Smart Structures and Systems, *Vol. 13, No. 1 (2014) 081-098*
DOI: http://dx.doi.org/10.12989/sss.2013.13.1.081

Predicting the buckling load of smart multilayer columns using soft computing tools

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(Received June 17, 2012, Revised March 9, 2013, Accepted March 25, 2013)

Abstract. This paper presents the elastic buckling of smart lightweight column structures integrated with a pair of surface piezoelectric layers using artificial intelligence. The finite element modeling of Smart lightweight columns is found using ANSYS® software. Then, the first buckling load of the structure is calculated using eigenvalue buckling analysis. To determine the accuracy of the present finite element analysis, a compression study is carried out with literature. Later, parametric studies for length variations, width, and thickness of the elastic core and of the piezoelectric outer layers are performed and the associated buckling load data sets for artificial intelligence are gathered. Finally, the application of soft computing-based methods including artificial neural network (ANN), fuzzy inference system (FIS), and adaptive neuro fuzzy inference system (ANFIS) were carried out. A comparative study is then made between the mentioned soft computing methods and the performance of the models is evaluated using statistic measurements. The comparison of the results reveal that, the ANFIS model with Gaussian membership function provides high accuracy on the prediction of the buckling load in smart lightweight columns, providing better predictions compared to other methods. However, the results obtained from the ANN model using the feed-forward algorithm are also accurate and reliable.

Keywords: smart columns; buckling load; artificial neural network; fuzzy inference system; adaptive neuro fuzzy inference system; ANSYS

1. Introduction

There is an increasing interest to reduce the thickness of the structural elements in aerospace engineering, civil engineering, mechanical and even bio-engineering. Buckling phenomena will be an important design constraint for these structures. The effective flexural stiffness of such elements under compression, in-plane edge loads or shear loads suffer from buckling failure. Nevertheless, contrary to the impression, the buckling phenomenon is not always undesirable. For example, the shape morphing using the bi-stable method applies shape changing by the use of active forces to transform buckling into a target structural element. The buckled element can produce useful deflections and thereby the desired shape will be obtained. Despite the many studies carried out on

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

the buckling and post-buckling of elastic structures, limited researches have been performed within the field of buckling analysis of smart structures. The smart material is used in adaptive beams, plates and shells by bonding patches to host structures to enhance the performance of the structural components, such as its load carrying capacity, buckling behaviors and to compensate for the stiffness reduction due to using reduced thicknesses.

A helical spring form of shape memory alloys externally attached to host core is utilized to enhance the elastic stability of beams by an axial compressive load at the rate of 0.0917 N/s. The results showed that controlled buckling load increases to three times the uncontrolled amount by activating the shape memory alloy (Baz and Tampe 1989, Baz *et al*. 1991). The potential of piezoelectric material to increase the load bearing strength of imperfection sensitive composite columns loaded in compression are examined by (Thompson and Loughlan 1995). They applied a controlled voltage to the actuators to induce a reactive moment at the column centre thereby the lateral deflections will be removed and they will enforce the column to behave in a perfectly straight manner. Their experimental analysis shows that the buckling load of graphite-epoxy strips can be increased from 19.8% to 37.1% by using PZT(Lead-Zirconate-Titanate) actuators. (Meressi and Paden 1993) showed that the buckling of a flexible beam could be postponed beyond the first critical load by the means of feedback using piezoelectric actuators. Also, the spillover problem is considered. (Berlin 1994) examined the use of induced-strain actuation to control the buckling of a thin steel column and obtained an increase of 5.6 times in the load bearing capacity of the column.

In another study, the effectiveness of networked arrays of MEMS-based sensors and filamentary PZT actuators to control the buckling instability of a column is shown (Berlin et al. 1998). The initial buckling of smart beams and plates was studied using an electromechanically coupled formulation in combination with an 8-node FE (Varelis and Saravanos 2002). The critical buckling load was examined by altering the electrical conditions. (Oh *et al*. 2000, 2001) presented a layer wise formulation for the static buckling analysis of smart plates subjected to thermal and electrical effects. They have used the Newton–Raphson iterative method to solve the nonlinear governing equations.

The buckling temperature is determined through the reduction of the problem to an eigenvalue one. (Kapuria and Achary 2006) developed a coupled zigzag theory for static buckling analysis of the hybrid piezoelectric plates. A global third order variation across the thickness with a layer wise linear variation is assumed for the in-plane displacement components. Static post-buckling analysis for imperfect plates with fully covered or embedded piezoelectric actuators subjected to thermal and thermo-electro-mechanical loads based on Reddy's higher-order shear deformation plate theory are studied in separate studied by (Shen 2001). A mixed Galerkin-perturbation technique was used to determine the thermal buckling temperature and the post-buckling equilibrium paths. A similar study on the thermo-dynamic buckling analysis of smart plates is conducted by (Shariyat 2009). (Kundu *et al*. 2007) investigated the nonlinear post buckling of piezoelectric laminated doubly curved shells based on the first order shear deformation theory and using the finite element method. A total Lagrangian approach associated to the arc-length method was used to solve the equilibrium equations. (Chase and Bhashyam 2001) derived optimal design equations to actively stabilize laminated plates loaded in excess of the critical buckling load using a large number of sensors and actuators. The mechanical buckling of a homogeneous Engesser-Timoshenko beam with piezoelectric actuators subjected to axial compressive loads was studied by (Nezamabadi and Korramabadi 2010). The results showed that the critical buckling loads of a homogeneous Engesser-Timoshenko beam under an axial compressive load generally increase with the relative increase of thickness. Also, (Maurini *et al*. 2007) have investigated the effect of distributed multi-parameter actuation on a simple supported straight beam with end-shortening. They derived finite dimensional models for the buckling and post-buckling analysis. The effect of the bending actuation on the bistable buckled beam was analyzed by a reduced order 2 d.o.f. the results showed that the axial actuation plays the same role as a buckling parameter. In another work, (Gurel 2007) studied the buckling of slender prismatic circular columns with multiple non-propagating edge cracks by use of the transfer matrix method. The columns were modeled as an assembly of sub-segments connected by massless rotational springs whose flexibilities depend on the local flexibilities introduced by the cracks. The obtained results showed that the buckling loads are affected considerably by the depths, locations and number of cracks.

