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Recurrence plot entropy for machine defect severity assessment

Ruqiang Yan*[1], Yuning Qian[1], Zhoudi Huang[1] and Robert X. Gao[2]

[1]*School of Instrument Science and Engineering, Southeast University, Nanjing, China*
[2]*Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269, USA*

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Abstract. This paper presents a nonlinear time series analysis technique for evaluating machine defect severity, based on the Recurrence Plot (RP) entropy. The RP entropy is calculated from the probability distribution of the diagonal line length in the recurrence plot, which graphically depicts a system's dynamics and provides a global picture of the autocorrelation in a time series over all available time-scales. Results of experimental studies conducted on a spindle-bearing test bed have demonstrated that, as the working condition of the bearing deteriorates due to the initiation and/or progression of structural damages, the frequency information contained in the vibration signal becomes increasingly complex, leading to the increase of the RP entropy. As a result, RP entropy can serve as an effective indicator for defect severity assessment of rolling bearings.

Keywords: recurrence plot; phase space reconstruction; defect diagnosis; severity assessment

1. Introduction

The continued demand for high-quality, low-cost products and safe production has changed the strategy for machine maintenance from corrective over preventive to condition-based maintenance, for which real-time fault detection, diagnosis and remaining service life prognosis are needed. The past decades have seen a worldwide increase in the installation of wind turbines and related machine systems for renewable energy extraction from natural resources. Accordingly, condition monitoring and health diagnosis of rotating machines have taken on new significance (Jardine *et al*. 2006). Such efforts have promoted the continued advancement of sensing as well as signal-processing technologies. In addition to commonly used time (statistical) and frequency (spectral) domain techniques, advanced signal processing methods such as bi-spectrum (Sinha 2006), wavelet transforms (Wang *et al*. 2011)and time-frequency analysis (Shi *et al*. 2004), have been investigated for defect diagnosis in rotating machines. Due to instantaneous variations in friction, damping, or loading conditions, machine systems are often characterized by non-linear dynamic behaviors (Wang *et al*. 2006, Inoue *et al*. 2011). Therefore, techniques for non-linear time series analysis provide a good approach to extracting defect-related features hidden in the measured signals for rotating machine defect severity evaluation. Various nonlinear time-series analysis techniques, such as correlation dimension (Yan *et al*. 2010), partial correlation integral (Janjarasjitt *et al*. 2008), Lyaponov exponent (Tao *et al*. 2007), and complexity measure (Yan and

∗Corresponding author, Professor, E-mail: ruqiang@seu.edu.cn

Gao 2004), have been investigated previously to characterize nonlinear dynamic behavior of rotating machines during their service life. The recently developed recurrence plot (Eckmann *et al.* 1987) and related recurrence quantification analysis(Webber and Zbilut 1994) have become a new research focus in nonlinear time series analysis due to its short data length requirement and good anti-noise ability (Marwan and Meinke 2004). Their applications have been seen in bifurcation tracking (Gao 1999), pre-epileptic seizure characterization in rats (Li *et al.* 2004), determinism detection in noisy signals (Zbilut 1998), and performance evaluation of production and logistics networks (Donner *et al.* 2008). Furthermore, it has been shown that the quantitative measures extracted from the recurrence plots is more sensitive to identify changing dynamics than those obtained using linear approaches, such as frequencies (Webber and Zbilut 1994). In manufacturing, recurrence quantification analysis has been reported for determining transient and steady-state cutting in face milling operations (Mhalsekar *et al.* 2009), characterization of cycle-to-cycle pressure oscillations, heat release in engines (Grzegorz *et al.* 2007, Asok *et al.* 2008), and detection of damage-induced changes of a rectangular steel plate (Nichols *et al.* 2006). Motivated by the prior studies, this paper focuses on the utility of recurrence plot entropy, which is one of the measures derived from the recurrence quantification analysis, for rolling bearing defect severity evaluation. After introducing the analytical background of phase space reconstruction for nonlinear time series analysis in Section 2, a surrogate data testing approach is introduced to identify a signal's nonlinearity, often seen in vibration signals measured on rotating machine elements, such as rolling bearings. Section 3 describes the recurrence plot and recurrence quantification analysis, where the RP entropy is discussed in detail. In Section 4, vibration signals from rolling bearings with pre-seeded structural damage under different working conditions are analyzed using the RP entropy, and the degree of severity of the structural damage is quantified. Finally, conclusions are drawn in Section 5.

