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Sensor placement for structural health monitoring of Canton Tower

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Abstract. A challenging issue in design and implementation of an effective structural health monitoring (SHM) system is to determine where a number of sensors are properly installed. In this paper, research on the optimal sensor placement (OSP) is carried out on the Canton Tower (formerly named Guangzhou New Television Tower) of 610 m high. To avoid the intensive computationally-demanding problem caused by tens of thousands of degrees of freedom (DOFs) involved in the dynamic analysis, the three dimension finite element (FE) model of the Canton Tower is first simplified to a system with less DOFs. Considering that the sensors can be physically arranged only in the translational DOFs of the structure, but not in the rotational DOFs, a new method of taking the horizontal DOF as the master DOF and rotational DOF as the slave DOF, and reducing the slave DOF by model reduction is proposed. The reduced model is obtained by IIRS method and compared with the models reduced by Guyan, Kuhar, and IRS methods. Finally, the OSP of the Canton Tower is obtained by a kind of dual-structure coding based generalized genetic algorithm (GGA).

Keywords: structural health monitoring (SHM); canton tower; optimal sensor placement (OSP); generalized genetic algorithm (GGA); model reduction

1. Introduction

The rapid development of civil structures to new horizons enhances the role of structural engineers in assuring a safe and habitable environment. The Canton Tower located in Guangzhou, China, assures a place among the supertall structures worldwide by virtue of its total height of 610 m (Ni *et al*. 2009). It consists of a main tower (454 m) and an antennary mast (156 m). As shown in Fig. 1, this structure comprises a reinforced concrete inner structure and a steel lattice outer structure. The outer structure consists of 24 concrete-filled-tube (CFT) columns uniformly spaced in an oval configuration and inclined with respect to the vertical direction. The oval cross-section of the outer structure varies along the height of the tower; it decreases from 50 m × 80 m at ground level to the minimum dimensions of 20.65 m × 27.5 m at the height of 280m, and then increases to 41 m × 55 m at the top of the tube (454 m). The inner structure is also an oval with constant dimension of 14 m × 17 m. The centroid differs from that of the outer structure. This hyperbolic shape makes the structure interesting and attractive from an aesthetics perspective, and it also makes it structurally complex.

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In order to enhance the safety of the Canton Tower, researchers have made use of significant technological advances in various disciplines of civil engineering. Gu *et al*. (2009) divided the entire tower model into 19 sections and tested each section model in different turbulent flow fields in wind tunnel. The wind-induced displacement, acceleration, internal force responses as well as equivalent static wind loads were computed using the complete quadratic combination (CQC) method based on the results of force balance test. These results were adopted for wind-resistant design of the tower. Considering the insufficient space for installing tuned mass damper (TMD) in the Canton Tower, Tan *et al*. (2009) studied wind-resistant dynamic reliability of TMD with the limited spacing. Numerical simulation showed that the limited spacing design was necessary to prevent the collision between the TMD and the main structure. Pan *et al*. (2008) designed a scale model of the Canton Tower and tested it on the shaking table under 49 types of earthquake ground motions, and the experimental results demonstrated that the prototype can meet the requirements of aseismic design under the frequently occurring earthquakes, designed earthquakes and rare earthquakes. Guo *et al*. (2010) carried out an investigation on multi-column out-plane buckling in ground level open-space region of the Canton Tower, and based on the experimental and numerical analysis, the simplified effective length values proposed for the columns in open-space region of the Canton Tower were obtained. Guo *et al*. (2008, 2009) studied the stability behavior of the waist section of the Canton Tower, and the ultimate bearing capacity and the failure mode when different horizontal loads were acquired. Xiong *et al*. (2010) compared the insert type and the transfer truss type connection system between the antenna and the main structure of the Canton Tower, of which the results indicated that the insert type connection system was superior to the transfer truss type. Additionally, Chen *et al*. (2009) and Tong *et al*. (2010) studied the seismic behavior and bending rigidity of the steel column-beam-brace joints of the Canton Tower, respectively.

(a) Bird's-eye view (b) Outer structure (c) Inner structure and connecting floors