Artificial neural networks (ANNs) are known as soft computing tools with the capability of maintaining the experience and learning. They do not assume any fixed relationship between the input and output data and therefore, they have been recently used for engineering aims. Also, fuzzy inference system (FIS) has recently been employed in different engineering subjects. FIS can be used to predict uncertain systems and its application does not require having any knowledge on the underlying physical process as a precondition. However, it has some deficiencies. In order to improve the results obtained through this method, the neuro-fuzzy methods such as Adaptive Neuro Fuzzy Inference System (ANFIS), which is a combination of ANN and FIS, were defined.

As far as the authors are aware of, there is no study on the buckling of smart columns using ANN and FIS methods. However, (Bilgehan 2011) has investigated the application of ANFIS and neural network to predict the critical buckling load of axially loaded compression passive rods.

The results showed that the architectures of the ANFIS and NN established in the study perform sufficiently in the estimation of critical buckling loads. Also, (Cevik *et al*. 2009) investigated the application of an optimal ANN model for the strength prediction of heat-treated extruded aluminum alloy columns failing by flexural buckling. Their investigations showed that the applied ANN model is more accurate than the previous analytic expressions and codes. In another study, (Sheidaii and Bahraminejad 2012) have developed an ANN model by using the backpropagation training algorithm to model the nonlinear relationship between load and displacement. By predicting this relationship, they were able to perform an extensive parametric study on the buckling behavior of a steel compression member. The obtained results showed that by using the developed network, it is possible to conduct an accurate study on the effects of various parameters at the critical load. Also, the idea of data compression by means of the backpropagation neural network replicators were applied to the analysis of the buckling load of axially compressed cylindrical shells with initial geometrical imperfections by (Waszczyszyn and Bartczak, 2002). Finally, buckling analysis of slender prismatic columns with a single non-propagating open edge crack subjected to axial loads has been presented by (Bilgehan *et al*. 2012). They were used the transfer matrix method and multi-layer feedforward artificial neural networks with backpropagation learning algorithm. The final results showed that the proposed methodology may constitute an efficient tool for the estimation of elastic buckling loads of edge-cracked columns.

This paper deals with the prediction of the buckling load in a thin smart column structure integrated by a pair of surface piezoelectric layers using ANN, FIS, and ANFIS soft computing tools. A numerical finite element modeling is constructed using ANSYS software. Buckling loads of simple supported smart columns are obtained by the eigenvalue buckling analysis of ANSYS in which the theoretical buckling strength of an ideal elastic structure is predicted by computing the structural eigenvalues for the given system loading and constraints. The buckling loads are compared with literature. Finally, some models were developed based on the mentioned soft computing tools to predict the buckling load of a thin smart column and then the efficiency of the models are compared.

2. Finite element modeling and buckling analysis

For any piezoelectric material the charge developed due to strain in the material is known as the *Direct Effect* and the deflection caused by the applied electric field is known as the *Converse Effect*. Here, a three layer composite column consisted of one long elastic core and two surface piezoceramics patches is scrutinized which acts as the extensive actuation mechanism. An adequate mathematics or numerical model to evaluate the smart columns based on their converse effect is necessary. In this paper, the finite element modeling of a smart lightweight column is carried out with ANSYS®. The piezoelectric material is chosen to be PZT5H. The geometrical and material properties used for the finite element analysis (FEA) are similar to (Alghamdi 2001) in which the column is made up of an aluminum core with E_a=70 GPa, L=2 m, w =50 mm, t =10 mm. Also, the PZT type 5H has the thickness of 1 mm, the strain coupling (d_{31}) = 274e-12 m/V, and an elasticity module equal to 64 GPa.

There are some assumptions in our FE simulation as listed below:

- The SOLID 45 element is used to model the elastic core of the column.
- The SOLID5 element is used to model the piezoelectric actuators.
- The elastic materials (the core layer) are isotropic but the piezoelectric materials (the outer layers) are orthotropic or transversely isotropic.
- The polarization and electric field intensity vectors are parallel and both are normal to the neutral axis of the smart piezoelectric stack in order to establish the axial mode of the actuator dynamics.
- The metal layers for the electrodes are not taken into account in the simulation because they have negligible thicknesses (about 100 nm), compared with the thick piezoelectric layers.

The simple supported condition is chosen and the first three buckling loads of the structure are calculated via the buckling analysis option of ANSYS® software. The first buckling loads are 1268 N, 5045 N, and 11614 N where corresponding buckling modes are shown in Figs. 1(a)-1(c), respectively. The first buckling load is rather similar to that of (Alghamdi 2001), i.e., P_{cr}=1200 N.

