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A non-destructive method for elliptical cracks identification in shafts based on wave propagation signals and genetic algorithms

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Abstract. The presence of crack-like defects in mechanical and structural elements produces failures during their service life that in some cases can be catastrophic. So, the early detection of the fatigue cracks is particularly important because they grow rapidly, with a propagation velocity that increases exponentially, and may lead to long out-of-service periods, heavy damages of machines and severe economic consequences. In this work, a non-destructive method for the detection and identification of elliptical cracks in shafts based on stress wave propagation is proposed. The propagation of a stress wave in a cracked shaft has been numerically analyzed and numerical results have been used to detect and identify the crack through the genetic algorithm optimization method. The results obtained in this work allow the development of an on-line method for damage detection and identification for cracked shaft-like components using an easy and portable dynamic testing device.

Keywords: cracked shafts; elliptical cracks identification; wave propagation; non-destructive evaluation; genetic algorithms

1. Introduction

For any mechanical component diagnosis systems it is necessary to allow early detection of any damage and to enable maintenance and repair work at the initial damage phase, such that the structural safety and reliability are guaranteed with a minimum of costs. The tools for diagnosis systems are the techniques of non-destructive evaluation, among them, the most frequently methods used are radiography, visual inspection, thermal analysis, magnetic field methods or ultrasonic testing, which can detect and evaluate the damage in a mechanical component (Farrar and Lieven 2007, Achenbach 2009). In addition, one can find other methods based on experimental tests which analyse the changes of dynamic and static characteristics caused by the appearance of the defect.

In the case of rotating shafts, fatigue cracks usually grow up following transversal planes to their longitudinal axis, due to the fatigue produced by the cyclic loads to which they are submitted, which are a potential source of catastrophic failures (Bachschmid *et al.* 2010, Ishida 2008). Therefore, as inspection of shafts for damage is vital for making decisions about their repair or replacement, it is

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very important to have inspection methods, which reveal not only the crack position, but also their depth and shape. The increasingly importance of problems related with safety and costs derived from the catastrophic failures have pushed the researchers in the field of damage detection to look for and develop methods for the detection and identification of defects since the 70's decade of 20th century. Researchers have developed different direct crack non-destructive diagnosis methods for shaft transmission systems health monitoring, such as ultrasonic inspection (Fasel *et al.* 2009, Zimmer *et al.* 2010), acoustic emissions (Al-Balaushi and Samanta 2002, Toutountzakis *et al.* 2005, Li *et al.* 2010), active thermography (Uhl *et al.* 2008) or magnetic fields (Ryua *et al.* 2002, Mani *et al.* 2006), these techniques usually require the damaged section to be known a priori and accessible to direct measurement (Cademi and Greco 2006). Otherwise, the identification of a crack in a shaft can also be approached according to other two different approaches. The first one is based on the modification of the dynamic responses of the cracked rotor during its rotation, and the second one is based on the fact that the presence of a crack in a mechanical element produces an increase in its flexibility which is accompanied by a change in modal properties (natural frequencies and modes shapes) (Sabnavis *et al.* 2004, Sinou and Lees 2005) These methods are successfully applied when the presence of crack causes changes in some of the lower natural frequencies and mode shapes, because if changes are produced at higher frequencies the errors of the method are bigger. This problem can be avoided by increasing the order of the model, however, such an approach leads to an increase in computational effort. Moreover, when the damage is small, the modal parameters changes are also small, they often have the same level as measurement error, so that the method should not be apply (Krawczuk 2002).

Aforementioned reasons lead to, during the last few years a lot of work has been done in order to derive procedures which allow to detect and identify damages in rotating shafts based on other methodologies. Among them, elastic wave propagation models have already successfully applied in several engineering fields, such models are well suited for detection of even very small defects as they are very sensitive to changes in local dynamic impedance (Krawczuk 2002, Tenenbaum *et al.* 2011). These theories have been extensively applied in many areas, such as seismic research (Yan *et al.* 2007, Resende *et al.* 2010) , civil engineering (Algernon *et al.* 2008, Huang *et al.* 2010) or nondestructive evaluation (NDE) of structures for crack detection, especially made of composite materials (Mitra and Gopalakrishnan 2007, Glushkov *et al.* 2010), and also for metallic mechanical components (Tian *et al*. 2003, LaBerge 2007).

In a typical NDE process, actuators are used to generate elastic wave propagation in the component. When cracks are encountered the wave will be reflected and the corresponding response of the structure must be measured using a system of sensors (Bao and Wang 2009). Several researchers have studied cracks detection in shaft-like parts (beams, shafts or rods) by means of stress wave propagation techniques. Some of these works apply numerical methods, some of them using spectral finite elements, which were formulated for predicting high frequency structural vibrations in composite materials (Krawczuk 2002), and which have also been used later to model wave propagation in other mechanical components (Grabowska *et al.* 2008, Krawczuk *et al.* 2004). Recently, one can find different studies relating to the application of traditional finite elements in order to solve 3D wave propagation problems (Idesman *et al.* 2010, Idesman *et al.* 2011, Bou Matar *et al.* 2012). Also, experimental studies can be found, in Tian *et al*. 2003 and Grabowska *et al.* 2008, authors use flexural waves to detect cracks in a cantilever beam, measuring strains and accelerations respectively. Otherwise, there are various works that make use of axial waves, within these, in Meia *et al.* 2006 the transmission and reflection matrices of axial waves passing through a crack are calculated by means of numerical methods and in Zaczmarek (2008) the transfer function is experimentally calculated by measuring strains due to tensile stress. On the contrary, elastic wave propagation techniques have seldom been applied to shafts monitoring. In LaBerge (2007) one can find a diagnosis system for shafts based on wave propagation, in this case tensile stress is not caused by an external load but by the stress changes generated by the breathing crack during shaft rotation.

