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Some precautions to consider in using wavelet transformation for damage detection analysis of plates

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Abstract. Over the last two decades Wavelet Transformation (WT) method has been widely utilized for the damage identification of structures. The main objective of this paper is to discuss and present some of common shortcomings and limitations of mathematical software, as well as other precautionary measures that need to be considered when using them for wavelet analysis applications. Due to popular usage of MATLAB® comparing to other mathematical tools among researchers for data processing of structural responses through WT analysis, this software was chosen for specific study. To the best of the authors' knowledge, these limitations and observations have not been previously identified or discussed in the literature. In this work, a square plate with a severe damage, in form of a crack, parallel to the left edge of the plate is selected for a pilot study. The steady state harmonic response is used for measuring the deflection shape across the line parallel to one edge and perpendicular to the damage. Several criteria and cases such as the smallest size damage that can be detected, correlation between the crack width and the number of sampling points, and the influence of the damage thickness on the accuracy of the result are investigated.

Keywords: damage detection; wavelet transform; harmonic excitation; plate

1. Introduction

Over the last 15 years numerous efforts have been carried out to develop reliable and optimum methodologies for 'Structural Health Monitoring' (SHM). Among them, Wavelet Transform (WT) method has been recognized as a powerful tool in damage identification.

In contrast with other techniques, such as modal characterization method, WTM does not require the response of the healthy structure. Moreover, it is less sensitive to the errors in input data measurement. In this method, the response of the damaged structure, i.e. the space domain signals, are analyzed by WT. The response is transformed by WT and the position of the damage is estimated by the variation of the spatial response signal via the high resolution capability of the wavelet transform.

The wavelet transform is capable of describing a signal in a localized time (or space) and/or frequency (or scale) domain. In the simplest term, WT transfers a 2D function f(x), into a 3D space with parameters x and 1/x, i.e., f(x, 1/x). In a time domain signal, x stands for the time and 1/x

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stand for the frequency. In a space domain signal (like static response deflection of a beam or a plate), this mathematical analysis identifies the location of the singularities, for instance caused by a crack or a point load, along the response of the structure through a sudden change in the graphs of WT coefficients. The above property makes this method potentially reliable and cost-effective for structural health monitoring applications.

1.1 Mathematical aspect of wavelet transformation

The ability to present both time and frequency contents are two important properties of wavelet functions. A function $\psi(x)$ is a wavelet if and only if its Fourier transform $\Psi(\omega)$ satisfies

$$\int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|^2} d\omega < +\infty \tag{1}$$

This condition implies that

$$\int_{-\infty}^{+\infty} \psi(x)\, dx = 0 \tag{2}$$

which means that a wavelet is an oscillatory function with a zero average value. This basic wavelet function $\psi(x)$ that is localized in both time and frequency domains is called the mother wavelet and can be used to create a family of wavelets $\psi_{a,b}(x)$ as

$$\Psi_{a,b(x)} = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right) \tag{3}$$

where ‘a’ and ‘b’ are real numbers that dilate (scale) and translate the function $\psi(x)$, respectively. The continuous wavelet transforms CWT of function f(x), where independent variable ‘x’ is time or space, is defined as

$$\mathrm{CWT}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \psi\left(\frac{x-b}{a}\right) dx = \int_{-\infty}^{+\infty} f(x)\, \psi_{a,b}(x) dx \tag{4}$$

In translating Eq. (4) one might recognize the inner product of $f(x)$ with scaled and translated versions of the original wavelet function. In other words, the continuous wavelet transform (CWT) is the sum over the entire time span of the signal multiplied by a scaled and shifted version of a mother wavelet. Large values of scale 'a' correspond to large wavelets and thus coarse features of $f(x)$, while low values of scale 'a' correspond to small wavelets and fine details of $f(x)$. If instead of using a continuum of dilations and translations discrete values of 'a' and 'b' are used one can define Discrete Wavelet Transform (DWT) as discussed in detail in Daubechies (1992).The results of the CWT are wavelet coefficients that show how well a wavelet correlates with the analyzed signal. Any singularity in the signal creates wavelet coefficients with large amplitudes. These singularities are easily detected in a graph of the coefficients of CWT or DWT. Wavelets are usually used to analyze signals in time domain, however, by replacing the time with space or length, spatially distributed signals can also be analyzed by WT.

1.2 Brief literature review

A relatively large volume of studies have been carried out on the utilization of Wavelet

Transforms for damage detection. Following works that have been cited are some of the major research effort carried out in this area. Hou and Noori (1999) and Hou *et al*. (2000) proposed a wavelet-based approach for structural damage detection and health monitoring. Hansang and Melhem (2004) presented the state-of-the-art of wavelet-based damage detection in structures. The sensitivity of WT to singularities caused by sudden changes of stiffness and its ability to increase the resolution of presenting a signal in time–frequency domain has led to considerable developments in utilizing wavelet-based damage detection methodologies.

Deng and Wang (1998) investigated the possibility of determining the presence and the location of damage in beams using WT (spatial wavelet). They used a discrete Haar WT to analyze the response signal of a simply supported beam under a static point load and a cantilever beam under a dynamic impact load. Quek *et al*. (2001) studied the utilization of suitable wavelet functions and the effect of size and shape of the damage. In the work reported by Chang and Chen (2003) the location of damage on a cantilever Timoshenko beam was identified by using a Gabor (Morlet) wavelet on the dynamic response (first and third modes). Ovenesova and Suarez (2003) utilized both CWT and DWT and used finite element method to calculate the dynamic and static response of a fixed-end beam. Gentile and Messina (2003) discussed the use of an analytical approach to model the damaged beam (a notch with a high *d*/h ratio equal to 0.5 simulated the damage). Recently Zhong and Oyadiji (2011) proposed a new approach based on the difference of the CWTs of two sets of mode shape data of a cracked simply-supported aluminum beam. They provided a better crack indicator than the result of the CWT of the original mode shape data. The analytical and experimental results demonstrated that the proposed method had great superiority with respect to the methods which require the modal parameter of an uncracked beam as a baseline for crack detection. In experimental studies performed by Wu and Wang (2011), a high resolution laser profile sensor was employed to measure the static deflection profile of a cracked cantilever aluminum beam subjected to a static displacement at its end. De-noise processes of the original deflection signal given by the laser sensor and the surface polishing treatment of the beam was critical for an effective detection. The spatial wavelet transform on the beam with different crack depths was proven to be effective in identifying the damage area even for a minor crack. Damage detection in plates using WT could be found in the works of several researchers. Lee (1992) used the Rayleigh–Ritz method to obtain fundamental frequencies of annular plates having internal concentric cracks. Lee and Lim (1993) investigated the vibration behavior of a rectangular plate with a centrally located crack. Wang and Deng (1999) extended the use of wavelet analysis to a cracked plate with a through-thickness crack. Khadem and Rezaee (2000) developed an analytical approach for crack detection in rectangular plates. In another study by Douka *et al*. (2003), mode shapes were used as a basis for WT analysis. The advantage was attributed to the sensitivity of the mode shapes to small-size damages in comparison with the natural frequency variations. Schulz *et al*. (2003) showed that small damage can be detected more accurately using determined patterns for dynamic excitation resulting in constrained vibration deflection shapes. Khatam *et al*. (2007) also demonstrated that vibration deflection shapes resulting from a constrained dynamic excitation can detect damage location with high accuracy and high efficiency and they are more suitable to be utilized than static deflection shapes. In recent years applicability of 2-D WT to identify locations and shapes of damages in plate-type structures were examined. A concept of isosurface of 2-D wavelet coefficients was proposed by Fan and Qiao (2009) for damage identification in plats. The indication of the location and approximate shape or areas of the damages were inspected by using 2-D CWT-based approach. This algorithm was applied to the numerical vibration mode shapes of damaged cantilever plates to illustrate its effectiveness and viability. It demonstrated that the proposed algorithm was superior in noise immunity and robust with limited sensor data. In the other study performed by Huang *et al*. (2009), the spatially distributed 2-D CWT algorithm for a sensor network to automatically detect the local features around the damage sites was used. This method was suitable for both dense and sparse sensor networks. The advantageous features of this algorithm were its reliance on local data and limited communication and computation requirement.

