. Under the ground level one meets two additional levels, B1 at the bottom and B2, with an inter-storey height of 3.65 m. The steel yielding stress is 345 MPa. The masses to be considered are $5.32x10^5$ kg at the ground level, $5.52x10^5$ kg for all the intermediate levels and $5,84x10^5$ kg at the top level. The total mass over the ground level is 1, 11 $x10^7$ kg.

The original frame was subjected to several ground acceleration time histories. For the purposes of this paper, it is enough to consider a single accelerogram, i.e., the N-S component of the acceleration recorded during the El Centro seismic event of May 18, 1940. The peak acceleration is 3.417 m/sec^2. The dynamic analyses carried out introduce a time discretization of step 0.02 sec.

After the same horizontal displacement is given to all the nodes of the same storey, the reduced (via balanced truncation) model requires 20 states variables in Eqs. from (10) to (12) (Othori *et al*. 2004, Casciati and Faravelli 2014), which means that the number of sensors to be deployed according to the proposal of this paper is 10.

3.1 Number of sensors equal to the reduced model order

Assume first to decide to deploy 20 accelerometers, each at a different storey. If they read the acceleration response computed by the reduced order model, the readings can be organized in a vector of length 20 which at each time step assumes the values obtained by multiplying the associated sub-matrix of ***C*** and the vector of the 20 state variables at the same time step. Thus inverting the sub-matrix the state variables can be accurately estimated: the implemented Matlab code would result in a value of δ of $2.34*10^{-8}$ which can be regarded as a numerical tolerance. But this just occurs without round-off error: the computed matrix determinant is quite close to zero and replacing the reading with the actual response, i.e., the accelerations one obtains from the full size model (with 792 state variables!), the calculation becomes inaccurate and the value of δ jumps up of several order of magnitude. The situation does not change, for such an inverse problem, by placing 6 sensors for each storey (i.e., a total of 120 sensors!) and using their average as storey input, average which in (Casciati and Faravelli 2014) is shown to be consistent with the 20-states storey response.

The same reasoning applies when deploying relative displacement sensors. The numerical tolerance associated with the readings of the reduced order model response is $6.16*10^{-6}$, but again the selected rows of matrix ***C*** result in a singular squared matrix.

A feasible problem inversion can be found by mixing acceleration and velocity sensors, or displacement and velocity sensors, but the solution comes with an associated error and this error should be minimized along a search pattern. The approach proposed in previous section:

(1) assumes that the velocity in a location is estimated from either the absolute acceleration or the relative displacement in that location;

(2) accepts the additional error coming from this velocity estimation;

(3) pursues a sub-optimal solution under these additional constraints.

3.2 Positioning 10 sensors on 10 different stores

The main results obtained by the approach proposed in the previous section are summarized in Fig. 1 for both the two ways of selecting the sensors devices, reading either accelerometers or displacements.

Three remarks apply.

(1) The solutions have a sub-optimal character: indeed, the optimization tool in (Casciati 2008) is driven by some tolerances and stop options which actually suggest not to proceed further;

(2) The sensor location is fully defined by the storey level: indeed the monitored degree of freedom is the horizontal one and all the nodes in the storey have the same horizontal displacement in the reduced numerical model to which the balanced truncation applies;

(3) When presenting this results to the audience of a conference, an attending senior professional designer stand up and said: “the sensor locations you found are exactly the ones I would have proposed on the basis of my expertise”. The authors consider this a positive comment of their proposal. Nevertheless, entering more complex systems, as done in the last section of this paper, even a deep expertise would not cover the full understanding of the structural problem.