Regarding to detection of cracks in transmission shafts is necessary to take into account that most of the researchers in the field of fracture mechanics and rotordynamics assume straight fronted cracks (Papadopoulos and Dimaragonas 1987), but experience shows that fatigue cracks in rotating shafts propagate with an elliptical shape front, as many authors consider (Zhou *et al.* 2005, Rubio *et al.* 2011). None of the above mentioned works dedicated to damage detection in mechanical components through elastic waves models has considered the case of circular sections with elliptical cracks, which are found in rotating shafts. It is important that this fact is considered in developing methods for crack determination and identification in order to set up effective maintenance plans in machinery.

Furthermore, inverse problems are recently playing an important role in the area of non destructive methods in order to establish fault diagnosis systems for machine maintenance. As opposed to the conventional forward problems, inverse problems estimate the cause from the result. Most of the formulations of inverse problems proceeds directly to the setting of an optimization problem. In particular, genetic algorithms (GA) optimization techniques have been successfully included in fault detection methods applied to mechanical systems, for instance, in recent times, (Buezas *et al*. 2011, Wang *et al*. 2011, Amiri 2011).

In this work, a non-destructive method for the detection and identification of cracks in shafts based on elastic wave propagation is proposed. The wave propagation in a cracked shaft has been numerically analyzed to establish a non-destructive method for the detection and identification of cracks using a simple dynamic test, and in which not only position and depth of the crack, but also the shape of the front will be taken into account. Cracks with increasingly depths and shape fronts have been considered and studied. To solve the inverse problem of detecting and identifying fatigue cracks in the shafts, the numerical results previously obtained have been used in an optimization genetic algorithm which has been developed within the MatLab environment. This would allow the development of an on-line method for damage detection and identification for cracked mechanical elements using an easy and portable dynamic testing device together with a very effective and reliable optimization technique like genetic algorithms.

2. Simulation

2.1 Simulation model

A 3D Finite Element Method (FEM) model, using Commercial Code ABAQUS/ Explicit, of a shaft with a transversal crack has been carried out.

A free-free cracked shaft model 700 mm long and 20 mm diameter, has been submitted to boundary conditions of velocity. The material of the shaft is steel with the following mechanical properties:

· Young’s Modulus: E=210 GPa

· Poisson ratio: υ=0.3

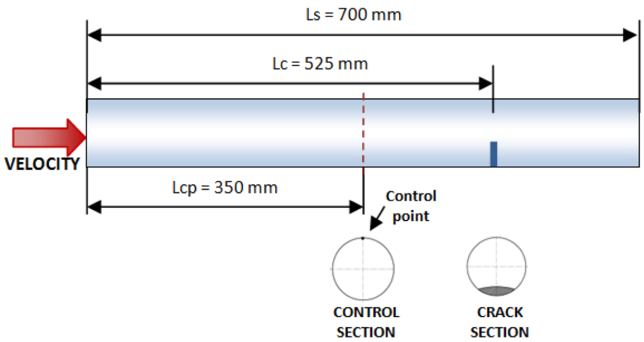

Fig. 1 Crack and test point location

· Density: ρ=7850 Kg/m^3

Consequently, the wave propagation rate c_o takes the value of 5172,2 m/s. Being

$$c_0 = \sqrt{\frac{E}{\rho}} \quad (1)$$

In the model, the crack is located 525 mm far away from the end of the shaft, and the one-dimensional stress wave propagation has been recorded in the control point situated at the mid-span of the shaft, as can be seen in Fig. 1. The situation of the control point around the surface of the shaft has been chosen according to previous works, in which the results for the top control point have been considered to be the clearest and easiest to treat (Rubio *et al.* 2009).

The characteristic parameters of the elliptical crack are the following (see Fig. 2):

· $\alpha=a/D$ characteristic depth of the crack

· $\beta=a/b$ shape factor of the crack (β=1 corresponds to a semicircular crack and β=0 corresponds to a straight crack)

The initial crack considered has a relative depth, α=0.05. Cracks with increasingly depths have been also considered and studied up to a value of 0.5 with increments of 0.05. In order to determine the influence of the shape of the crack front, different models have been analyzed with increasing values of the aspect ratio parameter β, varying from 0 to 1 with increments of 0.1. So, an amount of 110 simulations have been carried out using a FEM model of the shaft.

In relation with the finite elements properties, when an implicit integration scheme is used, it is necessary to establish the mesh density so as to get the stability of the solution. The elements size must be such that the distance traveled by the wave has to be smaller than the element length. So,

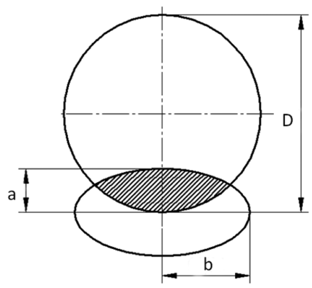

Fig. 2 Parameters of elliptical crack

there is a critical element length (l_c), function of a maximum integration time step (t_s), which leads to the analysis stability (Kim *et al*. 2008) as Eq. (2).

$$t_s = \frac{l_c}{c_o} \tag{2}$$

In order to verify the previous condition, in the developed model, t_s=1 μs and the largest element length is l=1 mm. On the other hand, the element type chosen is a 8-node linear brick element. In general, at the same number of degrees of freedom, quadratic elements yield better accuracy than linear elements. However, the ABAQUS Theory Manual (ABAQUS 2009), in the chapter dedicated to “mesh design for dynamics”, explains that for simulations of impact, first order elements must be used due to their formulation is better to model the effect of stress waves. Also, in the literature, several authors have employed these kind of elements to solve wave propagation problems, for instance, (Zeinoddini *et al.* 2008, Kiernan *et al*. 2009, Kang *et al.* 2011).

In Fig. 3 a detail of the finite element half-model is shown.