1.3 Research significance

Given that the utilization of WT for damage detection of plates has been a growing area of research, in this paper some observations and valuable suggestions are presented on the limitations and the scope of WT application for the structural health monitoring of plates. The practical guidelines and suggestions presented in this paper should also be taken into account for the application of WT in the analysis of other structures. In particular, since WT toolbox in MATLAB is widely used for the transformation of responses resulting from the finite element analysis of damaged structures comparing to other similar software, as will be discussed in this paper the spatial characteristics of input data are very important. In the work presented in this paper, first the sensitivity of WT coefficients in the variation of sampling point interval is illustrated through a simple mathematic example. As shown, the extraction and compaction of input data causes the occurrence of singularity in the spatial variation of WT coefficient which can be observed as a sudden disturbance in the graph of WT coefficients. This disturbance may be mistakenly perceived as the damage. Using sampling point intervals that are vastly different from the crack width, however, leads to the identification of deep cracks instead of shallow cracks. Subsequently, other important issues that have not been presented in the literature such as the lower limit of damage severity, which can be detectable, and the interaction between the crack width and reducing sampling points, and finally the influence of plate thickness on the accuracy and the resolution of the results are presented in this research. The motivation for this work has been to develop a thorough understanding of the feasibility, and the advantages, of introducing dynamic response of structures as an input response function in a wavelet analysis for detecting damages in plates. The dynamic response of a cracked plate is transformed by WT and the position of the crack is estimated by the variation of the spatial response signal along a line vertical to the crack resulting from the high resolution property of the wavelet transform.

The other objective of the present work is to study the ability of the WT method in detecting a small damage in a plate and how to increase the efficiency and the accuracy of wavelet-based damage detection in structures. To the best of the authors' knowledge, this issue has not been addressed in the literature. As stated earlier, in this study, instead of utilizing the static response deflection or the mode shapes, the response of the plate subjected to a harmonic loading (with specified frequency and location) is used as the response signal in the wavelet analysis. This work is the extension of an earlier work in damage detection of beams, where a similar approach had demonstrated successful results (Beheshti-Aval 2011). It is concluded that the utilization of this harmonic response approach for damage detection purposes, utilizing WT, has a few advantages over the application of static response. These advantages include less sensitivity to noise, convenience of application, and better suitability in practical applications, as demonstrated, and they yield more reliable results in comparison with other methods.

Before presenting the main outcome of this research, it is necessary to illustrate a simple mathematical concept which explains the influence of contraction and dilation of input signal on WT graph of coefficients.

2. The limitations and the advantages of using software tools

In this paper common shortcomings and limitations of mathematical software, as well as other precautionary measures that need to be considered when using them for wavelet analysis applications are discussed. Due to popular usage of MATLAB comparing to other mathematical software tools for data processing of structural responses through WT analysis, this software was chosen for specific study. In fact, utilizing mathematical software such as MATLAB toolbox for transforming responses into wavelet integral, in order to find the coefficients for various scales, has been a common approach in the research carried out in this area. In this work Finite Element Method (FEM) is used as a tool to obtain the dynamic responses of a plate and subsequently these responses are used as the input signal in MATLAB and are transformed into wavelet framework. Our findings show that the sampling point intervals that are selected should be the same as the crack width. Otherwise, the maximum coefficient in the graphs of CWT that implicates on crack depth will not be correct. Moreover, should sampling points intervals and crack depth be different, acquired coefficients will be greater than the actual coefficient. Unfortunately, this important and critical criteria, which is a serious limitation of the MATLAB toolbox, is not mentioned anywhere in the MATLAB software. Since this software is widely used by the researchers who study damage identification, it is probable that some of the published research published in the literatures that have used MATLAB for this application may not be reliable.

For the sake of clarity, let us explain this phenomenon via a simple example. We know frequency means the rate of change of a function with respect to a changing variable. If the function (physical or mathematical) is sensitive to the changing variable, its frequency is high. If the variations are slow, the frequency is called low and if there is no variation at all, the frequency will be zero. The following example explains the aforementioned shortcoming in MATLAB. Whenever the periodic function, $f(x) = Sin(\pi x)$ with a period [0.2], is compacted along the x axis, by preserving its amplitude, its domain diminishes to [0,1]. This compaction results in a new function $g(x) = Sin(2\pi x)$ with a frequency twice that of the frequency of $f(x)$ .In other words, function $g(x)$ which has a higher frequency reaches the same maximum value (amplitude) of $f(x)$ in smaller intervals as compared with $f(x)$ (Fig. 1).