2. Phase space reconstruction and nonlinearity testing

Most of the non-linear time series analysis techniques, including the recurrence plot, have been developed from the concept of phase space (Takens 1981), which does not require *a priori* information on the underlying dynamics of the system being investigated for its implementation. In practice, the actual phase space of a physical system is difficult to obtain, and only time series are available (through measurement from the physical system) (Kantz and Schreiber 1997). To solve this problem, a time-delayed coordinate approach based on the Takens embedding theorem (Kantz and Schreiber 1997) has been developed to reconstruct the phase space from a measured time series. The main idea of the Takens embedding theorem (Kantz and Schreiber 1997) is that the phase space can be directly reconstructed from the lagged time series measured from the system. Specifically, from a measured time series $\{x(1), x(2), \ldots, x(N)\}$, a reconstructed phase space can be generated as

$$\begin{cases} X(1) = \{x(1), x(1+\tau), \ldots, x(1+(m-1)\tau)\} \\ \quad \ldots \\ X(i) = \{x(i), x(i+\tau), \ldots, x(i+(m-1)\tau\} \\ \quad \ldots \\ X(N-(m-1)\tau) = \{x(N-(m-1)\tau), x(N-(m-2)\tau), \ldots, x(N)\} \end{cases} \tag{1}$$

where $i=1,2,\ldots,\ N-(m-1)\tau$ and $N_m = N-(m-1)\tau$ is the total number of points in the reconstructed phase space. The parameters τ and m are the time delay and the embedding dimension of the reconstructed phase space, which are determined by using mutual information (Fraser and Swinney 1986) and False Nearest Neighbors (FNN) (Kennel *et al.* 1992) approaches in this study, respectively.

The mutual information in a reconstructed phase space is defined as

$$MI(\tau) = \sum P(X(i), X(i+\tau)) \log_2 [\frac{P(X(i), X(i+\tau))}{P(X(i))P(X(i+\tau))}] \tag{2}$$

where $P(X(i))$ and $P(X(i+\tau))$ are the probabilities of $X(i)$ and $X(i+\tau)$ in the reconstructed phase space, respectively, and $P(X(i), X(i+\tau))$ is the joint probability of $X(i)$ and $X(i+\tau)$. The optimal value of the time delay τ is selected when the mutual information $\mathrm{MI}(\tau)$ reaches its first minimum. The embedding dimension can be determined using the FNN approach, where for each point $X(i)$ in the m dimensional phase space, its nearest neighbor $X(j)$ is identified and their Euclidean distance is calculated as $\|X(i)-X(j)\|_m$. Subsequently, the ratio $r_i(m)$ of the Euclidean distance between these two points in the m and m+1 dimensional phase spaces is obtained as $\|X(i)-X(j)\|_{m+1} / \|X(i)-X(j)\|_m$. If the ratio $r_i(m)$ is greater than a pre-defined threshold r_{thr}, the identified neighbor is called a false neighbor. The embedding dimension m can then be determined as the one whose summation of all indentified false neighbors approaches zero.

Before any nonlinear time series analysis technique is applied to processing the measured data, it is important to recognize the existence of nonlinearity in the data series. Surrogate data testing is one of the widely used techniques for nonlinearity identification (Theiler *et al.* 1992). In essence, it is a statistical hypothesis testing technique, and the main idea is based on the assumption that the data measured came from a linear Gaussian random process. The surrogate data, which have the same mean value, standard deviation, and/or power spectrum as the measured data, can be computationally generated from the measured data. Some discriminant statistic indicators, such as correlation dimension and Lyapunov exponent, are then calculated from both the measured and the surrogate data. The existence of nonlinearity can subsequently be studied by formulating a hypothesis testing problem, in which the null hypothesis is that the measured data and surrogate data are from the same linear Gaussian random process and their statistical indicators are within the same distribution. If such a null hypothesis is rejected with a desired level of significance (normally 95%), it indicates that the measured data are not generated from the linear Gaussian random process. This, in return, implies some (assumed large) degree of confidence that the measured data are produced by a nonlinear dynamic system (e.g., a bearing test system as exemplified in this study), and nonlinearity could exist in the data series.

Fig. 1 illustrates how the surrogate data technique is applied to identifying the existence of nonlinearity in a measured data through a null hypothesis test. The phase randomized Fourier transform is first applied to generating a series of surrogate data from the measured data (Schreiber and Schmitz 2000). Mathematically, for a given time series

$\{x(t)\}, t = t_0, t_1, \ldots t_{N-1} = 0, \Delta t, \ldots (N-1)\Delta t$
, applying Discrete Fourier Transform leads to

$$X(f) = F\{x(t)\} = \sum_{n=0}^{N-1} x(t_n) e^{i2\pi f \Delta t} = A(f) e^{i\phi(f)} \tag{3}$$

where $A(f)$ and $\phi(f)$ represent amplitude and phase of the transformation results, respectively. Performing phase randomization of Eq. (3) results in

$$\tilde{X}(f) = A(f) e^{i[\varphi(f) + \phi(f)]} \tag{4}$$

where $\varphi(f) \in [0, 2\pi)$, and the condition $\varphi(f) = -\varphi(-f)$ is required to ensure that the results of performing the Inverse Fourier Transform on $\tilde{X}(f)$ will be real values, as given by