Fig. 1 Canton tower

Despite many attempts as mentioned above to enhance the properties of the Canton Tower, generally these have led to improvements in the safe operation and preventative maintenance of the structure only to a certain extent. This is because large-scale civil engineering infrastructures like the Canton Tower often serve for a long period of several decades or even more than one hundred years. During the service life, they inevitably suffer from environmental corrosion, material aging, fatigue and coupling effects with long-term load and extreme load. The induced damage accumulation and performance degeneration would reduce the resisting capacity of the structures against natural disasters, and even result in collapse with the structural failure under extreme loads. The health monitoring and condition assessment of the Canton Tower has become a requisite requirement. Generally, a complete structural health monitoring (SHM) system serves several purposes (Yi *et al*. 2011). For example, it can provide structural response data, allowing the as-built performance to be checked against design criteria. Over a long period, the monitoring can also provide the opportunity to identify anomalies that may signal unusual loading conditions or changes in structural behavior. A further benefit is to provide data for calibrating the design codes. However, owing to economic reasons concerning the cost related to data acquisition and analysis, and practical reasons such as the inaccessibility of some locations, responses are usually recorded in a number of locations which is smaller than the total number of DOFs of the structure. Therefore, a crucial issue in design and implementation of an effective SHM system in the Canton Tower is how to select sensor locations from a set of possible candidate positions. Placement strategies for sensor systems for the SHM has been a well-studied subject, as demonstrated by Maul *et al*. (2007), who made lengthy reviews of the existing literature in this area. For example, a few methods suggested maximizing the area of coverage per sensor using geometrical and physical constraints (Liu *et al*. 2009). Others focused on enhancing the detection efficiency and minimizing the uncertainty in decision-making based on data acquired from the sensor networks (Field and Grogoriu 2006). Papadimitriou *et al*. (2004) presented a method based on the concept of information entropy as a measure of effectiveness. Kammer *et al*. (1991, 2004) proposed an effective independence (EI) method, which tended to maximize the trace and determinant and minimize the condition number of the Fisher information matrix corresponding to the target modal partitions. Yi *et al*. (2011) proposed a sensor placement method based on the simplified FE model which was able to avoid distinguishing the translational and rotational DOFs. Salama *et al*. (1987) used modal kinetic energy (MKE) as a means of ranking the importance of candidate sensor location. Li *et al*. (2007) analyzed the relation between the EI Method and the MKE Method. Meo *et al*. (2005) compared the capabilities of various methods by assessing the results of their use in the modal identification of a bridge.

It is well known that the sensors installed in the Canton Tower is sparse compared to the DOFs of a structure. To avoid the intensive computationally-demanding problem caused by tens of thousands of DOFs and considering that physically the sensors can only be arranged in the translational DOFs of the structure, the model reduction method is proposed in this paper. A kind of the improved genetic algorithm (GA), called generalized genetic algorithm (GGA), is adopted to get the optimal sensor placement (OSP) of the Canton Tower. The layout of the paper is as follows: Section 2 describes the FE model of the Canton Tower and evaluates the model reduction results based on different method. Section 3 gives the basic theory of the improved GA including the selection of the fitness function, the coding system and genetic operator. Section 4 shows the performance of this algorithm for the OSP of the Canton Tower. In Section 5, some overall conclusions are drawn and discussions for further study are stated.

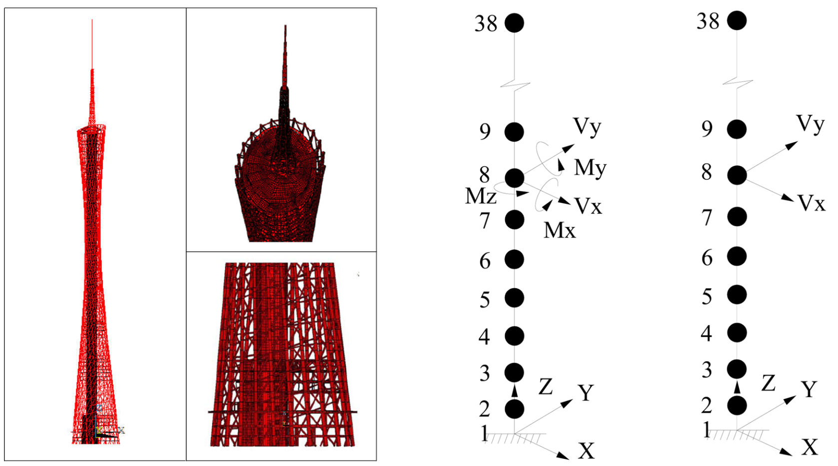
2. Numerical model for Canton Tower

For a structure that has simple geometry, or smaller number of DOFs, the experience and a trial-and-error approach may suffice to solve the OSP problem. For a large-scale complicated structure like the Canton Tower, whose FE model may have tens of thousands of DOFs, a systematic and efficient approach is needed to solve such a computationally demanding problem. Here, the model simplification method and model reduction method are used to solve this problem.

2.1 Full and simplified FE model

In order to provide the input data for the OSP method, a precise three-dimensional FE model of the Canton Tower has been developed (Ni *et al.* 2012). The construction of a FE model, capable of accurately replicating the behavior of the real world structure, was undertaken using the ANSYS software, as shown in Fig. 2(a). The 3D full-order model contains 122,476 elements, 84,370 nodes, and 505,164 DOFs in total. In the model, the PIPE16 and BEAM44 (2-node 3D beam elements with six DOFs at each node) are employed to model the outer structure, antenna mast, and the connection girders between the inner and outer structures. Four-node and three-node shell elements with six DOFs at each node are used to model the shear walls of the inner structure and the floor decks.