This concept can be generalized in the case of compacting or expanding a function in a smaller interval of its domain, as follows. Whenever a function is compacted (or expanded) in distinct intervals of its domain, in that interval it will have frequencies that are different from the frequencies in other intervals. This process causes a singularity in this interval that can be detected using WT.As an example, signal function of $f(x) = Sin(\pi x)$ is sampled by 100 equal interval points discretely. The period of this function is equal to 2. But if from all these points, point 51 (N51) is deleted and the value of function in point 52 (N52) is replaced with its value in point 51 (N51) (new signal $g(x)$), since the compaction has occurred between n50 and n52 in the signal of $f(x)$, a sudden increase in the frequency will occur between N50 and N51in the signal of $g(x)$ that is different from the frequency of the main signal ($f(x)$). Unfortunately, this kind of compacting occurs if the crack width is different from the sampling point's intervals. This is due to the fact that MATLAB software is not able to receive the signal matrix in 2D state. As mentioned earlier, this phenomenon causes a singularity in this interval that can be observed after applying WT. Thus, even though a crack does not exist, by sampling of points in the same interval except in one or several points, location of these intervals appear as several peaks in graphs of CWT. These peaks can be interpreted, by mistake, as the existence of a crack. These singularities, however, are caused by the compaction (or dilation) of sampling points and are resulted from the limitation of MATLAB software to differentiate the compaction or the dilation of input responses.

In the following, the aforementioned argument is articulated through studying the response signal of a plate with 100 sampling points. In Fig. 2 graph of mid-sectional displacement response of an undamaged plate with the dimensions of $1000 \times 1000 \times 10\ mm$ is presented. This signal is captured by 100 sampling points. Fig. 3 shows 3D and 2D graphs of CWT for this signal. As can be expected, no perturbation is observed on the graphs, which indicates a healthy plate.

Fig. 4 shows the response of the same plate in the state that the 31st point and the 32nd point are deleted and thus the distance between the 30th point and the next point is threefold of other adjacent points. Nonetheless, the resulting signal is plotted based on the *x* and *y* coordinates of the available points. Fig. 5 is the same signal presented in Fig. 4, however, the distance between the 30th point and the adjacent point are intentionally assumed to be equal. In other words, the value of the response signal at the 33rd point as shown in Fig. 2 has been replaced with value of the response signal at the 31st point. Literally speaking, this is the same mistake which commonly occurs while entering the data into MATLAB software. As discussed, MATLAB software has received an input array in a one-dimensional form and interval points are assumed to be equal. Due to this shortcoming, MATLAB makes a compaction (or singularity) on *x* axis in the neighborhood of point 31 that appears as part of the signal with a higher frequency.

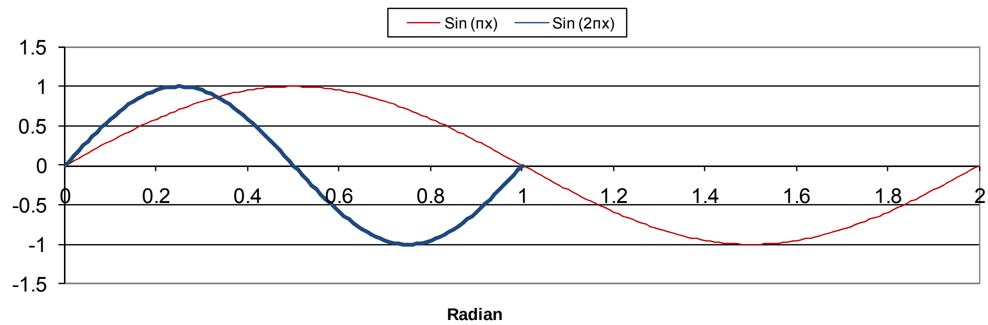


Fig. 1 Compression of function $Sin(\pi x)$ and transformation to function $Sin(2\pi x)$

25	26	27	28	29	30	31	32	33	34	35	36
0.32	0.329	0.338	0.347	0.355	0.363	0.371	0.378	0.385	0.391	0.397	0.403

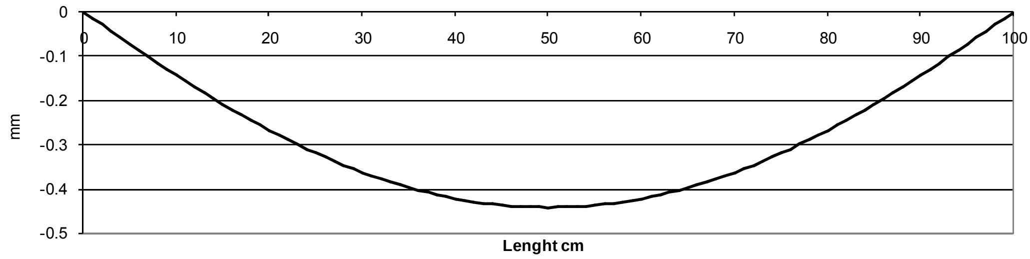


Fig. 2 Numerical representations of segmental displacement response with equal interval of sampling points for undamaged plate along with static response curve

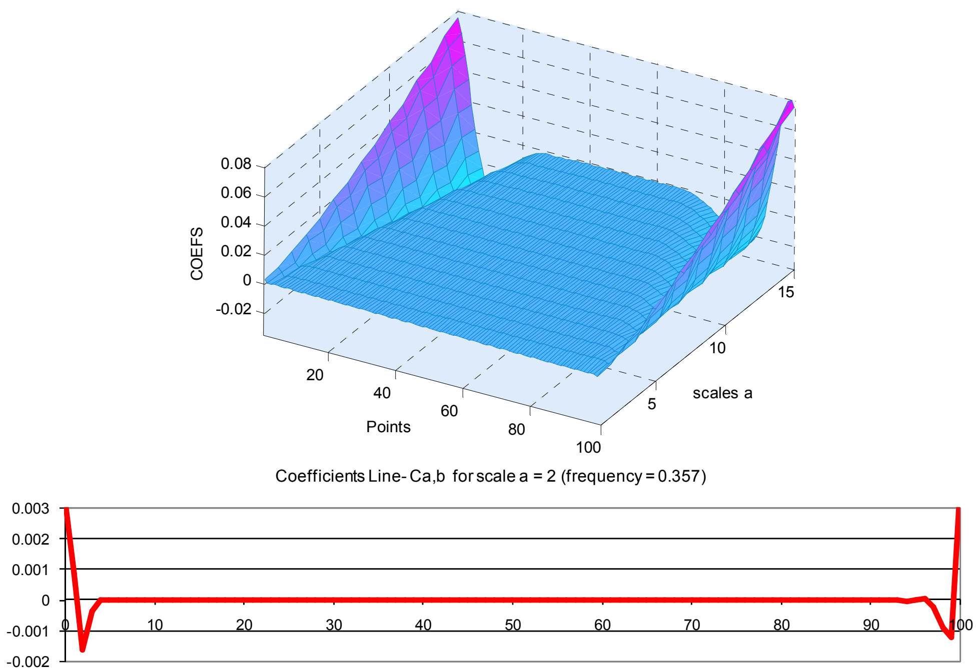


Fig. 3 3D and 2D graph of WT coefficients of response for undamaged plate with Sym 4