$$\tilde{x}(t) = F^{-1}\{\tilde{X}(f) e^{i\varphi(f)}\} \tag{5}$$

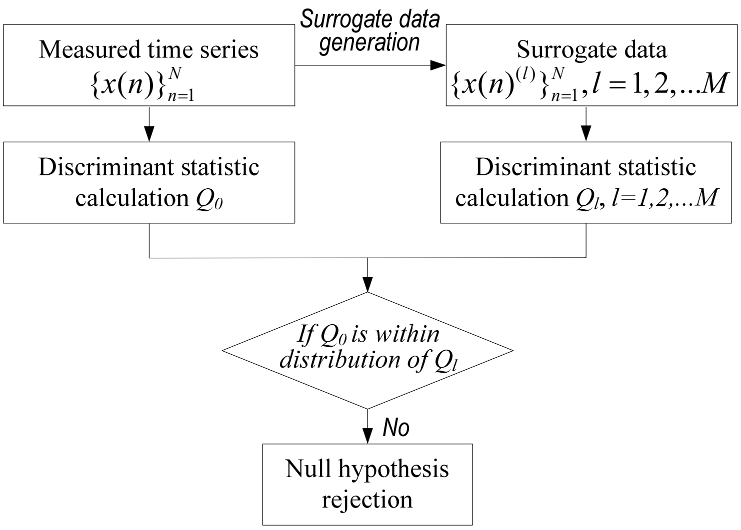
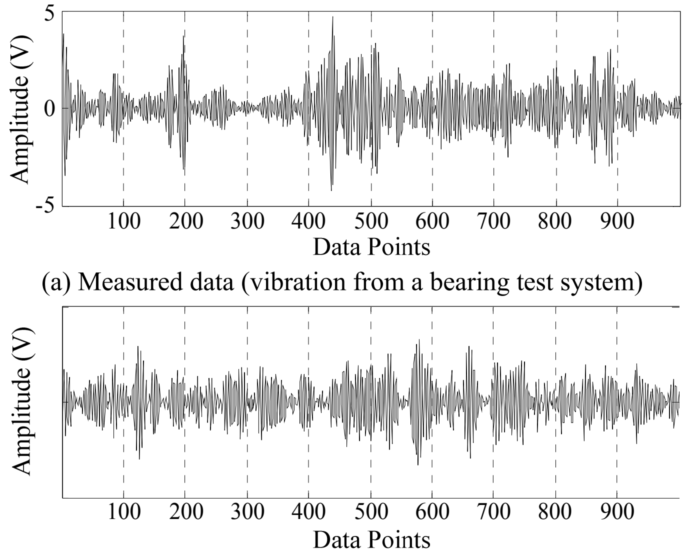


Fig. 1 Illustration on surrogate data testing

In Eq. (5), $\tilde{x}(t)$ are the generated surrogate data, which have the same power spectrum as the original time series.

With the surrogate data generated, the next step is to calculate the discriminant statistic Q_0 and Q_l ($l=1,2,\ldots M$) from the measured data and the surrogate data, respectively. Subsequently, hypothesis testing is performed by evaluating if Q_0 is within the distribution of Q_l. This can be realized using a one-side rank-order test approach (Theiler *et al*. 1992, Schreiber and Schmitz

2000), in which a probability α of a false rejection is selected, which corresponds to $(1-\alpha)\times 100\%$ level of significance. For the one-side rank-order test investigated in this study, it is required that $M = k/\alpha - 1$ (with *k* being a positive integer) surrogate data series need to be generated. Normally, a large *k* value provides more robust test result than when *k=1*, as more surrogate data series are involved in the test. However, *K =1* has been considered as a sufficient value for conducting rank-order test (Schreiber and Schmitz 2000). Furthermore, it minimizes the computational cost on surrogate data generation. Thus, to achieve 95% level of significance in one-side rank-order test, a total of 19 (=1/0.05-1) surrogate data series are required. If the discriminant statistic Q_0 calculated from the measured data is less than all Q_l (*l=1,2,…M*), obtained from *M* surrogate data , and it is outside of the distribution of Q_l, then the null hypothesis will be rejected. This suggests a different explanation for the measured data. We assume (as do many others) that Q_0 outside the distribution of Q_l implies some (assumed large) degree of confidence that the measured data were produced by a nonlinear system.



(a) Measured data (vibration from a bearing test system)

(b) Surrogate data

Fig. 2 Measured data and examplary surrogate data used in non-linearity test

As an example, Fig. 2 shows vibration signals of a damage bearing measured from a motor bearing test system, together with one of its corresponding 19 surrogate data. Fig. 3 illustrates the scaling behavior of measured data and one of 19 surrogate data for correlation dimension estimation, where the embedding dimension m=2-20 and time delay $\tau=2$ are used. It can be seen that as embedding dimension increases, the estimated correlation dimension approaches a stable value. Furthermore, the correlation dimension values and their distribution of both the measured data and all 19 surrogate data are illustrated in Fig. 4. It can be seen that the correlation dimension of the measured data is smaller than those of the surrogate data, and is also outside of their distribution. This indicates the existence of non-linearity in the measured bearing vibration signal.