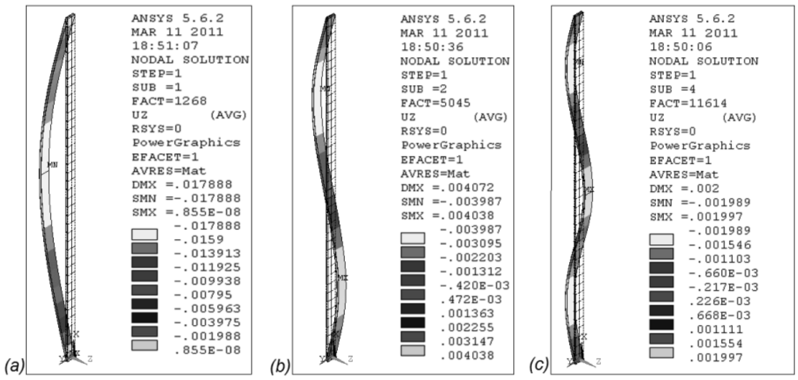

Fig. 1 The first three buckling loads and associated modes of smart simple supported columns

In the follow up, the parametric analyses are performed with variations in the geometrical parameters of the smart column and the first buckling load is determined. The length and width of the column, the thickness range of the elastic core and the piezoelectric layer were changed from 1.5 m to 2.5 m with 0.1 m intervals, 0.01 m to 0.09 m with 0.01 m intervals, 0.01 m to 0.019 m with 0.001m intervals and 0.001 m to 0.009 m with 0.001 m intervals. A brief of the results is presented in Tables 1- 3.

Table 1 The pattern of variation in piezoelectric thickness and its corresponding buckling load of the smart column

Length of Column (m)	Elastic Core Thickness (m)	Elastic Core Width (m)	Piezoelectric Thickness (m)	Buckling Load (N)
2.500				811.29
2.300				958.52
2.100				1149.80
2.000	0.01	0.05	0.001	1267.70
1.900				1404.60
1.700				1754.50
1.500				2253.60
2.500				1288.80
2.300				1522.70
2.100				1826.50
2.000	0.01	0.05	0.002	2013.70
1.900				2231.40
1.700				2787.10
1.500				3579.80
2.500				1920.50
2.300				2269.80
2.100				2721.80
2.000	0.01	0.05	0.003	3000.70
1.900				3324.80
1.700				4139.70
1.500				5333.30
2.500				2729.10
2.300				3224.50
2.100				3884.10
2.000	0.01	0.05	0.004	4280.40
1.900				4723.60
1.700				5901.70
1.500				7579.20
2.500				3737.40
2.300				4416.00
2.100				5267.30
2.000	0.01	0.05	0.005	5969.80
1.900				6473.20
1.700				8081.10
1.500				10380.00

Table 2 The pattern of variation in core thickness and its corresponding buckling load of the smart column

Length of Column (m)	Elastic Core Thickness (m)	Elastic Core Width (m)	Piezoelectric Thickness (m)	Buckling Load (N)
2.500				1031.10
2.400				1118.80
2.300				1218.20
2.100				1461.30
2.000	0.011	0.05	0.001	1611.00
1.900				1785.00
1.700				2229.90
1.600				2517.10
1.500				2863.90
2.500				1287.30
2.400				1396.70
2.300				1520.90
2.100				1824.30
2.000	0.012	0.05	0.001	2011.30
1.900				2228.50
1.700				2783.70
1.600				3142.40
1.500				3575.40
2.500				1586.20
2.400				1717.20
2.300				1869.80
2.100				2249.20
2.000	0.013	0.05	0.001	2473.10
1.900				2743.30
1.700				3422.40
1.600				3863.40
1.500				4395.60
2.500				1920.00
2.400				2083.30
2.300				2268.40
2.100				2720.90
2.000	0.014	0.05	0.001	2999.80
1.900				3323.80
1.700				4151.70
1.600				4686.80
1.500				5332.40
2.500				2302.00
2.400				2497.80
2.300				2719.70
2.100				3262.40
2.000	0.015	0.05	0.001	3596.70
1.900				3985.20
1.700				4977.80
1.600				5619.30
1.500				6393.60

Table 3 The pattern of variation in column width its corresponding buckling load of the smart column

Length of Column (m)	Elastic Core Thickness (m)	Elastic Core Width (m)	Piezoelectric Thickness (m)	Buckling Load (N)
2.500				162.117
2.400				175.907
2.300				191.535
2.100				229.788
2.000	0.01	0.01	0.001	252.656
1.900				280.658
1.700				350.285
1.600				395.351
1.500				450.226
2.500				324.418
2.400				352.013
2.300				383.287
2.100				459.762
2.000	0.01	0.02	0.001	506.883
1.900				560.907
1.700				701.543
1.600				791.955
1.500				901.046
2.500				486.689
2.400				528.092
2.300				575.005
2.100				689.736
2.000	0.01	0.03	0.001	760.427
1.900				842.570
1.700				1052.500
1.600				1188.100
1.500				1351.800
2.500				648.981
2.400				704.183
2.300				766.744
2.100				919.736
2.000	0.01	0.04	0.001	1014.000
1.900				1124.000
1.700				1403.500
1.600				1584.300
1.500				1802.600
2.500				811.294
2.400				880.313
2.300				958.520
2.100				1149.800
2.000	0.01	0.050	0.001	1267.700
1.900				1404.600
1.700				1754.500
1.600				1980.600
1.500				2253.500

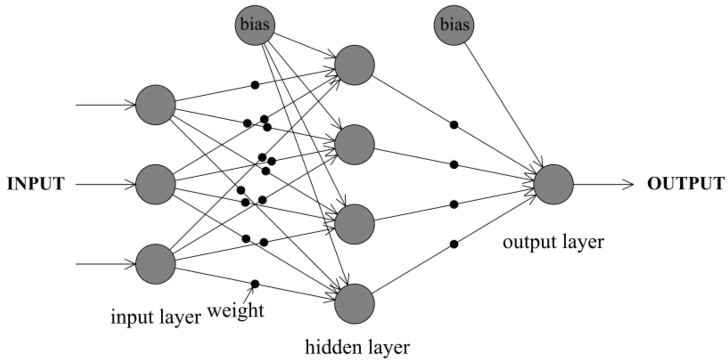

Fig. 2 Schematic representation of the three-layer feed forward artificial neural network