25	26	27	28	29	30	33	34	35	36	37	38
0.32	0.329	0.338	0.347	0.355	0.363	0.385	0.391	0.397	0.403	0.409	0.414

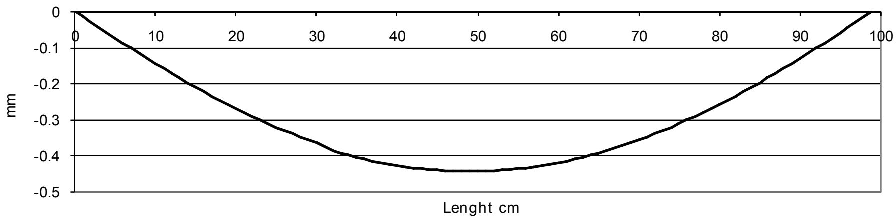


Fig. 4 Numerical representations of segmental displacement response with unequal interval of sampling points for undamaged plate along with plotted curve by 2D array

25	26	27	28	29	30	31	32	33	34	35	35
0.32	0.329	0.338	0.347	0.355	0.363	0.385	0.391	0.397	0.403	0.409	0.409

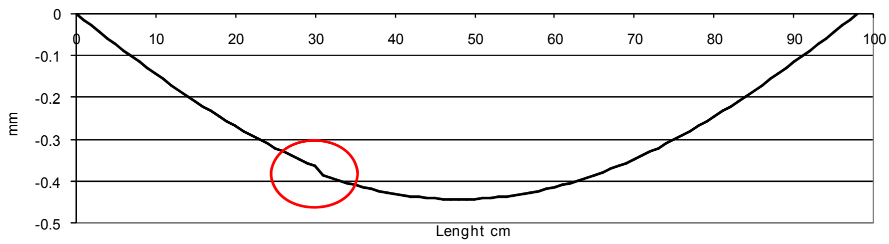


Fig. 5 Numerical representations of segmental displacement response with unequal interval of sampling points for undamaged plate along with plotted curve by 1D array

This point has been shown in Fig. 5 with a circular marker. By analyzing this input signal with symlet4, the location of variation in frequency can be observed as perturbations in the graphs of wavelet coefficient in Fig. 6 in 3D and 2D ($a = 2$).

It is noteworthy, as explained earlier, that despite the presence of any crack, location of this disharmonic distance of adjacent points appears as several peaks in the graphs of CWT. This is due to one or several singularities occurred as a result of compaction (or expansion) of the sampling points and because of the shortcoming the MATLAB as discussed earlier. This limitation indicates that when using MATLAB wavelet toolbox for the damage detection sampling points

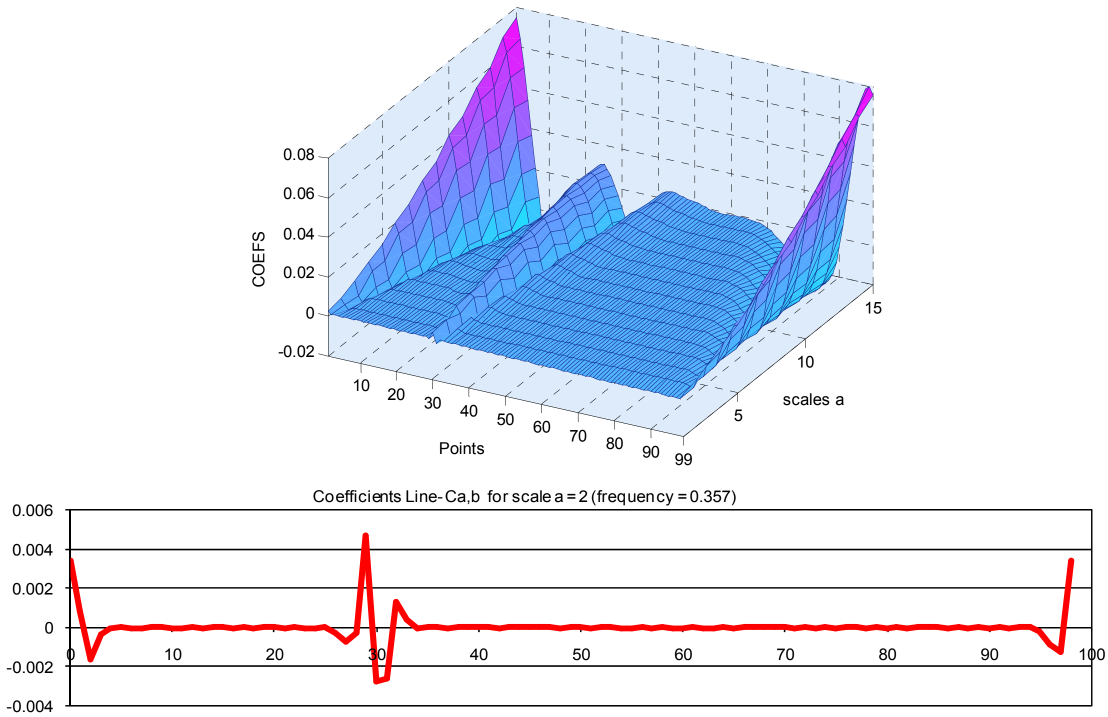


Fig. 6 3D and 2D Graph of WT coefficients of undamaged plate with Sym 4

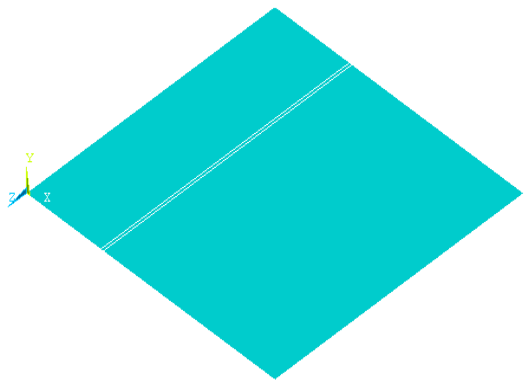
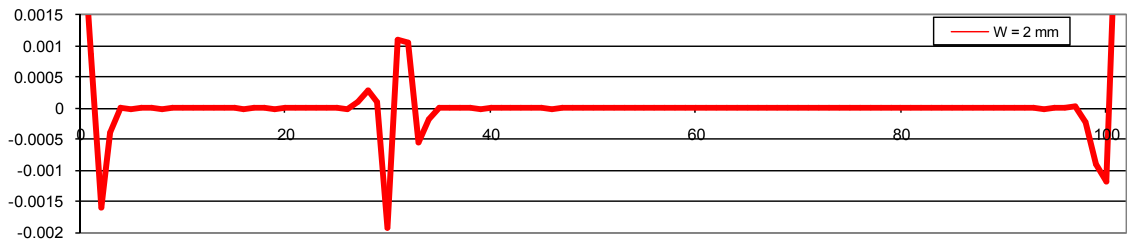