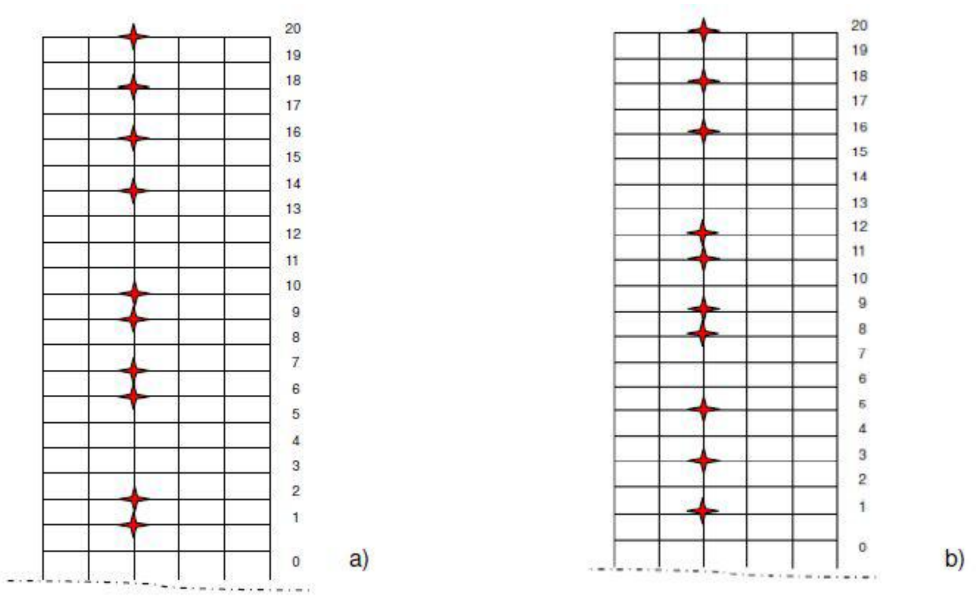

Fig. 1 Results for the case study; reduced model of order 20 (i.e., 10 sensors are required):)a) absolute acceleration sensors $\delta_{opt} = 0.1050$ with monitored stores 1, 2, 6, 7, 9, 10, 14, 16, 18, 20 and (b) relative displacement sensors $\delta_{opt} = 0.0587$ with monitored stores 1,3, 5, 8, 9, 11, 12, 16, 18, 20

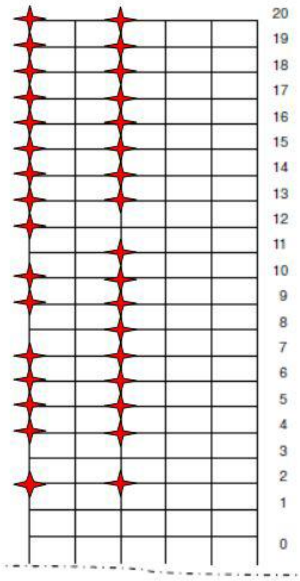

Fig. 2 Results for the case study; reduced model of order 66 (i.e., 33 sensors are required): relative displacement sensors are adopted with $\delta_{\mathrm{opt}} = 0.2271$ and monitored potential sensor locations 2, 4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40

3.3 Positioning 33 sensors

The previous section was using Eqs. from (10) to (12). In this section one moves to Eqs. from (4) to (6), where the direct balanced truncation results in a reduced order model of order 66. This means that one has to place 33 sensors in any node detecting accelerations or displacements for the two in-plane directions, as well as for the rotation. After several numerical trials it became evident that once again only sensor of the horizontal motion were significant, which would result in selecting 33 positions among 120 potential locations. Indeed the configuration of the structure involves that the response in nodes of the same storey is the same on the internal columns while different from that one records on the external columns. This suggested to reduce the number of potential locations to 40, namely the 20 nodes (from the bottom to the top) in the left external column and 20 nodes on the third column from the left (where the master nodes were located to obtain the reduced model discussed in the previous subsection).

Despite the drastic simplification of the search problem, a trivial application of the process was leading to value of δ very high, i.e., significant errors in the state variables reconstructions. The errors were in amplitude when relative displacement sensors are considered and in outset when acceleration sensors are considered. Indeed the measured signals (i.e., the response of the full model) are rich of frequencies and must be processed in order to obtain consistent results. The data processing suggests to use a low pass filter for displacement measurements and a high pass filter for acceleration measurements.

The results obtained after cutting the frequencies above 5 Hz on the full model relative displacements at the sensor locations (which are regarded as recorded displacements) are summarized in Fig. 2.

4. A 3D numerical example

The finite element package Marc-Mentat (MSC 2013) offers the option to save the structural matrices in a file. This allows one to build a finite element mode of any complexity in the Mentat pre-processor resulting in the assemblage of mass, stiffness and damping matrices, which can be then exported into Matlab to pursue the model reduction step. In that follows, one just consider a hexahedral medium (the soil) supporting a slender block simulating a pylon or an antenna: it is a very common model any designer meets in studying soil-structure interaction (Casciati and Borja 2004). The soil is discretized into five by five 3D elements in each of three layers (75 elements and 144 nodes), while the antenna comes with 4 elements along the vertical direction (4 elements and 20 nodes, with 4 of them common to the soil surface; see Fig. 3). The damping matrix is given by assigning 2% to the damping ratio for the firth and fifth mode. Homogeneous materials are assumed with different density and elastic properties. The horizontal ground acceleration only acts, along direction *x*, on the bedrock supporting the system.