The size of the 3D full model is too large for the OSP and related studies. Based on the full model, a simplified 3D beam model is established by Ni *et al.* (2012). Three assumptions are made in developing the simplified 3D beam model: (1) The floor systems are assumed as rigid bodies; (2) Each segment between two adjacent floors is modeled as a linear elastic beam element; (3) The mass of each story is lumped at the corresponding floor. Consequently, the whole structure is modeled as a cantilever beam with 37 beam elements and 38 nodes. The main tower consists of 27 elements and the mast is modeled as 10 elements. As shown in Fig. 2(b), the nodes are numbered from 1 at the fixed base to 38 at the free top end. As the geometric centroid of each floor varies, the beam of the simplified model aligns with the centroidal axis of the tower mast. The vertical



(a) Full order FE model (b) Simplified FE model (c) Reduced FE model

Fig. 2 Finite element (FE) model for Canton Tower

Table 1 Dynamic characteristics of different numerical model

Mode	Full order model	Simplified model		Reduced model							
	Frequency (Hz)	Frequency (Hz)	Difference (%)	Guyan (Hz)	Difference (%)	Kuhar (Hz)	Difference (%)	IRS (Hz)	Difference (%)	IIRS (Hz)	Difference (%)
1	0.110	0.110	0.09	0.111	0.91	0.110	0.00	0.110	0.00	0.110	0.00
2	0.159	0.159	0.13	0.159	0.00	0.159	0.00	0.159	0.00	0.159	0.00
3	0.347	0.346	0.17	0.348	0.58	0.348	0.58	0.346	0.00	0.346	0.00
4	0.368	0.369	0.13	0.369	0.00	0.369	0.00	0.369	0.00	0.369	0.00
5	0.400	0.399	0.01	0.401	0.50	0.401	0.50	0.399	0.00	0.399	0.00
6	0.461	0.461	0.01	0.477	3.47	0.477	3.47	0.461	0.00	0.461	0.00
7	0.485	0.485	0.00	0.738	52.16	0.737	51.96	0.485	0.00	0.485	0.00
8	0.738	0.738	0.01	0.904	22.49	0.904	22.49	0.738	0.00	0.738	0.00
9	0.902	0.903	0.02	0.989	9.52	0.988	9.41	0.903	0.00	0.903	0.00
10	0.997	0.997	0.01	1.038	4.11	1.038	4.11	0.997	0.00	0.997	0.00
11	1.038	1.037	0.04	1.184	14.18	1.179	13.69	1.037	0.00	1.037	0.00
12	1.122	1.122	0.01	1.484	32.26	1.482	32.09	1.124	0.18	1.122	0.00
13	1.244	1.244	0.01	1.730	39.07	1.722	38.42	1.244	0.00	1.244	0.00
14	1.503	1.503	0.00	1.982	31.87	1.981	31.80	1.503	0.00	1.503	0.00
15	1.726	1.726	0.00	1.997	15.70	1.996	15.64	1.760	1.97	1.726	0.00

displacement is disregarded in the simplified model and thus each node has 5 DOFs, i.e., two horizontal DOFs and three rotational DOFs. As a result, each element has 10 DOFs and the entire model has 185 DOFs in total. The floor mass is added to the corresponding node. Half mass of the segment between two adjacent nodes is added to the upper node and the remaining half added to the lower node. The equivalent rotational inertia with respect to the node at each floor is also calculated. The element stiffness matrix of the simplified model is formulated by the displacement method.

The dynamic characteristics obtained from the full model and the simplified are listed in Table 1. Their relative errors to compare the accuracy of two models are calculated by

$$E = \frac{[f_f - f_s]}{f_f} \times 100\% \tag{1}$$

where f_f and f_s are the frequencies of the full model and simplified model, respectively. One can see that the dynamic characteristics of the two models are in very good agreement; the maximum frequency difference among first 15 modes is only 0.17%.

2.2 Rotational DOF reduced model

Although the simplified model has not so many DOFs compared to full order model, only the translational DOFs could be considered for possible sensor installation in this study, as the rotational DOFs are usually difficult and expensive to measure. For the differentiation between translational and rotational modes, generally they can be accurately selected by the modal participation mass ratios.

However, it's difficult to accurately distinguish them for the Canton Tower with closely spaced modes. In this study, a new method of taking the horizontal DOFs as the master DOFs and rotational DOFs as the slave DOFs, and reducing the slave DOFs by the model reduction is proposed.

From the mathematical point of view, the model reduction could be considered as a kind of physical coordinate transformation. One of the oldest and most popular reduction methods is the Guyan (or static) reduction (Guyan 1965). The state and force vectors, x and f, and the mass and stiffness matrices, M and K, are split into sub-vectors and sub-matrices related to the master DOFs, which are retained, and the slave DOFs, which are eliminated. If no force is applied to the slave DOFs and the damping is negligible, the equation of motion of the structure can be expressed as

$$\begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{x}_m \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{Bmatrix} f_m \\ 0 \end{Bmatrix} \tag{2}$$

Here, the subscripts m and s relate to the master and slave coordinates respectively. Neglecting the inertia terms for the second set of equations gives

$$K_{sm}x_m + K_{ss}x_s = 0 \tag{3}$$

which can be used to eliminate the slave DOFs so that

$$\begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{bmatrix} I \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \{x_m\} = T_S x_m \tag{4}$$

where T_s denotes the static transformation between the full state vector and the master coordinates.

The reduced mass and stiffness matrices are then given by

$$\begin{cases} M_R = T_S^T M Ts \\ K_R = T_S^T K T_S \end{cases} \tag{5}$$

where M_R and K_R are the reduced mass and stiffness matrices.