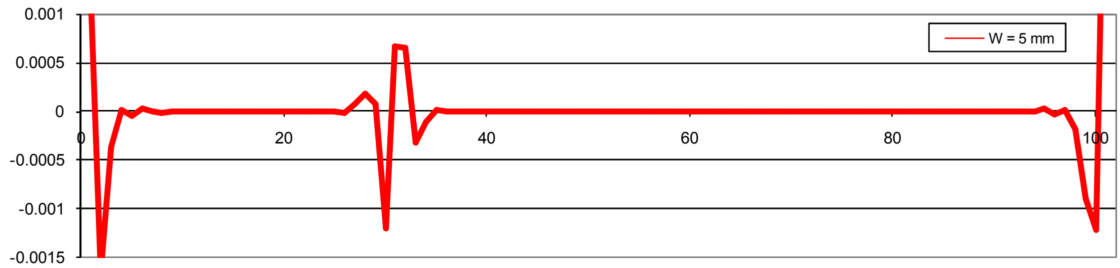
Fig. 7 Plate with size of $1000 \times 1000 \times 10$ mm with crack at $x_d = 300$ mm

with equal intervals have to be considered or instead of utilizing MATLAB other software that have the capability of using sampling points with unequal intervals need to be considered.

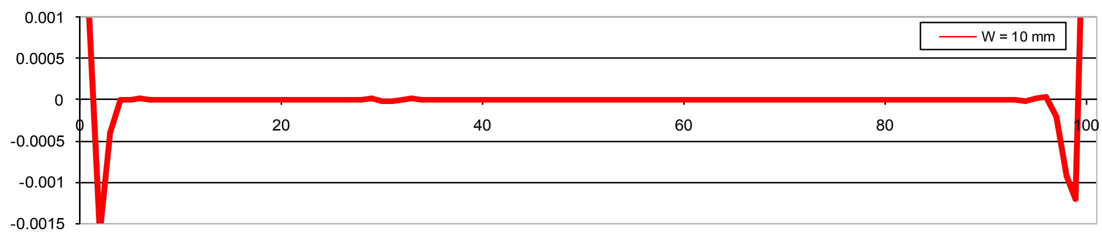
In the following example, a similar study is presented demonstrating the error in calculating the graph of CWT for a damaged plate model. The steady state harmonic response is used for measuring the deflection shape. As shown in the previous study (Deng and Wang 1998), using the harmonic response is more versatile and more effective in comparison with the static deflection response, especially in the presence of noise. The plate structure in this case is an elastic square plate, simply supported along all four edges, with the dimensions of $1000 \times 1000 \times 10\ mm$. It also has a continuous crack running alongside of one edge, as shown in Fig. 7. The plate contains a rectangular notch of width w = 10 mm and a relative depth to plate thickness of d/h = 0.1 at a distance of, x_d = 300 mm away from the left-hand edge of the plate. This is considered to be an open crack. The location of the input excitation force is x_v = 500 mm. The plate is analyzed using a frequency of 0.5 Hz. The crack depth is varied during multiple analyses. The material properties are as follows: Young's Modulus E = 200 GPa, mass density $\rho = 7860$ kg/m^3, and Poisson ratio $\nu = 0.3$. Damping coefficient is assumed to be 0.5%, which is a reasonable value for steel.



(a) w = 2 mm



(b) w = 5 mm



(c) w = 10 mm

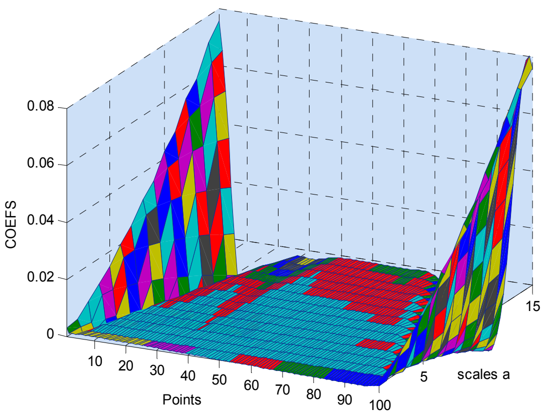
Fig. 8 Graph of coefficients of damaged plate for a = 2 (Sym 4), w = 10 mm

Distances of sampling points along the line perpendicular to the crack and from mid-plate, before and after the crack were considered 10 mm, however, the distance between the two points on the two sides of the crack in three separated analysis were assumed to be w = 2 mm, 5 mm, 10 mm. The graphs of the coefficients for the three cases described above are shown in Fig. 8. As discussed, MATLAB maintains its accuracy and efficiency if all sampling points are sampled in equal intervals. Obviously, and as a result, Figs. 8(a) and (b) are not accurate or reliable and only Fig. 8(c) shows the coefficients accurately.

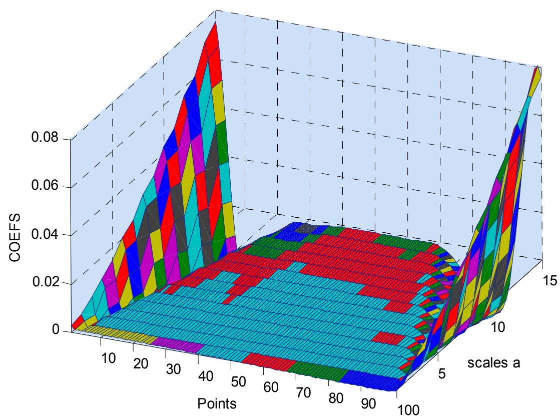
3. Lower limitation of detectable damage

Subsequent to the above discussion, investigation about the lower limitation of a detectable damage seems necessary. Obviously, the smallest detectable damage with WT, or any other method, depends on the intensity of the error measurement of the quantities considered as the response of the damaged structure. If the exact structural response can be computed, there is no restriction on the severity of detectable damage. However, it is difficult, if not impossible, to have the exact structural response and in vast majority of the cases only a measured response of a system (often continuous quantities such as displacements, accelerations, stresses and strains), are available. In the majority of the published work finite element method is used to calculate the response of a structure. The inherent approximation of FEM in evaluating the nodal responses as input signals, and subsequently processing them by utilizing WT in low scales, causes disturbances in the graph of CWT's coefficients. Therefore, even if no intentional error function is added to the response signal, depending on the limited accuracy of the FEM there is lower limitation for the detectable damages.