3. Artificial neural networks (ANNs)

An artificial neural network is an information-processing system that has certain performance characteristics in common with biological neural networks. It is one of the artificial intelligence techniques where the intelligence results from the interaction between different neurons (Jain and Deo 2006). It is also a useful tool for solving different engineering problems because it can approximate a desired behavior without the need to specify a particular function. This is a big advantage of artificial neural networks compared to that of multivariate statistics (Wieland and Mirschel 2008). A neural network is characterized by (1) its pattern of connections between the neurons (called its architecture), (2) its method of determining the weights on connections (learning algorithm), and (3) its activation function (Fausett 1994).

Among the applied neural networks, the feed forward neural networks (FFNN) are the most commonly used method in solving various engineering problems. The FFNN technique consists of a layer being fully connected to the preceding layer by weights (Rajaee *et al*. 2009). Fig. 2 illustrates the common three-layer feed forward type of an artificial neural network.

Learning of these ANNs is performed by first or second order learning algorithms. Back propagation, adaptive learning rate and the steepest descent are first-order methods as they use the first derivative of error (slope) and follow the gradient descent approach. Quick Prop, the Gauss-Newton method, and the Levenberg-Marquardt method are second-order methods and they rely on both the first and second derivatives of error (slope and curvature) in the search for the optimum weights (Samarasinghe 2007). In the present study, the Levenberg-Marquardt (LM) algorithm was chosen because of its high-performance and fast convergence. It minimizes a predetermined error function (E) of the following form

$$E = \sum_P \sum_p (y_i - t_i)^2 \tag{1}$$

Where y_i is the ith component of the ANN output vector $\boldsymbol{Y}$, t_i is the ith component of the target output vector $\boldsymbol{T}$, p is the number of output neurons and P is the number of training patterns.

The LM algorithm uses the following formula to calculate weight (W) in subsequent iterations

$$W_{new} = W_{old} - \left[J^T J + \gamma I\right]^{-1} J^T E(W_{old}) \tag{2}$$

Where J is the Jacobian of the error function E, I is the identity matrix, and γ is the parameter used to define the iteration step value. In this method, γ is chosen automatically until a downhill step is produced for each epoch. Starting with an initial value of γ, the algorithm attempts to decrease its value by increments of $\Delta\gamma$ in each epoch. If the E is not reduced, γ is increased repeatedly until a downhill step is produced (Samarasinghe 2007). Several forms of activation functions have been used in ANNs, such as linear, binary sigmoid, bipolar sigmoid, hyperbolic tangent, etc. The hyperbolic tangent function, which was used in this paper, is given as

$$f(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} \tag{3}$$

More details on the ANN can be found in (Fausett 1994) and (Samarasinghe 2007).

4. Fuzzy inference systems (FISs)

The fuzzy inference system (FIS) is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. The basic structure of a fuzzy inference system consists of three conceptual components: (1) a rule base, which contains a selection of fuzzy rules. The general form of a fuzzy if-then rule is as follows: *if X is A then Y is B*. The first part is often called the antecedent or premise, while the other part is called the consequence or conclusion; (2) a database, which defines the membership functions used in the fuzzy rules; and (3) a reasoning mechanism, which performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion. There are three types of fuzzy inference systems in the literature that have been widely employed in various applications: Mamdani, Sugeno (TSK), and Tsukamoto fuzzy inference systems. The difference between these three fuzzy inference systems lies in the consequents of their fuzzy rules. Although the fuzzy inference system has a structured knowledge representation in the form of fuzzy if-then rules, it lacks the adaptability to deal with changing external environments. Thus, neural network learning concepts in fuzzy inference systems have been incorporated by various authors, resulting in neuro-fuzzy modeling (Jang *et al*. 1997).

An adaptive neuro-fuzzy inference system (ANFIS) is a first order Sugeno type FIS in which the premise and consequence parameters of fuzzy if-then rules are optimized by a five-layer artificial neural network. For a first-order Sugeno fuzzy model, a common rule set with two fuzzy if-then rules and three inputs is as follows:

Rule 1: If x_1 *is* A_1 *and* x_2 *is* B_1 *and* x_3 *is* C_1*, then* $f_1 = p_1x_1+q_1x_2+r_1x_3+s_1$,

Rule 2: If x_1 *is* A_2 *and* x_2 *is* B_2 *and* x_3 *is* C_2*, then* $f_2 = p_2x_1+q_2x_2+r_2x_3+s_2$.

Fig. 3(a) illustrates the reasoning mechanism for this Sugeno model. The corresponding equivalent ANFIS architecture is as shown in Fig. 3(b). Every node in the first layer is an adaptive node. The output of the layer is the involvement degree of linguistic variables A_i, B_i, and C_i. In the second layer, every node is a fixed node. This layer calculates the firing strength for each rule, whose output is the algebraic product of all the input signals. In the third layer the *i*th node calculates the ratio of the *i*th rule's firing strength to the sum of all rule's firing strengths. Every node in this layer is a fixed node. Outputs of the layer are called normalized firing strengths. In the fourth layer the output of an adaptive node is obtained from multiplying the normalized firing strength by $f_i = p_ix_1+q_ix_2+r_ix_3+s_i$. The fifth layer, which has a fixed node, computes the overall output as follows

$$f = \sum_i \varpi_i f_i = \frac{\sum_i \omega_i f_i}{\sum_i \omega_i} \tag{4}$$

A hybrid learning algorithm is used for the learning of neural network. The hybrid learning algorithm consists of two passes. In the anterior pass, node outputs go forward until layer 4 and the consequent parameters are identified by the least squares method. In the posterior pass, when the consequent parameters are fixed, the error signals propagate backward and the premise parameters (membership functions' parameters) are updated by gradient descent. More detailed information on ANFIS can be found in (Jang *et al*. 1997).