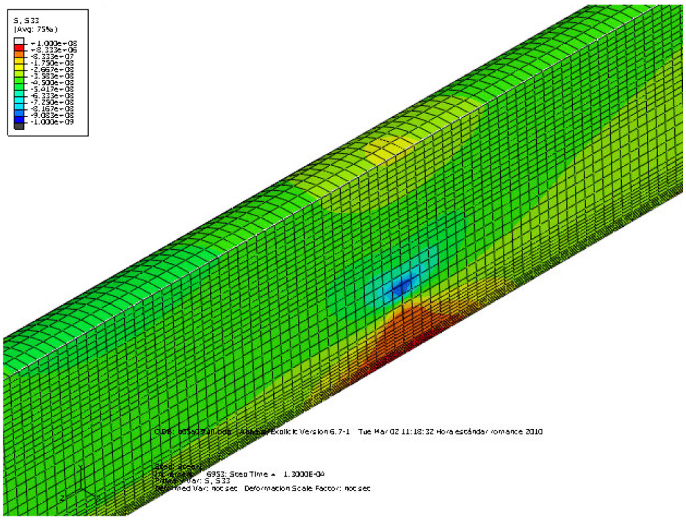

Fig. 3 Detail of the half-model of the shaft

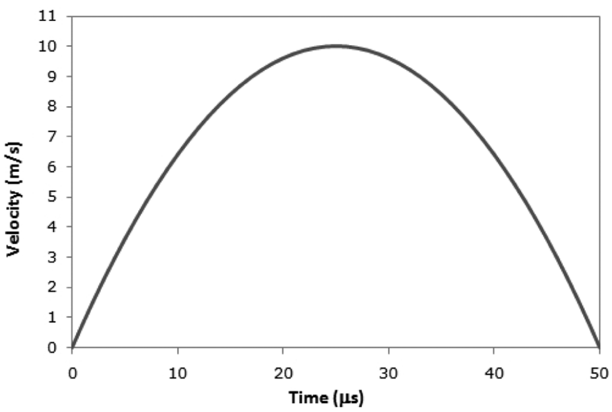

Fig. 4 Initial velocity at the free end of the shaft

Last, the numerical tests have been developed and analyzed corresponding to an applied parabolic velocity of maximum amplitude v=10 m/s and duration of 50 μs as one can see in Fig. 4.

2.2 Simulation results

In Fig. 5(a), in the case of an undamaged shaft, the incident and reflected waves measured at the control point have been plotted, and in Fig. 5(b), one can see the measured waves for damaged shafts with different crack parameters values, specifically, β=0.1 and different crack depths. The Fig. 5 represents the propagation of the stress wave starting with the compression signal followed by the tensile signal produced as the stress wave reflects at the free end of the bar. When the wave propagates through a cracked bar, see Fig. 5(b), a small tensile signal appears between these two aforementioned signals. This small tensile signal is produced when part of the incident wave reflects at the free face of the crack.

2.2.1 Model verification

For the purpose of checking the validity of the finite element model, the results of displacement,

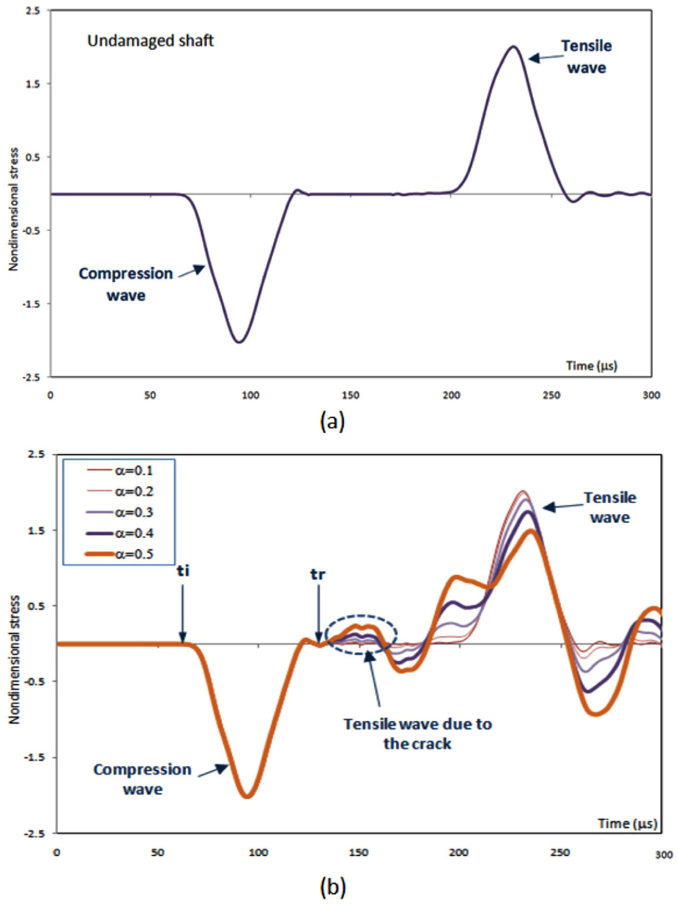

Fig. 5 Incident and reflected waves measured. (a) Undamaged shaft and (b) damaged shafts, β=0.1 and different crack depths, α

velocity, acceleration and stress, obtained from the finite element model of an undamaged shaft, have been compared with the ones calculated by analytical methods.

The equation of motion for a shaft is the well-known wave equation, see Eq. (3).

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_o^2}\frac{\partial^2 u}{\partial t^2} \tag{3}$$

Where u is the displacement in the longitudinal axis direction (x) and t is the time variable. The D'Alambert solution to Eq. (3) is

$$u(x,t) = f(x - c_o t) + g(x + c_o t) \tag{4}$$

Thus, longitudinal waves propagate at the velocity c_o and without distorsion.

According to Graff (1975), we can obtain

$$v(x,t) = -\frac{c_o}{E}\sigma(x,t) \tag{5}$$

being σ the axial stress and v the particle velocity.

From Eq. (5) one can obtain the relationship between the velocity and the load applied in one the edges of the shaft, $P(x,t)$.

$$P(x,t) = -\frac{EA}{c_o}v(x,t) \tag{6}$$

where A is the area of the cross section.