In Fig. 9, 3D-graphsof CWT coefficients prepared with Sym 4 for relative depths 0.05, 0.10 are shown. In case of d/h = 0.1, damage location at x_d = 300 mm is detectable but for d/h = 0.05, resolution has decreased. Fig. 10 shows the graph of the coefficients in scale a = 2 for relative depths of d/h = 0.01, 0.05, 0.1 and 0.2. As can be seen, (i) Cracks with relative depths lower than 0.05 are not detectable. (ii) Although increasing the crack depth causes augmentation of CWT



(a) d/h = 10%



(b) d/h = 5%

Fig. 9 Graphs of coefficients for Sym 4

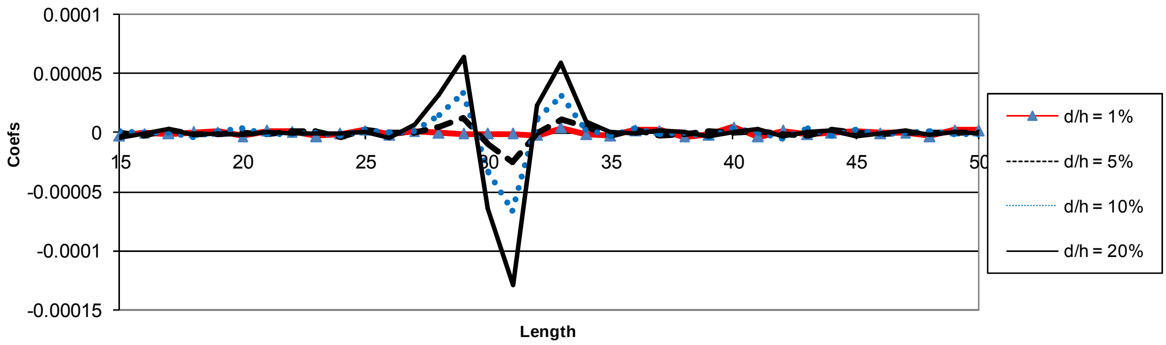


Fig. 10 Graph of WT coefficients for a = 2 (Sym 4), with different ratios of d/h

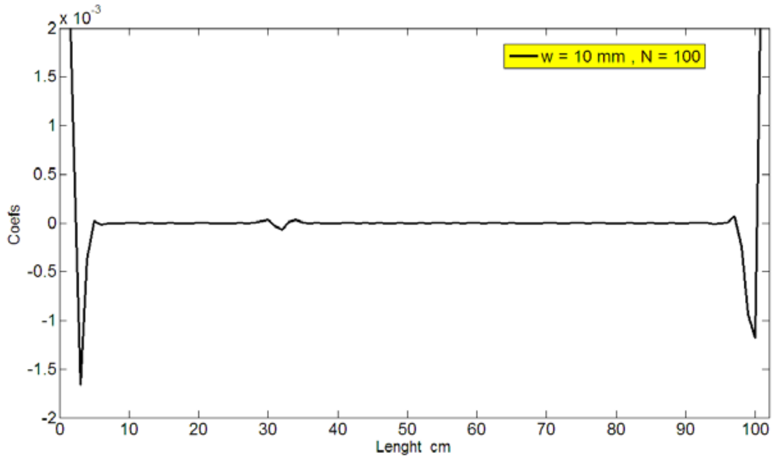
coefficient values at the damage location, there is no linear relation between them. It is noteworthy that this low limitation has been reported in the absence of noise contamination.

4. Concurrent influence study of crack width and reducing sampling points

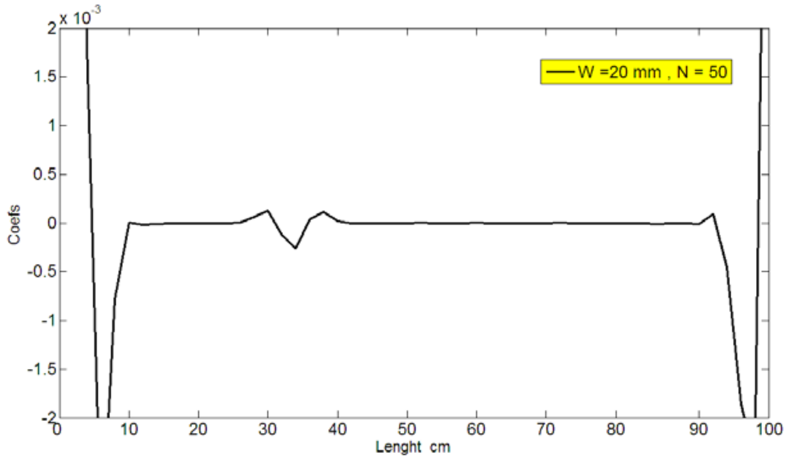
In the vast majority of the research work reported so far, the influence of crack width and reduction in sampling points have been considered as two separated parameters. Even in some case sit has been assumed that the ability of WT to identify crack location is only affected by the damage depth. However, the influence of the damage depth in the magnitude of the coefficients and thus, the identification of the damage location is undeniable. What is ignored in these studies is the attention to the important fact that with the reduction in the sampling points crack width should not be assumed constant. Considering the inability of MATLAB in sampling points with unequal intervals, it is clear that dealing with such parameters poses a question in the accuracy of these investigations. Subsequently, it seems that the influence of the reduction in the sampling points and the crack width should be studied simultaneously. Because, otherwise, only with a variation in the crack width, or only a reduction in the number of sampling points, crack width will be different from the distance between the sampling points. Therefore, the accuracy of the result, as argued before, is questionable. Thus, whenever MATLAB is used for the wavelet analysis of the response signal, this limitation needs to be taken into consideration, i.e., the ratio of the crack width (w) to the interval distance of sampling points should be kept the same.

Fig. 11 shows the wavelet transform of the response signal of the plate with the dimensions of $1000 \times 1000 \times 10\ mm$ and with 200, 100, 50, and 20 sampling points. In this case, the aforementioned limitation has been taken into account, i.e., $(w/dist = 1)$. For example in 100 sampling points, crack width has been assumed 10 mm and in 50 sampling points the crack width has been assumed 20 mm. The damage location has been considered at x_d = 300 mm from the left support and the relative depth has been assumed as d/h = 20%. So far, it has assumed that increasing the crack width has no significant influence on the graph of the coefficients and it decreases the coefficient values at the crack location.