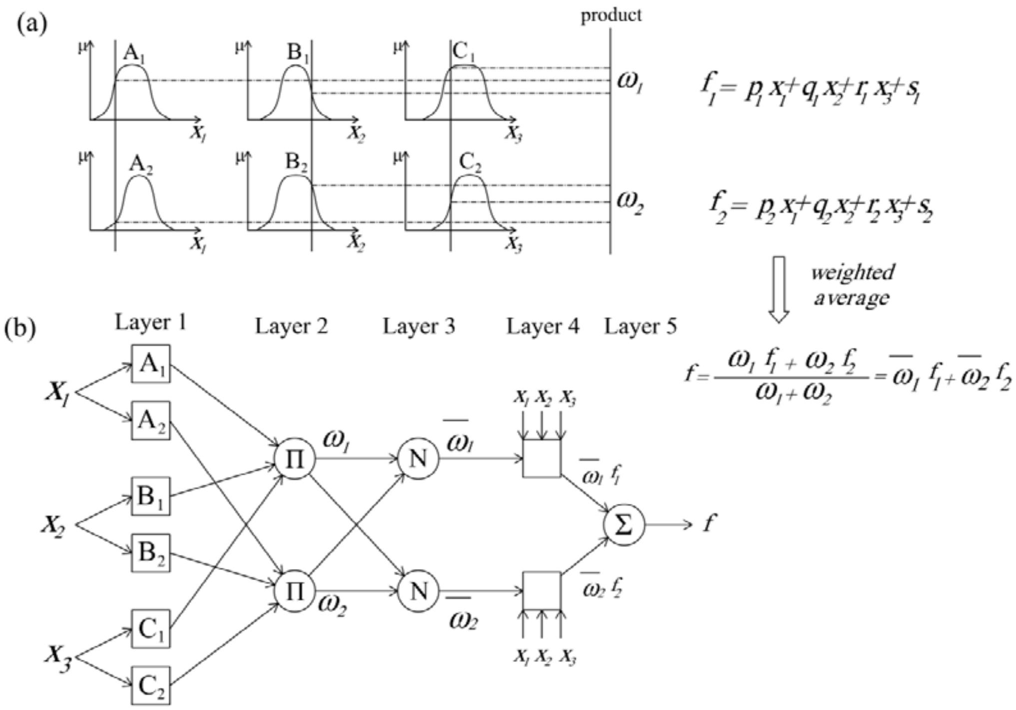

Fig. 3 A first-order Sugeno fuzzy model with two rules (three inputs) (a);equivalent ANFIS architecture (b)

There are many various membership functions such as triangular, trapezoidal, bell, and the Gaussian function can be applied in fuzzy modeling. In this study, since the majority of natural phenomena follow the Gaussian probabilistic distribution, the Gaussian membership function is used as follows

$$\mu(x) = \exp\left[-\left(\frac{x-c}{a}\right)^2\right] \tag{5}$$

Where $\mu(x)$ is the membership function. a and c are the membership functions' parameters that change the shape of the membership function. These parameters are referred to as the premise parameters.

In this paper, in order to develop a FIS model with a minimum number of fuzzy rules, a subtractive clustering method is used. In the subtractive clustering method (Chiu 1994), each data point is considered as a potential cluster center and then a measure of the potential for each data point is defined. A data point with many neighboring data points will have a higher potential value. The data point with the highest potential value is selected as the first cluster center. Then, the potential of the data points whose distance from a selected cluster center is less than a pre-specified value (cluster radius) are subtracted and the potential values are then updated. This procedure continues until holding some conditions.

5. Data set, results and discussion

The data set used in this paper is the numerical data obtained from the modeling of a thin column structure using ANSYS® software. For modeling, the data set is divided into three parts: training, checking and testing sets. The training and testing data sets are respectively used for learning and evaluating the developed models. The checking data set is part of the training data set used to reduce over-training. In order to predict the buckling load, 220 data points out of a total 297 data points were used as training sets, 17 data points were used as checking sets and the remaining as testing sets. In Table 4, the statistical characteristics of the training and testing data set used in predicting the buckling load of the smart column are presented.

Table 4 The statistical characteristics of data points used in predicting the buckling load

			Training data (numbers = 220)		
	Length of Column (m)	Elastic Core Thickness (m)	Elastic Core Width (m)	Piezoelectric Thickness (m)	Buckling Load (N)
Min	1.5	0.01	0.01	0.001	162.12
Max	2.5	0.019	0.09	0.009	28323
Average	1.999	0.012	0.05	0.002	4479.7
Standard Deviation	0.3176	0.0028	0.0146	0.0024	4729.29
			Testing data (numbers = 60)		
	Length of Column (m)	Elastic Core Thickness (m)	Elastic Core Width (m)	Piezoelectric Thickness (m)	Buckling Load (N)
Min	1.5	0.01	0.01	0.001	209.34
Max	2.5	0.019	0.09	0.009	17942
Average	2.02	0.011	0.049	0.002	3792.48
Standard Deviation	0.3272	0.0025	0.0169	0.0023	4033.54

First, an artificial neural network was developed using training data to predict the buckling load of the smart column. Before learning the ANN, the training input and output values are normalized within the range of -1 to 1, using the following equation

$$x' = 2\frac{x - x_{\min}}{x_{\max} - x_{\min}} - 1 \tag{6}$$

Where $x_{\min}$ and $x_{\max}$ denote the minimum and maximum of the data set.