On the other hand, acceleration can be obtained from velocity

$$a(x,t) = \frac{\partial v(x,t)}{\partial t} \tag{7}$$

Fig. 6 shows the comparison of two methods, analytical and numerical, in the control point for 200 μs.

As can be seen in Fig. 6, the analytical results show a very good agreement with the numerical ones. Displacement, velocity and stress graphics made by both methods are very similar, only in the case of the acceleration, errors are slightly higher. In any case, according to these results, the finite element model can be considered good enough.

3. Methodology

From the analysis of the acquired signals of the tests, it is possible to localize the crack along the length of the shaft and the influence that the depth and the shape of crack front have in the shape and the amplitude of the wave signals. The relation among them will allow the identification of the cracks. The results are dimensionless with the characteristic stress of the shaft given by Eq. (8). Similar waves to that of Fig. 5(b) would be obtained for different shape ratios of the crack front.

$$\sigma_0 = \frac{\rho c_0 v}{2} \tag{8}$$

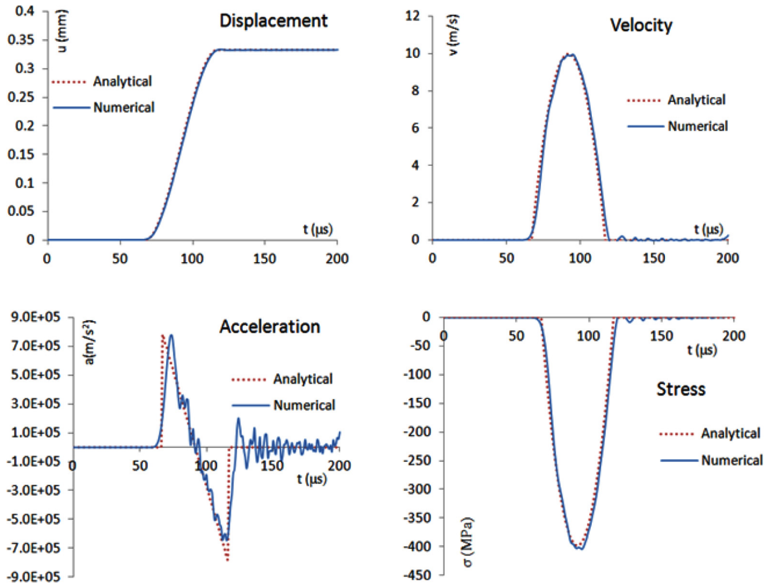

Fig. 6 Comparison of analytical and numerical methods. Calculations in the control point for an undamaged shaft

3.1 Detection methodology

The three parameters needed to detect and identify cracks in mechanical elements like shafts are the location of the crack along the shaft, the depth of the crack and the shape of the front of the crack. To determine them, results like the ones shown in Fig. 5(b) are used.

The location of the crack, L_{ce}, is determinated taking the time at which the incident wave appears, t_i, and the time at which the reflected-in-the-crack wave starts to rise, t_r, and using the wave propagation rate, c_o, the estimation of the location is immediate (Rubio *et al.* 2009).

$$Ⱡ_{ce} = \frac{c_0}{2} \cdot (t_i + t_r) \tag{9}$$

3.2 Identification methodology

In order to identify the crack once it has been detected, it is necessary to determine the depth and the shape of the front. For that, the tensile wave, produced as a reflection of the incident one on the crack face, dashed indicated in Fig. 5(b), has been studied. Some of those results are shown in Figs. 7 and 8.

As the amplitude of the curves increase with the crack depth and with the flattened of the front, it seems to be able to establish a relation between some parameters of the wave (the part of it corresponding to the reflected-in-the-crack wave) and the depth and shape of the crack. These parameters could be the area below the curve or the amplitude of the curve. In Fig. 9 the procedure to obtain those parameters is shown. Both variables have been dimensionless, respectively, with the area below the curve and the amplitude of the initial compression wave, giving de final parameters *AC* and *AM* for each considered.