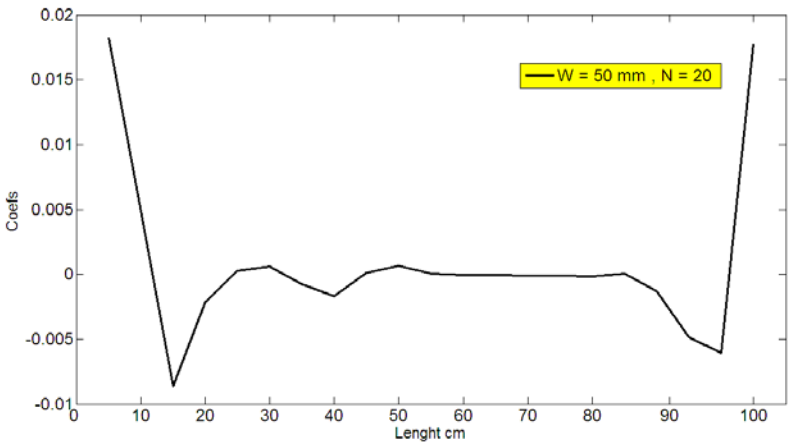
As can be seen in Fig. 11, however, increasing of the crack width and thus increasing the distance of other sampling points (and thus reducing the number of sampling points) not only does not affect the graph of the coefficients but also it increases the coefficients at the crack location. A



(a)



(b)



(c)

Fig. 11 Graph of WT coefficients for a = 2 (Sym 4), with different cracks widths and different number of sampling points

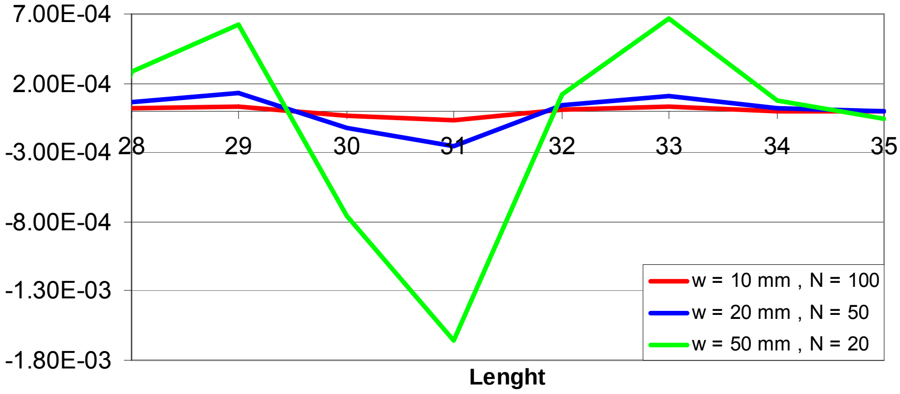


Fig. 12 Merging graphs of Fig. 11 in one graph

closer review of Fig. 11 in Fig. 12 reveals that a reduction in the number of sampling points to 20 points causes a decrease in the spatial resolution of the damaged location.

5. Influence of plate thicknesses on the resolution of the results for a constant relative crack depth

If we apply WT to damaged plates with different thicknesses and keep the relative crack depth constant we can observe that WT coefficients at the crack location depend solely on the selected thickness of the plate. This means that the lower the plate thickness, the larger the WT coefficients will be. For example, two plates with the dimensions of $1000\times1000\times10\ mm$ and $1000\times1000\times5\ mm$ were considered. For both plates, the crack location was assumed at x_d = 300 mm from the left support. Relative crack depths were kept constant and equal to 30 percent of the plate thickness. Fig. 13 shows the graph of WT coefficients of the response signal for both plates. As can be seen under the same loading for both plates, the WT coefficient values of the thicker plate at the damaged location are less. This is due to higher displacement values of the slim plate as compared with the thicker one. Should the values of the input signal be larger, the values of the output WT coefficients will be subsequently greater.

6. Influence of the support type on the CWT coefficient

Comparing the graph of CWT coefficient for a plate with two boundary conditions, i.e. simple and fixed supports, prepared with Sym 4 as shown in Fig. 14(a), can demonstrate two important differences: (i) First difference is attributed to the reduction of the perturbation near the supports in the case of a plate with fixed supports, as compared with the simply supported plate. Simple supports cause severe discontinuity as compared with the fixed supports. This can be clearly observed on the graph of CWT coefficient. It can be concluded that in the case of a fixed-supported plate the detection of the damage near the supports is performed with more accuracy than a simply supported plate. In practice the likelihood of failure occurring in fixed support plate

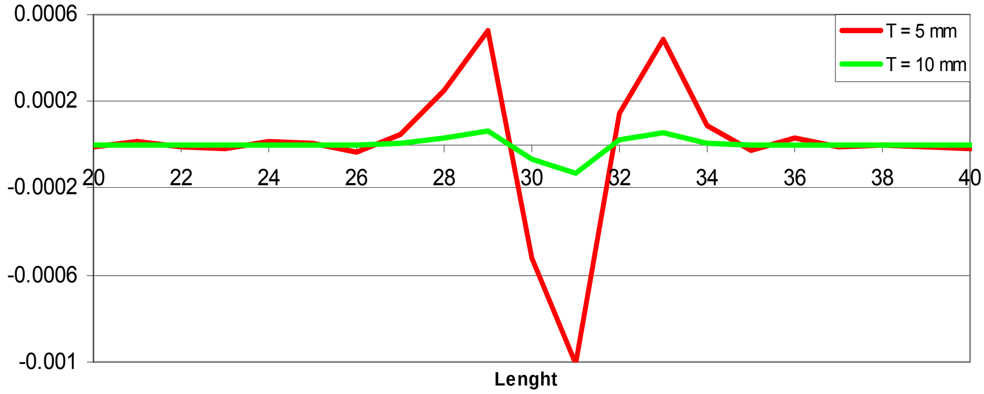


Fig. 13 Graph of WT coefficients for a = 2 (Sym 4), for plate with size of $1000 \times 1000 \times 10\ mm$ and $1000 \times 1000 \times 5\ mm$ with d/h = 0.3