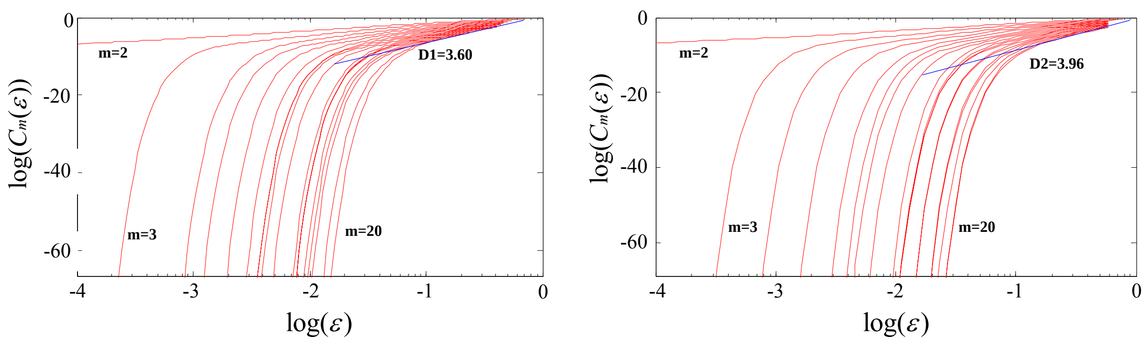


Fig. 3 Scaling behavior of measured data and surrogate data (m=2-20 and τ=2)

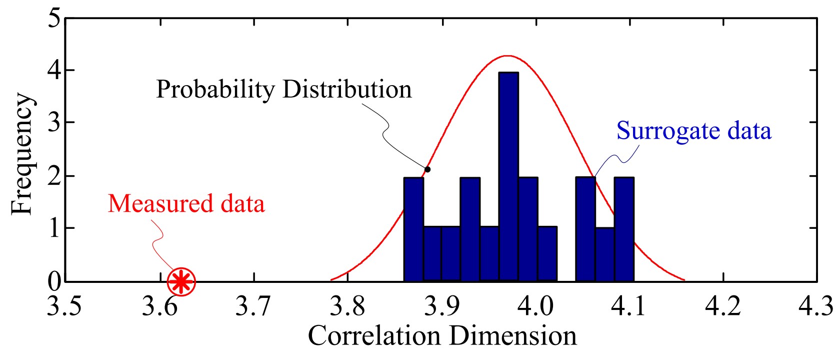


Fig. 4 Null hypothesis test result indicating a system being inherently nonlinear

3. Recurrence plots and recurrence plot entropy

3.1 Recurrence Plot

The recurrence plot has been introduced to graphically describe the dynamical properties of a time series in a qualitative manner, by revealing time-correlated information existing in a physical system (Marwan *et al*. 2003). Given a time series $\{x(1), x(2), ..., x(N)\}$ measured from the physical system, the phase space can be reconstructed using Eq. (1), where each vector $X(i)$ is a point in the m-dimensional reconstructed space and represents the state of system in time i. Next, the recurrence matrix can be formed as

$$R_{ij} = \Theta(\varepsilon - \| X(i) - X(j) \|) \tag{6}$$

Where $\Theta(\bullet)$ represents a Heaviside function (Marwan 2003), and ε denotes a threshold value. Depending on the value of the element R_{ij} being zero or one, a blank space is left or a dot (i, j) is drawn in a two dimensional space (Marwan *et al*. 2003). The result is called recurrence plot, and the dynamical properties of a physical system can then be characterized by the line structure and point density in such a plot.

It should be noted that the selection of threshold value ε is critical to characterizing the dynamical structure of the system. A value that is too small will not enable successful identification of the dynamical changes experienced by the system (Nichols *et al*. 2006). Several methods for the choice of the threshold ε have been investigated in the literature, such as a certain percentage of the maximum phase space diameter (Koebbe and Mayer-Kress 1992, Zbilut and Webber 1992), the recurrence point density (Zbilut *et al*. 2002), or composition of the real signal and some observational noise with standard deviation (Thiel *et al*. 2002). In the current study, 10% of the maximum phase space diameter, computed from the time series with the embedding parameters dimension and time delay, has been used to find an appropriate threshold value, (Norbert *et al*. 2007).

3.2 Recurrence plot entropy

A recurrence plot provides a qualitative description of the dynamics embedded within a time series. As a means of quantifying various features of a recurrence plot, Zbilut and Webber formulated a procedure called recurrence quantification analysis (RQA), by introducing several parameters based on the line structure and point density in the recurrence plot. The RQA method was further extended by Marwan (Marwan 2003) with the introduction of additional parameters, such as recurrence rate (RR), determinism (DET), laminarity (LAM), trapping time (TT), and entropy (ENTR). In the present study, a parameter describing the entropy, termed RP entropy, is investigated for defect severity evaluation in rolling bearings.