A reduced model of 20 states was found as suitable by the balanced truncation method and the placement of 10 sensors along one of the four vertical vertices was investigated. The optimal solution is given in Fig. 3.

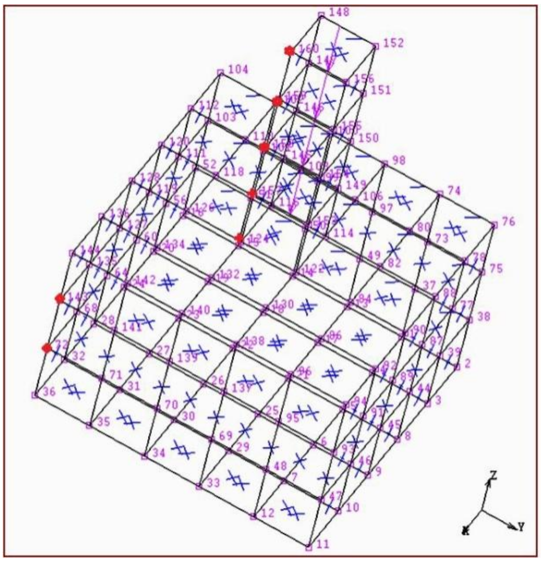

Fig. 3 Result for the 3D case study when displacement sensors are used: reduced model of order 20 (i.e., 10 sensors are required) and δ_{opt} = 0.2534, with monitored degrees of freedom 1, 3, 8, 10, 11, 12, 13, 14, 15, 16. (diamond is for measurements along x; four-pointed star is for measurements along z and six-pointed star means measurements along x and z)

5. Conclusions

This paper provides the developments by which the application of a model order reduction (MOR) scheme leads one to selecting the number and the locations of sensors in view of the structural system monitoring.

In the numerical examples, the technique is applied to a plane frame subject to base ground excitation and a 3D problem. The focus is then on the selection of the potential sensor sites. The numerical results are supporting the theoretical treatment of the problem in section 2. Nevertheless, a full exploitation of the proposal in this paper still requires that more attention is paid to the numerical optimization tool. Future developments will therefore be focused on the selection of the more convenient scheme among the several today proposed in the literature which covers heuristic optimization methods.

Acknowledgments

The authors gratefully acknowledge the financial support provided by the corresponding Athenaeum Research Grants.

References

Borzì, A. and Schulz, V. (2011), *Computational optimization of systems governed by partial differential equations*, SIAM, Philadelphia, (ISBN 978-1-611972-04-7)

Calberg, K., Bou, Mosleh and Farhat, C. (2011), “Efficient non-linear model reduction via a least squares Petrov-Galerkin projection and compressive tensor approximations”, *Int. J. Numer. Meth. Eng.*, **86**(2),155-181.

Casciati, F., Casciati, S., Faravelli L. and Franchinotti, M. (2012), “Model order reduction vs. structural control”, *Proceedings of the 5th EACS*, Genoa, July.

Casciati, S. (2008), “Stiffness identification and damage localization via differential evolution algorithms ”, *Struct. Control Health Monit.*, **15**(3), 463-449.

Casciati, S, and Borja, R.L. (2004), “Dynamic FE analysis of south memnon colossus including 3D soil-foundation-structure interaction” , *Comput. Struct.*, **82**(20-21), 1719-1736.

Casciati, S. and Faravelli, L. (2014), “Quantity vs. quality in the Model Order Reduction (MOR) of a linear system”, *Smart Struct. Syst.*,**13** (1), 99-109.

MSC (2013), Marc 2013 User's Guide , http://www.mscsoftware.com.

Ohtori, Y., Christenson, R.E. and Spencer, B.F. (2004), “Benchmark control problems for seismically excited nonlinear buildings”, *J. Eng. Mech. - ASCE*, **130**, 366-385.

Papadimitriou C. (2004), “Optimal sensor placement methodology for parametric identification of structural systems”, *J. Sound Vib.*, **278**(4), 923-947

Schilders, W.H.A., van der Vorst, H.A. and Rommes, J. (2008), *Model Order Reduction : Theory, Research Aspects and Applications*, Springer, Berlin.

Yang , X.S. (2008), *Nature-inspired metaheuristic algorithms*, Luniver Press.

CY

This paper is the text of the ASEM13 keynote lecture given by Prof. Casciati at the International Conference on Smart Structure and Systems (ICOSS13) hosted in Jeju (Korea) in September 2013 under the big umbrella of 2013 World Congress on Advances in Structural Engineering and Mechanics (ASEM13).