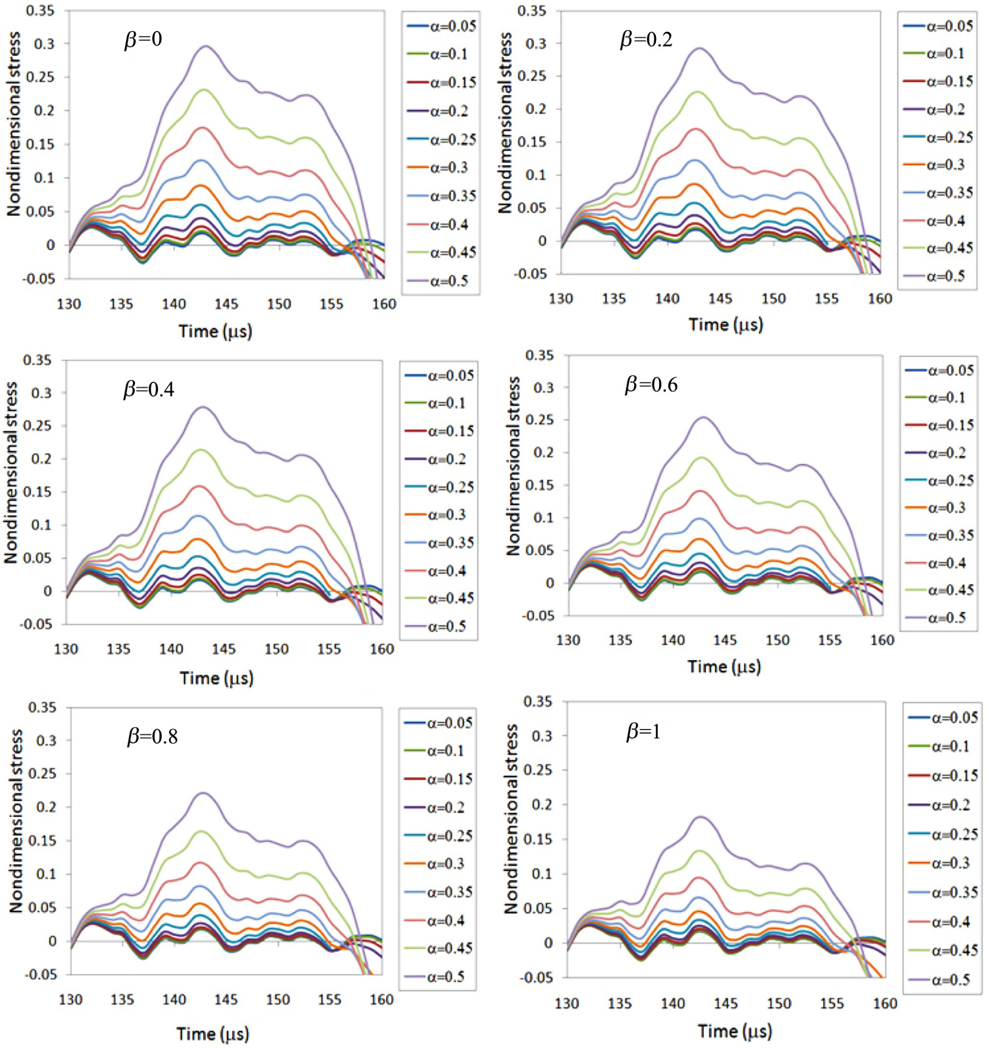

Fig. 7 Zoom of the tensile wave reflected-on-the-crack for different values of α at fixed values of β

One of the main achievement of this work is, as Fig. 5 shows, that to apply the proposed method, it is not necessary to know the measured stress wave for a undamaged shaft.

3.2.1 Analysis of the propagation wave

Having the values of *AC*, *AM*, α and β for the 110 numerical cases, carried out in this study polynomial fittings for *AC* and *AM* can be done

$$AC = \sum_{i=0}^{p} \sum_{j=0}^{q} C_{ij}^{cc} \alpha^i \beta^j \tag{10}$$

$$AM = \sum_{i=0}^{p} \sum_{j=0}^{q} C_{ij}^{mm} \alpha^i \beta^j \tag{11}$$

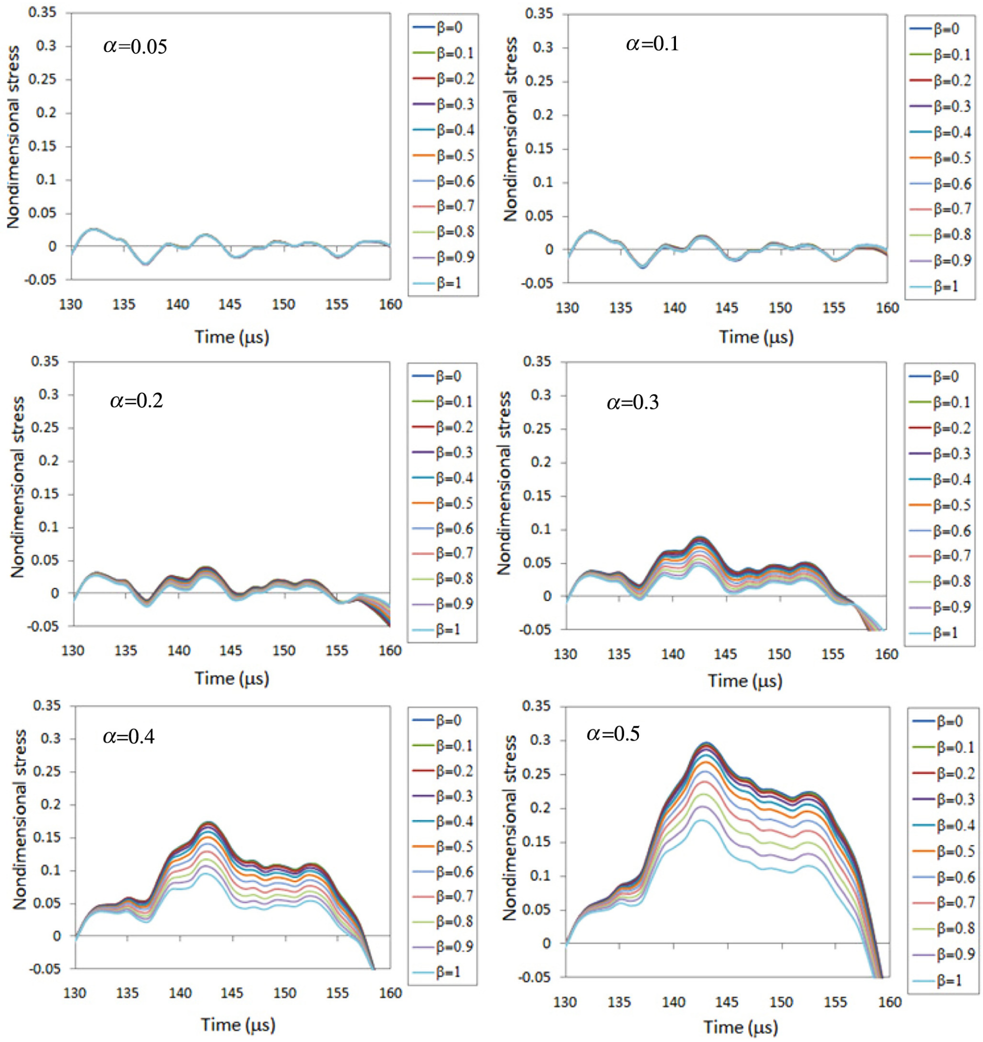

Fig. 8 Zoom of the tensile wave reflected-on-the-crack for different values of β at fixed values of α