Suppose $p(l)$ stands for the frequency distribution of the lengths of the diagonal lines in a recurrence plot, which is expressed as

$$p(l) = N_l \,/ \sum_{\alpha=l_{\min}}^{N} \alpha N_\alpha \tag{7}$$

where l_{min} is the minimum length of the diagonal line, which is different for different systems. In this study, $l_{min} = 2$. N_l denotes number of lines with length l.

The RP entropy is then defined as

$$ENTR = -\sum_{l=l_{\min}}^{N} p(l) \ln p(l) \tag{8}$$

It can be seen in Eq. (8) that entropy is defined as the Shannon entropy based on the frequency distribution of the diagonal line length. Furthermore, it is a measure for the complexity of a deterministic structure in a dynamical system. For example, if all diagonal lines are of the same length l=4, then only $p(l=4)=1$, others are equal to zero (i.e, $p(\alpha)=0$, $\alpha=l_{min},\ldots,N$ and $\alpha\neq4$). As a result, the RP entropy is zero, which means the system dynamics are not complex. In contrast, if the diagonal lines are spread with a broad range of lengths, the calculated RP entropy will be high, which implies an increased complexity of the system dynamics. Therefore, the RP entropy can be used for identifying the changing dynamics of the system, i.e., characterizing the degradation of a dynamic signal measured from that system, which can be represented by the deterioration of a bearing's working condition. A flow chart for RP entropy calculation is shown in Fig. 5, where the average mutual information, false nearest neighbor techniques and the maximum phase space diameter have being applied to determing the time delay, embedding dimension and threshold value, respectively. Using this procedure, the RP entropy value is obtained from the recurrence plot for bearing defect severity evaluation.

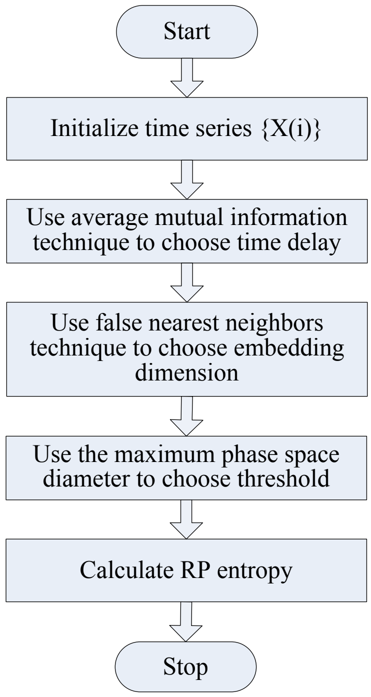


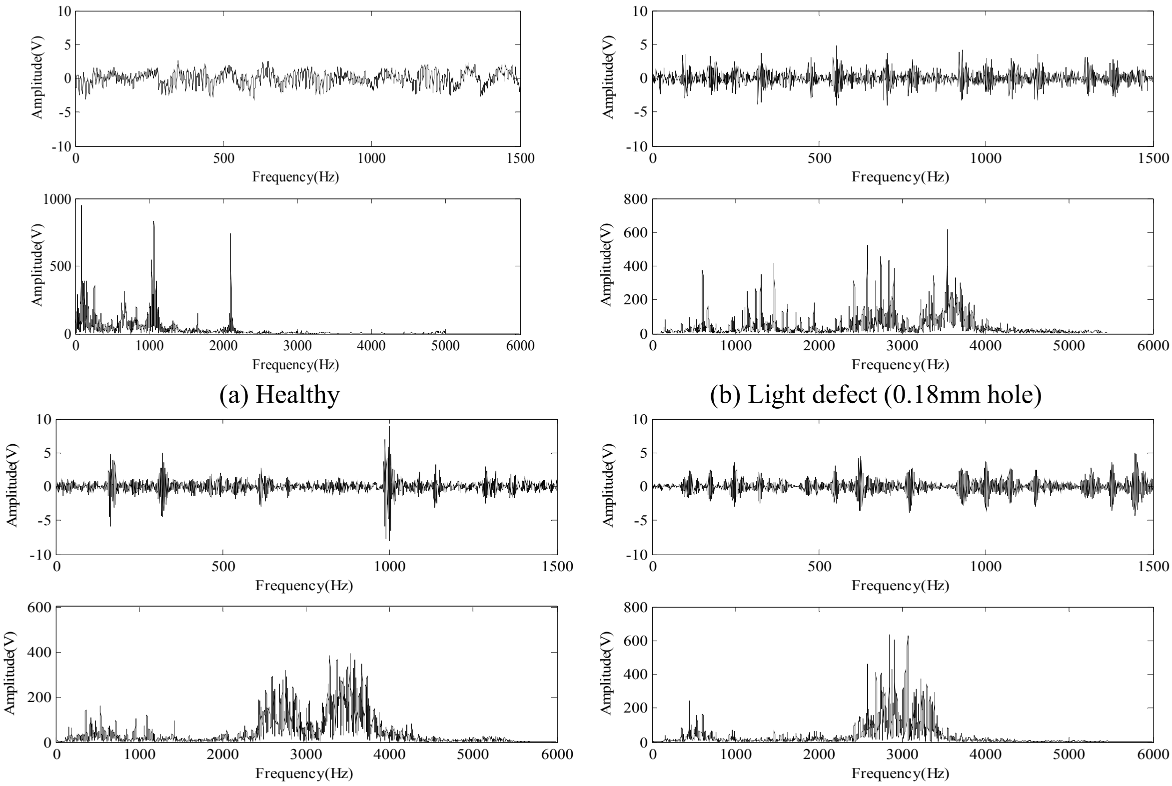
Fig. 5 Flow chart of recurrence plot entropy calculation