The Guyan reduction method, although it is exact for the reduction of static problems, introduces large errors when applied to the reduction of dynamic problems. Any frequency response functions generated by the reduced matrices in Eq. (5) are exact only at zero frequency. The various methods proposed as modifications of the Guyan reduction were developed, such as the Kuhar (or dynamic) reduction (Kuhar and Stahle 1974); the improved reduced system (IRS) (O'Callahan 1989); the iterated improved reduced system (IIRS) (Friswell 1995).

To investigate the effectiveness of different reduction methods, the rotational DOFs of the simplified FE model are reduced by the above four methods respectively. The results are shown in Table 1. It can be seen from the table that for the classical Guyan reduction method, since the influence of the inertia force is ignored, it causes the big frequency difference between the reduced model and simplified model, of which the frequency of the 7th mode reaches 52.16%; compared with the Guyan method, the Kuhar method reflects the dynamic characteristics of structures to certain extent and the precision has been certainly improved; the IRS method is the improved method based on the Guyan method, the influence of the inertia force is considered by adding additional items in the static reduction and the precision is much improved, of which the largest frequency difference of first 15 modes is only 1.97%. For the structure concerned, the eigenvalues obtained after 275 iterations by the IIRS method are identical with those of simplified FE model. Therefore, the model obtained by the IIRS method is used as the final computation model.

3. Sensor placement using genetic algorithm

The sensor placement optimization can be generalized as "given a set of n candidate locations, find m locations, where $m \ll n$, which could provide the best possible performance". The problem is a kind of combinatorial optimization problem. The number of all different sensor configurations involving m sensors is

$$C = \frac{n!}{m!(n-m)!} \tag{6}$$

This for most cases of practical interest can be an extremely large number. Thus, an exhaustive search over all sensor configurations for the computation of the optimal sensor configuration is largely time-consuming and in most cases prohibitive even for models with a relatively small number of DOFs. The use of intelligent algorithms is therefore justified.

3.1 Generalized genetic algorithm

Countless methods of optimization have been developed. Inspired by the Darwin's theory of evolution, Holland (1975) first proposed the genetic algorithm (GA) used as an effective numerical method to find an optimal (or sub-optimal) solution to a complicated multi-parameter optimization problem. An artificial GA starts with a discrete set (also called generation) of design vectors (also called individuals, chromosomes, or strings) and through some transformation operations it modifies the current set towards a better generation of design points. Similar to natural genetics, there are three main operations in a GA: selection, crossover, and mutation. In every generation, the fittest chromosomes are given a greater chance of survival and also of passing their genes to the next generation. The GA has been proved to be a powerful tool to the OSP, but it also has some faults needed to be improved. When the GA is used to solve the OSP, the general crossover and mutation operators may generate chromosomes which don't satisfy the constraints. For example, one location should not be placed with two or more sensors or sensor number is equal to a certain number. Therefore, some improvements need to be carried out in order to overcome these faults. Here, an improved GA called generalized genetic algorithm (GGA) is introduced to find the optimal placement of sensors. The GGA is based on some famous modern biologics theories such as the genetic theory by Morgan (Dong 1998), the punctuated equilibrium theory by Eldridge and Gould (Dong 1998), and the general system theory by Bertalanffy (Dong 1998), so it is superior in biologics to the classical GA. For the sake of completeness, a brief discussion of the GGA is given here. For more details the reader can refer to the standard introduction to the subject (Dong 1998).

The GGA may be described that applying the crossover on the two parents selected from the initial population; remaining two individuals with best fitness values from the 4 individuals, of which 2 were created before crossover and 2 after crossover (i.e., two quarters selection); eliminating the two individuals with worst fitness values; applying the mutation on the two remains and using two quarters selection on the 4 individuals according to the fitness values; and finally the next generation is produced by $N/2$ times by repeating the above steps.

The difference between the GGA and simple genetic algorithm (SGA) mainly reflects in the evolutionary process. In short, the evolutionary process of the SGA is:

Two parents selection $\rightarrow$ crossover $\rightarrow$ mutation $\rightarrow$ survival selection $\rightarrow$ next generation.

While the process of the GGA is:

Two parents selection $\rightarrow$ crossover $\rightarrow$ a family of four $\rightarrow$ two quarters selection $\rightarrow$ mutation $\rightarrow$ a family of four $\rightarrow$ two quarters selection next generation.

It can be found that the two quarters selection is introduced in the GGA. The parents are allowed to compete with the children during the process of crossover and mutation and only the best one could enter next competition, which can ensure the stability of iterative procedure and complete the function of realizing global optimum.

In addition, the crossover and mutation of the GGA have a little difference with the SGA. In the SGA, the crossover may operate according to a certain probability; while for the GGA, since the parents are able to join in the competition, the crossover probability remains 1.

Generally, the GGA has the following distinctive features:

(1) The selection of the GGA uses the two quarters selection which allows the parents to compete with the children and the better ones enter the next competition. It can be found from the nature that the fitness of some individuals may be improved by the crossover and mutation while that of the other individuals may be gradually reduced. Therefore, the individuals with lower fitness values must be gradually eliminated under the natural selection; while the ones entering the next competition should be the winners of children and parents.

(2) The crossover and mutation are considered as necessary events, but not incidental events in every cycle of the GGA evolutionary process, which provides more chances to create better generation and ensures the diversity of species.