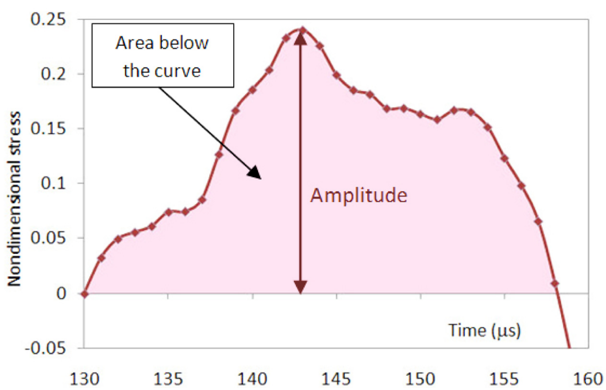

Fig. 9 Characteristic parameters of the wave: area below the curve, *AC*, and amplitude, *AM*

Table 1 C_{ij}^{cc}, fitting coefficients for *AC*

p \ q	0	1	2	3	4	5
0	0.00193	-0.02048	0.1366	-0.3047	0.2661	-0.09129
1	-0.04606	0.183	-0.4346	0.7797	-0.0001213	0
2	0.6825	-1.709	-2.361	-1.739	0	0
3	11.04	7.042	6.884	0	0	0
4	-30.26	-8.62	0	0	0	0
5	18.33	0	0	0	0	0

Table 2 C_{ij}^{mm}, fitting coefficients for *AC*

p \ q	0	1	2	3	4	5
0	0.01097	-0.005459	0.021836	-0.0587	0.07354	-0.04024
1	0.1103	0.09074	-0.07057	0.007173	0.1453	0
2	-1.57	-0.4296	-0.03613	-0.4517	0	0
3	8.528	0.3292	0.2815	0	0	0
4	-13.43	0.2857	0	0	0	0
5	7.954	0	0	0	0	0

where C_{ij}^{cc} and C_{ij}^{mm} are the coefficients of the fittings for the dimensionless amplitude and area of the curve, given in Tables 1 and 2. Further, p is the polynomial grade in α, depth of the crack, and q is the polynomial grade in β, shape factor of the crack. The coefficients of the fittings have been calculated by using of multiple regression techniques.

The fittings were performed considering cracks up to α=0.5 of the diameter. Different shapes of the front crack, varying from straight front to semicircular front, have been also considered in the fittings as the cracks in shafts generally propagate with this shape. For those conditions, the best fittings were reached with grade 5 (p=5) polynomials in α and with grade 5 (q=5) in β with a

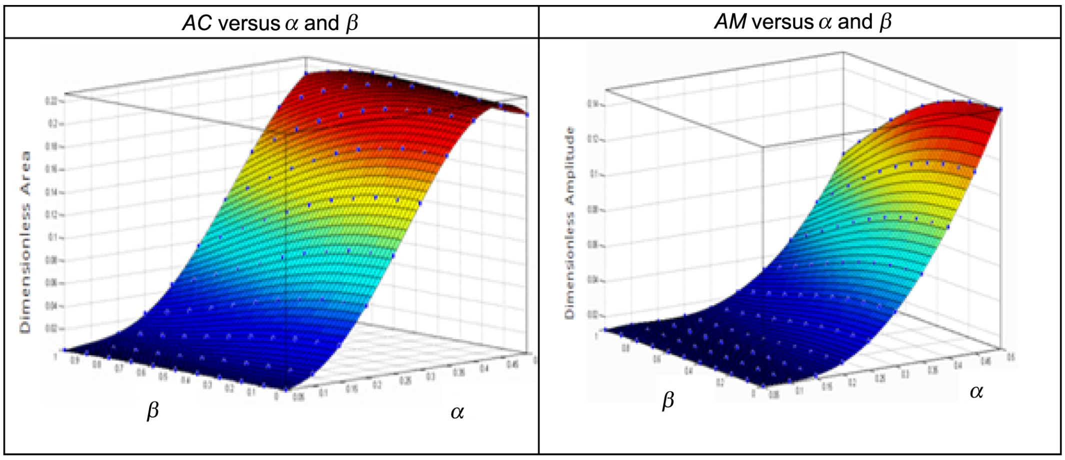

Fig. 10 *AC* and *AM* vs α and β

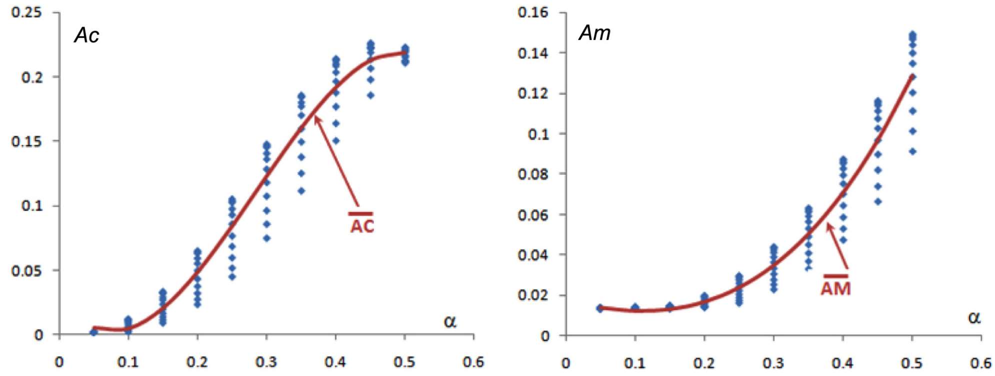

Fig. 11 α vs $\overline{AC}$ and $\overline{AM}$

coefficient R^2=0.9999 for both area and amplitude.

In Fig. 10, the dimensionless amplitude and area of the curve are plotted versus crack depth α and front shape ratio β. Surfaces produced can be resumed saying that for a given value of the shape of the crack front, both of them, AC and AM, increase with the depth of the crack. However, for a given value of α, AC and AM remain practically constant with β.

Since AC and AM vary with the crack depth but they are constant with the shape front, in this work, it has also been found the relationship between α and the average values of AC, $\overline{AC}$, and AM, $\overline{AM}$, see Fig. 11.