4. Experimental study

To verify the feasibility and practicality of the recurrence plot entropy in defect severity evaluation for rolling bearings, two case studies are conducted and the results are discussed below.

4.1 A. Case Study I: motor bearing seeded defect test

In this case study, vibration signals measured on a motor bearing test system are analyzed (Loparo 2011). Single point faults are introduced to the inner raceway of the test bearing (model 6205-2RS JEM SKF) using electro-discharge machining with fault diameters of 0.18 mm, 0.36 mm and 0.53 mm, respectively. Figure 6 illustrates vibration signals and their Fourier spectrum results of four bearings (including a healthy bearing) with different severities of damage under the running condition of 1,772 RPM and 1 HP motor load. From the waveforms in time domain and amplitude spectra in frequency domain, it is difficult to distinguish the level of bearing damage severity.



(a) Healthy

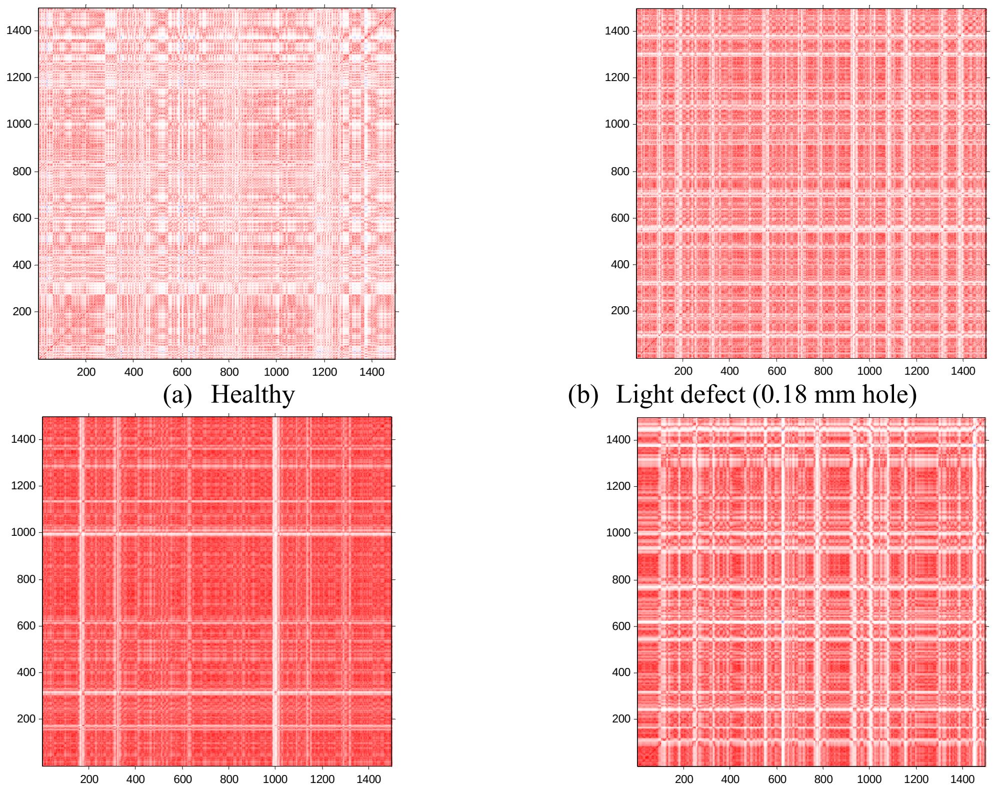
(b) Light defect (0.18mm hole)

(c) Medium defect (0.36mm hole)

(d) Severe defect (0.53mm hole)

Fig. 6 Vibration signals of bearings under different levels of damage
(Speed: 1,772 rpm, Load: 1 hp)

Following the flow chart in Fig. 5, the time delay τ=2 and embedding dimension m=6 are first determined. Next, the phase space is reconstructed from the four vibration signals. With the threshold value ε chosen as 0.36 through the maximum phase space diameter approach, the recurrence plots of the four vibration signals are drawn in Fig. 7. The vertical and horizontal lines in the plot indicate the presence of laminarity and intermittency in the vibration signals, whereas the diagonal line and board structure signify the existence of regular oscillation. Generally, the structures of recurrence plot can be classified into different patterns (Gao and Cai 2000, Marwan *et al*. 2007). A homogeneous type feature, corresponding to the healthy bearing, can be seen in Fig. 7(a), while blocks type features are shown in defective bearings. The blocks reflect abrupt changes in the dynamics of the bearing system.