After examining different topologies with the tangent hyperbolic activation function, the best topology for all models was found to be $4 \times 9 \times 1$ (neurons in the input × hidden × output layers). The four input values are the length and width of the adaptive column, thickness of the elastic core and the piezoelectric layers. The output layer refers to the buckling load of the adaptive column. After learning, the developed ANN is evaluated using the testing data. The comparison between the observed and predicted buckling load using the testing data is shown in Fig. 4. As can be seen, the results obtained from the ANN model are very accurate and reliable.

The other prediction model developed is the ANFIS model. Firstly, by the use of the subtractive clustering method and the training data including the length and width of the adaptive column, thickness of the elastic core, and thickness of the piezoelectric layers as input parameters, a FIS model was developed. The developed FIS model was then used as an initial FIS for the ANFIS model.

After developing the FIS and ANFIS models, testing data were used to evaluate the accuracy of the developed models. Figs. 5 and 6 respectively show the comparison between the observed and predicted buckling load of the smart column using the generated FIS and ANFIS models. These mentioned results are for the testing data. As can be seen, again, like the ANN model, the results obtained from the developed FIS and ANFIS models are very accurate and reliable.

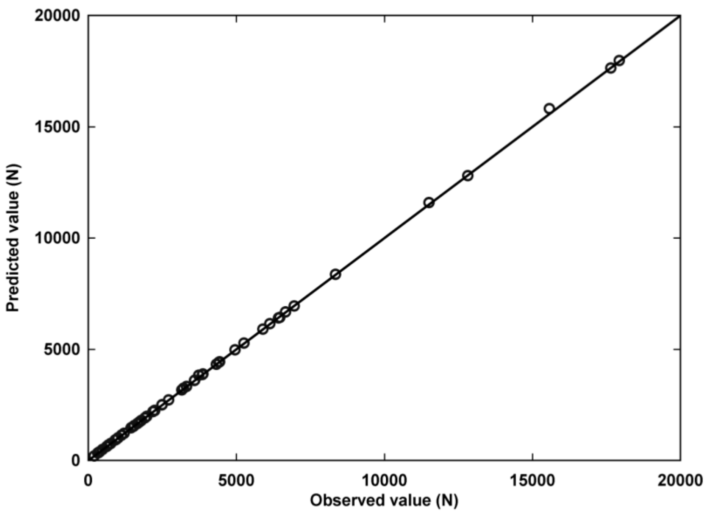

Fig. 4 A comparison between the observed and predicted values obtained from a ANN model used for the prediction of buckling loads

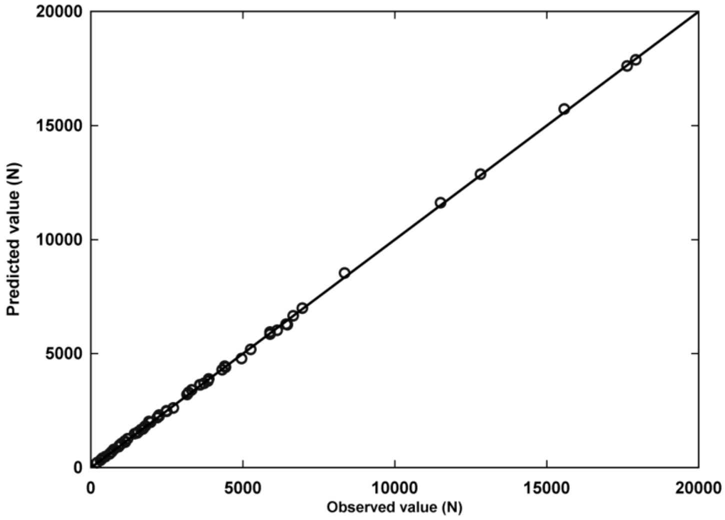

Fig. 5 A comparison between the observed and predicted values obtained from a FIS model used for the prediction of buckling loads

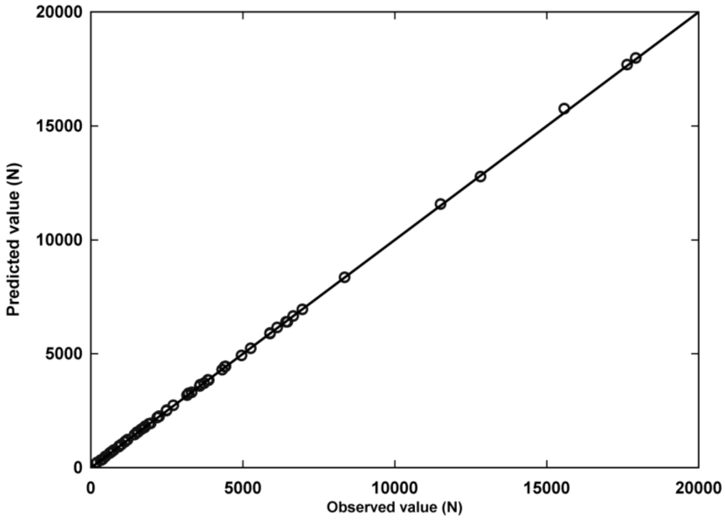

Fig. 6 A comparison between the observed and predicted values obtained from a ANFIS model used for the prediction of buckling loads

Fig. 7 shows the initial and Fig. 8 shows the final membership functions of the input variables. It is seen that there is a considerable change in the shape of the membership functions of the elastic core thickness and piezoelectric thickness after training. As a result, these two parameters are very important and sensible in predicting the buckling load of a smart column. Two parameters, in the shape of membership functions, also showed some change, but they are much less notable than those of the parameters for the thickness of elastic core and of the piezoelectric.