(3) The evolutionary process of the GGA has two stages: the gradual change and sudden change. During the gradual change, the local optimum could be achieved mainly by crossover and selection in which the single point crossover and swap mutation are generally used and the sequence of operations is first crossover and then mutation. In the sudden change, the global optimum may be reached mainly by the mutation and selection which realize the escape from one local optimum to better local optimum, in which the uniform crossover and inversion mutation are mainly used and the sequence of operations is first mutation and then crossover.

(4) In the GGA, the whole evolutionary process is mainly by the gradual change and assisted by the sudden change. The gradual change will be automatically transferred to the sudden change by the feedback element when the evolutionary process tends to the local convergence; as well, once the leader of population is changed, the sudden change may be automatically transferred to the gradual change by the feedback element.

3.2 Dual-structure coding method

From the view of mathematics, the OSP is a kind of particular knapsack problem, and it places specified sensors at optimal locations to acquire more structural information. Its mathematic model is a 0-1 programming problem, if the value of the j-th gene code is 1, in which it denotes that a sensor is located on the j-th DOF. In contrast, if the value of the j-th gene code is 0, it denotes that no sensor is placed on the j-th DOF. The total number of 1 in a chromosome is equal to the sensor

Table 2 Dual-structure coding method

Append code	$s(1)$	$s(2)$	$s(3)$	...	$s(i)$	...	$s(n)$
Variable code	$x_{s(1)}$	$x_{s(2)}$	$x_{s(3)}$	...	$x_{s(i)}$	...	$x_{s(n)}$

Table 3 Example of a dual-structure coding method

4	3	5	8	6	10	2	7	9	1
0	1	1	0	1	0	0	1	0	1

number. To implement the GA, it is necessary to devise a general coding system for the representation of the design variables first. Most commonly the design variables are coded by the simple one dimension binary coding method that is very simple and intuitionistic. However, the number of 1 will be changed in the crossover and mutation (i.e., the number of sensors will be changed), which cannot meet the demand of the OSP. Here, a dual-structure coding GA is adopted to overcome this problem (Yi *et al*. 2011).

The dual-structure coding method is shown in Table 2. The individual chromosome is composed of two rows, the upper row $s(i)$ denotes the append code of x_j, and $s(i)=j$. The lower row represents the variable code $x_{s(i)}$ corresponding to append $s(i)$. When coding a certain individuals, the shuffle method is firstly used to produce the stochastic $\{s(i), (i=1,2...n)\}$ and listed on the upper row, then the variable code (0 or 1) is generated randomly.

For example, for an OSP case with 10 sensors, the randomly generated order of append code is (4, 3, 5, 8, 6, 10, 2, 7, 9, 1), thus the dual-structure code can be presented as shown in Table 3. It corresponds to a feasible solution, namely the sensors are located on the third, fifth, sixth, seventh and first DOFs.

3.3 Fitness function

In the case under investigation the fitness function is a weighting function that measures the quality and the performance of a specific sensor location design. This function is maximized by the GA system in the process of evolutionary optimization. The fitness functions presented in this paper is the modal strain energy (MSE). The objective of the MSE is to find a sensor placement, which maximizes the measure of the MSE of the structure. The reason is that the signal to noise ratio (SNR) of the measured response data is larger on the DOFs which have the larger MSE and it helps parameter identification when the sensors are placed on these locations. Let the mode shape matrix be $\Phi=[\phi_1, \phi_2,...,\phi_n]$ (subscript n is the number of mode shape vector) and the number of the measured points is m, the MSE fitness function can be given by

$$f=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{r,s\in m}\sum_{r,s\in m}|\phi_{ri}k_{rs}\phi_{sj}| \tag{7}$$

where k_{rs} represents the stiffness coefficient between the rth DOF and sth DOF, the element corresponding to the rth row and sth column in the stiffness matrix; ϕ_{ri} denotes the deformation of the rth element in the ith mode and ϕ_{sj} indicates the deformation of the sth element in jth mode. r, $s\in m$ denotes that r and s are all included in the total measured point set.

3.4 Genetic operators

3.4.1 Initial population

The first step in the GGA is to create an initial population of randomly generated individuals. In this paper, the “group in group” scheme is used, i.e., for the initially generated population of N individuals, a small size population of K individuals (called leading population) which are the best ones in current population is simultaneously defined; to ensure the diversity of the leading population, the individuals of the leading population are different from each other; and the residual N-K individuals are called supporting population.

3.4.2 Selection scheme

In different population, the different selection mechanism is used. In leading population, the roulette wheel selection (RWS) scheme is adopted. Conceptually, each member of the population is allocated a section of an imaginary roulette wheel. Unlike a real roulette wheel, the sections are in different sizes, proportional to the individual's fitness, such that the fittest candidate has the biggest slice of the wheel and the weakest candidate has the smallest. If there is a particularly fit member of the population, we may expect it to be more successful in producing offspring than a weaker rival. Using this technique, it is possible that one or more individuals are selected multiple times. As for in supporting population, the stochastic universal sampling (SUS) method is adopted. The SUS works by making a single spin of the roulette wheel. The selection process proceeds by advancing all the way around the wheel in equal sized steps, where the step size is determined by the number of individuals to be selected. This method ensures that the observed selection frequencies of each individual are in line with the expected frequencies. In the evolutionary process of each generation, select one parent from the leading and supporting population and then create two children by two quarters selection, which repeats (N-K)/2 times, and then K individuals of leading population directly enter the next generation, which can realize the elitist preservation mechanism.