(a) Healthy

(b) Light defect (0.18 mm hole)

(c) Medium defect (0.36 mm hole)

(d) Severe defect (0.53 mm hole)

Fig. 7 Recurrence plots of vibration signlas from four test bearings
(Speed: 1,772 rpm, Load: 1 HP)

The RP entropy is subsequently calculated and shown in Fig. 8. It is seen that the RP entropy value of the defective bearings are larger than that of the healthy bearing. The more severe the damage is, the larger the RP entropy has shown to be. This can be explained that vibration signals of defective bearings contain more frequency components than that of the healthy bearing. Accordingly, the complexity of the signals increases, leading to a higher value of the PR entropy (Yan and Gao 2007). As an example, structural damage within a bearing would cause the contact pressure of the bearing components to change, leading to a modulation of the vibration amplitude or frequency such that more harmonics emerge. The result is an increased complexity of the vibration signal and the system's entropy.

In order to verify the advantage of RP entropy for defect severity assessment, a comparison study has been investigated, and two other statistical parameters (Kurtosis and Fano factor) , are calculated from the same data. These two parameters are defined as

$$Kurtosis = \frac{\sum_{i=1}^{N}(x(i)-\mu_x)^4}{\sigma_x^{\ 4}} \tag{9}$$

$$Fano = \frac{\sigma_x^{\ 2}}{\mu_x} \tag{10}$$

where $x(i)$, i=1,2,3,…N, represents the vibration data, and μ_X, σ_X, are the mean value and standard deviation of the vibration data, respectively. As can be seen from the results listed in Table 1, except for RP entropy, kurtosis and Fano factor are not effective indicators for characterizing the defect severity of bearings

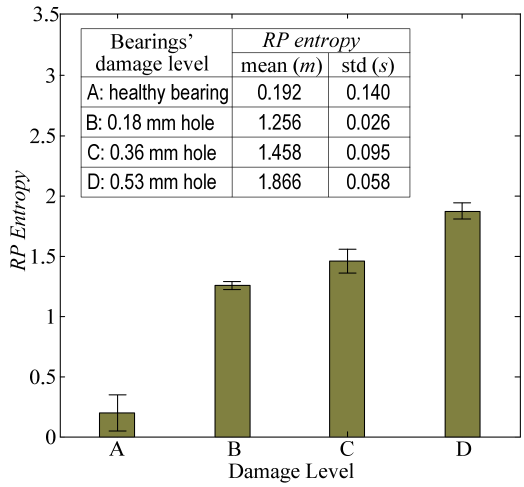


Fig. 8 RP entropy of vibration signals from four test bearings (Speed: 1,772 rpm, Load: 1 HP)

Table 1. Comparison between RP entropy and some commonly used methods (Speed: 1,772 rpm, Load: 1 HP)

Damage level	Kurtosis		Fano factor		RP entropy	
	Value	Change	Value	Change	Value	Change
A: Healthy bearings	2.931		0.338		0.192	
B: 0.18 mm hole	5.542	↑	14.782	↑	1.256	↑
C: 0.36 mm hole	22.084	↑	7.611	↓	1.458	↑
D: 0.53 mm hole	7.667	↓	64.167	↑	1.866	↑

To confirm the analysis results, another set of vibration signals under different operation conditions (speed : 1,750 rpm; load: 2HP) are analyzed. As Fig. 9 and Table 2 indicate, similar trend can be identified to that in Figure 8 and Table 1. This verifies the effectiveness of RP entropy, compared with some other commonly used statistical parameters, for damage severity assessment in motor bearings.