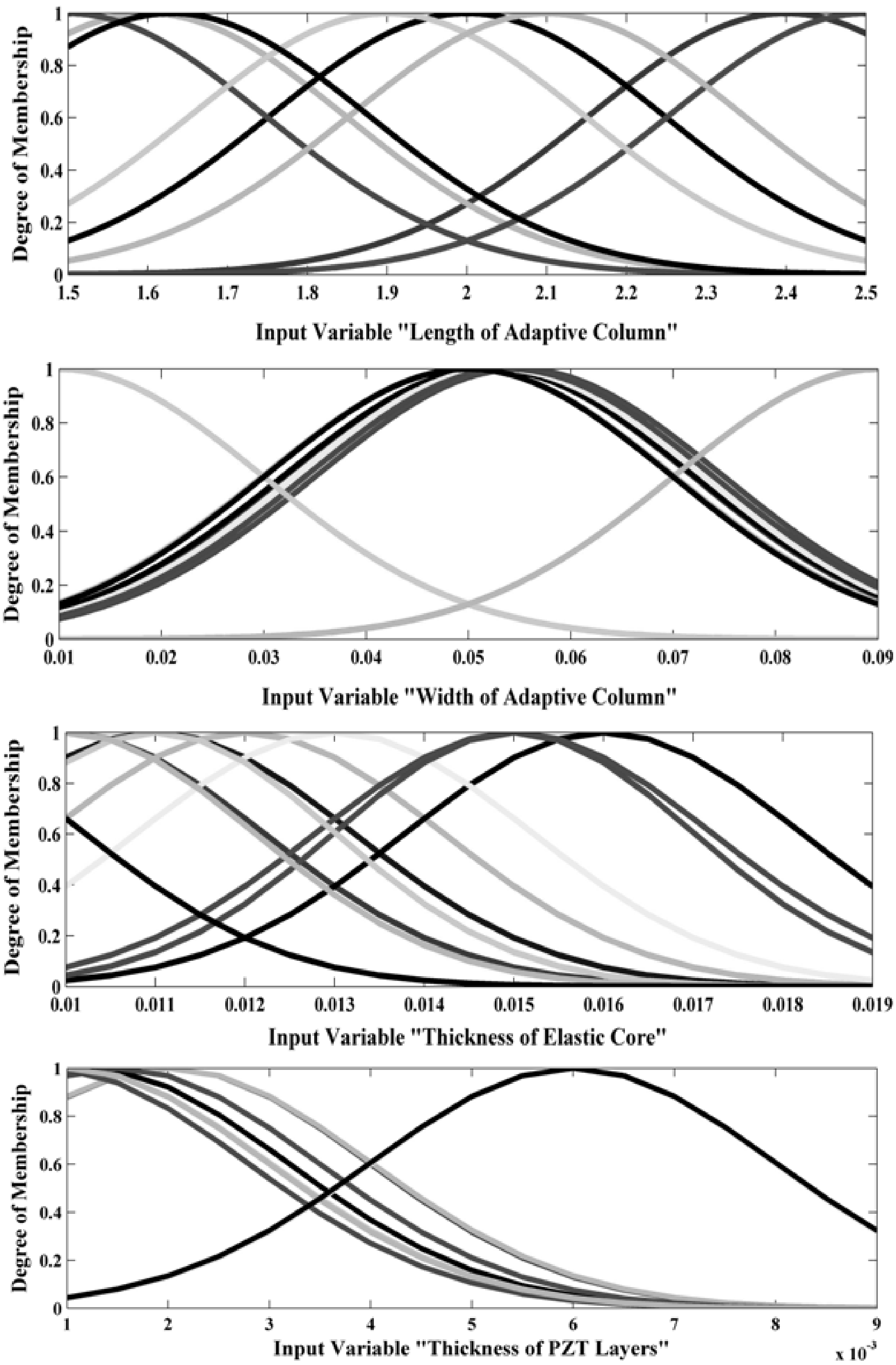

Fig. 7 Initial membership functions of input variables on the prediction of the buckling load imposed on the adapted column