During each evolutionary process, the individuals are sorted according to the fitness values in descending order. When the relative error of the fitness values between last individual and that of their parents' generation is smaller than predefined value C, the algorithm is considered to achieve a local optimum and then it enters the sudden change stage. In this stage, the algorithm is able to find better individual by changing the mutation method. Once the leader of the population is changed, the algorithm enters the gradual change stage of another local optimization process. The gradual stage mainly highlights the local search capability of the algorithm, while the sudden stage enhances the transfer capability among local optimum solutions. The optimum solution of the GGA will be automatically determined if the continuous L sudden changes happen.

3.4.3 Crossover strategy

In the GA, each pair of parents produces a number of newborns which replace the weak chromosomes of same number. In this paper the order crossover (OX) method is adopted. The OX involves two parents creating two children at the same time. This operator allows the order in which the parts are placed into the creation vat to be changed. After determining the parents for the process, the basic algorithm is as follows:

(1) Two random cut points are determined (where the dotted lines denote the cut points).

(2) Then the string positions between the cuts are placed directly into the child.

(3) Starting after the second cut point and proceeding to the end of the string and when the end of

Step 1

4	3	5	8	6	10	2	7	9	1
0	1	1	0	1	0	0	1	0	1

1	5	10	9	2	4	8	6	7	3
1	1	0	0	1	1	0	0	1	0

Step 2

*	3	5	8	6	10	*	7	*	1
0	1	1	0	1	0	0	1	0	1

1	5	*	9	2	4	*	*	7	3
1	1	0	0	1	1	0	0	1	0

Step 3

6	10	7	*	*	*	1	3	5	8
0	1	1	0	1	0	0	1	0	1

2	4	7	*	*	*	3	1	5	9
1	1	0	0	1	1	0	0	1	0

Step 4

6	10	7	9	2	4	1	3	5	8
0	1	1	0	1	0	0	1	0	1

2	4	7	8	6	10	3	1	5	9
1	1	0	0	1	1	0	0	1	0

Fig. 3 Order crossover for dual-structure code

the string is reached, the copying process is continued at the beginning of the string and continues until all positions are filled.

(4) The symbols are copied from the second parent into the first child, and from the first parent into the second child. In this process any symbols already presented in the child are omitted from this copying operation.

The detailed operation process is shown in Fig. 3. From Fig. 3, one can easily find that the OX method used is different from the normal OX method, in which in dual-structure coding, the OX method only operates on upper append code and the lower variable value of offspring is fixed. That means the number of sensors could be unchanged.

3.4.4 Mutation mechanism

The crossover enables the method to extract the best genes from different individuals and recombine them into potentially superior children. The mutation children are created by introducing random changes into a single parent within a feasible range. The mutation process is added to the diversity of a population, and thus reduces the chance that the optimization process will become trapped in local optimal regions. Considering that there are gradual change and sudden change in the GGA, the swap mutation and inversion mutation are used herein, as shown in Figs. 4 and 5 (random positions are 4 and 7), respectively. In the stage of gradual change, the swap mutation is