Then, polynomial fittings for $\overline{AC}$ and $\overline{AM}$ as function of α can be done

$$\overline{AC} = -6.024\alpha^3 + 5.139\alpha^2 - 0.6785\alpha + 0.02681 \qquad (12)$$
$$R^2=0.9$$

$$\overline{AM} = 0.6886\alpha^3 + 0.2698\alpha^2 - 0.08429\alpha + 0.01753 \qquad (13)$$
$$R^2=0.9$$

3.2.2 Inverse problem. Genetic algorithms

Once the direct problem is known, the crack identification can be considered as inverse problem. The direct problem, in this case, has consisted in obtaining the propagation waves providing the knowledge of the crack properties. The inverse problem will be then, obtaining the properties of the crack, α and β, knowing the tensile wave and its properties. In the proposed methodology, the genetic algorithm optimization technique is applied to solve the inverse problem of crack identification.

As well known, GA is a general-purpose optimization algorithm based on Darwin's theory of evolution and natural selection that searches a solution space for the optimal solution to a problem. In a GA, the potential solutions of the problem at hand are coded in chromosomes. Initially, the algorithm creates a population of possible solutions and lets them evolve over multiple generations to find better and better solutions. A fitness value, derived from the problem's objective function is assigned to each member of the population, so that, in each new generation new individuals appear from the information of the best adapted individuals of the previous generation, therefore, the selection process is analogous to the survival of the fittest in the natural world.

A general scheme of a GA can be described by following steps (Fig. 12):

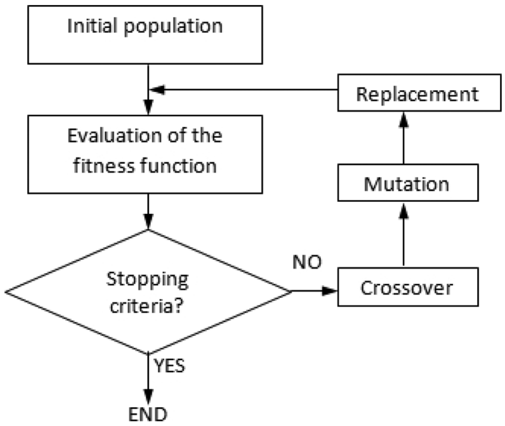

Fig. 12 Basic scheme of a GA

1. Initialisation of the population (set of randomly generated individual).
2. Evaluation of the fitness function for all individuals.
3. Selection of the parents.
4. Children are generated from parents using two genetic operators: crossover and mutation.
5. Completion of the new population from the new children and selected members of the old population.
6. End the algorithm and report the best solution.

The genetic crossover operator recombines two parents with given probability to produce a pair of children, it is done by the exchange of two subparts of the parent chromosomes, defined by the crossover point, to produce two child chromosomes, crossover represents a way of moving through the space of possible solutions based on the information gained from the existing solutions. On the other hand, the mutation operator makes small random changes in the individuals in the population, it aims to achieve some stochastic variability of GA in order to get a quicker convergence. At the last, it is necessary to specify a percentage of the population to replace each generation.

Based on the aforementioned theory, the proposed GA is developed under the MatLab environment. To begin the GA, an initial population of 250 individuals are randomly generated. Every individual consists of 2 genes, α and β, encoded as a binary string (chromosome), and each gene is made up of 10 alleles. In summary, one individual includes 20 alleles, which can be 0 or 1.

According to the GA toolbox of MatLab, the fitness function, f, should be minimize. It is calculated taking into account two kinds of conditions. The first one tries to minimize the differences between the calculated and measured values for AC, f_1, and AM, f_2

$$f_1 = AC_c - AC_m \tag{14}$$

$$f_2 = AM_c - AM_m \tag{15}$$

Where AC_c and AM_c are the calculated dimensionless area of the curve and dimensionless amplitude, respectively, by means of Eqs.(10) and (11), AC_m and AM_m are the measured values of the same variables.

In the same way, the second kind of conditions related to the fitness function is obtained minimizing the differences between the calculated and measured values for $\overline{AC}$, f_3, and $\overline{AM}$, f_4.

$$f_3 = \overline{AC}_c - AC_m \tag{16}$$

$$f_4 = \overline{AM}_c - AM_m \tag{17}$$

Where $\overline{AC}_c$ and $\overline{AM}_c$ and AC and AM are the calculated values of $\overline{AC}$ and $\overline{AM}$, respectively, by means of Eqs. (12) and (13).

Therefore, from Eqs. (10), (11), (12) and (13), the fitness function, f is

$$f = \sqrt{f_1^2 + f_2^2 + f_3^2 + f_4^2} \tag{18}$$

In relation to the crossover technique, the two points operator has been employed, in this case two points of the chromosome are randomly chosen in both individuals and then the alleles falling in-between the two points are swapped to give two new offspring. On the other hand, mutation method is used, which randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation. Last, 80 is the percentage of the population to replace each generation.

4. Numerical experiments

To show the procedure of the method, three numerical experiments have been carried out. The three experiments have been chosen considering the fact that the front of incipient cracks is practically circular while when a crack grows its front becomes straight (Carpinteri 1993). Genetic algorithm input data in the three cases can be seen in Table 3. To guarantee that the algorithm converges to the same solution, each case has been run five times during 250 generations. Table 4 shows the obtained results, where the estimated values of α and β have been calculated as the average of the five repetitions of the same case.

In all cases the error is calculated as

Table 3 Numerical experiments input data

	Case 1 (α = 0.1, β = 0.8)	Case 2 (α = 0.25, β = 0.5)	Case 3 (α = 0.5, β = 0)
AC	0.004421	0.086219	0.211130
AM	0.013208	0.023857	0.14901

Table 4 α and β estimation results

Variable	True value	Estimated value	Error (%)
Case 1			
α	0.1	0.101	1
β	0.8	0.812	1.55
Case 2			
α	0.25	0.25	0
β	0.5	0.495	-1
Case 3			
α	0.5	0.5	0
β	0	0	-

Table 5 Results of the crack dimensionless location estimation

True value	Estimated value	Error (%)
0.75	0.745	-0.67

$$error(\%) = \frac{estimated\ value\ \text{-}\ true\ value}{true\ value} \times 100 \tag{19}$$

Regarding the crack location estimation, it is calculated once directly from Eq. (9), this estimation does not depend on α and β, it only depends on the material properties of the shaft. Table 5 shows the result for the crack dimensionless location, with respect to the length of the shaft.