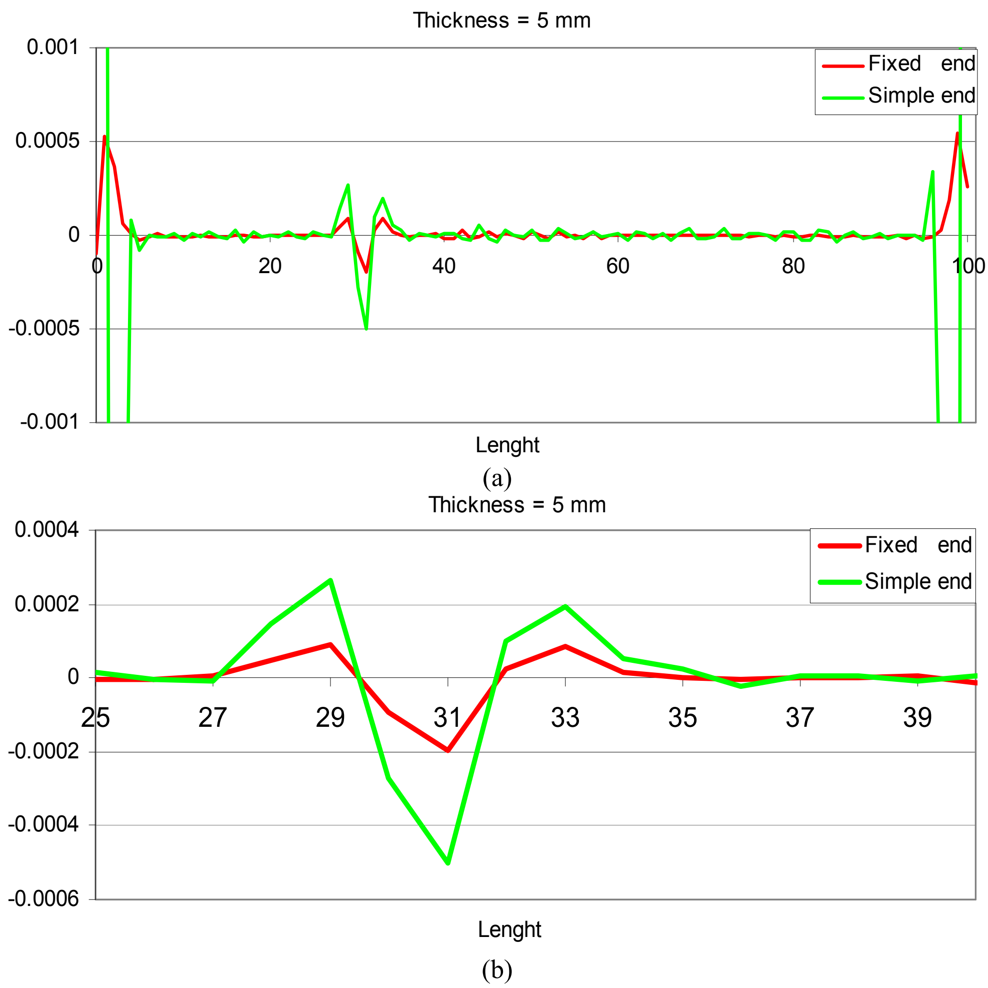


Fig. 14 (a) Graph of WT coefficients (a = 2 / Sym 4) for plate (h = 5 mm) with different boundary conditions and (b) Close-up view of WT coefficients at damage locations

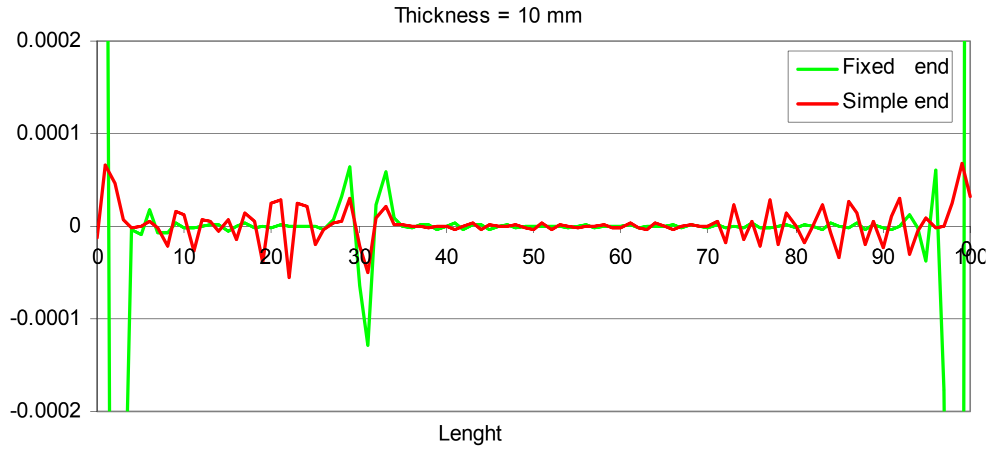


Fig. 15 Graph of WT coefficients (*a* = 2 / Sym 4) for plate (h = 10 mm) with different boundary condition

is higher than simple supports. (ii) The second observation corresponds to the peak value of the coefficients in the location of the damage. As it can be seen, the peak value of the coefficients for simply supported plate is almost 2.5 times greater than the same value in the fixed boundary conditions. This difference is due to the higher curvature of the deflection curve of the simply supported plate as compared with the fixed support (Fig. 14(b)). (iii) Additional noises are appeared in the graph of CWT coefficients for the simply supported plate as compared with the fixed ones. These noises are the damped part of the high discontinuity experienced adjacent to the simple supports. Greater thickness causes CWT coefficients to be contaminated by the higher noise (Fig. 15).

7. Conclusions

In this study the WT tool box in MATLAB, as a powerful mathematical tool for crack identification in plate structures, has been critically studied. The limitations with using this toolbox have been presented first through a simple mathematical example and subsequently through using the response of a cracked plate containing a crack equal to the dimension and parallel to one edge of the plate as an input function in a WT analysis. Damages in the plate induce certain perturbation features in the structural response along the lines perpendicular to the crack. Such perturbation features that are not obvious from the response data are, however, demonstrated as a singularity by analyzing the spatial response with continuous wavelet transform and with the desired resolution. Therefore, as concluded in this paper, certain cautions need to be taken into account in using WT tool box in MATLAB for processing the signal response of structures. Based on this study, it can be concluded that

1. MATLAB software is reliable for this type of analysis only if the sampling points are collected with equal interval sizes.
2. Even if WT is applied using the response signal of an undamaged plate, location of unequal interval distance of sampling points appears as abrupt changes in the graph of the wavelet coefficients. These changes should not be mistaken as representing the presence of a crack

3. Cracks with relative depths lower than 0.05 are not detectable in plates of this study.
4. Ratio of a crack width to the interval distance of sampling points should be equal to one.
5. Increasing of the crack width, and thus increasing of the interval distance of other sampling points, (thus, reducing the number of sampling points), causes an increase in the value of the coefficients at the crack location and also decreases the spatial resolution of the damage location.
6. Wavelet transform can detect more accurately the damages that are further away from the simple support, while the opposite is true for the case of a fixed support plate.

It is important to indicate, however, that the present investigation cannot be considered as a conclusive work. Further investigations are needed to verify the effect of unequal interval distance of sampling points in other structures.

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