Step 1

4	3	5	8	6	10	2	7	9	1
0	1	1	0	1	0	0	1	0	1

Step 2

4	3	5	2	6	10	8	7	9	1
0	1	1	0	1	0	0	1	0	1

Fig. 4 Swap mutation for dual-structure code

Step 1

4	3	5	8	6	10	2	7	9	1
0	1	1	0	1	0	0	1	0	1

Step 2

4	3	5	2	10	6	8	7	9	1
0	1	1	0	1	0	0	1	0	1

Fig. 5 Inversion mutation for dual-structure code

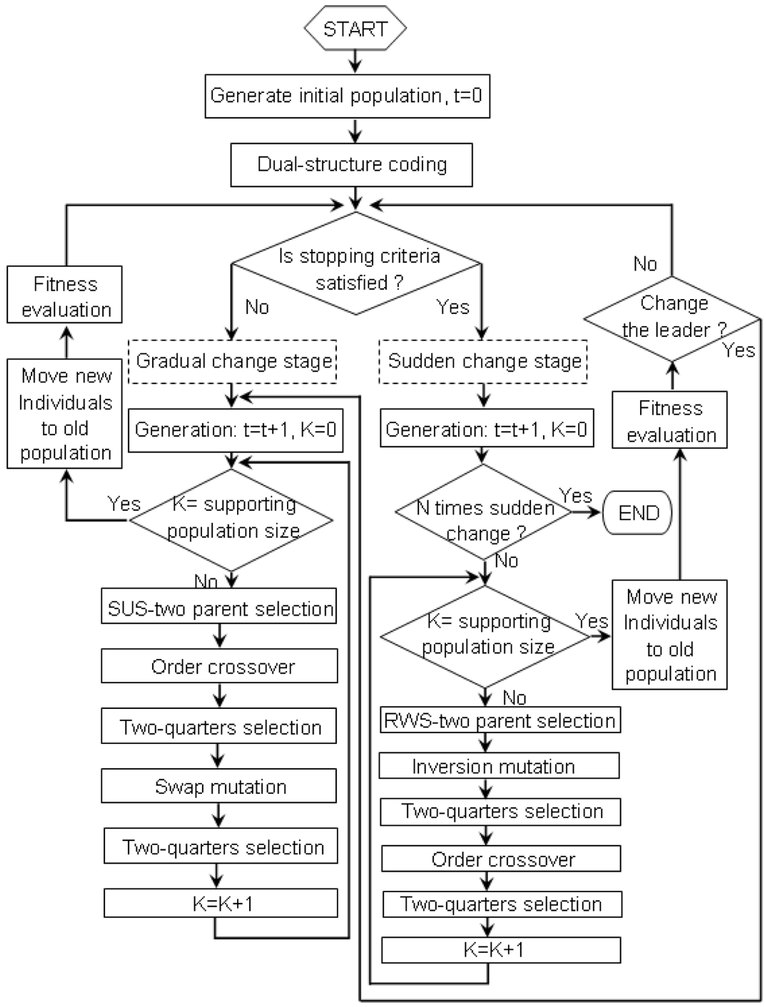


Fig. 6 Flowchart of the dual-structure coding based GGA

applied, which has little operation on genes and the computation needed is simpler, but the precision of the local optimum is lower; while in the sudden change, the inversion mutation is applied, which has lots of operations on genes and the precision of the local optimum is higher, and also the computation is more complex. The same to crossover strategy, the variable code in mutation mechanism has never changed.

To sum up, the whole flowchart of the genetic search to find the optimal sensor locations presented in this paper is shown in Fig. 6.

4. Parametric analysis and discussion

The GGA has a number of parameters that are specific problem and need to be explored and tuned so that the best algorithm performance is achieved. These parameters are the population size, leading population size and number of sudden change. To find out the most appropriate population size for minimal computational cost, various simulations with different population sizes have been first done and population size of 200 has been found to be adequate. In the GGA process, the leading population size is one quarter of the population size and the predefined relative errors is 1.0e-6. When the relative error of the fitness values between last individual and that of their parents' generation is smaller than 1.0e-6, the algorithm is considered to achieve a local optimum and then it enters the sudden change stage. Here, a relatively large number N of sudden changes is selected to avoid redundant iteration. The genetic process will be stopped automatically if the sudden change continuously happenes N times.

Suppose that there are 20 one-dimension accelerometers needed to be installed. For the problem at hand, the size of the searching space is the number of DOFs on the reduced FE model excluding the constrained node. There are 37 candidate nodes that could be used to place sensors. Since each node has two horizontal DOFs; thus, 74 DOFs define the searching space for the GGA process. Wang *et al*. (2007) analyzed the dynamic characteristics of the Canton Tower by the ANSYS software; he found that at least 10 modes should be considered according to the modal mass participation ratio when the vibration response of the structure was investigated. In order to improve the results of optimal sensor layout, the first 50 modes of the Canton Tower are selected for calculation.

It should be noticed that, due to the nature of the GA's method, the results are usually dependent on the randomly generated initial conditions, which means the algorithm may converge to a different result in the parameter space. For the problem considered here, the GGA process for each case has been run for 5 times, and the best results are selected in Table 4. From the Table 4, one can easily find that these values are very close to each other. Obviously, the evolution progress of the best GGA run is the case 8. In order to evaluate the reliability of the above results, all the fitness convergence cures of different cases are shown as Fig. 7, where both the fitness progress of the best individual found by the algorithm as well as the average and worst fitness of the entire population at each generation are plotted. The optimization in the entire GGA population can be seen from the general increase of the average population fitness, despite of the numerous fluctuations caused in the search process through the genetic operators of crossover and mutation. It is obvious that all the best fitness values tend to a constant quickly and the average fitness value steadily tends to the maximum along with increasing number of generations. It shows a good

Table 4 Experimental results for different values of parameter

Case number	Population size	Leading population size	Number of sudden change	Generation number	Best fitness value
1	40	10	5	70	24892432376229196.00
2	40	10	10	109	25258694708402768.00
3	40	10	20	56	25186548210261200.00
4	100	25	5	66	25931975240031752.00
5	100	25	10	68	26697814451014852.00
6	100	25	20	115	27939361992959988.00
7	200	50	5	67	26515116086821920.00
8	200	50	10	68	28065570971298872.00
9	200	50	20	99	27246166917873536.00