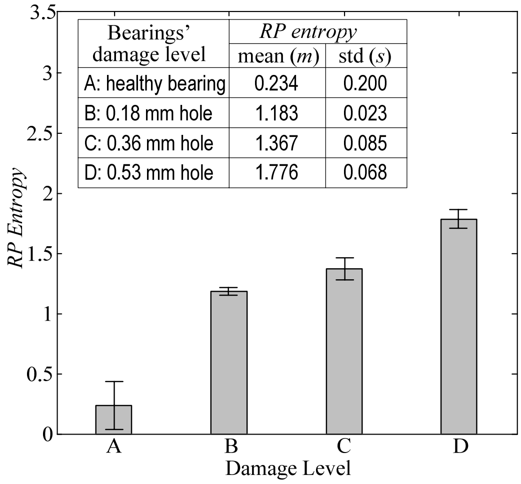


Fig. 9 RP entropy of vibration signlas from four test bearings (Speed: 1,750 rmp; Load: 2HP)

Table 2. Comparison between RP entropy and some commonly used methods (Speed: 1,750 rpm, Load: 2 HP)

Damage level	Kurtosis		Fano factor		RP entropy	
	Value	Change	Value	Change	Value	Change
A: Healthy bearings	2.925		0.325		0.134	
B: 0.18 mm hole	5.564	↑	19.703	↑	1.183	↑
C: 0.36 mm hole	21.686	↑	7.222	↓	1.367	↑
D: 0.53 mm hole	8.058	↓	69.704	↑	1.776	↑

4.2 B. Case study II: spindle bearing run-to-failure test

For the second case study, run-to-failure testing was conducted on an 1100KR ball bearing under a radial load of 5,498 N and a rotational speed of 2,000 rpm. An initial 0.27 mm-wide groove was introduced across the outer raceway to accelerate the defect propagation process. After approximately 2.7 million revolutions, the bearing came to a seizure, with spalling across the entire raceway. Vibration signals were collected during the run-to-failure test at an interval of every 7 minutes. Fig. 10 illustrates the trend of the vibration amplitude (measured by accelerometer that output a voltage signal) along the process of defect propagation. Three sample waveforms are displayed, which were measured during the interval when the bearing was physically examined.

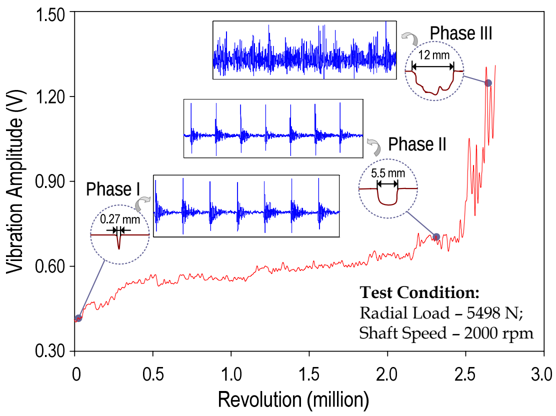


Fig. 10 Amplitude as a function of bearing revolution

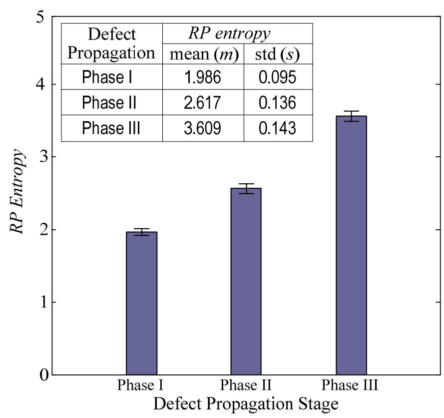


Fig. 11 RP entropy of vibration signals at different run-to-failure test stages

For each vibration signal obtained from the run-to-failure test, 40 non-stationary data segments of 1000 data points are chosen to calculate the RP entropy values. Applying the same procedure illustrated in Fig. 5, phase spaces corresponding to the different test stages are reconstructed from each data segment, and RP entropy values are calculated from the recurrence plot formulated in the reconstructed phase spaces. As the results in Fig. 11 demonstrate, an increasing trend of the RP entropy values is seen, as the defect severity increases (i.e., increasing defect size). This verifies again the effectiveness of RP entropy for bearing defect severity assessment.

5. Conclusions

One of the quantitative measures derived from the recurrence quantification analysis, termed RP entropy, has been investigated for its effectiveness in serving as an indicator for defect severity assessment in mechanical systems. The process starts with first performing a surrogate data testing to sensor signals in the time domain to identify the existence of non-linearity in the signal. Case study using vibration signals measured on healthy and defective rolling bearings has been performed, and the results demonstrate that the severity of structural defect can be effectively diagnosed, based on the increase of the RP entropy values (e.g., 330% increase when 0.18 mm hole was introduced into the inner raceway of a healthy bearing under the running condition at 1,772 RPM and 1 HP load). From the run-to-failure test, the RP entropy has shown to be an effective measure for characterizing degradation of the spindle bearing system. However, the limitation of this technique is that the reason which causes the damage can not be identified from the RP entropy, and this needs further research. Furthermore, the trial-and-error approach for threshold parameter determination should be replaced by a more general and quantitative approach in order to extend the usage of the recurrence quantitative analysis in other applications. It is envisioned that besides rolling bearings, the developed technique is applicable to a wide range of machine systems (such as motors and gearbox in wind turbines) where timely assessment of the degree of structural damage in rotating machines is critical to enabling condition based, intelligence maintenance strategy for more reliable and economical operations.

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