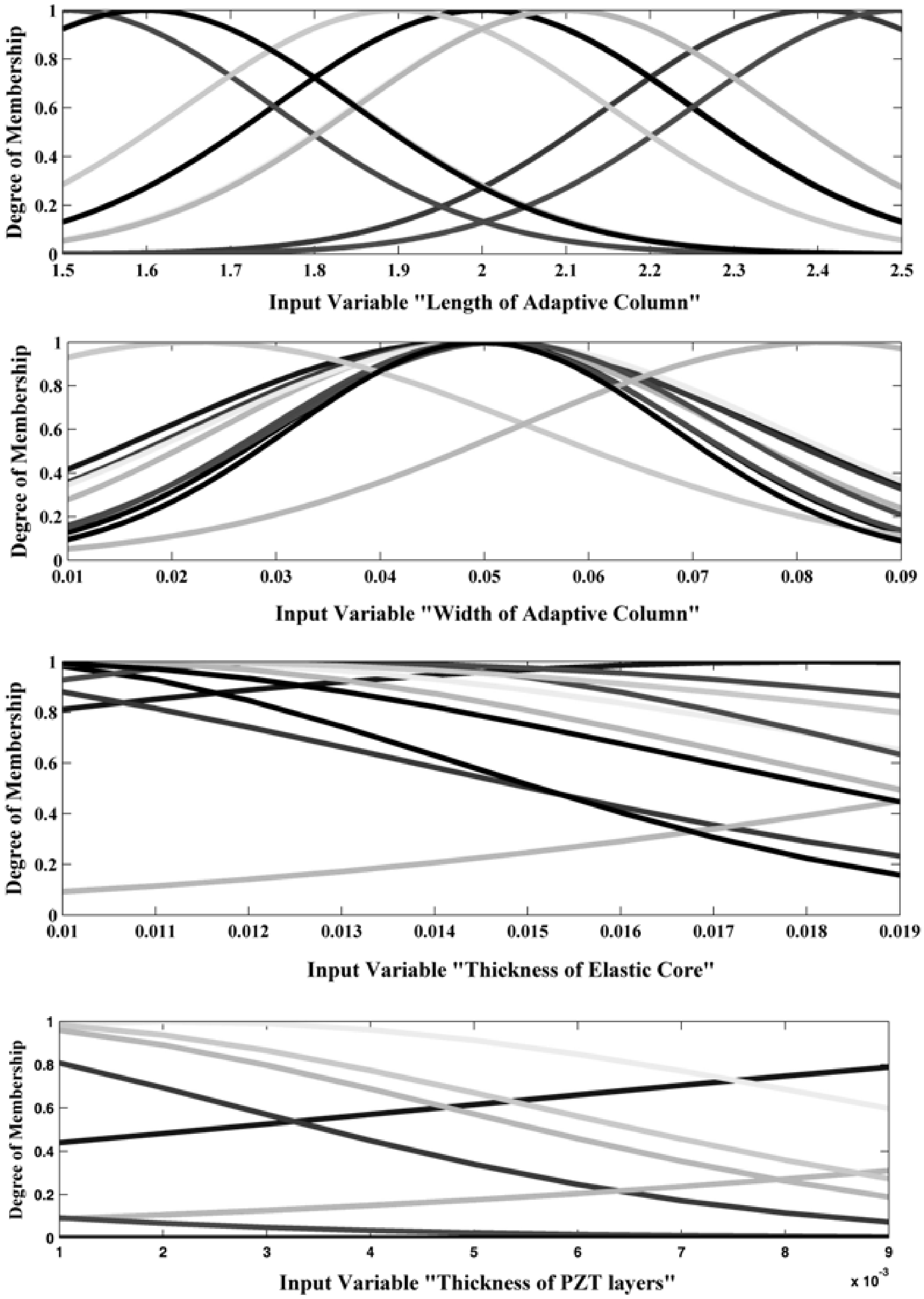

Fig. 8 Final membership functions of input variables on the prediction of the buckling load imposed on the adapted column

To complete the study, a statistical comparison between the observed and predicted parameters of the prediction was carried out to evaluate the developed soft computing models. The use of bias, Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Scatter Index (SI) and Correlation Coefficient (CC) are defined as follows

$$bias = \frac{1}{N}\sum_{i=1}^{N}(y_i - t_i) \tag{7}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|y_i - t_i| \tag{8}$$

$$RSME = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - t_i)^2} \tag{9}$$

$$SI = \frac{RSME}{Average\ Observed\ Value}\times 100 \tag{10}$$

$$CC = \frac{\sum_{i=1}^{N}(t_i - \bar{t}_m)(y_i - \bar{y}_m)}{\sqrt{\left(\sum_{i=1}^{N}(t_i - \bar{t}_m)^2\right)\left(\sum_{i=1}^{N}(y_i - \bar{y}_m)^2\right)}}\times 100 \tag{11}$$

Where N is the number of observations; t_i is an observed value obtained from numerical modeling; y_i is a predicted value; $\bar{t}_m$ is the observed mean value; and $\bar{y}_m$ is the predicted mean value.

Table 5 shows the error statistics of the proposed ANN, FIS and ANFIS models. These mentioned errors belong to the testing data. As shown, the error statistics of the FIS model is larger than those of the ANN model except for bias. After training the FIS model, the errors of the obtained ANFIS model is lower than the FIS model. A comparison between the errors of ANN and ANFIS models show that except for the MAE value, errors of the proposed ANFIS model are lower than those of the proposed ANN model. In other words, the proposed ANFIS model is more accurate than the proposed ANN and FIS models. The bias of ANFIS model is equal to 1.14 meaning that it overestimates the buckling load. However, all of the three developed soft computing models in this study are accurate and reliable. As a result, employing soft computing tools is very useful and effective in predicting the buckling load of a smart column.

Table 5 Statistics of the predicted buckling load of the smart column by the use of testing data

Methods	Average observed value (N)	Average predicted value (N)	Bias (N)	MAE (N)	RMSE (N)	SI (%)	CC
ANN	3792.48	3798.29	5.81	12.82	34.95	0.92	0.999967
FIS	3792.48	3790.75	-1.73	44.27	65.51	1.73	0.999866
ANFIS	3792.48	3793.62	1.14	19.75	31.96	0.84	0.999971

6. Conclusions

In this study, some models were developed to predict the elastic buckling load of smart thin column structures using soft computing tools such as ANNs, FIS and ANFIS. With the purpose of developing these models, the buckling analysis was established using ANSYS and the numerical results were gathered. For each method, a separate model was established with the output of the first buckling load of the defined column. The inputs were length, width, thickness of the elastic core, thickness of the outer piezoelectric layer, material properties and piezoelectric matrices of thin smart column components.

A comparison between the proposed ANN, FIS and ANFIS models indicate that the error of the ANFIS model in predicting the buckling load is less than those of the other methods.

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