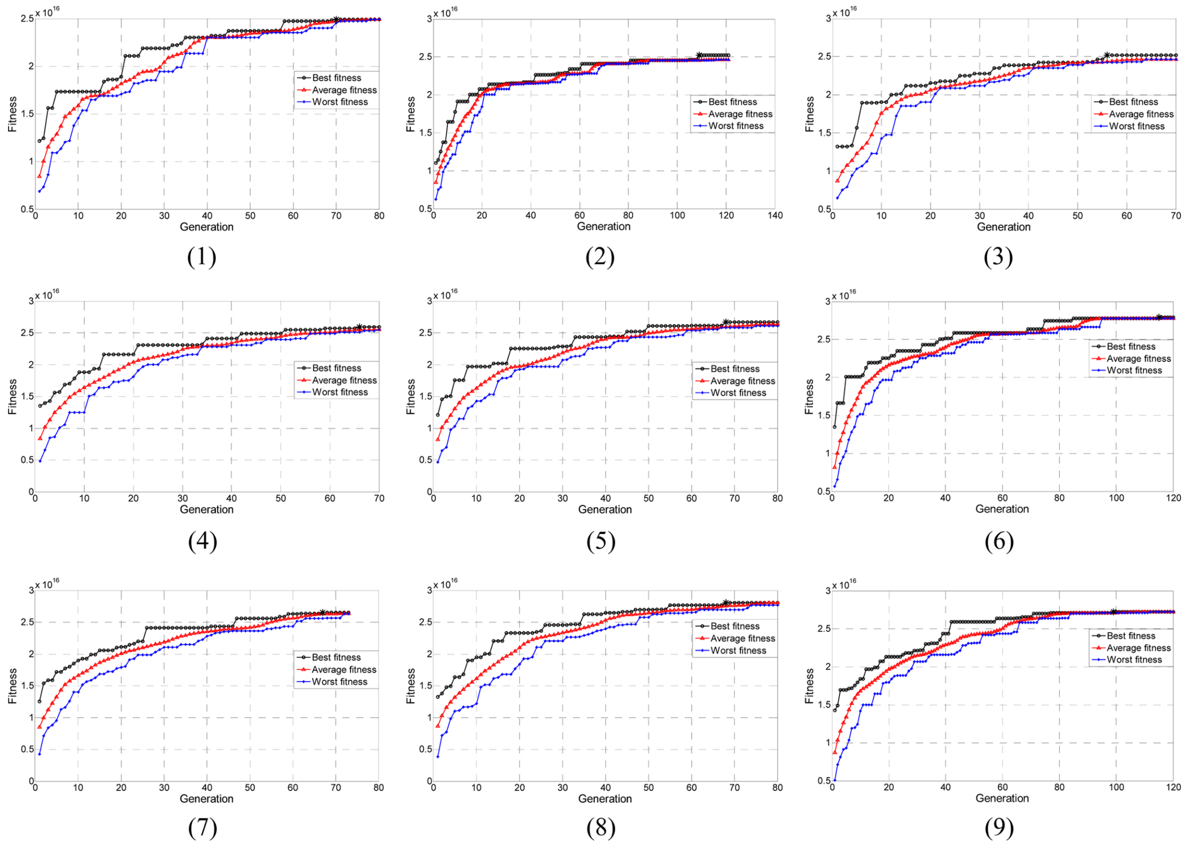


Fig. 7 Fitness curves of GGA with different parameters

Table 5 Sensor placement result of the Canton Tower

Sensor number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
X Direction					9	16		19	20		21		24		26					36
Y Direction	1	2	4	6			16			20		22		24		28	30	31	35	

characteristic of convergence. Numbers of the convergence generations are compared in Table 4. Sensor placement results of the Canton Tower obtained from the case 8 are shown in Table 5.

5. Conclusions

A health monitoring system should ideally have sensor information at all the DOFs of a FE model used for monitoring the integrity of the structure. However, due to cost, weight and accessibility issues, a limited number of locations could be instrumented. The present study investigates scheme for selecting optimal sensor locations designed to monitor the health condition of the Canton Tower. With the case analysis, some conclusions and recommendations are summarized as follows:

(1) The method of taking the horizontal DOFs as the master DOFs and rotational DOFs as the slave DOFs, and reducing the slave DOFs by the model reduction is proposed. This reduction as well as

model simplification in optimization process could result in a dramatic reduction in the required computational effort caused by tens of thousands of DOFs. In addition, the method allows a test engineer to specify a set of locations that they absolutely want or don't want to have in the final sensor configuration. Especially, if sensor resources are scarce, it is important to have the initially selected sensor set optimal in some sense with respect to the final locations.

(2) Considering the characteristics of the OSP techniques in the large-scale structures, some innovations in GGA such as the dual-structure coding method instead of binary coding method are proposed in this paper to obtain the solution. Accordingly, the dual-structure coding based selection scheme, crossover strategy and mutation mechanism are derived and given in detail. This method operates only on upper append code and the lower variable value of offspring is kept fixed that guarantee the number of sensors could be unchanged.

(3) The optimal results were shown to be robust with respect to the selection of the initial sensor set obtained by the model simplification and reduction method. The GGA is particularly effective in solving the combinatorial optimization problem such as OSP problem when the performance tradeoffs are not unbearable and when the number of combinations is too large to preclude enumeration. The parameter analysis indicates for the GGA, the population size much larger, the diversity of the individual in the leading and supporting groups much higher, the probability of generating new and high-fitness individuals much larger and the results much better. If the population size is much smaller, the group may focus on searching one small region of the population, therefore the precociousness phenomenon may occur in the evolutionary process. The population size should be determined according to the specific problems and hardware configuration of computers used.

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