Tables 4 and 5 show the estimations for the crack location and properties. They are rather good as the errors are kept, in any case, under 1.6\%. The comparison of the true values with the estimated ones obtained with the proposed technique, reveals that the last is very useful to detect and identify elliptical cracks in shafts, which are one of the most dangerous defects in shafts.

5. Sensitivity of the crack identification method

The final aim of the explained methodology is to apply it to real structures. Fig. 13 shows the steps of the measurement procedure.

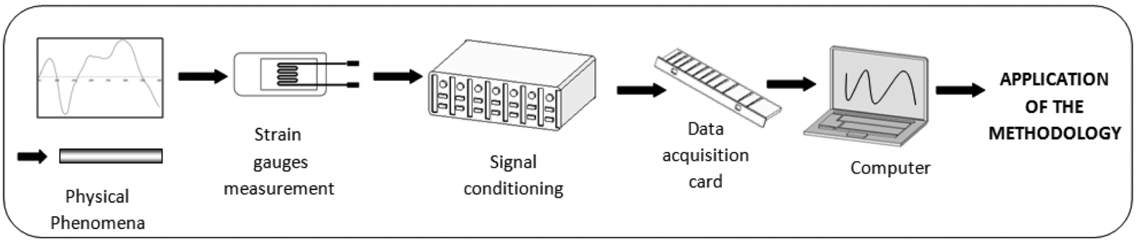

Fig. 13 Experimental measurement chain

Table 6 Sensitivity analysis for α=0.1 and β=0.8 (Case 1)

Variable	True value	Estimated value	Error (%)
	random noise: 1%		
α	0.1	0.101	1
β	0.8	0.815	1.87
	random noise: 3%		
α	0.1	0.101	1
β	0.8	0.816	2
	random noise: 5%		
α	0.1	0.1	0
β	0.8	0.801	0.12
	random noise: 10%		
α	0.1	0.103	3
β	0.8	0.787	-1.62

In any experimental measurement there are always many types of possible errors, normally due to the limited precision of metrology instruments, or the people improper operation. The next study of this work is related with the sensitivity of the developed method. In order to simulate actual measurements, sensitivity to the errors has also been studied.

For this second analysis the same beam in Fig. 1 and the same technique has been used. Moreover, in order to simulate experimental measurements, the GA input values are the nominal ones used in the previous study plus different percentages of random noise (1%, 3%, 5% and 10%). Tables 6, 7 y 8 show the calculated results for the three prior cases and every value of random noise.

As can be seen, in this study, the worst estimations are obtained for the parameter β when the crack depth is very large, although the parameter α is perfectly estimated. However, the estimations of α and β are very good for medium and small depths as the calculated errors are kept, in any case, under 2%. The goodness of the proposed method has been proved as errors in the measurements have been simulated by the addition of random errors.

Table 7 Sensitivity analysis for α=0.25 and β=0.5 (Case 2)

Variable	True value	Estimated value	Error (%)
	random noise: 1%		
α	0.25	0.25	0
β	0.5	0.496	-0.8
	random noise: 3%		
α	0.25	0.25	0
β	0.5	0.496	-0.8
	random noise: 5%		
α	0.25	0.25	0
β	0.5	0.494	-1.2
	random noise: 10%		
α	0.25	0.246	-1.6
β	0.5	0.498	-0.4

Table 8 Sensitivity analysis for α=0. 5 and β=0 (Case 3)

Variable	True value	Estimated value	Error (%)
	random noise: 1%		
α	0.5	0.5	0
β	0	0.149	-
	random noise: 3%		
α	0.5	0.5	0
β	0	0.075	-
	random noise: 5%		
α	0.5	0.5	0
β	0	0.073	-
	random noise: 10%		
α	0.5	0.5	0
β	0	0.275	-

6. Conclusions

In this work, a non-destructive method to detect and identify cracks in shafts has been developed. The method is based on the study of the behaviour of the stress wave generated along the shaft when a impulse load is applied in one of its edges. In this study, the transversal crack has been supposed to present an elliptical front, which is seldom found in the literature but that is a more real configuration for a fatigue crack in a shaft. The proposed method could be implemented in an on-line method for damage detection and identification for cracked shaft-like components using an easy and portable dynamic testing device, like a strain gauges and signal analysis system.

To get the results, first of all a finite element model has been developed in order to obtain numerical results for different shapes and depths of the crack. From them, the forward problem has been set up. This has been performed, using a multiple regression technique fitting, to calculate polynomial expressions of the stress wave parameters as a function of the crack characteristics, depth and shape. The next step has been the formulation of the appropriate inverse problem, which can be seen as an optimization exercise which has been solved through the GA method.

The proposed GA has been validated for three different cases taking into account the growth behaviour of a real fatigue crack, namely, that while the crack is shallow the front is practically circular and as it gets deeper the front becomes straighter. The obtained results confirm the effectiveness of the developed methodology, so the depth crack, the shape front and the crack position are estimated with very tiny errors. Moreover, the validation has been completed with a sensitivity to the errors study, in order to simulate experimental measurements. The last analysis conclusions can be summarized saying that the results are specially good when the depth crack is small or medium, in these cases the errors between actual and estimated values of the crack characteristics are less than 2%. This is a very important conclusion because the cracks should be identified as they arise, just when they are at the first stages and before a hazardous situation or an unexpected malfunction occurs. Moreover, it must be highlighted that, in order to detect and identify a crack, it is not necessary to measure previously the stress wave along an undamaged shaft.

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